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Parametric Excitation of Alfvén Waves in the Ionosphere

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A novel method for the excitation of ELF waves in the ionosphere is presented. It utilizes the nonlinear mixing of two ground- or satellite-produced radio frequency waves with a compressional Alfvén wave. Parametric amplification thresholds are estimated to be between 10 and 20 MW peak power for 5-MHz pumps producing waves in the ELF range. Potential applications to ELF communications are discussed.

Introduction

The purpose of this report is to present a way by which the ionospheric plasma can act as a nonlinear active medium to transform HF waves radiated from the ground into ELF waves with efficiency sufficient to excite the transverse electric (TEM) mode of the 'waveguide' formed by the earth and the ionosphere. Excitation of this mode by ground-based ELF antennas has been considered as an ideal mode for Navy communications due to its low propagation losses in the waveguide (≈ 1 dB/Mm) and its low absorption properties in the seawater [*Bernstein et al.*, 1974]. Since the free space wavelength of the ELF waves is of the order of 3000 km, ground-based antennas are very inefficient radiators and require enormous antenna systems. If the power generated by our nonlinear scheme can be coupled to the waveguide, it can provide a viable alternative to the ground antenna systems.

Our proposed scheme is based on the interaction of two HF waves [*Papadopoulos*, 1975] in a manner analogous to the conventional stimulated forward Brillouin scattering [*Liu*, 1976]. The interaction is in a stimulated (i.e., lasing) fashion so that the efficiency achieved can exceed the efficiency of conventional mode mixing by several orders of magnitude. The excitation of the waveguide differs from ground-based systems in two respects. First, the waveguide is excited from the ionosphere rather than the ground, allowing more flexibility in location and focusing properties. Second, the antenna is 'wireless,' since the nonlinear currents are excited in the ionospheric plasma rather than in wires. It is shown that the power available from RF heating installations is sufficient for experimental tests of these concepts.

The nonlinear mechanism considered here is the parametric decay of a high-frequency radio wave ($\omega_0 \gg \omega_e$), into a low-frequency compressional Alfvén wave and a high-frequency sideband. The density perturbations associated with the Alfvén wave (ω, $\mathbf{k}$) couple to the drift velocity of the electrons due to the pump to produce a current driving the sideband. The sideband, in turn, couples with the pump to produce a low-frequency ponderomotive force, driving the instability. The frequency of the excited low-frequency wave is fixed by simply enhancing the sideband energy from the ground with a transmitter at a frequency $\omega_1 = \omega_0 - \omega$ and wave number $\mathbf{k}_1 = \mathbf{k}_0 - \mathbf{k}_1$. For the frequencies of interest $\omega \ll \omega_0$, $|\mathbf{k}| \ll |\mathbf{k}_0|$; therefore the process corresponds to forward stimulated scattering off Alfvén waves. We proceed below to estimate quantitatively the constraints and the efficiency of the process.

Basic Equations

We derive first the mode coupling strength for a homogeneous plasma. Consider a high-frequency electromagnetic wave $\hat{\mathbf{e}}_y E_0 \exp[-i(\omega_0 t - \mathbf{k}_0 \cdot \mathbf{x})]$ propagating in the $x - z$ plane in the ionosphere; the z axis is in the direction of the earth's magnetic field, while the x axis lies in the vertical plane. For pump frequency $\omega_0 > \Omega_e$, where Ω_e is the electron cyclotron frequency, the electron high-frequency response is unmagnetized. We examine the decay of the pump into an Alfvén wave (ω, $\mathbf{k}$) and two high-frequency sidebands ($\omega_\pm$, $\mathbf{k}_\pm$), where $\omega_\pm = \omega \pm \omega_0$, $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0$ and $\mathbf{k}$ lies in the $x - z$ place. The linear response of the electrons at the high frequency may be written as

$$\mathbf{v}_j = \hat{\mathbf{e}}_y \frac{eE_j}{im\omega_j} \qquad j = 0, 1, 2 \tag{1}$$

where $-e$ and m are the charge and mass of the electrons.

