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Lower Hybrid Turbulence at Cometary Bow Wave and Acceleration of Cometary Protons

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The wave measurements at the spacecraft encounters with comet Halley have shown intense wave activity at the lower hybrid frequency. The excitation of the lower hybrid instability by the pickup cometary ions (protons and water group) in the bow wave region and the quasi-linear diffusion of the ions in these fluctuations are discussed. The quasi-linear diffusion of the pickup protons takes place over a scale length shorter than that of the heavier water group ions. This enhances damping of the waves by protons, and when the pickup proton density is large enough, it can result in the saturation of the instability as this damping balances the heavy ion driven growth. The observed electric field amplitude and the scale length of proton relaxation are in agreement with the theory. For small pickup proton density the instability can saturate due to the wave energy cascade arising from the modulation instability of the large-amplitude lower hybrid waves. This saturation mecahnism leads to electron acceleration and suprathermal tail formation.

1. INTRODUCTION

The mass spectrometer and wave measurements at the comet Halley encounter with the Giotto [*Neugebauer et al.*, 1987, 1989, 1990] and VEGA [*Galeev et al.*, 1986; *Klimov et al.*, 1986] spacecraft have shown very effective cometary proton heating which is well correlated with the peak of wave activity at the lower hybrid frequency. The cometary proton heating occurs within several heavy ion gyroradii or approximately 5×10^4 km of the estimated "shock front." The magnetohydrodynamic (MHD) turbulence driven by the mass loading of the solar wind with heavy cometary ions is the dominant source of fluctuations in the transition region and develops very far away from the comet at distances of 5-10 $\times$ 10^6 km [*Johnstone*, 1990]. However the MHD turbulence may not be effective for proton heating because of the low spectral energy density of these waves resonating with the protons. In the upstream region of the mass-loaded solar wind the lower hybrid turbulence is quite weak as the growth rate scales as $\gamma \sim \omega(v_A/u)^3$, as is evident from the following (6), and the ratio of the Alfven to the solar wind velocities v_A/u in the undisturbed solar wind is usually small, $\sim$ 0.2. Also the isotropization of the cometary ion velocity distribution and the formation of the velocity shell from the initial ring distribution due to pitch angle scattering caused by MHD waves [*Gary et al.*, 1986; *Sagdeev et al.*, 1986] lead to the depletion of the source of the lower hybrid instability and its stabilization. Far upstream of the cometary bow shock this isotropization is ineffective because of a very low level of MHD turbulence. As the Giotto measurements have shown, the boundary at which the cometary ions picked up by the solar wind acquire a shell-like distribution is at 2.5–3 $\times$ 10^6 km from the comet Halley [*Coates et al.*, 1990], which is quite far upstream. The situation changes closer to the bow wave, where the ratio v_A/u approaches unity. The cometary ion velocity isotropization and formation of the shell distribution are due to their pitch angle diffusion caused by MHD waves as it was originally proposed by *Sagdeev et al.* [1986]. The rate at which the solar wind is mass-loaded by the cometary ions is proportional to $1/r^2$, r being the distance from the comet, and consequently much closer to the bow shock there are a significant number of freshly born ions whose velocity distribution has not been isotropized. The typical scale for isotropization is at least $\sim (u/\omega_{ci})(B^2/\Delta B^2)$, where the Larmor radius of the pickup ion is $u/\omega_{ci} \sim 10^4$ km at comet Halley and the relative level of MHD turbulence is $\Delta B^2/B^2 \sim 0.1$. The presence of freshly born ions with ring distribution could be easily masked by the presence of the dominant population of isotropized ions convected by the solar wind from the upstream region, but their presence is an important feature of the solar wind mass-loading process. The ions with the ring distribution can excite lower hybrid turbulence with typical space scale of its development of the order of several heavy ion gyroradii. Thus, it is important to examine whether the heating of cometary protons is associated with the development of lower hybrid turbulence. An alternative explanation is the heating of cometary protons by quasi-linear diffusion in the MHD turbulence [*Gombosi*, 1988], in which case the typical space scale for the heating mechanism should be of the order of 10^6 km, which is larger than the observed scale lengths. The diffusion lengths in the MHD waves are the same

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for the protons and the heavy ions. This arises as the quasi-linear diffusion coefficient is proportional to $(m_i/m_p)^2$, where m_i and m_p are the masses of the cometary heavy ions and protons respectively, and to the spectral energy density of the modes in resonance with the particles. The cometary protons are accelerated with modes that are in resonance, i.e., $ku\cos\theta = \omega_{cp}$, where θ is the pitch angle and ω_{cp} is the proton gyrofrequency. As a result the acceleration is due to very short wavelength ($k \geq \omega_{cp}/u$) oscillations, and with $|E_k|^2 \sim k^{-2}$, the gain in the diffusion coefficient from the mass ratio of the ions is compensated by the decrease in the wave spectral density [*Galeev et al.*, 1991].

