THE LOWER-HYBRID-DRIFT INSTABILITY AS A SOURCE OF ANOMALOUS RESISTIVITY FOR MAGNETIC FIELD LINE RECONNECTION

J. D. Huba*, N. T. Gladd and K. Papadopoulos

Naval Research Laboratory, Washington, D. C. 20375

Abstract. It is shown that the lower-hybrid-drift instability can be unstable over a large region of the magnetotail where collisionless reconnection is expected to occur. For typical quiescent tail parameters the instability threshold is much lower than the ion acoustic instability. Preliminary considerations indicate that it can produce anomalous resistivity substantially larger than classical, reconnection rates of the order 10^{-1} V_A, electromagnetic noise is the frequency range $\omega_{LH} < \omega \ll \Omega_e$ and field aligned relativistic electron fluxes.

Introduction

Magnetic field reconnection is one of the most fascinating and important aspects of magnetospheric and space physics. It is believed to be responsible for the rapid release of energy stored in the magnetic field that produces dissipative events, such as solar flares and magnetospheric substorms. While substantial progress has been achieved in understanding the process on the macroscopic level (Vasyliunas, 1974), key processes related to the microscopic interactions have been inadequately identified. Since Coulomb collisions cannot provide sufficiently rapid field dissipation in low density space plasmas, these microscopic processes are crucial in determining both the time scale and the size of the regions where energy can be released. Coroniti and Eviatar (1976) elaborated on earlier suggestions (Syrovatskii, 1972) that the unmagnetized ion acoustic instability can provide the necessary "anomalous" resistivity (Papadopoulos, 1977), and produced a rather comprehensive reconnection model. However, the ion acoustic instability has several limitations since it is operative only very close to the neutral line $\lesssim (c/\omega_{pe})$ and strong currents are required to excite it. The purpose of this letter is to demonstrate that for realistic magnetic field and density profiles, cross-field current driven instabilities of the lower-hybrid-drift type (Krall and Liewer, 1971) can operate over a much larger area ($\sim r_{Li}$) and can provide the necessary resistivity for slow and explosive reconnection.

The lower-hybrid-drift instability is a cross-field current driven instability which is an important source of anomalous transport in several plasma confinement devices (Commisso and Griem, 1976; Bretz and DeSilva, 1973). The free energy necessary to drive this instability is provided by the cross-field current and inhomogeneities in the plasma and magnetic field; features which are inherent in the configuration of the reconnection process. Typically, the lower-hybrid-drift instability is characterized at maximum growth by

$$\omega_r \approx \omega_{LH},\ \gamma \lesssim \omega_{LH},\ kr_{Le} \gtrsim 1 \text{ and } \vec{k} \cdot \vec{B} = 0$$

where $\omega = \omega_r + i\gamma$ and k are the complex frequency and wavenumber of the mode, respectively, $r_{L\alpha} = v_\alpha/\Omega_\alpha$ is the Larmor radius of species α and $\omega_{LH} = \omega_{pi}(1 + \omega_{pe}^2/\Omega_e^2)^{-1/2}$ is the lower hybrid frequency. Here, $\omega_{p\alpha} = (4\pi n_\alpha e_\alpha^2/m_\alpha)^{1/2}$ is the plasma frequency, $\Omega_\alpha = |e_\alpha|B/m_\alpha c$ is the cyclotron frequency and $v_\alpha = (2T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity, all of species α. An important advantage of this instability is that it can be excited by weak currents, $V_D/v_i > 2\,\Omega_i/\omega_{LH}$ (Friedberg and Gerwin (private communication)) where V_D is the drift current velocity. This low excitation threshold persists even in the regime $T_i > T_e$ and is much smaller than the current threshold required to excite the ion acoustic instability under similar conditions (i.e., $V_D/v_e \approx 1$). Moreover, it has been shown to produce substantial anomalous resistivity (Davidson and Gladd, 1975). Recently, a comprehensive analysis of this instability has been performed by Davidson et al. (1977) whose theory is fully electromagnetic and accounts for both resonant and non-resonant ∇B orbit modifications of the electrons. We base our subsequent analysis on this theory and only review its basic assumptions and results at this time, referring the reader to Davidson et al. (1977) for details.