The high-frequency waves produce a low-frequency (ω, $\mathbf{k}$) ponderomotive force $\mathbf{F}_p = -e\mathbf{E}_p$ on the electrons, given by

$$\mathbf{E}_p = \frac{i\, e\, \mathbf{k}\,(\mathbf{E}_0 \cdot \mathbf{E}_- + \mathbf{E}_0^* \cdot \mathbf{E}_+)}{2m\omega_0^2} \tag{2}$$

which drives a low-frequency current,

$$\mathbf{J} = \boldsymbol{\sigma}_e \cdot (\mathbf{E} + \mathbf{E}_p) \tag{3}$$

and density fluctuations

$$n = \frac{-\mathbf{k} \cdot \boldsymbol{\sigma}_e \cdot (\mathbf{E} + \mathbf{E}_p)}{e\omega} \tag{4}$$

where $\boldsymbol{\sigma}_e$ is the electron conductivity tensor and $\mathbf{E}$ is the self consistent electric field. Using (3), the low-frequency wave is given by

$$-k^2\mathbf{E} + \mathbf{k}\,\mathbf{k} \cdot \mathbf{E} + \frac{\omega^2}{c^2}\left(\mathbf{I} + \frac{4\pi i}{\omega}\boldsymbol{\sigma}_i\right) \cdot \mathbf{E} = -\frac{4\pi i\omega}{c^2}\boldsymbol{\sigma}_e \cdot (\mathbf{E} + \mathbf{E}_p) \tag{5}$$

where $\boldsymbol{\sigma}_i$ is the ion conductivity tensor.

The nonlinear current at the sidebands is given by $\mathbf{J}_+^{NL} = -\frac{1}{2}ne\mathbf{v}_0$, $\mathbf{J}_-^{NL} = -\frac{1}{2}ne\mathbf{v}_0^*$. Using these expressions for $\mathbf{J}_\pm^{NL}$

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and (4) and (5), the wave equation for the sidebands gives

$$\mathbf{E}_+ = \frac{2\pi e}{m\omega D_+}\mathbf{k}\cdot\left(\boldsymbol{\sigma}_i + \frac{\omega}{4\pi i}I\right)\cdot \mathbf{E}\,\mathbf{E}_0 \tag{6}$$

$$\mathbf{E}_- = \frac{2\pi e}{m\omega D_-}\mathbf{k}\cdot\left(\boldsymbol{\sigma}_i + \frac{\omega}{4\pi i}I\right)\cdot \mathbf{E}\,\mathbf{E}_0^* \tag{7}$$

where

$$D_\pm = \omega_\pm^2 - k_\pm^2 c^2 - \omega_e^2$$

Using (2), (6), and (7) in (5), we obtain

$$\mathbf{E} = \boldsymbol{\alpha}\cdot\mathbf{E} \tag{8}$$

where

$$\boldsymbol{\alpha} = \frac{4\pi^2 V_0 V_0^*}{c^2}\left(\frac{1}{D_+}+\frac{1}{D_-}\right)\left[\frac{\omega^2}{c^2}\boldsymbol{\varepsilon} - k^2\,\mathsf{I} + \mathbf{k}\,\mathbf{k}\right]^{-1} \cdot [\boldsymbol{\sigma}_e\cdot\mathbf{k}\,\mathbf{k}\cdot\boldsymbol{\sigma}_i] \tag{9}$$

$\boldsymbol{\varepsilon}$ is the low-frequency dielectric tensor. Equation (8) immediately gives the nonlinear dispersion relation $|\mathsf{I} - \boldsymbol{\alpha}| = 0$, which to the first order in V_0^2 becomes

$$1 = \alpha_{xx} + \alpha_{yy} + \alpha_{zz} \simeq \frac{k_x^2 V_0^2}{4c^2}\,\frac{\omega_i^4\,(k^2 - \omega^2/V_A^2\cos^2\theta)}{(\omega^2-\Omega_i^2)(k^2-\alpha_1)(k^2-\alpha_2)} \cdot\left(\frac{1}{D_+}+\frac{1}{D_-}\right) \tag{10}$$

where $V_A^2 = cB^2/4\pi nM$ is the Alfvén speed, $\theta = \tan^{-1} k_x/k_z$, and

$$\alpha_{1,2} = \frac{\omega^2\Omega_i^2[-(1+\cos^2\theta) \pm (\sin^4\theta + (4\omega^2\cos^2\theta/\Omega_i^2))^{1/2}]}{2V_A^2(\omega^2-\Omega_i^2)\cos^2\theta} \tag{11}$$

We should mention that $k^2 - \alpha_1 = 0$ represents the compressional Alfvén branch which continues to frequencies above Ω_i, taking over as the whistler mode. The $k^2 - \alpha_2 = 0$ corresponds to the shear Alfvén wave, which exists only below the ion cyclotron frequency. We consider below only the compressional Alfvén branch.