The key role of the lower hybrid turbulence was first recognized by *Hizanidis et al.* [1988], who developed a self-consistent model of the resonant lower hybrid instability based on the quasi-linear diffusion of cometary ions. The saturation of the lower hybrid turbulence due to quasi-linear diffusion alone leads to fluctuation levels that are higher than the observed values. In this paper a self-consistent model of the lower hybrid instability based on the stabilization of the wave growth due to two mechanisms is presented. For sufficiently large density of the pickup cometary protons ($n_p/n_i > m_p/m_i$, n_p and n_i being the densities of the cometary protons and heavy cometary ions respectively), the stabilization is attributed to a smoothing of the bump on the cometary proton velocity distribution due to diffusion and the consequent damping of the waves, while the heavier water group ions continue to destabilize the waves. For smaller densities of cometary protons the stabilization is attributed to a wave energy cascade caused by the modulational instability of the lower hybrid waves. The characteristic values of the wave amplitudes for lower hybrid waves obtained in both cases are in good agreement with the observations. Furthermore, the computed spatial development of the cometary proton velocity distribution agrees with the observations.

2. LOWER HYBRID INSTABILITY AND QUASI-LINEAR DIFFUSION

In the cometary bow wave region a number of plasma modes are excited in the solar wind by the cometary ions (see the review by *Tsurutani* [1991]). The lower hybrid waves are excited by the ring distribution of the cometary ions and can be considered electrostatic. The frequency of these electrostatic oscillations is given by

$$\omega^2 = \omega_{LH}^2\left[1 + \frac{m_p}{m_e}\frac{k_\parallel^2}{k^2}\right],$$

where $\omega_{LH} = \sqrt{\omega_{ce}\omega_{cp}}$ is the frequency of the lower hybrid resonance, ω_{ce} is the electron cyclotron frequency, and the wave number k has the component $k_\parallel$ parallel to the magnetic field. For linearly unstable waves $k_\parallel/k < \sqrt{m_e/m_p}$ as electron Landau damping can quench the instability for larger $k_\parallel$, that is, when the longitudinal phase velocity $\omega/k_\parallel < \sqrt{m_p/m_e}\, v_A$ is small enough. The frequency of the excited oscillations is close to the lower hybrid resonance frequency, which is ~ 5-10 Hz for the comet Halley bow wave. The assumption of the waves being electrostatic implies that $kc >> \omega_{pe}$, ω_{pe} being the electron plasma frequency [*Hizanidis et al.*, 1988]. For oscillations with $k \sim \omega_{LH}/v_{Tp}$, v_{T_p} being the solar wind proton thermal velocity, the damping by the solar wind protons prevails over the wave growth. This condition may be rewritten as $k^2c^2/\omega_{pe}^2 \sim 1/\beta > 1$, for $\beta < 1$, where $\beta = 8\pi n_o T/B^2$ is the ratio of the energy densities of the solar wind plasma and the magnetic field. It should be noted that the solar wind forms the bulk of the plasma, consisting of the solar wind and cometary plasmas, and thus the relevant β for determining the wave properties is that of the solar wind plasma. At the encounter with comet Halley the β for the solar wind plasma, which supports the waves, can be comparable to unity [*Johnstone et al.*, 1987]. However, the finite β effects which contribute to the electromagnetic nature of the waves do not change qualitatively the picture of wave-particle interaction [*Sagdeev et al.*, 1987].

The quasi-linear set of equations for the instability describes the evolution of the wave energy density and the particle distribution function, and has been discussed extensively elsewhere (see the review by *Shapiro and Shevchenko* [1988]). It consists of the equation for the spectral wave energy density, in which the wave growth due to their interaction with the cometary ions is balanced by wave convection by the solar wind ($\partial/\partial t \rightarrow u\partial/\partial x$),

$$u\frac{\partial |E_{\vec{k}}|^2}{\partial x} = \frac{4\pi^2 e^2}{k^2}\frac{\omega^3}{\omega_{pp}^2}\sum_\alpha \frac{k_\perp}{m_a}\int d\vec{v}\cdot cos\phi \frac{\partial f_\alpha}{\partial v_\perp} \times \delta(k_\perp v_\perp cos\,\phi - \omega)|E_{\vec{k}}|^2 \tag{1}$$

and the quasi-linear diffusion equation for each of the cometary ion species α given by

$$(u - v_\parallel cos\lambda)\;\frac{\partial f_\alpha}{\partial x} = \frac{\pi e^2}{m_\alpha^2}\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\sum_{\vec{k}} |E_{\vec{k}}|^2 cos^2\phi \times \delta(kv_\perp\, cos\phi - \omega)v_\perp\frac{\partial f_\alpha}{\partial v_\perp}. \tag{2}$$

