Nonlinear stabilization of beam plasma interactions by parametric effects

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The nonlinear stabilization of the kinetic stage of electron beam plasma instabilities by parametric effects is investigated. It is found that within a definite range of plasma parameters parametric instatilities induced by the beam generated waves can stabilize the system at a level of wave energy density substantially lower than expected by quasi-linear theory. This occurs because at a certain level of beam-generated plasma waves, the transfer rate of wave energy outside the spectral region in resonance with the beam exceeds the beam plasma instability growth rate. A model of a quasi-steady state for the case of continuous beam injection is proposed. The possibility of utilizing ultrarelativistic electron beams for achieving ignition temperatures in a tokamak is discussed.

I. INTRODUCTION

The interaction of low density beams with a plasma has been studied extensively with the neglect of nonlinear effects[1,2] due to the ambient plasma. In this approximation the energy of the oscillations excited by the beam is comparable to the initial beam energy and the role of nonlinear interactions reduces to the transformation of the spectrum appearing in the final stage. A similar point of view was taken in some recent computer simulations[3–5] of the nonresonant beam plasma interaction in which case the beam plasma interaction was stabilized first by trapping the beam electrons and the nonlinear effects due to the plasma simply transformed the final spectrum.

In this paper, it is shown that stabilization of beam instabilities is possible in a region of parameter space due to similar nonlinear effects such as discussed in Refs. 3 and 4. The result will be that the beam may pass through the plasma without any large broadening in velocity space and with little energy loss. In the presence of constant electric fields the beam can actually be accelerated. This has several important consequences in the utilization of relativistic electron beams injected in toroidal geometries[6] to provide the additional heating required to go from Ohmic to ignition.[7] In addition it provides a natural explanation for several nonthermal features of space phenomena such as the type III radiobursts,[8] particle and field distributions, and anomalous resistivity of the auroral plasma,[9,10] etc.

The physics behind the stabilization process is that above a certain level of beam-generated plasma oscillations in the region of phase velocity in resonance with the beam, the spectrum in itself becomes unstable and transfers energy into phase velocity regions nonresonant with the beam. When such a process takes place in a time interval less than the characteristic time for the generation of the beam waves, the level of wave energy resonantly interacting with the beam remains low at all times and the beam state does not change much.

Tsytovich and Shapiro[11] investigated the influence of induced (nonlinear) scattering of plasma waves by thermal ions (nonlinear Landau growth) as a possible mechanism for nonlinear stabilization of the beam instability, while Rudakov[12] proposed that induced scattering on thermal electrons might be dominant for the case of ultrarelativistic electron beams. However, the validity of these results is restricted by the conditions of validity of weak turbulence theory which implies that they are correct only if $W/nT \ll (k_0\lambda_D)^2$. Here W and nT are the wave and plasma energies per unit volume, k_0 is the wave number typical of the spectrum, and λ_D is the Debye length. The nonlinear transfer rates found by such restrictive conditions are, in general, too small to affect the beam-plasma interactions in most situations of interest. This is especially true in the case of continuous beam injection which results in an even higher level of plasma waves.[2] When $W/nT > (k_0\lambda_D)^2$, the nonlinear broadening of the frequency spectrum $\Delta\omega/\omega_e \approx W/nT$ becomes more important than the dispersive properties of the plasma waves due to their kinetic energy, which is of the order of $\Delta\omega_{\rm kin}/\omega_e \approx \frac{3}{2}(k_0\lambda_D)^2$. This can be seen from the dispersion relation $\omega^2 = \omega_e^2 + 3k^2V_e^2$, since the presence of density fluctuations such that $\delta n/n \approx (k_0\lambda_D)^2$ introduces turning points trapping the plasma waves in the low density regions. Since $T\delta n = W$ or $\delta n/n \approx W/nT$, oscillations with short wavelengths $k\lambda_D \approx \sqrt{W/nT}$ can be produced. These waves have lower phase velocities and can interact with the ambient particles.

This picture is physically similar to the recently discussed parametric instabilities.[13] Parametric instability theory has revealed, in addition to the usual decay type instabilities where a plasma wave decays into another plasma wave of higher phase velocity and a sound wave,[2,13] the existence of aperiodic instabilities where the new waves are a plasma wave with lower phase velocity and a nonlinearly modified purely growing ion density fluctuation.[13] This last interaction is actually the dominant process if the frequency of the pump wave is near the plasma frequency (ω_e) as is usually the case with beam plasma instabilities. Notice that in this case the new plasma waves due to the lowering of their phase velocities can interact more efficiently with the ambient plasma and produce anomalous heating or tails.

The aperiodic instability, usually called the oscillating two-stream instability can be related to the induced ion scattering since both result from the interaction of two plasma waves with ions. In analogy with the beam plasma interaction[1] one can view the induced scattering as having two stages, the kinetic and the hydrodynamic. The kinetic stage corresponds to a resonance of the beats of two plasma waves with a small group of resonant plasma ions ($\omega_{ek1} - \omega_{ek2} = (\mathbf{k}_1 - \mathbf{k}_2 \cdot \mathbf{V})$ (nonlinear Landau growth) and it is usually called induced scattering. The hydrodynamic stage of this interaction describes the case where all the plasma particles nonresonantly participate in the interaction. It is this stage that is described as the oscillating two-stream or modified decay instability or when the computation is performed in real instead of wavenumber space as spikon or plasma soliton formation. In order to avoid confusion in nomenclature we will keep the name oscillating two stream for the second process and only refer to the first one as induced scattering. Induced scattering on ions or electrons, which has been considered as a possible nonlinear stabilization mechanism[12,14] is a thresholdless process, with small growth rate $\gamma_{nl} < \Delta\omega$, kc_s ($\Delta\omega$ is the width of the spectrum as seen by the ambient plasma, and c_s is the ion sound speed) and results in wave transfer to higher phase velocity or simply in change in the direction of the phase velocity of the wave[14] (i.e., tends to isotropise the spectrum). Oscillating two-stream-like processes including the modulational and the modified decay instabilities occur at pump amplitudes larger than a threshold. The value of the threshold depends on the damping rate of the daughter waves and the frequency mismatch between the pump and the daughter waves. As shown in Ref. 15 and 16 the growth rate $\gamma_{nl} > \Delta\omega$ and the direction of spectral transfer is toward lower phase velocities where the waves can interact resonantly with particles and create fast tails. We should also mention that although the warm beam plasma interactions create a spectrum with large width Δk, the frequency spread as seen by the plasma is $\Delta\omega \simeq v_{gr}\Delta k$ which is small since $v_{gr} \ll V_e$. Therefore, the coherence condition between pump waves and ambient particles (i.e., $\gamma_{nl} > \Delta\omega$) can be satisfied without requiring very large pump amplitudes. An extensive discussion of these points can be found in Ref. 15 and 16.