For the resonant decay, it is sufficient to consider only the lower sideband, since the other one is off resonance, i.e., $D_- \ll D_+$. Expanding $\omega = \omega + i\gamma_0$, $\gamma_0 < \omega$, the growth rate for the parametric instability turns out to be

$$\gamma_0^2 = \frac{k_x^2 V_0^2\omega_i^2}{16\omega\omega_0}\,\frac{(\omega^2-\Omega_i^2)^2}{\Omega_i^4}\left(1 - \frac{k_z^2 V_A^2}{\omega^2}\right) \cdot\left\{(1+\tfrac{1}{2}\tan^2\theta)\left(\sin^4\theta + \frac{4\omega^2}{\Omega_i^2}\cos^2\theta\right)^{1/2} + \frac{\sec^2\theta}{2}\left[\frac{2\omega^2(\omega^2-3\Omega_i^2)}{\Omega_i^4}\cos^2\theta - \sin^4\theta\right]\right\}^{-1} \tag{12}$$

In the two limits of $\omega \ll \Omega_i$ or $\omega \gg \Omega_i$, (12) reduces to

$$\gamma_0^2 \simeq \frac{k_x^2 V_0^2\omega_i^2}{16\omega\omega_0} \tag{13}$$

It should be noted here that for this channel of resonant decay, the nonlinear dispersion relation (12) can be easily obtained by writing the x and y components of (5) (neglecting E_z),

$$-k_z^2 E_x + \frac{\omega^2}{c^2}(\varepsilon_{xx}E_x + \varepsilon_{xy}E_y) \simeq 0 \tag{14}$$

$$-k^2 E_y + \frac{\omega^2}{c^2}(\varepsilon_{xx}E_y - \varepsilon_{xy}E_x) = -\frac{2\pi\omega e\,k_x}{m\omega_0^2c^2}\sigma_{exy}E_0E_1 \tag{15}$$

and rewriting the wave equation for the sideband as

$$(\omega_-^2 - \omega_e^2 - k_-^2c^2)E_1 = 2\pi i\omega_0\,neV_0^* \simeq \frac{2\pi i\omega_0}{\omega}\sigma_{exy}k_xE_yV_0^* \tag{16}$$

Equations (14) and (15) can be combined to obtain

$$E_y = -\frac{(\omega^2\varepsilon_{xx}/c^2 - k_z^2)}{(k^2-\alpha_1)(k^2-\alpha_2)}\,\frac{2\pi\omega e\,k_x\sigma_{exy}}{m\omega_0^2c^2}E_0E_1 \tag{17}$$

Equations (16) and (17) yield the nonlinear dispersion relation, viz., (12).

We give below an order of magnitude estimate of the growth and a justification for the neglect of damping. A ground-based transmitter of power W produces, at a height R, a power density $P = GW/4\pi R^2$, where $G = 4\pi R^2/L^2$ is the gain of the antenna and L is the transverse extent of the pump. For a 10-MW, 5-MHz transmitter with $G = 10^3$ and $R = 120$ km, one obtains $P = 5\times10^{-2}$ W/m², $E_0 \simeq 6.6$ V/m, $V_0 \simeq 4\times10^6$ cm/s, $\gamma_0 \simeq 10^2$ s^{-1} at $\omega \simeq 10^3$ rad s^{-1}. It may be mentioned here that the linear damping rates Γ, Γ_1, of the decay waves require a threshold for the instability $\gamma_0 > (\Gamma\,\Gamma_1)^{1/2}$ which turns out to be quite small.