In (1) the summation over α is over the mass m_α and the distribution function f_α of both species of cometary ions (protons, p, and heavy ions, i), and $\omega_{pp} = (4\pi e^2 n_o/m_p)^{1/2}$ is the proton plasma frequency in the solar wind, n_o being the solar wind proton density. The waves are assumed to be polarized in the plane perpendicular to the magnetic field. The x axis is oriented along the sun–comet line in the cometward direction, λ is the angle between this axis and the magnetic field, ϕ is the azimuthal angle in the velocity space, and the index $\parallel$ denotes the direction antiparallel to the magnetic field. Both types of ions execute cycloidal trajectories under the $\vec{E} \times \vec{B}$ force in the solar wind, while their velocity along the magnetic field remains equal to the cometary gas expansion velocity, which is much smaller than the solar wind velocity. Consequently the initial distribution function of the cometary ions in the solar wind frame is well approximated by

$$f_\alpha(0, v_\perp, v_\parallel) = \frac{n_\alpha}{2\pi u_\perp}\delta(v_\perp - u_\perp)\delta(v_\parallel + u_\parallel),$$

where $u_\perp = c\,E/B$ and $u_\parallel$ are the components of the solar wind velocity perpendicular and antiparallel to the magnetic field. Equation (2) describes the diffusion of the cometary ion perpendicular velocity due to their interaction with the lower hybrid waves. The parallel component of the particle velocity does not change significantly as $v_\parallel \simeq -u_\parallel \simeq u\cos\lambda$. Also, in the plane perpendicular to the magnetic field, the velocities are assumed to be phase mixed over the azimuthal angle ϕ so that

$$f_\alpha = f_\alpha(t, v_\perp, v_\parallel).$$

By changing the summation over the wave vector $\vec{k}$ into an integral and integrating over ϕ, (2) becomes

$$u\sin^2\lambda\frac{\partial f_\alpha}{\partial x}=\frac{e^2}{4\pi^2 m_\alpha^2}\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\int dk_\perp k_\perp dk_\parallel\frac{\omega^2}{k_\perp^2} \times|E_k|^2\frac{1}{v_\perp}\frac{1}{\sqrt{k_\perp^2 v_\perp^2-\omega^2}}\frac{\partial f_\alpha}{\partial v_\perp} \tag{3}$$

with $k_\perp \geq \omega/v_\perp$. Further the ϕ integration in the right-hand side of the equation for the wave spectral energy density can be carried out to yield

$$u\frac{\partial|E_{\vec{k}}|^2}{\partial x}=\frac{2\pi\omega^3}{n_o k^2}\int dv_\parallel\int dv_\perp\frac{\omega}{\sqrt{k_\perp^2 v_\perp^2-\omega^2}} \times\left\{\frac{\partial f_p}{\partial v_\perp}+\frac{m_p}{m_\iota}\frac{\partial f_\iota}{\partial v_\perp}\right\}|E_{\vec{k}}|^2. \tag{4}$$

The left-hand side of this equation equals $2\gamma|E_{\vec{k}}|^2$, where γ is the growth rate.

Another type of interaction between the particle distribution and the waves is the pitch angle diffusion caused by turbulent Alfven waves. This interaction starts at large distances from the comet and results in the transformation of the initial ring into the isotropic shell distribution:

$$f_\alpha(v_\perp, v_\parallel)\rightarrow f_\alpha(v).$$

It is readily seen from (4) that such a transformation results in the stabilization of lower hybrid instability since the integrand in the growth rate in (4) can be rewritten using the spherical velocity coordinates (v, θ) with $\cos\theta \geq \omega/k_\perp v$ and integration over θ yields

$$\gamma=\frac{\pi\omega^4}{k^3}\int_{\omega/k}^{\infty}dv\frac{\partial f_\alpha}{\partial v}\arcsin\sqrt{1-\frac{\omega^2}{k_\perp^2 v^2}}. \tag{5}$$

This gives a negative value of γ even when the shell distribution has a maximum at a certain $v = v_o$. Because of the arcsin the main contribution to the integrand is from the region $v > v_o$,, where $\partial f/\partial v < 0$. Thus the lower hybrid instability is excited only in the region quite close to the bow wave, where as a result of the very fast implantation of new ions into the solar wind, their distribution has had insufficient time to isotropize and thus maintain the ring distribution. This can occur when either the distributions of both the heavy ions and protons are close to the ring distribution, or the heavy ions have ring distribution and dominate over the contribution from the protons with shell distribution [*Neugebauer et al.*, 1989]. The growth rate can be obtained from (3) as

$$\gamma=\frac{1}{2}\frac{\Delta m}{m}\frac{\omega^4}{k^3u_\perp^3}\left(\frac{m_p}{m_\iota}\right)^2+\frac{\pi}{n_o}\frac{\omega^4}{k^3}\int\frac{1}{v_\perp}\frac{\partial f_p}{\partial v_\perp}dv_\perp dv_\parallel \tag{6}$$

where it has been assumed that $\omega/kv_\perp \sim v_A/u << 1$. The first term in this growth rate is the input from heavy ions, $\Delta m/m$ being the relative increase in the mass density of the solar wind due to the mass loading, with $\Delta m \simeq n_i m_i$ and $m \simeq n_o m_p$. For the heavy ions the velocity diffusion due to the lower hybrid waves is weak and their distribution function remains close to the initial δ function, as was assumed while obtaining the first term in the right hand side of (6). The second term in (6) is the contribution of cometary protons and becomes important for $n_p > n_i\, m_p/m_i$. It is easily seen from (6) that the growth rate γ is proportional to $(\omega/ku)^3$ $\sim (v_A/u)^3$, as was stated in the introduction. Considering the heavy ion contribution to the instability, the characteristic length L_{inst} for the development of lower hybrid instability is given by

$$L_{\rm inst}\simeq 10\frac{u}{\omega_{LH}}\frac{m}{\Delta m}\frac{u_\perp^3}{v_A^3}\left(\frac{m_\iota}{m_p}\right)^2. \tag{7}$$