In this paper we shall examine the possibility of nonlinear stabilization of the resonant beam plasma interactions by ambient ion nonlinearities. We treat the nonlinear mode coupling effects by using the hydrodynamic equations for the ambient plasma and averaging over the "fast time"[15] ω_e^{-1}. This type of description contains the oscillating two-stream and the decay instability, but not the induced scattering which is consistent with the philosophy of the paper.

Since our purpose is to demonstrate the physical mechanisms which are operating, we are considering the interaction of a nonrelativistic beam in a homogeneous one-dimensional space which implies either a strong longitudinal magnetic field or that the transverse modes which grow very fast have been stabilized by a small transverse temperature spread.[4] Extension of these results to the case of relativistic beams with finite angular spread is presented at the end of the paper. We again stress that we are interested in the case where the beam has a substantial thermal spread $\Delta V_b / V_b > (n_b/n_p)^{1/3}$, where V_b, ΔV_b are the beam and speed and thermal spread and n_b, n_p are the beam and plasma densities. In case this condition is not satisfied at the beginning, our theory describes the second stage of the interaction, namely any energy exchange between the beam and the plasma after the cold beam instability is stabilized by trapping.[3,4]

In Sec. II we present our basic mathematical model and reduce it to a more tractable set. Section III examines the stability properties of a spectrum of long-wavelength plasma oscillations. The conditions for stabilization of the beam plasma instability in one shot injection are derived in Sec. IV. The case of a steady state injection and its consequences form the subject of Sec. V. Section VI deals with application of the theory to space and laboratory experiments. Section VII examines the possibility of utilizing beams for heating of toroidal devices. Finally, we summarize our results in Sec. VIII.

II. BASIC NONLINEAR EQUATIONS

Consider the interaction shown in Fig. 1. The evolution of this system including mode coupling terms can be described by the following set of equations[15–17]:

$$\frac{\partial F_b}{\partial t} = \frac{\partial}{\partial v}\left(D_b \frac{\partial F_b}{\partial v}\right), \tag{1}$$

$$\left[i\frac{\partial}{\partial t} - \frac{3}{2}\omega_e (k\lambda_D)^2\right] E(k) = i(\gamma_k - \nu_k)E(k) + \frac{\omega_e}{2n_p}\int dk'\, \delta n(k-k')\, E(k'), \tag{2}$$

$$\left(\frac{\partial^2}{\partial t^2} + \nu_{ik}\frac{\partial}{\partial t} + k^2 c_s^2\right)\delta n(k) = -\frac{k^2}{16\pi M}\int dk'\, E(k-k')\, E^*(-k'), \tag{3}$$

$$\frac{\partial}{\partial t}F_e = \frac{\partial}{\partial v}\left(D\frac{\partial F_e}{\partial v}\right), \tag{4}$$

$$\gamma_k = \frac{\pi}{2}\,\omega_e \frac{\omega_e^2}{|k|^3} k \left(\frac{\partial}{\partial v} F_b\right) v = \frac{\omega_e}{k} \quad , \tag{5}$$

$$D_b = \frac{16\pi^2 e^2}{m^2}\int dk\, \delta(\omega_{ek} - kv)\frac{|E_k^2|}{8\pi} \quad , \tag{6}$$

$$D = \frac{e^2}{m^2}\int \frac{\alpha_k E_k^2 d}{(\omega_{ek} - kv)^2 + \alpha_k^2} \quad , \tag{7}$$

$$\omega_{ek} = \omega_e + \tfrac{3}{2}\,\omega_e (k\lambda_D)^2, \tag{8}$$

where ν_k, ν_{ik} are the total (collisional and collisionless) electron and ion damping, $m(M)$ is the electron (ion) mass, $\delta n(k)$ is the ion density fluctuation, and the c_s is the sound speed. We proceed to describe the physics and the assumptions involved in the previous set of equations. Equation (1) is simply the usual quasi-linear interaction of the beam with the waves it generates, with the resonant diffusion coefficient D_b given by Eq.

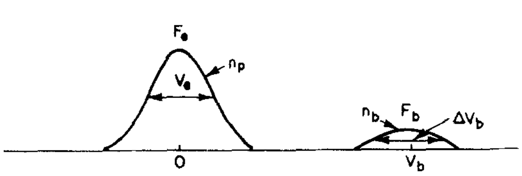

FIG. 1. Beam plasma interaction geometry in velocity space.

(6). It is derived under the assumption that the particles see random phased waves, which is readily satisfied for the resonant instability waves.[1] Equation (2) is simply the equation for the amplitude of mode k with phenomenological damping ν_k, growth rate γ_k given by Eq. (5), and mode-coupling terms. Notice that γ_k is different from zero only for waves in resonance with the beam. Equation (3) describes the evolution of the ion density fluctuations. In deriving Eqs. (2) and (3) we have averaged over the fast time ω_e^{-1}, neglected the electronic nonlinearity,[15] and described the motion of the electrons and ions on the basis of hydrodynamic equations.[13,15,16] These equations properly describe the oscillating two stream and decay interactions but not the nonlinear Landau damping, although they can easily be extended to include it. (For a more detailed discussion of these assumptions see Refs. 15–17.) Finally, Eq. (4) is added to describe the possibility of resonant diffusion of ambient electrons if the oscillating two-stream operates and transforms the spectrum to low phase velocities. The value of α_k in Eq. (7) simply describes the resonant width of the interaction in the Dupree–Weinstock sense.[18] At this point we should mention that in a numerical solution of the system Eqs. (1), (4), and (6) can be replaced by one diffusion equation of the form

$$\frac{\partial}{\partial t}F=\frac{\partial}{\partial v}\left(D\,\frac{\partial F}{\partial v}\right),$$

with $F=F_e+F_b$ and D given by Eq. (7), but with α_k being interpreted as the larger of the γ_k or the resonant width. The only assumption involved in this description is that particle trapping is nowhere important. We are actually in the process of finding numerical solutions of the set for situations of interest. However, the predominant physics can be found by the following simplified analysis.