EFFECT OF PLASMA INHOMOGENEITY AND NONUNIFORM PUMP

On account of the vertical density gradient in the ionosphere, the three wave resonance conditions are satisfied only locally, and the instability could be absolute or convective depending on the horizontal extent of the pump. To investigate these effects, we rotate our frame of reference to designate x axis as the vertical ($\nabla n \parallel \hat{x}$) and z axis as the horizontal. The wave number matching conditions are satisfied at $x = 0$. Expressing

$$\omega \to \omega + i\frac{\partial}{\partial t} \qquad \mathbf{k} \to \mathbf{k} - i\frac{\partial}{\partial \mathbf{x}}$$

$$V_AE_1/c = \varepsilon_1 \qquad (\omega_0/\omega)^{1/2}E_y = \varepsilon$$

Equations (15) and (16) can be written as [*Rosenbluth*, 1972]

$$\frac{\partial\varepsilon}{\partial t} + \mathbf{v}_g\cdot\frac{\partial\varepsilon}{\partial\mathbf{x}} = \gamma_0(z)\,\varepsilon_1\exp(-ik'x^2/2) \tag{18}$$

$$\frac{\partial \varepsilon_1}{\partial t} + \mathbf{v}_{g1} \cdot \frac{\partial \varepsilon_1}{\partial \mathbf{x}} = \gamma_0(z)\varepsilon \exp\left(\frac{ik'x^2}{2}\right) \tag{19}$$

where $\mathbf{v}_g$ $(= V_A^2\ \mathbf{k}/\omega)$ and $\mathbf{v}_{g1}(= c^2\mathbf{k}_-/\omega)$ are the group velocities of the decay waves and $k' = (\partial/\partial x)\ (k_x - k_{0x} - k_{-x})$. Equations (18) and (19) have the standard form of two-dimensional nonlinear interaction. Following *Reiman* [1978], we reduce these equations into the form of one dimensional interactions. Introducing a set of new variables, $t' = t$, $x' = z$, $z' = x - \alpha z - ut'$ with $\alpha = (v_{gx} - v_{g1x})/(v_{gz} - v_{g1z})$, $u = v_{gx} - \alpha v_{gz}$ and expressing,

$$\varepsilon = a \exp i\, \frac{k'}{2} \left\{ z' \left[-\frac{z'}{2} + 2\,\phi\,(x' - v_{gz}t') \right] - u\,(x' - v_{gz}t')^2\, \phi/v_{gz} \right\} \tag{20}$$

$$\varepsilon_1 = a_1 \exp -i\, \frac{k'}{2} \left\{ z' \left[-\frac{z'}{2} - 2\,\phi_1\,(x' - v_{g1z}t') \right] - u\,(x' - v_{g1z}t')^2\, \phi_1/v_{g1z} \right\} \tag{21}$$

$\phi = v_{g1x}/(v_{gz} - v_{g1z})$, $\phi_1 = v_{gx}/(v_{gz} - v_{g1z})$, equations (18) and (19) can be written as

$$\frac{\partial a}{\partial t'} + v_{gz}\frac{\partial a}{\partial x} = \gamma_0(x')\, a_1 \exp\left(-ik_{\text{eff}}'\,\frac{x'^2}{2}\right) \tag{22}$$

$$\frac{\partial a_1}{\partial t'} + v_{g1z}\frac{\partial a_1}{\partial x'} = \gamma_0(x')\, a \exp\left(+ik_{\text{eff}}'\,\frac{x'^2}{2}\right) \tag{23}$$

where $k_{\text{eff}}' = k' v_{gx} v_{g1x}/v_{gz} v_{g1z}$. It must be mentioned here that for a uniform pump, the instability is convective and the convective amplification is given by

$$\lambda = \pi\, \gamma_0^2/v_{gx} v_{g1x}\, k' \tag{24}$$

Taking $k_x \sim L_n$ (density scale length) and using the expression for γ, (24) may be rewritten as

$$\lambda \approx \pi \frac{V_0^2}{V_A^2}\left(\frac{\Omega_i L_n}{c}\right)^2 \frac{c^4}{V_A^3 v_{g1x}} \frac{\Omega_i^2}{16\omega\omega_0}$$