In obtaining L_{inst} we have taken the logarithm of the ratio Λ of the wave to the thermal fluctuation energy densities to be given by $\ln\Lambda \simeq 10$. For the Giotto encounter with comet Halley near the bow wave where $v_A/u \simeq 0.5$ and $\Delta m/m \simeq 0.3$ [*Johnstone et al.*, 1987], L_{inst} - 5×10^4 km.

The contribution of the cometary protons to the growth rate, the second term in (6), is important when their density is sufficiently large. Initially this contribution is positive, but as the instability develops, the sign of this term changes as the particle distribution function changes its form due to the turbulent diffusion. That could result in the stabilization of the instability. The proton velocity spread Δv resulting from such diffusion can be expressed in terms of the scale length L and the coefficient D as

$$(\Delta v)^2\simeq\frac{DL}{u\sin^2\lambda}.$$

Using $L \simeq L_{inst}$, expressing the diffusion coefficient from (4) through the electric field energy density

$$W=\frac{E^2}{8\pi}=\frac{1}{32\pi^3}\int dk_\perp k_\perp dk_\parallel|E_{\vec{k}}|^2$$

and with the help of the approximate relations $\omega/k \approx V_A$, $\omega \simeq \omega_{LH}$ and $v_\perp \simeq u\sin\lambda$, we obtain

$$n_o m_p(\Delta v)^2\simeq 2W\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{v_A^3}{u_\perp^3}\frac{\omega_{LH}L_{\rm inst}}{u_\perp\sin^2\lambda} \tag{8}$$

The total wave energy of the lower hybrid oscillations can be obtained as

$$W_{\rm Tot}=\frac{\partial(\omega\varepsilon)}{\partial\omega}W$$

where $\varepsilon = 1 + \omega_{pe}^2/\omega_{ce}^2 - \omega_{pp}^2/\omega^2$ is the dielectric constant transverse to the magnetic field, for unmagnetized ions ($\omega >> \omega_{ci}$) and magnetized electrons ($\omega << \omega_{ce}$). For electrostatic oscillations $\varepsilon = 0$ and hence

$$W_{\rm Tot}=2\frac{\omega_{pe}^2}{\omega_{ce}^2}W.$$

The initial bump in tail for the distribution of cometary protons is thermalized when $\Delta v \sim u_\perp$. Also the level of the lower hybrid wave field at which cometary proton diffusion leads to the saturation of the instability can be calculated by using L_{inst} given by (7). This yields

$$W\frac{\omega_{pe}^2}{\omega_{ce}^2}\simeq\frac{1}{20}n_o m_p u_\perp^2\frac{\Delta m}{m}\left(\frac{m_p}{m_\iota}\right)^2\sin^3\lambda. \tag{9}$$

For the parameters of the VEGA encounter with comet Halley namely, $\omega_{pe}^2/\omega_{ce}^2 \sim 10^4$, $n_o \sim 10$, $u_\perp \sim 350$ km/s and $\Delta m/m \sim 1/3$, it follows from that $E \sim (1-2)$ mV/m$\sqrt{\rm Hz}$, which is quite close to observed values in the bow wave region [*Galeev et al.*, 1986; *Klimov et al.*, 1986].

3. Cometary Proton Distribution and Lower Hybrid Wave Energy Density

The velocity distribution of the cometary protons due to the quasi-linear relaxation can be obtained by solving (3). Assuming, as in the case of the calculation of the growth rate, that $\omega << kv_\perp$ and using the dimensionless length variable

$$\xi = 25\int_o^x \frac{\omega_{pe}^2}{\omega_{ce}^2}\sum_k \frac{|E_{\vec{k}}|^2}{8\pi}\frac{\omega_{LH}^4}{k^3u_\perp^4\sin\lambda}\frac{dx'}{\frac{1}{2}n_o m_p u_\perp^2} \tag{10}$$

and the normalized energy variable $w = v_\perp^2/u_\perp^2$, (3) can be reduced to the following standard form [*Galeev*, 1967; *Davidson*, 1972]:

$$\frac{\partial f_p}{\partial \xi} = \frac{4}{25}\frac{\partial}{\partial w}\left(\frac{1}{\sqrt{w}}\frac{\partial f_p}{\partial w}\right). \tag{11}$$

