Formation of positive slope on electron runaway distribution in tokamaks

1977 Nucl. Fusion 17 1087

(http://iopscience.iop.org/0029-5515/17/5/020)

REFERENCES

[1] INOUE, S., ITOH, K., Nucl. Fusion 16 (1976) 867.
[2] KADOMTSEV, B.B., POGUTSE, O.P., Nucl. Fusion 11 (1971) 67.

(Manuscript received 3 May 1977
Final version received 6 June 1977)

FORMATION OF POSITIVE SLOPE ON ELECTRON RUNAWAY DISTRIBUTION IN TOKAMAKS

K. PAPADOPOULOS, B. HUI*, N. WINSOR
(Naval Research Laboratory, Washington, D.C.,
United States of America)

1. INTRODUCTION

Runaway electrons are a basic ingredient of hot toroidal plasmas. In the present generation of tokamaks, their implications are more evident in the so-called low-density regime [1–6] ($n \lesssim 10^{13}\ cm^{-3}$). Their presence can have a significant effect on the tokamak operation since besides constituting a large energy loss via synchrotron and X-ray emission [7, 8] they have often led to liner destruction [3]. Understanding of the runaway behaviour is, therefore, of significant practical importance not only because of its possible use as a diagnostic tool, but also since it can help us learn ways for reducing radiation losses and even delivering part of their energy for heating the main plasma. The latter possibility seems to be born out from the recent observations in Alcator [1], T-6 [2] and T-M3 [4, 5] tokamaks, where the production of runaways has been found to contribute to ion heating. The main thrust of these observations was that reduction of the plasma density resulted in inhibition of the energy of the runaways as seen by X-ray measurements, while leading to simultaneous excitation of lower hybrid waves, followed by intense ion heating [1, 4, 5]. Understanding of the physics and the scaling of such collisionless phenomena is very important, not only for the low-density regime, but also for the hot high-density tokamaks.

To determine the collective modes that can be excited by the runaways a knowledge of their parallel distribution function

$$f(v_z) = \int d\vec{v}_\perp f(\vec{v})$$

is required. In the absence of experimental information, one has to use analytic or computational means. On the basis of the Fokker-Planck models [6, 9] an almost flat, monotonically decreasing (with the parallel velocity) distribution function is expected as is shown by the solid line of Fig.1. Stability of such a distribution function [1, 10–12] indicates that unless a substantial fraction of the runaways is accelerated to velocities larger than 10–20 MeV, low-frequency waves cannot be excited. At the same time it was shown in Ref. [13], by analysis and particle simulations, that by assuming a beam-like distribution function with a positive slope for the runaways, the observed behaviour can be easily understood. It is the purpose of this letter to demonstrate that collisionless effects can result in forming a positive slope on distribution of the type of Fig.1, within time scales of interest.

2. QUALITATIVE PHYSICAL CONSIDERATION

We present next a brief summary of the physics of the relevant collisionless interaction which in the presence of an electric field can transform a flat tail into a bump. This will be followed by a numerical computation of the evolution of a tail in the presence of an enhanced field. The latter uses a Fokker-Planck code, modified to include collisionless effects due to plasma instabilities.

The stability of distribution functions with runaway electrons (Fig.1) has been examined in detail

* Science Applications, Inc., McLean, Virginia, USA.

in Refs [10–12]. It was shown that low-frequency oscillations ($\omega_k \ll \Omega_e$) of the type

$$\omega_k^2 = \omega_i^2 + \left(\frac{k_z}{k}\right)^2 \omega_e^2 \tag{1}$$

can be excited independently of the existence of a positive slope in the distribution function, under anomalous Doppler effect conditions

$$\omega_k = k_z v_z - \Omega_e \tag{2}$$

where ω_i and ω_e are the ion and the electron plasma frequency, respectively, and Ω_e is the electron cyclotron frequency. Excitation of these oscillations is possible only when the Cherenkov damping due to the ambient electrons can be neglected. This imposes a restriction on the wave number k_z given by

$$\frac{\omega_k}{k_z} \gg v_{th} \tag{3}$$

From Eqs (2) and (3) we notice that only frequencies $\omega_k \gg (v_{th}/v_R)\Omega_e$, where v_R is the maximum velocity in the tail, can be excited. Therefore, excitation of frequencies approaching the ion plasma frequency, ω_i, would require a substantial number of runaways at enormous energies ($\gtrsim$10 MeV). In addition, since the instability is convective it can be shown that the growth length for exciting these frequencies becomes much larger than the system radius. Therefore, only high frequencies close to ω_e will be excited. Another important conclusion from Eqs (2) and (3) is the fact that only particles with velocities $v_z > v_m \equiv (\Omega_e/\omega_e)\, v_{th}/k_z\lambda_D$ can be resonant with the excited waves. It can be easily shown [10] that for $(\Omega_e/\omega_e) \gg 1$ this interaction leads to pitch angle scattering along nearly constant energy lines for particles with $v_z > v_m$. Therefore, in the presence of an electric field, runaway electrons with velocities $v_z < v_m$ will be accelerated faster than particles with $v_m > v_z$ and a pile-up of particles will occur near v_m. As a consequence, the runaway tail will develop a positive slope near $v = v_m$. This is the simplified physical picture of the formation of a positive slope on the runaway distribution. It is very hard to derive analytic solutions for the evolution of the distribution function in the presence of a constant electric field and Fokker-Planck type collisions. For this reason, we use numerical means to confirm the process and examine the sensitivity of the positive-slope formation to the system parameters (ω_e/Ω_e, n_T/n_0, E_0, etc.).