For $\Omega_i \sim 2 \times 10^2$ rad s^{-1}, $c/V_A \simeq 10^3$, $L_n \simeq 10$ km, $f_0 \simeq 5$ MHz, the threshold ($\lambda \gtrsim 1$) comes out to be $W \simeq 10$ MW at $G \simeq 10^4$. However, with a nonuniform pump, the instability could become absolute. Expressing a, a_1 as

$$a = \frac{A\, e^{\gamma t'}}{|v_{gz}|^{1/2}} \exp\left[-\frac{x'}{2}\,\gamma\left(\frac{1}{v_{gz}} - \frac{1}{|v_{g1z}|}\right)\right] \tag{25}$$

$$a_1 = \frac{A_1\, e^{\gamma t'}}{|v_{g1z}|^{1/2}} \exp\left[-\frac{x'}{2}\,\gamma\left(\frac{1}{v_{gz}} - \frac{1}{|v_{g1z}|}\right)\right] \tag{26}$$

$$\eta = \frac{\gamma}{2}\left(\frac{1}{v_{gz}} + \frac{1}{|v_{g1z}|}\right)$$

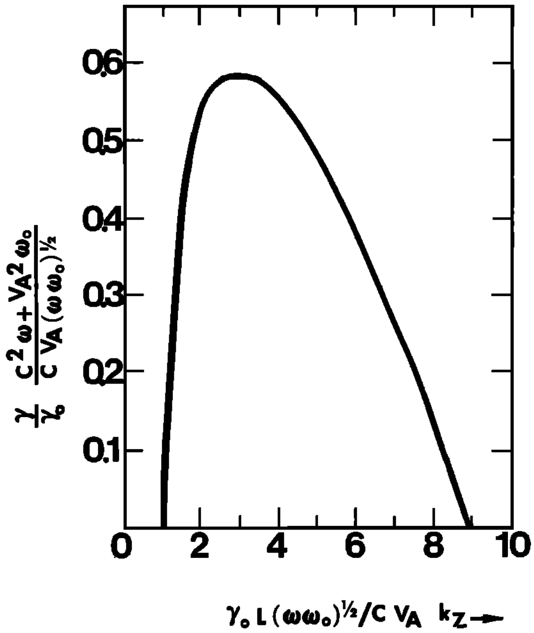

Fig. 1. Growth rate of the fastest growing Alfven wave for a Gaussian bump in an inhomogeneous plasma. The parameters are RF power density ~ 0.5 W/m^2, $f_0 \sim 5$ MHz, $f \sim 10^2$, $\Omega_i \sim 2 \times 10^2$ rad s^{-1}, $c/V_A \sim 10^3$, $L_n \sim 3$ km.

Equations (22) and (23) turn out to be

$$\left(\frac{\partial}{\partial x'} - \eta\right) A = -\frac{\gamma_0 A_1}{|v_{g1z}\, v_{gz}|^{1/2}} \exp -\left(\frac{ik'x'^2}{2}\right) \tag{27}$$

$$\left(\frac{\partial}{\partial x'} + \eta\right) A_1 = \frac{\gamma_0 A}{|v_{gz}\, v_{g1z}|^{1/2}} \exp\left(\frac{ik'x'^2}{2}\right) \tag{28}$$

For a Gaussian pump $\gamma_0 = \gamma_0\, e^{-x'^2/L^2}$, (27) and (28) can be solved numerically to obtain the normal modes. The growth rate of the largest growing mode as a function of the extent of the pump L is displayed in Figure 1 for $\lambda = 2$ (which corresponds to the RF power density $\simeq 0.5$ W/m^2 at $f_0 \sim 5$ MHz, $f \sim 10^2$, $\Omega_i \sim 2 \times 10^2$ rad s^{-1}, $c/V_A \simeq 10^3$, $L_n \simeq 3$ km). It is obvious from the figure that the instability is absolute for values of $\gamma_0 L(\omega\omega_0)^{1/2}/cV_A k_z$ between 1 and 9 and possess largest growth rate

$$\gamma_{\max} \simeq 0.5\ \gamma_0\, \frac{cV_A(\omega\omega_0)^{1/2}}{c^2\omega + V_A^2\,\omega_0} \simeq 0.2\ \gamma_0$$

at $\gamma_0 L(\omega\omega_0)^{1/2}/cV_A k_z \simeq 3$. This is achievable by a 20-MW transmitter having large gain $G \simeq 10^4$ (i.e., $L \simeq 3$ km).