Review of Instability Theory

The plasma geometry under consideration is shown in Fig. 1. We assume the only spatial variation is in the x-direction. The ambient magnetic field is in the z-direction while the equilibrium electric field is in the x-direction. The electrons drift in the y-direction with a mean fluid velocity $(m_e \to 0)$ $V_{ey} = V_E + V_{De}$ where $V_E = -cE_x^0/B$ is the equilibrium $\vec{E} \times \vec{B}$ velocity, $V_{De} = -(v_e^2/2\Omega_e)\partial \ln n/\partial x$ is the electron diamagnetic drift velocity and $n = n_e \approx n_i$ is the density. Equilibrium force balance on an ion fluid element in the x direction requires $V_{iy} = V_E + V_{Di}$ where $V_{Di} = (v_i^2/2\Omega_i)\partial \ln n/\partial x$ is the ion diamagnetic drift velocity. We consider the case where the ions carry no current in the y-direction so that

*NRL/NRC Research Associate

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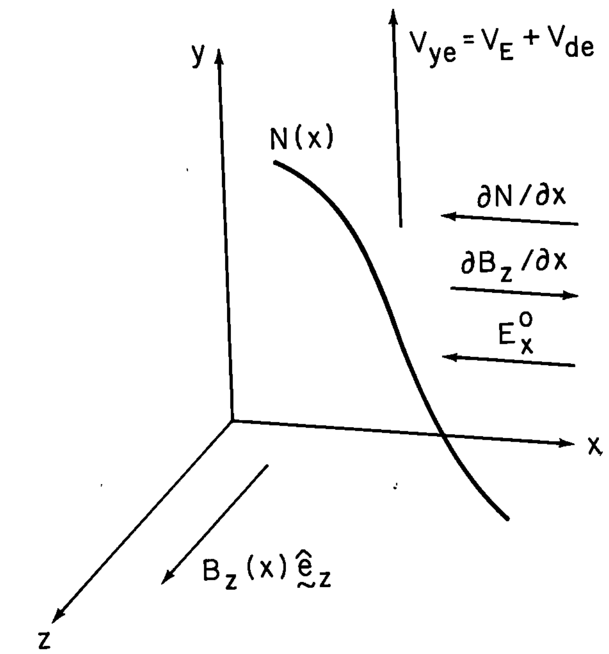

Fig. 1. Slab geometry and background configuration. For the magnetotail, x is in the south-north direction, y is in the dusk-dawn direction and z is in the earthward direction.

$V_E = - V_{Di}$ or $E_x^o = -(1/en)\partial(nT_i)/\partial x$. Thus, the equilibrium electric field acts to balance the ion pressure in the x direction. We consider only flute perturbations ($\vec{k}\cdot\vec{B} = 0$) and assume that $k_y^2 \gg k_x^2 \gg (\partial \ell n\, n/\partial x)^2$, $(\partial \ell n\, B/\partial x)^2$ which justifies the use of the local approximation (Krall, 1968). Furthermore, the electrons are assumed to be magnetized while the ions are treated as unmagnetized. This is reasonable since we are considering waves such that $kr_{Li} \gg 1$ and $\Omega_i \ll \omega \ll \Omega_e$. Finally, the weak inhomogeneity approximation is used (i.e., $r_{Le}^2\,(\partial \ell n\, n/\partial x)^2 \ll 1$ and $r_{Le}^2\,(\partial \ell n\, B/\partial x)^2 \ll 1$) and we assume $\partial T_e/\partial x = \partial T_i/\partial x = 0$.