The solution of this equation with the initial condition corresponding to the ring distribution function is

$$f_P(\xi, w) = \frac{5n_P}{2u_\perp^2}F(\xi, w) \tag{12}$$

where

$$F(\xi, w) = \frac{1}{\xi}w^{3/4}\exp\left[-\frac{1+w^{5/2}}{\xi}\right]I_{-3/5}\left[2\frac{w^{5/4}}{\xi}\right]$$

and $I_{-3/5}$ is the modified Bessel function of order -3/5. The evolution of the distribution function for ξ = 0.2, 0.55 and 5.0 is shown in Figure 1. With the increase of ξ, which corresponds to the increase of the wave energy, the bump of the velocity distribution of the picked-up protons $v_\perp = u_\perp$ spreads and finally disappears. Similar behavior have been observed in the measurements of the distribution function on the Giotto spacecraft as it traverses the bow wave region of comet Halley (*Neugebauer et al.*, 1987). It may however be noted that the observed disappearance of the bump in the distribution function may also be due to the fact that the cometary proton peak cannot be distinguished from the larger peak of the heated and compressed solar wind protons (*M. Neugebauer*, personal communication, 1992). For $\xi >> 1$ the distribution function has the self-similar solution that decreases monotonically with energy [*Sagdeev and Galeev*, 1973] as

$$f \simeq \frac{5n_P}{2u_\perp^2\xi^{2/5}}\exp\left[-\frac{w^{5/2}}{\xi}\right]. \tag{13}$$

With the help of the distribution function (12) the growth rate given by (6) can be rewritten as

$$\gamma = \frac{\omega_{LH}^4}{k^3u_\perp^3}\frac{\Delta m}{m}\frac{m_p}{m_i} \times\left\{\frac{1}{2}\left(\frac{m_p}{m_i}\right) + \pi\frac{5n_P}{2n_i}\int_o^\infty \frac{dw}{\sqrt{w}}\frac{\partial}{\partial w}F(\xi, w)\right\} \tag{14}$$

Using the parameters of the encounter with comet Halley, the plot of $\gamma(\xi)$ as a function of x, which is related to ξ by (10), is shown in Figure 2 for different values of the n_p/n_i ratio. It follows from this figure that for $n_p/n_i > 3\times 10^{-2}$ the velocity spread of the proton distribution due to quasi-linear diffusion changes the sign of γ and hence leads to the stabilization of the lower hybrid instability. Also, it is readily seen that the estimate for the wave energy given by (9) corresponds to the values $\xi \sim 1$.

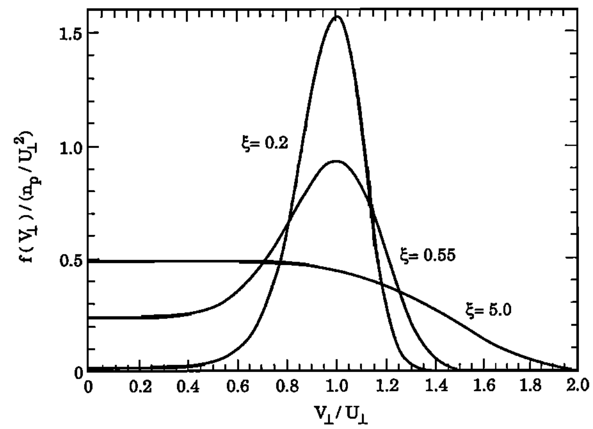

Fig. 1. The evolution of the cometary proton distribution function for different values of the dimensionless variable ξ. The forms of the distribution agree with observations [*Neugebauer et al.*, 1987].

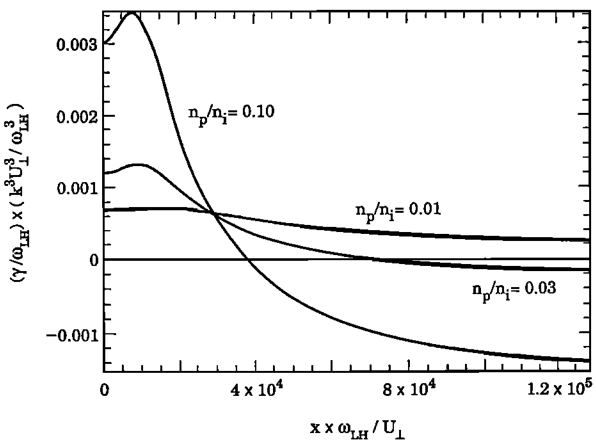

Fig. 2. The normalized growth rate of lower hybrid waves as a function of x given by (16). The different cases corresponding to the different mass loading by the cometary protons are shown.

More accurately, wave amplitudes for the quasi-linear saturation mechanism under consideration could be obtained from (4). On substituting for $\gamma(\xi)$ from (14) into (4), with

$$u\frac{\partial}{\partial \xi}|E_k|^2 = 2\gamma|E_k|^2/\frac{d\xi}{dx}$$

carrying out the summation over $\vec{k}$ and integrating over ξ, the electric field energy density is given by

$$W = \frac{1}{50}\sin^2\lambda\frac{\omega_{ce}^2}{\omega_{pe}^2}n_o m_p u_\perp^2\,\frac{\Delta m}{m}I(\xi)\frac{m_p^2}{m_i^2} \tag{15}$$

where we have used the notation

$$I(\xi) = \xi + \frac{5\pi n_p}{n_i}\frac{m_i}{m_p}\int_o^\xi\int_o^\infty \frac{dw}{\sqrt{w}}\frac{\partial}{\partial w}F(\xi, w)$$