We consider the wave spectrum as being composed of two groups of plasma waves and ion waves W_s. Group 1 consists of waves resonant with the beam which can be amplified. Group 2 consists of nonresonant waves which can grow only due to the mode coupling terms. Then, neglecting collisions we can use instead of Eqs. (1)–(4) the following set:

$$\frac{\partial W_1}{\partial t}=2\gamma_1 W_1-2\gamma_2(W_1)W_2\,, \tag{9}$$

$$\frac{\partial W_2}{\partial t}=2\gamma_2(W_1)W_2\,, \tag{10}$$

$$\frac{\partial W_s}{\partial t}=2\gamma_2(W_1)W_s\,, \tag{11}$$

$$\frac{\partial}{\partial t}F_b=\frac{8\pi e^2}{m^2}\,\frac{\partial}{\partial v}\left(\frac{W_1}{v}\,\frac{\partial F_b}{\partial v}\right), \tag{12}$$

$$\gamma_1=\frac{n_b}{n_p}\left(\frac{V_b}{\Delta V_b}\right)^2\omega_e\,, \tag{13}$$

with $W=\int dk\,|E_k|^2/8\pi$. $W_{1,2}$ represents the wave energy density in groups 1 and 2. The value γ_1 represents the average growth of group 1 due to the beam, $\gamma_2(W_1)$ represents the transfer rate from 1 to 2. For the time being from Eqs. (9)–(12) and assuming $\gamma_2(W_1)$, as known we can determine the value of W_1 at which the beam instability is nonlinearly stabilized and from Eq. (12) the new time scale for quasi-linear diffusion. The calculation of γ_2 will be the subject of the next section. Before closing this section we should point out that the values of γ and W are defined in an average sense for each wave group.

III. LINEARIZED THEORY

The transfer rates $\gamma_2(W_1)$ and the region of spectral transfer can be found by considering the stability of a spectrum of plasma oscillations (Fig. 2) on the basis of Eqs. (2) and (3), neglecting collisional effects and assuming thermal noise in the rest of the spectrum. The dispersion relation for such a system was found in Ref. 16 and is given by

$$(\omega^2-k^2c_s^2)+\frac{3}{4}\,\frac{m}{M}\,\frac{(k\lambda_D)^4}{nT_e}\,\omega_e^4\int dk'\,W_1(k')$$
$$\times\{\tfrac{9}{4}\omega_e^2(k\lambda_D)^4-[\omega-3(k\lambda_D)(k'\lambda_D)\omega_e]^2\}^{-1}=0. \tag{14}$$

Given the spectrum $W_1(k')$ one can find, by numerical means, the rates and regions of spectral transfer. This is in itself very useful for the evaluation of turbulent spectra, and it will be presented elsewhere.[16] For the purpose of the present paper it is sufficient to notice that if $\omega=\omega_0+i\gamma$ as long as

$$\tfrac{9}{4}\omega_e^2(k\lambda_D)^4+\gamma^2>[\omega_0-3(k_1\lambda_D)^2\omega_e]^2\,, \tag{15}$$

one can neglect the dependence of the denominator in the integration parameter of Eq. (14). This is equivalent to taking the spectrum $W(k')=\delta(k')$. The dispersion relation then becomes

$$(\omega^2-k^2c_s^2)+\tfrac{3}{4}(k\lambda_D)^4\,\frac{m}{M}\,\frac{W_1}{nT}\;\omega_e^4\,[\tfrac{9}{4}\omega_e^2(k\lambda_D)^4-\omega^2]^{-1}=0, \tag{16}$$

which is the same as the one found in the theory of parametric instabilities[13,19] with a pump at ω_e in the dipole approximation. In this sense one can consider (15) as the condition under which the dipole approximation can be applied to the stability of a spectrum of plasma oscillations. Equation (16) only has purely growing solutions, with typical growth rates given by

$$\gamma_2(W_1)\approx\left(\frac{m}{M}\right)^{1/2}\left(\frac{W_1}{n_pT}\right)^{1/2}\omega_e\,. \tag{17}$$

From Eqs. (15–(17) we find that for $\Delta k_1<k_1$ this result is correct as long as

$$\frac{W_1}{n_pT}>\left(\frac{\Delta k_1}{k_1}\right)(k_1\lambda_D)^2,\;\frac{m}{M}\,. \tag{18}$$

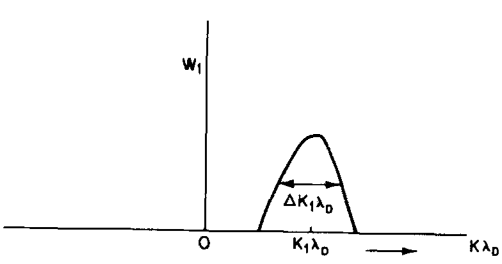

FIG. 2. Spectral distribution of primary plasma waves generated by the beam.

The inequality (18) can be considered as a modified threshold condition for the spectral transfer of the plasma waves to be toward the low phase velocity region with transfer rate given by (17). Since for any beam plasma interaction of interest Eq. (18) is easily satisfied, we further restrict our analysis to this regime [notice that typically $(k_1\lambda_D)^2 \ll 10^{-3}$]. For unusual cases where Eq. (18) is not satisfied, one should use Eq. (14) to determine the spectral transfer properties.

IV. NONLINEAR STABILIZATION

We proceed to examine the dynamics of beam plasma instabilities for the time dependent case (one shot injection) on the basis of the results of Secs. II and III. Assuming that we are in the regime given by Eq. (18), Eqs. (9) and (10) become

$$\frac{\partial W_1}{\partial t} = \alpha W_1 - \epsilon W_1^{1/2} W_2 , \tag{19}$$

$$\frac{\partial W_2}{\partial t} = \epsilon W_1^{1/2} W_2 . \tag{20}$$

In Eqs. (19) and (20) we are using the following dimensionless parameters:

$$t \to \omega_e t,\ W \to \frac{W}{n_p T}, \quad \epsilon = 2\left(\frac{m}{M}\right)^{1/2},$$

$$\alpha = 2\ \frac{n_b}{n_p}\left(\frac{V_b}{\Delta V_b}\right)^2 \delta n \to \frac{\delta n}{n_0} .$$

If we introduce a new time variable defined as

$$\tau = \int_0^t W_1^{1/2}(t)\, dt, \tag{21}$$

Eqs. (19) and (20) become

$$\frac{\partial W_1}{\partial \tau} = \alpha W_1^{1/2} - \epsilon W_2(0) \exp(\epsilon\tau), \tag{22}$$

$$W_2(\tau) = W_2(0)\exp(\epsilon\tau), \tag{23}$$

where $W_2(0)$ is the initial noise level.