DISCUSSION

We have demonstrated that with moderate RF technology ELF waves can be generated in the ionosphere in a stimulated fashion (i.e., absolute instability). In this sense the process can be considered as an equivalent MASER at ELF (ELFASER). The theoretical upper limit of the efficiency as given by the Manley-Rowe relations is

$$\eta = \frac{\omega}{\omega_0}\frac{c}{V_A} \approx 10^{-2}$$

which for the parameters discussed above gives an upper limit on ELF power in the ionosphere of the order of 200 kW. We, therefore, expect that an efficiency of even a few percent of the theoretical limit will produce substantial ELF power in the ionosphere. A discussion of the efficiency with which this power is coupled to the waveguide and its optimization lies beyond the scope of this short note and will be discussed elsewhere. We note, however, that studies of ELF transmitting satellites [*Kelly et al.*, 1976] indicate that 10 kW of ELF-radiated power is sufficient for long-range waveguide excitation. Our analysis was based on the ponderomotive force pressure being the dominant nonlinear force in our interaction region. A similar analysis can be carried for the case where the dominant force is from ohmic heating nonlinearities [*Perkins*, 1974; *Duncan and Benhinke*, 1978]. The ratio of the two forces is

$$\frac{P_0}{P_R} = \frac{1}{k_\perp^2 r_e^2 + 3k_2^2\lambda^2}$$

where λ is the electron mean free path and r_e is electron gyroradius. For F region parameters ($\lambda \approx 1.5 \times 10^4$ cm) this ratio is of order unity, and our results are valid to within a factor of 2.

Before closing we should comment on the role of competing parametric phenomena such as ion acoustic decay [*Fejer and Kuo*, 1973] and self-focusing [*Perkins and Goldman*, 1981]. Even if these phenomena occur simultaneously, since their conversion efficiency is again controlled by the Manley-Rowe relations, they are not expected to lead to pump depletion, suppressing the ELF generation. However, in a practical application the Alfvén wave decay can be easily made the dominant nonlinear mode by enhancing the lower sideband (i.e., $\omega_0 - \omega$) from the ground with power of 10–20 kW. This issue will also be discussed in detail elsewhere.

In conclusion, we have demonstrated the feasibility of exciting ELF waves in the ionosphere in a lasing mode by utilizing high-frequency pumps launched from a ground-based transmitter. The threshold power at 5 MHz is around 10 MW, which could be considerably reduced by including the effects of plasma inhomogeneity on the propagation of the pump and the sideband. It should be noted that the present method for ELF excitation is completely different from the method of modulating preexisting dc ionospheric currents [*Stubbe and Kopa*, 1977], using modulated ionospheric RF heaters.

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REFERENCES

Bernstein, S. L., M. L. Burrows, J. E. Evans, A. S. Griffiths, D. A. McNeil, C. W. Niessen, I. Richer, D. P. White, and D. K. William, Long range communications at ELF, *Proc. IEEE, 62,* 292, 1974.

Duncan, L. M., and R. A. Behinke, Observations of self-focusing of electromagnetic waves in the ionosphere, *Phys. Rev. Lett., 41,* 988, 1978.

Fejer, J., and Y. Y. Kuo, Structure in the nonlinear saturation spectrum of parametric instabilities, *Phys. Fluids, 16,* 1490, 1973.

Kelly, F., Waveguide-mode power budget for an ELF/VLF transmitting satellite, *Memo. Rep. 8032* Nav. Res. Lab., Washington, D. C., 1976.

Liu, C. S., Parametric instabilities in inhomogeneous unmagnetized plasmas, *Adv. Plasma Phys., 6,* 83, 1976.

Papadopoulos, K., Nonlinear production of ELF waves by two high frequency waves, internal report, Nav. Res. Lab., Plasma Phys. Div., Washington, D. C., March 1975.

Perkins, F. W., A theoretical model for short-scale field-aligned plasma density striations, *Radio Sci., 9,* 1065, 1974.

Perkins, F. W., and M. V. Goldman, Self-focusing of radio waves in an underdense ionosphere, *J. Geophys. Res., 86,* 600–608, 1981.

Reiman, A., Parametric decay in a finite width from pump including the effects of 3-dimensional geometry and inhomogeneity, *Phys. Fluids, 21,* 1000, 1978.

Rosenbluth, M., The parametric instabilities in inhomogeneous media, *Phys. Rev. Lett., 29,* 565, 1972.

Stubbe, P., and H. Kopka, Modulation of the polar electrojet by powerful HF waves, *J. Geophys. Res., 82,* 2319, 1977.

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