Using the definition of ξ and assuming as before that for the main part of the lower hybrid waves the wave phase velocity $\omega_{LH}/k \simeq v_A$, the equation connecting ξ and x is found to be

$$\int_o^\xi \frac{d\xi}{I(\xi)} = \frac{\Delta m}{m}\left(\frac{m_p}{m_i}\right)^2\frac{v_A^3}{u_\perp^3}\frac{x\omega_{LH}}{u} \tag{16}$$

Equations (15) and (16), in the parametric form, define the dependence of the electric field energy W on x. The plots of this dependence for different values of the n_p/n_i ratio are shown in Figure 3. The growth of the electric field energy W saturates in the described quasi-linear model only for higher values of n_p/n_i and remains monotonic for smaller values of n_p/n_i, as shown by the γ versus x behavior in Figure 2. In Figure 4 the maximum value of the electric field energy is shown as a function of the ratio n_p/n_i. Only a small a part of the solar wind energy, $\sim 10^{-4}$, goes into the lower hybrid oscillations. The numerical values are close to those given by the approximate relation (9), which agree well with the observations. In the case when the instability saturates due to quasi-linear diffusion of the pickup ions [*Hizanidis et al.*, 1988], the waves grow to higher amplitudes. In the present case the damping by the quasi-linearly relaxed pickup protons leads to saturation at lower amplitudes. It is also important to note that the quasi-linear mechanism under consideration is effective for values of $n_p/n_i > 3 \times 10^{-2}$, which is always the case in the cometary bow wave region [*Neugebauer et al.*, 1990].

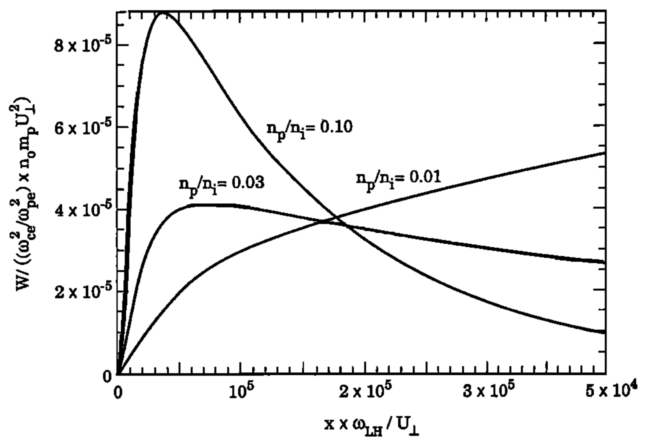

Fig. 3. The lower hybrid wave energy density as a function of x, the normalized space scale, with $\lambda = 50°$ and $v_A/u_\perp = 0.8$.

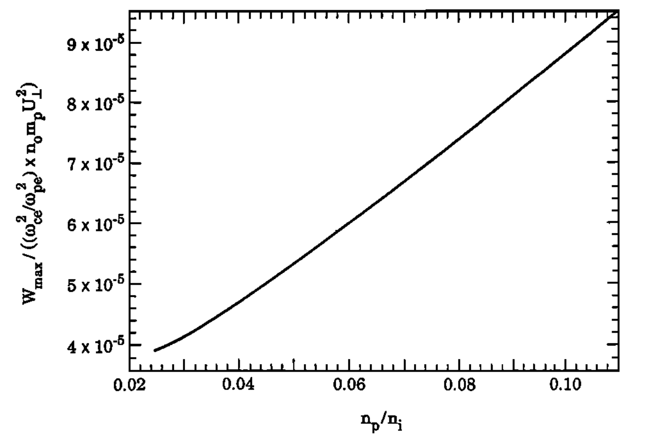

Fig. 4. The maximum of the wave energy density as a function of the pickup cometary proton density.

4. MODULATIONAL INSTABILITY OF LOWER HYBRID WAVES

Another nonlinear mechanism for the saturation of the lower hybrid instability is the wave cascade caused by the modulational instability. At comet Halley the fluctuation level at which this mechanism is effective is approximately the same as that given by the quasi-linear mechanism, and hence both mechanisms can contribute to the saturation process. An interesting feature of the modulational instability is the production of energetic electrons, as discussed below. Also, the wave cascade process could be important in other magnetospheric contexts; for example, the modulational instability of the lower hybrid waves has been observed in auroral regions of the Earth's ionosphere by rocket experiments [*Vago et al.*, 1992]. Modulational instability is a tendency of the lower hybrid waves to be localized in the density cavities with the plasma expelled out under the action of wave pressure. This process is described by the following set of equations [*Musher and Sturman*, 1975]:

$$-\frac{2i}{\omega_{LH}}\frac{\partial}{\partial t}\Delta\phi - R^2\Delta^2\phi + \frac{m_p}{m_e}\frac{\partial^2\phi}{\partial r_\parallel^2} = \frac{m_p}{i m_e}\frac{\omega_{LH}}{\omega_{ce}}[\nabla\phi \times \nabla\delta n]_\parallel \tag{17}$$

$$\frac{\partial^2 \delta n}{\partial t^2} - \frac{T}{m_p}\Delta\delta n = -\frac{1}{4\pi i m_p}\frac{\omega_{pe}^2}{\omega_{ce}\omega_{LH}}\Delta[\nabla\phi^* \times \nabla\phi]_\parallel. \tag{18}$$