Within the context of the above assumptions, the general dispersion equation describing the lower-hybrid-drift instability is (Davidson et al. 1976)

$$\left[1 + \frac{2\omega_{pi}^2}{k_y^2 v_i^2}\,(1+\xi_i Z(\xi_i)) + \frac{2\omega_{pe}^2}{k_y^2 v_e^2}\,(1-\Phi_3)\right] \times \left(1 + \frac{2\omega_{pe}^2}{k_y^2 c^2}\,\Phi_1\right) = - \frac{2\omega_{pe}^2}{c^2 k_y^2}\,\frac{2\omega_{pe}^2}{k_y^2 v_e^2}\,\Phi_2^2 \qquad (1)$$

where $\xi_i = \omega/kv_i$, $Z(\xi) = (\pi)^{-1/2}\int_{-\infty}^{\infty} dy \exp(-y^2)/(y-\xi)$ and Φ_1, Φ_2 and Φ_3 are defined by

$$\Phi_j = \int_0^\infty ds^2\, \frac{F_j \Lambda \exp(-s^2)}{\omega - k_y V_E - k_y \bar{V}_B s^2}\,,$$

$$j = 1,\, 2,\, 3$$

where $F_1 = (sJ_o'(\mu))^2$, $F_2 = sJ_o(\mu)J_o'(\mu)$, $F_3 = J_o^2(\mu)$, $\mu = k\, r_{Le}\, s$, $s = v_\perp/v_e$, $\Lambda = \omega - k_y V_E - k_y V_{De}$, $\bar{V}_B = -(v_e^2/2\Omega_e)\partial \ell n B/\partial x$ and $J_n(\mu)$ is the Bessel function of order n. An analytic solution to Eq. (1) can be found in the limit $V_E \ll v_i$, $T_i \gg T_e$, $|\omega - k_y V_E| \gg |k_y \bar{V}_B|$ and $\omega_{pe}^2 \ll c^2 k_y^2$. It can be shown then, that

$$\omega_r - k_y V_E = \frac{2\omega_{pe}^2}{k^2 v_e^2}\, k_y V_\Delta \quad \times$$

$$\left[1 + \frac{2\omega_{pi}^2}{k^2 v_i^2} + \frac{2\omega_{pe}^2}{k^2 v_e^2}\,(1 - I_o(b)\exp(-b))\right]^{-1} \qquad (2)$$

and

$$\gamma = -(\pi)^{\frac{1}{2}}\,\frac{T_e}{T_i}\,\frac{\omega_r}{k_y v_i}\,\frac{(\omega_r - k_y V_E)^2}{k_y V_\Delta} \qquad (3)$$

where

$$V_\Delta = I_o(b)e^{-b}\,\frac{v_e^2}{2\Omega_e} \quad \times$$

$$\left[\frac{\partial \ell n\, n}{\partial x} - \frac{\partial \ell n B}{\partial x}\,(1 - b(1 - I_1(b)/I_o(b))\right],$$

$b = k^2 r_{Le}^2/2$ and $I_n(b)$ is the modified Bessel function of order n. Clearly an instability is possible ($\gamma > 0$) when $\omega/k_y V_\Delta < 0$. However, in general, Eq. (1) must be solved using numerical techniques.

Application to the Magnetotail

To facilitate the application of the above theory to the magnetotail, we model the tail by a collisionless sheet pinch equilibrium. Assuming the ions carry no current in the y direction, it can be shown that the equilibrium profiles for a sheet pinch are (Harris, 1962, Hoh, 1966)

$$n(x) = n_o/\cosh^2(x/\lambda) \qquad (4)$$

$$B_z(x) = B_\infty \tanh(x/\lambda) \qquad (5)$$

and

$$E_x(x) = -\frac{V_E}{c}\, B_\infty \tanh(x/\lambda) \qquad (6)$$

where $\lambda = (c/V_E)(T_i/2\pi n e^2)^{1/2}(1 + T_e/T_i)^{-1/2}$, $n_o = n(x = 0)$ and $B_\infty = B_z(x = +\infty)$. We comment that Eqs. (4)-(6) are derived assuming $\partial T/\partial x = 0$ and $V_E/c \ll 1$. Since the stability theory requires the electrons to be magnetized, our analysis is restricted to the region $x > x_e$ where $x_e = (2r_{Le}\lambda)^{1/2}$ (Hoh, 1966). Physically, x_e is the distance a thermal electron travels from the neutral line before being reflected by the Lorentz force. Thus, electrons beyond x_e generally do not cross the neutral line and their orbits can be determined using the guiding center approximation.