On the basis of this set of equations we can sketch the evolution of the system in parametric space τ (Fig. 3). W_1 increases as $a^2\tau^2$ (this dependence corresponds to an exponential increase in t) until the contribution of the second term of Eq. (22) becomes significant. The wave energy W_1 reaches a maximum $W_{\max}$ at the value of τ^* given by

$$\exp(\epsilon\,\tau^*) = \frac{\alpha}{\epsilon}\ \frac{W_1}{W_2(0)} . \tag{24}$$

The value of this maximum is given by

$$W_{\max} = \frac{\alpha^2}{\epsilon^2}\left(\ln\frac{\alpha}{\epsilon}\ \frac{W_{\max}^{1/2}}{W_2(0)}\right)^2 \tag{25}$$

or

$$W_{\max} \approx \frac{\alpha^2}{\epsilon^2}\ \Lambda^2 , \tag{26}$$

where

$$\Lambda = \ln\frac{\alpha}{\epsilon}\ \frac{W_{\max}^{1/2}}{W_2(0)} . \tag{27}$$

Subsequently, the oscillation energy W_1 will decay exponentially while transferring energy to W_2. The eventual stationary state as described by (19)–(23) will be accomplished with $W_1 \to 0$, $W_2 \to W_{\max}$, and $W_s \to 0$. This, of course, is an erroneous conclusion due to the assumptions involved in deriving these equations. The first obvious defect is that we assumed that the pump waves W_1 are larger than W_2 which, of course, breaks down at the time τ_0 (Fig. 3) At this point the nonlinearity of the waves W_2 becomes equally important to W_1. In addition, ion fluctuations W_s increase in parallel with W_1 in accordance with Eq. (11). The presence of nonthermal density fluctuations results in an enhanced resistivity around the plasma frequency as discussed by Dawson and Oberman.[19] As we shall see in Sec. V, this can have a profound effect on the stationary state of the beam plasma interaction. Another defect is that the W_2 waves might start interacting with some ambient particles; i.e., the appropriate stationary state requires use of the complete set of equations (1)–(8). However, some extremely important conclusions with respect to the efficiency of the interaction of a beam with a plasma can be derived on the basis of the previous results with respect to the value of $W_{\max}$, for the case of a single shot injection. The results relevant to the stationary injection will be examined in the next section.

If the dynamics of the system were correctly described by quasi-linear equations until stabilization, the energy transfer would have been very efficient ($\eta > 0.3$), where the efficiency η is defined in terms of the beam energy ϵ_b as

$$\eta = \frac{W_1}{n_b \epsilon_b} . \tag{28}$$

The distribution function of the beam forms a plateau with $\Delta V_b / V_b \approx 1$, within a time $t_{qu} \approx n_p/n_b$ (in units of ω_e^{-1}) and $W_1^{qu} \approx \frac{1}{3} n_b\epsilon_b$ while another $\frac{1}{3} n_b\epsilon_b$ goes to oscillatory energy of the ambient particles and the rest remains with the beam particles. We can now draw the following obvious conclusions. For the case $W_1^{qu} < W_{\max}$, the nonlinear effects cannot stabilize the interaction and maximum transfer efficiency occurs with a value given by

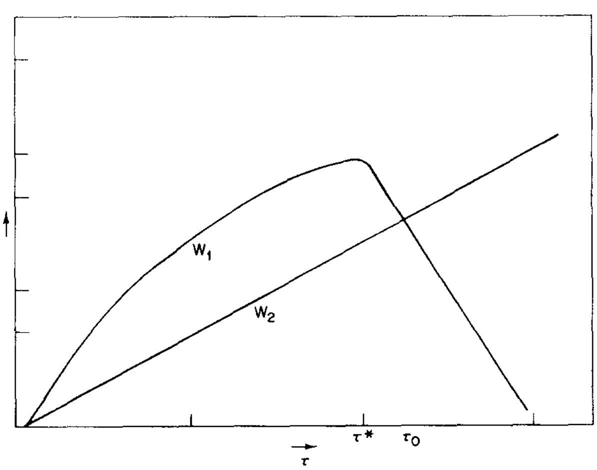

FIG. 3. Schematic of the evolution of W_1 and W_2 in parameter space t. (Notice semilog scale.)

$$\eta_0 = \frac{W_1^{qu}}{n_b \epsilon_b} . \tag{29}$$

However, for the opposite case ($W_{max} < W^{qu}$) the efficiency will be given by

$$\eta = \eta_0 \frac{W_{max}}{W_1^{qu}} . \tag{30}$$

In addition, one can find the thermal spread of the beam in this case. It will be given by

$$\frac{\Delta V_b}{V_b} \approx \frac{t^*}{t_{qu}} , \tag{31}$$

with t^* determined from τ^* and the inverse of Eq. (21),

$$t = \int_0^\tau \frac{dt'}{W_1^{1/2}(t')} . \tag{32}$$

We should note that the results of Eqs. (31) and (32) will not be influenced by the nonlinearity due to W_2 and the interaction of W_2 with the plasma particles which will only influence the distribution of the W_{max} between particles and waves and the final spectrum of turbulence.[20] Application of these results to specific situations in the laboratory and space will be discussed in the final section.

V. STEADY STATE INJECTION

The analysis of the previous section led to an unphysical asymptotic state due to the neglect of the nonlinear effects induced by the finite amplitude of the parametrically generated waves. Therefore, these results are relevant for the case of single shot experiments; that is, when the beam injection time t_0 is of the order of the nonlinear stabilization time t^*. However for stationary injection experiments ($t_0 \gg t^*$), most of the energy transfer occurs at times larger than t_0, when a stationary state is established. The purpose of this section is to examine possible stationary states with the inclusion of the nonlinearities due to the parametrically generated waves and the nonequilibrium state of the particles. A correct time dependent solution to this problem can only be provided by the numerical solution of Eqs. (1)–(8). Here, in the spirit of this presentation we shall attempt a simplified phenomenological approach, based on the marginal stability of the nonequilibrium system. We should point out that the nonlinear saturation mechanisms that will dominate have an extreme dependence on the parameters of the problem at hand. For this reason the analysis to be given should only be considered as an example of how a nonlinear state can be achieved if the particular mechanisms considered dominate, rather than a general state resulting in any beam plasma instability.