In these equations $\phi(t, \vec{r})$ is the complex amplitude of the lower hybrid wave potential: $\phi_{LH} = \frac{1}{2}\phi(t,\vec{r})\, e^{-i\omega_{LH}t} + \text{c.c.}$; $\delta n(t, \vec{r})$ is the low-frequency quasi-neutral density perturbation of the ion acoustic type; and R is a typical dispersion length, which is equal to electron Larmor radius in the case $\omega_{pe} >> \omega_{ce}$ under consideration. The details of the derivation of the set of equations (17) and (18) are given by *Shapiro et al.* [1992]. The temperature T is the sum of electron and proton temperatures in the solar wind. The initial space scales of the cavities created by the modulational instability can be obtained by comparing the dispersive and nonlinear terms in (17) as

$$l_\perp \sim R\sqrt{\frac{m_e}{m_p}\frac{n_o T}{2W}\frac{\omega_{ce}^2}{\omega_{pe}^2}}, \quad l_\parallel \sim \frac{l_\perp^2}{R}\sqrt{\frac{m_p}{m_e}}. \tag{19}$$

Thus the cavities are much more extended along the magnetic field than accross it. The modulation may not be stable as the wave pressure is negative, and it increases with the decrease of the density, thus leading to collapse until it reaches dimensions at which the resonant absorption of the wave energy by particles becomes important. The collapse is described by the self-similar law [*Sotnikov et al.*, 1978]

$$\phi = \phi\left(\frac{r_\perp}{\sqrt{t_o - t}}, \frac{r_\parallel}{t_o - t}\right)\exp\left[-i\int\frac{\lambda\left(\frac{r_\perp}{\sqrt{t_o-t}}, \frac{r_\parallel}{t_o-t}\right)dt}{t_o - t}\right]$$
$$\delta n = \frac{1}{t_o - t}\,\delta n\left(\frac{r_\perp}{\sqrt{t_o - t}}, \frac{r_\parallel}{t_o - t}\right), \tag{20}$$

t_o being the instant of collapse of the cavity. Due to the collapse the wave energy cascades to larger $k_\perp$ and $k_\parallel$, and as it proceeds faster in the parallel direction, the cascade to larger $k_\parallel$ is dominant. As the result of such a cascade the wave energy is transported to sufficiently small electron resonant velocities, $\omega/k_\parallel \lesssim v_A\sqrt{m_i/m_e}$, leading to the resonant absorption of waves by the electrons. This interaction of the very short scale length fluctuations with the electrons leads to the acceleration and formation of suprathermal tails.

The modulational instability can lead to the establishment of a quasi-steady turbulent state by the generation of short wavelengths and collapse, and then energy absorption mainly by resonant electrons. The rate of energy dissipation is characterized by the so-called effective collision frequency which can be as large as ω_{LH} [*Sotnikov et al.*, 1978]. Since the growth rate of the lower hybrid instability, which characterizes the rate of energy pumping into turbulence, is essentially smaller than ω_{LH} (see Figure 2), in the quasi steady state we expect that the saturation level of the energy of the lower hybrid wave will be close to the threshold for modulational instability, i.e.,

$$\frac{E^2}{8\pi}\cdot\frac{\omega_{pe}^2}{\omega_{ce}^2} \simeq 3n_o T\frac{m_e}{m_p}\frac{T_e}{m_p v_A^2} \tag{21}$$

For the typical values of wave numbers in the source region of the turbulence, the values corresponding to the lower hybrid instability $k \sim \omega_{LH}/v_A$ have been used in obtaining (21). Numerical values for the case of the comet Halley encounter are close to those obtained from (9), with $E \sim 1$ mV/m. For a sufficiently small number of cometary protons, $n_p/n_i < 3\times 10^{-2}$, saturation amplitudes due to the quasi-linear process obtained from Figure 4 are much smaller and the saturation due to modulational instability will be dominant. However, the observations show values above this threshold and hence these saturation mechanisms work concurrently.