We now evaluate Eq. (1) numerically based upon Eqs. (4)-(6) for a set of parameters typical of the quiescent magnetotail. ($T_e/T_i = .25$, $n_o = 10\text{cm}^{-3}$, $B_\infty = 2 \times 10^{-4}$G, $T_i/m_e c^2 = 10^{-3}$ and $V_E/v_i = 1.0$). Figure 2a shows a plot of n/n_o and B/B_∞ as a function of $x/(c/\hat{\omega}_{pe})$ where $c/\hat{\omega}_{pe} = c/\omega_{pe}(x = 0)$. Figure 2b presents

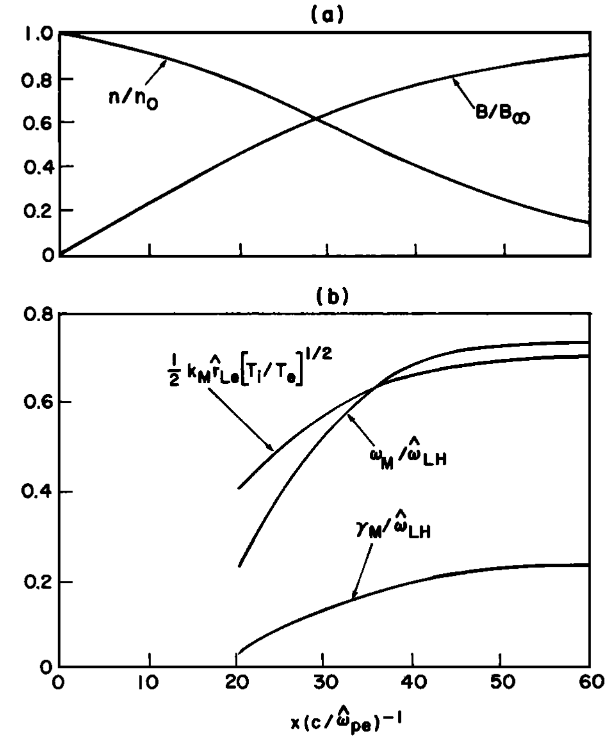

Fig. 2. Stability properties of the lower-hybrid-drift instability for parameters typical of the quiescent magnetotail. We assume that $T_e/T_i = .25$, $n_o = 10 cm^{-3}$, $B_\infty = 2 \times 10^{-4} G$, $T_i/m_e c^2 = 10^{-3o}$ and $V_E/v_i = 1.0$.

$\gamma_M/\hat{\omega}_{LH}$, $\omega_M/\hat{\omega}_{LH}$ and $1/2\ k_M \hat{r}_{LE}(T_i/T_e)^{1/2}$ as a function of $x/(c/\hat{\omega}_{pe})$ where γ_M is the maximum growth rate as a function of k, ω_M and k_M are the corresponding real frequency and wave number, $\hat{\omega}_{LH} = \omega_{LH}(x = \infty)$ and $\hat{r}_{LE} = r_{LE}(x = \infty)$. Although this instability is stabilized because of finite β effects (Davidson et al., 1976) at $x \approx 20c/\hat{\omega}_{pe}$ it is important to realize that there is instability over a wide region of the sheet pinch for $x \gg c/\hat{\omega}_{pe}$. We note that for these parameters $x_e \approx 5\ c/\hat{\omega}_{pe}$.