On the basis of the previous section the wave spectrum will be given by [Fig. 4(a)],

$$W(k, \omega) = W_1(k_1)\delta[\omega - \omega_e^{nl}(k_1)] + W^P(k, \omega) , \tag{33}$$

where $W^P(k, \omega)$ are the parametrically generated waves

$$W^P(k, \omega) = W_2(k_2)\delta\{\omega^2 - [\omega_e^{nl}(k_2)]^2\} + W_s(k_2)\delta\{\omega^2 - [\omega_s^{nl}(k_2)]^2\} , \tag{34}$$

where ω_e^{nl}, ω_s^{nl} are the nonlinear frequencies due to the presence of the finite amplitude waves (i.e., $\omega_{e,s}^{nl} = \omega_{e,s} + \Delta\omega_{e,s}^{nl}$, where $\Delta\omega^{nl}$ is the frequency shift). These and the values of W_1, W_2, and W_3 will be determined later. The nonequilibrium state of the particles will be given by Eq. (4). The nonequilibrium solution of this equation was discussed by Papadopoulos and Coffey,[9] where it was shown that the electron distribution function develops non-Maxwellian tails so that [Fig. 4(b)]

$$F_e(v) = F_e^M(V_e) + \beta F_T(v) , \tag{35}$$

where F_e^M is a Maxwellian and β is the normalized density of the tail. These results are quite insensitive to the exact parameters of the system when (18) is fulfilled in the initial stage. This can be seen from the following simple physical arguments. From the linear dispersion relation we can see that the growth rate (17) can be larger than the linear electron Landau damping only for $v_{ph} \gtrsim 3V_e$. Thus, since we have assumed that the trapping width is small, the total number of particles that will diffuse in velocity due to W_2 are $\beta \approx e^{-5} \sim 10^{-3}$, and they will exist only in regions where $W(k) \neq 0$, which in our case corresponds to $v_{ph} < V_b$, under the assumption that decay interactions which tend to populate the $v_{ph} > V_p$ region can be neglected.[21] We should notice that these results have subsequently been rigorously confirmed by Weinstock and Bezzerides.[22]

We proceed to examine the modification of the dispersion properties of the system due to the presence of the nonthermal features given by Eqs. (33)–(35). First, consider the high frequency waves. The presence of the parametrically generated waves $W^P(k_2, \omega)$ produces both a real frequency shift and a nonlinear damping of the electron plasma waves. They can be determined from the nonlinear dispersion relation given by[23] (in dimensionless units)

$$\epsilon^{nl}(\mathbf{k}, \omega) = \epsilon^l(\mathbf{k}, \omega) + \frac{1}{2}\int \frac{dk'}{2\pi}\int d\omega' S(k, \omega; k', \omega')W(k', \omega'), \tag{36a}$$

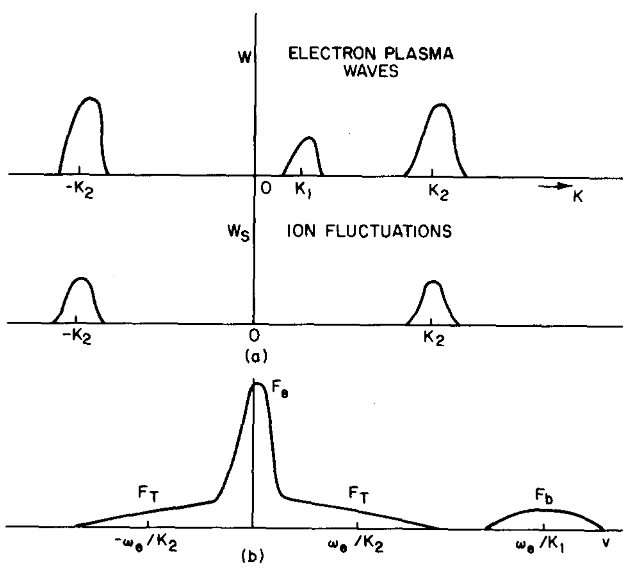

FIG. 4. Nonlinear quasi-steady state of beam-plasma system. (a) Spectral distributions of electron plasma waves (W) and ion fluctuations (W_s). (b) Particle distributions.

where $\epsilon^l(\mathbf{k}, \omega)$ is the linear dispersion relation and $S(k, \omega; k', \omega')$ is given by

$$S(k, \omega; k', \omega') = \frac{V(k, \omega; k', \omega')\, V(k-k', \omega-\omega'; k, \omega)}{\epsilon^{nl}(k-k', \omega-\omega')} + C(k, \omega; k', \omega')\,, \qquad (36b)$$

with

$$V(k, \omega; k', \omega') = \frac{1}{ik}\int dv (kv-\omega)^{-1} \frac{\partial}{\partial v}\{(k'v'-\omega')^{-1} + [(k-k')v-(\omega-\omega')]^{-1}\} F_e'(v)\,, \qquad (36c)$$

$$C(k,\omega;k',\omega') = \frac{1}{k}\int dv (kv-\omega)^{-1} \frac{\partial}{\partial v}\{[(k-k')v-(\omega-\omega')]^{-1} \frac{\partial}{\partial v}[(k'v'-\omega')^{-1}(kv-\omega)^{-1}]\} F_e'(v). \qquad (36d)$$

In Eqs. (36c,d) we have neglected the contribution of the ions since we are examining the high frequency waves.

In the present work we neglect the real frequency shifts in the high frequency oscillation since $\Delta\omega^{nl}/\omega_e \approx \frac{1}{8} W/nT \ll 1$, and since the relevant parameter is the frequency mismatch between them which is essentially unaffected. In addition, since both F_e' and F_i' vanish at $v=0$, nonlinear Landau damping between two plasma waves does not occur since their beat frequency is zero. In this way the predominant nonlinear effect is the coupling of the beam waves with the density fluctuations which scatters the waves to the nonresonant region. Then, from either Eq. (36a) or Dawson and Oberman[19] we find that in nondimensional units $\gamma_{nl} \simeq W_s/k_2^2$ and neglecting the spontaneous emission terms the rate equation for W_1 becomes

$$\frac{\partial W_1}{\partial t} = \left(2\gamma_b - \frac{W_s}{k_2^2}\right) W_1. \qquad (37)$$