An important feature of the modulational instability is the generation of energetic electrons [*Papadopoulos*, 1985; *Sagdeev et al.*, 1987]. The distribution function of the energetic electrons can be found by balancing the damping rate due to the absorption of lower hybrid waves on resonant electrons with the rate of nonlinear wave cascade due to collapse. This yields

$$\frac{\pi}{2}\frac{\omega_{ce}^2}{n_o k^2}\omega\frac{\partial f_e(\omega/k)}{\partial v_\parallel} + \frac{1}{\tau_{casc}} = 0 \tag{22}$$

where τ_{casc} is the typical time for cascade in the parallel wave numbers that is readily obtained from the self-similar law of collapse, i.e., (20):

$$\frac{1}{\tau_{casc}} \simeq \frac{1}{k_\parallel}\frac{dk_\parallel}{dt} \propto k_\parallel .$$

Then it follows from (22) that resonant absorption creates a prolonged (logarithmic) tail on electron distribution function

$$f_e(v_\parallel) \sim \ln(v^* - v_\parallel),\ v^* = \text{const} \tag{23}$$

The typical energies in the electron tail can be estimated from the diffusion equation, which governs the electron heating by lower hybrid waves. Since the electrons are magnetized, their diffusion occurs only in the direction of the magnetic field and the diffusion equation has the form

$$\frac{\partial}{\partial v_\parallel} D_\parallel^e \frac{\partial f_e}{\partial v_\parallel} = v_\parallel \frac{\partial f_e}{\partial r_\parallel}, \tag{24}$$

where the coefficient of electron diffusion along the magnetic field is given by

$$D_\parallel^e = \frac{e^2}{2\pi m_e^2}\frac{1}{v_\parallel}\int dk_\perp k_\perp |E_k|^2 \frac{k_\parallel^2}{k^2}$$

and $k_\parallel = \omega/v_\parallel$.

Assuming that electron diffusion occurs at distances L_{inst} where the lower hybrid turbulence is present, an estimate for the electron velocity is

$$v_\parallel^5 \simeq \frac{16\pi^2 e^2}{m_e^2} W \frac{v_A^2}{\omega_{LH}} L_{inst}$$

Substituting L_{inst} from (7), the typical energies in electron tail ε_e can be expressed as

$$\frac{\varepsilon_e}{m_e v_{Ae}^2} \sim \left[\frac{5\pi}{\sqrt{2}}\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{E^2}{B_o^2}\frac{u^4}{v_A^4}\sqrt{\frac{m_e}{m_p}}\sin^3\lambda\left(\frac{m_i}{m_p}\right)^2\right]^{2/5}. \tag{25}$$

Here $v_{Ae} = \sqrt{B_o^2/4\pi n_o m_e}$ is the electron Alfven velocity, and at the encounter with comet Halley $v_{Ae} \simeq 4\times 10^8\ s^{-1}$. The expression in the right hand side of (25) is of the order of unity and hence the typical energy in electron tail is 100–200 eV.

Finally the number of accelerated electrons can be estimated from the energy balance condition. Assuming that all of the energy pumped into the lower hybrid waves by the instability is carried away by energetic electrons, the energy balance gives

$$\gamma_b\frac{E^2}{8\pi}\frac{\omega_{pe}^2}{\omega_{ce}^2}L_{inst} \simeq \frac{1}{2}n' m_e v_e^3,$$

where n' is the number of accelerated electrons, v_e is the electron velocity and γ_b is the growth rate of the lower hybrid instability, which can be obtained from Figure 2. It follows from energy balance condition that

$$\frac{n'}{n_o} \sim \frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{E^2}{B_o^2}\cdot\frac{\gamma_b L_{inst}}{v_{Ae}}\frac{v_{Ae}^2}{v_e^2}. \tag{26}$$

For conditions of encounter with comet Halley $\gamma_b \sim 1\ s^{-1}$ and $E \sim 1\ mV/m$, yielding $n'/n_o \sim 10^{-2} - 10^{-3}$. These estimates are in agreement with the energetic electron data from the retarding potential analyzer instrument at the Giotto encounter with comet Halley, which gave $n'/n_o \sim 2\times 10^{-2}$ for 165–790 eV and $n'/n_o \sim 10^{-4}$ for 0.8–3.7 keV electrons [*Anderson et al.*, 1987].

5. Discussion and Conclusions

The lower hybrid instability plays an important role in the pickup ion relaxation and wave turbulence in the neighborhood of the bow wave. The instability is driven by the mass loading of the solar wind by both the cometary protons and heavy ions, and in spite of the low number density, the cometary protons play an important role because of their relatively lower mass. The quasi-linear relaxation of the cometary protons in the lower hybrid waves takes place over a scale length of $\sim 5\times 10^4$ km whereas the heavy ions are not significantly affected by these waves due to their heavier masses. The instability saturates when the damping by the thermalized protons balances the destabilization by the heavy ions. The fluctuation level calculated from this mechanism, $E \sim 0.5 - 1\ mV/m\sqrt{Hz}$, as well as the spatial evolution of the cometary proton distribution function agree with the observations. This mechanism yields lower values of the saturated electric fields compared to the case when the saturation is due entirely to the quasi-linear diffusion of the cometary ions [*Hizanidis et al.*, 1988].

When the density of the cometary protons is small, this saturation mechanism may not be effective and the saturation may then be due to the nonlinear wave cascade arising from the modulational instability. The waves excited by this process resonate with low-velocity electrons and can generate electron tails. The estimated wave amplitudes from this process in the case of comet Halley are of the same order as mentioned above. The typical energies of accelerated electrons are 100–200 eV, with the density $n'/n_o \sim 10^{-2} - 10^3$, and are in agreement with the observations.

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