As shown by Davidson and Gladd (1975) and Davidson (1976), the instability can produce substantial anomalous resistivity. For example in the limit $V_E \leqslant v_i$, $T_i \gg T_e$, $\omega_{pe}^2 \gg \Omega_e^2$ and $\omega_{pe}^2 \ll c^2 k^2$, the resistivity computed on the basis of quasilinear theory for momentum transfer is given by

$$\eta_{an} = 2\pi \sqrt{\frac{\pi}{2}}\ \frac{m_i}{m_e}\ \frac{\omega_{LH}}{\omega_{pe}^2}\ \frac{\epsilon_w}{nT_i} \tag{7}$$

where ϵ_w is the wave energy density. A plausible upper bound on the wave energy is obtained by equating it with the total available free energy.

$$[\epsilon_w]_{Max} = \frac{1}{2}\ n m_e V_E^2 \tag{8}$$

in which case the resistivity is

$$\eta_{an} \leqslant 2\pi \sqrt{\frac{\pi}{2}} \left(\frac{V_E}{v_i}\right)^2 \frac{\omega_{LH}}{\omega_{pe}^2} \tag{9}$$

Note that this is ten orders of magnitude larger than the classical value (Spitzer, 1962) for typical tail parameters. The influence of electromagnetic effects on the anomalous resistivity is presently under investigation and preliminary results indicate that the estimate given by Eq. (9) is not significantly modified. These results, along with a study of the role of specific non-linear processes such as resonance broadening, plateau formation and ion trapping, and two dimensional particle simulations, will be presented in a future detailed paper.

Discussion and Conclusions

A comprehensive analysis of the detailed role of the instability and its consequences with respect to reconnection lies beyond the scope of this preliminary letter. However it is worth mentioning the following facts connected to the present experimental and theoretical notions. (a) Observationally the instability will be detected as very low frequency electromagnetic noise. In order to compute the expected frequency range we should notice that ω_r as determined by Eq. (1) is in the ion reference frame ($V_{iy} = 0$), which implies $E_x^o = -(T_i/en)\ \partial n/\partial x$. Transforming to the reference frame where $E_x^o \equiv 0$, as the case seems to be in the plasma sheet, the observed frequency is given by the Doppler shifted frequency $\omega^{obs} = \omega_r + k_y V_E \approx 2k_y V_E$. For the parameters used in Fig. 2, with $V_E/v_i \approx 1$, $\omega^{obs} \gtrsim 1.1\ \hat{\omega}_{LH} \approx 15$ Hz. However, for larger values of V_E the observed frequency can increase by more than an order of magnitude. Observations of such low frequency electromagnetic turbulence have been reported in the plasma sheet (Scarf et al., 1974) and in the crossing of an interplanetary D-sheet (Formisano et al., 1974). (b) For the resistivity given by Eq. (9) the reconnection velocity $u = \eta c^2/4\pi L$ will be given by

$$u \leqslant \sqrt{\frac{\pi}{2}} \left(\frac{V_E}{v_i}\right)^2 \left(\frac{c}{L\omega_{pe}}\right) V_A \tag{10}$$

where L is the width of the diffusion region. To be consistent with the assumptions used to obtain Eq. (10) we require $L > r_{Li}/2$. Making use of this condition and the quiescent parameters in Fig. 2, we find that $u \leqslant .08\ V_A$ which is typical of the values suggested by solar and magnetospheric observations (Parker, 1973). (c) Although the analysis presented in this letter is restricted to waves such that $k \cdot B = 0$, we point out that this instability can also occur for finite values of $k_\parallel$ (Gladd, 1976). For this situation it has been shown by Lampe and Papadopoulos (1976) that the instability is capable of accelerating electrons parallel to the magnetic field to high energies ($\gtrsim$ MeV), which is important in understanding the acceleration mechanisms operating in the reconnection region.

In summary, we have demonstrated that the lower-hybrid-drift instability can operate over a substantial portion of the collisionless reconnection region and under conditions substantially less strenuous than the ion acoustic instability. Preliminary considerations indicate that it can produce anomalous resistivity ten orders of magnitude larger than classical, reconnection rates of the order of $10^{-1}V_A$, electromagnetic noise in the region between $\omega_{LH} < \omega \ll \Omega_e$ and field aligned relativistic electron fluxes. All these form the backbone of a reconnection theory consistent with observational requirements.

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