Similar considerations can be applied to the rate equation for W_2 with the following additional consideration. From Eq. (35) we see that these plasma waves can interact efficiently with the particles in the tail and thus can be damped either by direct Landau damping or by resonance broadening in case the tail is flattened quasilinearly. An accurate value of the damping rate can only be found for specific experimental cases, since the slope of F_T depends on the loss rate of the fast particles from the interaction region (to walls, etc.). For the present case we simply take the total damping rate as ν_e. In this way the rate equation for W_2 becomes

$$\frac{\partial W_2}{\partial t} = \frac{W_s}{k_2^2} W_1 - \nu_e W_2. \qquad (38)$$

While we could neglect the real part of the frequency shift for the plasma waves, this is not valid for the ion waves. As can be seen from the linear dispersion relation, Eqs. (14) or (16), the dominant effect of the hf fields on the ions is a negative pressure which balances the particle pressure $k^2c_s^2$ and appears in the dispersion relation as a downshift of the ion acoustic wave by $\Delta\omega = kc_s$, i.e., standing density waves. The pressure of the waves, W_2, will also reduce the particle pressure and thus reduce the threshold for undamped nonlinear ion waves.

A nonlinear dispersion relation for the low frequency branch in the presence of $W^P(k, \omega)$ was recently derived by Weinstock and Bezzerides,[24] which clearly demonstrates how the mode-coupling terms reduce the particle pressure and allow oscillating two-stream waves to spread to shorter wavelengths. The essential feature of the nonlinear dispersion relation is that Eq. (14) has an additional term similar to the second term, which corresponds to the waves[24] W_2. In order to determine our nonlinear marginally stable state we want to find the value of W_1 or W_2 which can keep ion waves with wavenumber k_2 undamped. This answer can be found from examining Eq. (14) separately for W_1 and W_2, and it is well known in the theory of parametric instabilities. If the dominant term is due to W_1, the threshold condition will be determined by the frequency mismatch for the high frequency modes with wavenumbers k_1 and k_2. This is given by[25]

$$W_1^T = (k_1 - k_2)^2 \approx k_2^2. \qquad (39a)$$

If the dominant term is due to W_2, the frequency mismatch is very small and the threshold will be determined by the damping ν_e and will be given by

$$W_2^T = 4\nu_e. \qquad (39b)$$

We expect that the threshold will be determined by the smaller of (39a) and (39b). On the basis of (37)–(39) we can look for an equilibrium state. This will be given by

$$2\gamma_b = \frac{W_s}{k_2^2}, \qquad (40)$$

$$\frac{W_s}{k_2^2} W_1 = \nu_e W_2, \qquad (41)$$

and the smaller of

$$W_1 = W_1^T \ \text{ or } \ W_2 = W_2^T. \qquad (42)$$

(a) If we assume that

$$k_2^2 > 4\nu_e,$$

we find that we can have a marginally stable state with

$$W_2^0 \approx 4\nu_e, \quad W_s^0 \approx 2k_2^2\gamma_b, \quad W_1^0 \approx 2\frac{\nu_e^2}{\gamma_b}. \qquad (43)$$

(b) If $k_2^2 < 4\nu_e$,

$$W_1^0 = k_2^2, \quad W_s^0 = 2k_2^2\gamma_b, \quad W_2^0 = \frac{\gamma_b}{\nu_e}k_2^2. \qquad (44)$$

The physics of the quasi-steady state can be summarized as follows (Fig. 4). The beam instability is stabilized since the waves W_1 scatter on W_s and are transferred into W_2 where they Landau damp on the tails. At the same time the pondermotive force due to W_2.(case a) or due to W_1 (case b) keeps the nonlinear ion fluctuations W_s at a steady level.

VI. EFFICIENCY OF ENERGY TRANSFER FOR RELATIVISTIC AND NONRELATIVISTIC BEAMS

On the basis of the results derived in the two previous sections we can answer the questions we set forth in the introduction. We first examine the one shot injection and find the parameter region for efficient energy transfer within scale lengths given by quasi-linear theory. In order to find some numerical estimates we take the

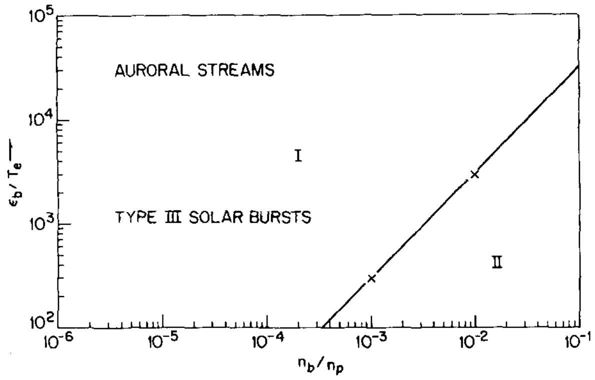

FIG. 5. Regions in parameter space where nonlinear stabilization (I) and quasi-linear stabilization (II) occurs for a nonrelativistic plasma. Notice the locations of auroral streamers and type III color bursts.

value of $\Lambda^2 \approx 50$. Since the dependence of Λ on the problem parameters is logarithmic, this value will not change by more than a factor of order unity for any case of interest in laboratory or space plasmas. A numerical check of this value should, however, be carried out *a posteriori*. Taking the mass ratio $M/m \approx 2\times 10^3$, we can plot Fig. 5, which summarizes the relevant parametric dependence. For plasma parameters in region I we shall have low efficiency transfer due to nonlinear stabilization, while in region II the stabilization in the case of single shot injection will be quasi-linear and the maximum efficiency transfer will occur on a very short timescale as given by Eq. (32) for t_{qu}. In Fig. 5 we have also marked the regions of two space phenomena, type III solar bursts[8] and high energy (10–20 keV) auroral streamers[9,10] which have been observed to propagate over large distances (10^{13} cm for the case of type III bursts and several thousands of kilometers for the precipitating electrons) without any significant energy loss. We can see (Fig. 5) that the location in parameter space of this space phenomena is in region I consistent with the observations. It is easy to check that in both cases.[8–10] $\Lambda^2 < 50$.

We can extend these results to the case of relativistic beams if we redefine the parameter α introduced in Sec. IV on the basis of the relativistically correct growth rate of the resonant beam plasma instability. As pointed out by Rudakov,[12] one should include the effect of the angular spread $\Delta\theta$ of the beam in the calculation of the growth rate. The value of the relativistic α will be given by[12,14]

$$\alpha_r = 2\frac{\gamma_1^r}{\omega_e} = 2\frac{n_b}{n_p}\frac{mc^2}{\epsilon_b}\frac{(\Delta\theta)^2 + mc^2/\epsilon_b}{[(\Delta\theta)^2 + (mc^2/\epsilon_b)^2\Delta\epsilon_b/\epsilon_b]^2}, \qquad (45)$$

where $\Delta\epsilon_b$ is the energy spread of the beam. It should also be noted that the spectrum of the beam waves created by a relativistic beam is given by $\Delta k_1/k_1 \approx (mc^2/\epsilon_b)^2\,\Delta\epsilon_b/\epsilon_b + (\Delta\theta)^2$, ,which as a rule is narrower than the nonrelativistic case, therefore facilitating the justification of a single wave parametric pump.

The maximum value of α is reached when $(\Delta\theta)^2 \le (mc^2/\epsilon_b)^2\,\Delta\epsilon_b/\epsilon_b$ and is given by

$$\alpha_r = 2\left(\frac{n_b}{n_p}\right)\frac{\epsilon_b}{mc^2}\left(\frac{\epsilon_b}{\Delta\epsilon_b}\right)^2 . \qquad (46a)$$

The dependence of the growth rate of the energy spread $\Delta\epsilon_b$ vanishes when $(\Delta\theta)^2 > (mc^2/\epsilon_b)^2\,\Delta\epsilon_b/\epsilon_b$ and if $(\Delta\theta)^2 > (mc^2/\epsilon_b)$, the value of α^r is given by

$$\alpha_r = 2\left(\frac{n_b}{n_p}\right)\frac{mc^2}{\epsilon_b}\,\frac{1}{(\Delta\theta)^2} . \qquad (46b)$$

In Fig. 6 we present results similar to those of Fig. 5, but on the basis of Eq. (46). Graph A corresponds to Eq. (46a) and graph B to (46b). Region I corresponds to low efficiency transfer for narrow spread $[(\Delta\theta)^2 \le (mc^2/\epsilon_b)^2\,\Delta\epsilon_b/\epsilon_b]$ but maximum efficiency for large angular spread $[(\Delta\theta)^2 < (mc^2/\epsilon_b)^2]$. Region II gives maximum efficiency in either case. Region III corresponds to maximum efficiency for narrow spread and low efficiency for large spread, while region IV always gives low efficiency. The expected efficiency in a single shot experiment can be estimated from Eqs. (26), (28), and (46). Experiments[26–32] relevant to our case in which the efficiency was substantially smaller than the quasi-linearly expected lie in region I. The only experiment where almost quasi-linear efficiency was achieved[27] is marked by a cross and its location is consistent with the theoretical prediction. The most recent publication in the literature[32] also indicates that the stabilization mechanism observed in this experiment is consistent with the strong turbulence theory notions first presented in Ref. 20 and elaborated on here.

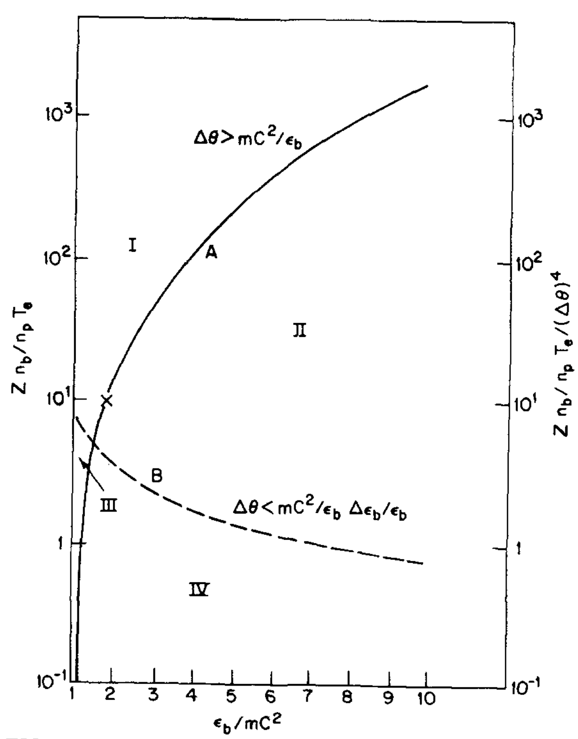

FIG. 6. Above the line A or B quasi-linear stabilization corresponding to appropriate parameter range, while below the lines nonlinear stabilization resulting in low efficiency transfer. Graph A corresponds to the scale right-hand while B to the left-hand scale. (Z is the atomic number of the ionic species.)

VII. RELATIVISTIC BEAM HEATING FOR THERMONUCLEAR PLASMAS

As an example of a concrete application of the theory presented previously we examine the feasibility of utilizing relativistic electron beams for tokamak heating.

As discussed by Sweetman,[7] in a thermonuclear ignition situation there is a gap between about 3 and 20 keV, over which total heat losses exceed the energy input due to Ohmic heating and heating due to the trapped fusion alpha particles. This energy can be supplied by an intense electron beam of short duration and a power level of 10^{12}–10^{13} W, which is presently available. The problem of the injection of the beam into a closed field configuration has been the topic of intensive experimental and theoretical research with promising results.[6] We do not concern ourselves here with this topic, but assume that the beam can be successfully injected into a toroidal device. Since we do not want to disrupt the equilibrium confinement, we should use a beam whose parameters lie in the weak coupling regions. On the basis of available high energy beams we consider, as a typical case, a beam with $\epsilon_b/mc^2 \approx 10$, $n_b/n_p = 10^{-3}$, and $\Delta\theta \approx \frac{1}{5}$, which is injected in a plasma with $T \approx 1$ keV and $n_p \approx 10^{14}$. From Fig. 6 we see that such a configuration lies well in the weak coupling region. The maximum value of wave energy $W_1^{\max}$ estimated from Eqs. (26) and (46b) is $W_1^{\max} \approx 7.5\times 10^{-2}$. The time scale for this to occur is given by Eqs. (25) and (30) and corresponds to times of a few nanoseconds. Within a few more nanoseconds the system will reach a quasi-steady state situation (Fig. 4) described in Sec. V. The value of k_2 can be estimated[16,32] to be $k_2 \approx \sqrt{W_1^{\max}}/6 \approx 0.1$ During this time the beam will lose about 1% of its energy. The quasi-steady state values and the subsequent evolution can be found from either Eq. (43) or (44) if we can estimate ν_e. The value of ν_e depends on the slope of the the tails F_T', which can be found only by balancing the effects of quasi-linear diffusion, resonance broadening, and particle convective loss to walls or out of the system. Since we know that $\beta \simeq 10^{-3}$, we can immediately put an upper limit on ν_e, namely, $\nu_e < 10^{-3}$. Since $k_2 \approx 0.1$, it turns out that $k_2^2 > \nu_e$ and the equilibrium will be found from Eqs. (43). In order to find some estimates of the turbulent levels, we assume that for a toroidal system we do not have losses of fast particles from the system, and a steady state distribution of the tails can be established by balancing quasi-linear diffusion with emission and absorption of waves due to collisions. For the tail distribution therefore,

$$\frac{\partial}{\partial t} F_T = \frac{\partial}{\partial v} D_2 \frac{\partial}{\partial v} F_T + \frac{\partial}{\partial v}\, \nu \left(vf + \frac{T_e}{m}\frac{\partial F_T}{\partial v}\right), \tag{47}$$

where ν is the binary collision frequency, $\nu \approx \lambda\, \omega_e^4/nv^3$ (λ is the Coulomb logarithm), and D_2 is the diffusion coefficient due to W_2. It is easy to solve Eq. (45) for the steady state since it leads to a sharp reduction in absorption.

$$\frac{\partial F_T}{\partial v} = \left(1 + D_2 \frac{m}{T_T \nu}\right)^{-1} \frac{\partial F_M}{\partial v}, \tag{48}$$

where F_M is a Maxwellian of temperature T_e. From Eq. (48) we find that

$$\nu_e = \beta(k_2^3)\, \frac{1}{W_2^0}\frac{\lambda}{n\lambda_D^3}. \tag{49}$$

Taking $k_2 \approx 0.1$, $\beta \approx 10^{-3}$, $n\lambda_D^3 \sim 10^6$ and, $\lambda \approx 20$, we find, for the effective collision frequency,

$$\nu_e \approx 10^{-11}\frac{1}{W_2^0}. \tag{50}$$

From Eqs. (45) and (50) we can determine the turbulence level for the marginal state (quasi-equilibrium). For the parameters used in the example we find $W_1^0 \approx 4\times 10^{-8}$, $W_2^0 \approx 2\times 10^{-6}$, and $W^0 \approx 5\times 10^{-5}$. These steady state levels of turbulence are quite low, and we do not expect the confinement of the plasma to be affected.

Given the turbulence levels we can estimate the slowing down lengths of the beam electrons due to their interaction with the high and low frequency waves. The relaxation time of the electrons due to the high frequency waves will be given by $t^H = c^2/D_1 \approx 10^{11}\,\omega_e^{-1}$, which is of the same order of magnitude as binary collisional processes. Nonresonant scattering, energy loss for sustaining of the tails, and friction amount to smaller contributions.

The predominant effect of the low frequency waves on the beam electrons is due to return current. This can be estimated from the relationship $(j^2/\sigma)\,t^E = n_b\epsilon_b$, use of the fact that $\sigma = ne^2/m\nu_0$, where ν_0 is an effective collision frequency, and noting that $en_b c = j$. Then

$$t^E = \frac{n_p}{n_b}\frac{\epsilon_b}{mc^2}\frac{\omega_e}{\nu_0}\,\omega_e^{-1}.$$

Assuming that $\nu_0 \approx \omega_e(W_s^0/nT)$, we find that

$$t^E = \frac{n_p}{n_b}\frac{\epsilon_b}{mc^2}\frac{\omega_e}{\nu_0}\,\omega_e^{-1} \approx 10^8\,\omega_e^{-1}.$$

In estimating the relaxation time t^E we assumed that the return current was stable either linearly or nonlinearly due to the enhanced level of W_s^0. For cases where an ion sound instability arises, the time t^E might be somewhat shorter.

These estimates seem to indicate that if beam injection in a toroidal device becomes feasible, relativistic beam heating becomes a very attractive method for maintaining thermonuclear temperatures.

VIII. CONCLUDING REMARKS

We have presented a theory for the stabilization of the resonant beam plasma instability due to the nonlinearity of the background ions. It has been shown that when the energy density of the beam-generated wave spectrum exceeds a threshold, it becomes unstable to other plasma waves with lower phase velocities and ion density fluctuations whose dispersion characteristics are determined by the nonlinear dispersion relation. Following time honored tradition,[3–5] we have called this process oscillating two-stream instablility with the generalized meaning of the instability of a plasma wave spectrum whose daughter waves have lower phase velocity. In this sense it includes the so called quasi-decays, modulational, and electrostatic self-focusing processes resulting in lower phase velocity plasma waves.

A numerical solution of the set of equations (1)–(8) presently in progress will provide a more complete solution to the problem for situations of interest. The emphasis here was in isolating and demonstrating the importance of various physical effects neglected in previous studies.[11,12,14] It was shown that:

(a) if the required level of wave energy density W_1 required for nonlinear stabilization at the initial stage is $W_1 > k_1^2$, the dominant stabilization is the oscillating two-stream instability which results in transferring energy to coupled lower phase velocity plasma waves and nonlinear ion waves.

(b) The consequences of such a stabilization mechanism instead of stimulated scattering on ambient particles are important with regard to the observational features and the eventual steady state of the beam plasma interaction. More specifically, it allows for the creation of symmetric electron tails, and for the propagation of weakly damped nonlinear ion waves even when $T_e/T_i \approx 1$. The presence of a nonthermal level of ion waves parametrically created by the beam itself can provide the necessary dissipation to stabilize the beam waves W_1 in a steady state. The absorption by the tails provides the damping for the stabilization of W_2, while the ponderomotive force exerted by the plasma waves on the ions allows for the existence of the weakly damped nonlinear ion waves.

There have been several experiments where such features as electron tails and plasma and ion waves not satisfying the linear dispersion relations have been observed.[26] In addition, we have provided estimates of the maximum possible energy transfer from the beam to the plasma for one shot experiments, examined the propagation of the beam under steady state conditions, and assessed the possibility of using electron beams for tokamak heating. More specific estimates will appear in future publications.

Before closing we should again point out our original assumptions in deriving Eqs. (1)–(8). Namely, we neglected:

(i) spontaneous emissions,

(ii) stimulated scattering,

(iii) mode conversion to electromagnetic waves, and

(iv) particle trapping.

In addition, in the results of Sec. VIII we neglected the decay interactions which will tend to produce some spectral density of plasma waves in regions with phase velocities larger than V_b.

The role of the above can only be assessed for the particular situations where the theory is applied.

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