Numerical solution

The equations that determine the evolution of the tail described previously are

$$\frac{\partial}{\partial t} f_T(\vec{v}) + \vec{E}\cdot\frac{\partial}{\partial \vec{v}} f_T(\vec{v}) = \left(\frac{\partial f}{\partial t}\right)_c + \frac{\partial}{\partial \vec{v}}\cdot \overleftrightarrow{D}^w \cdot \frac{\partial}{\partial \vec{v}} f_T(\vec{v}) \tag{4}$$

with the usual Fokker-Planck terms given by

$$\frac{1}{\Gamma}\left(\frac{\partial f}{\partial t}\right)_c = -\nabla\cdot(f\nabla h) + \frac{1}{2}\nabla\nabla : (f\nabla\nabla g)$$

where h and g are the Rosenbluth potentials and

$$\Gamma = \frac{4\pi Z^4 e^4}{m^2}\log\left[\frac{3k_B T}{2Z^2 e^2}\left(\frac{k_B T_e}{\pi N_e e^2}\right)^{1/2}\right] \tag{5}$$

$\overleftrightarrow{D}^w$ are the diffusion coefficients due to collisionless processes given by [12]

$$\begin{bmatrix} D^w_{\perp\perp} \\ D^w_{\perp z} = D^w_{z\perp} \\ D^w_{zz} \end{bmatrix} = \frac{e^2}{8\pi m^2}\int d\vec{k}\, |E^2(\vec{k},t)|^2 \begin{bmatrix} \sin^2\theta\, \delta(\omega_k - k_z v_z + \Omega_e) \\ -\sin^2\theta\left(\frac{k_z v_\perp}{\Omega_e}\right)\delta(\omega_k - k_z v_z + \Omega_e) \\ 4\cos^2\theta\, \delta(\omega_k - k_z v_z) \end{bmatrix} \tag{6}$$

and the electric field value is given by [11, 12]

$$\frac{d}{dt}|E(\vec{k},t)|^2 = \pi\alpha\frac{\omega_k^3}{\cos^2\theta}\frac{1}{k_z^2}\left[\cos^2\theta\frac{\partial f_T}{\partial v_z} + \sin^2\theta\frac{k_z}{\Omega_e} f_T\left(\frac{\omega_k + \Omega_e}{k_z}\right)\right]|E(\vec{k},t)|^2 \tag{7}$$

with ω_k taken from Eq.(1). Here $f_T(v_z) = \int d\vec{v}_\perp f_T(\vec{v})$, α is the tail-to-plasma density ratio and $\cos\theta = k_z/k$. Note that since the instability time scale is much faster than the Fokker-Planck one, we can eliminate Eq.(7) and use in Eq.(6) for $|E^2(\vec{k}, t)|^2$ its saturation value. The saturation of the instability by quasi-linear effects has been examined by Tekula and Bers [11] and Liu and Mok [14]. We have also examined the more plausible possibility of convective stabilization. In both cases, the characteristic spectral distribution turns out to be determined by the linear stage having a peak at an angle with $k_\perp/k_z = \sqrt{2}$ and extending between $(v_{th}/v_R)\,\Omega_e/\omega_e < k_z\lambda_D < 0.3$ with the maximum weighted towards the smaller values of k_z. Since, as we shall see, the formation of the positive slope depends basically on the existence of a cut-off value with $k_z\lambda_D \ll 1$ and not on the other spectral details, we assume that

$$\frac{|E(\vec{k})|^2}{8\pi} = \frac{A}{k_z^2}\,\delta\,(k_\perp - \sqrt{2}\,k_z)$$

$$0.3 \geqslant k_z\,\lambda_D \geqslant \frac{v_{th}}{v_R}\,\frac{\Omega_e}{\omega_e} \tag{8}$$

and zero elsewhere. Here, A is a normalization constant that depends on the wave energy density W_0/nT. Neglecting the drag force due to collective effects as small compared with pitch angle scattering terms involving $\partial f_T/\partial v_z$, owing to its initial small value, and using Eqs (6) and (8) in Eq.(5), we find

$$\frac{\partial}{\partial t}\,f_T(\vec{v}) + \vec{E}\cdot\frac{\partial}{\partial\vec{v}}\,f_T(\vec{v}) = \left(\frac{\partial f}{\partial t}\right)_c + \frac{3e^2}{4m^2}$$

$$\times\left(\frac{1}{v_\perp}\,\frac{\partial}{\partial v_\perp} - \frac{1}{v_z}\,\frac{\partial}{\partial v_z}\right)\left[\frac{W(v_z)}{v_z}\right]$$

$$\times\left(\frac{1}{v_\perp}\,\frac{\partial}{\partial v_\perp} - \frac{1}{v_z}\,\frac{\partial}{\partial v_z}\right)\,f_T(\vec{v}) \tag{9}$$

where $W(v_z) = \dfrac{1}{8\pi}\displaystyle\int d\vec{k}_\perp\,|E(\vec{k})|^2$.

Numerical results

When Eq.(9) is solved with the appropriate values of E_0, Ω_e/ω_e, $\alpha = nT/n_0$, T_e and W_0/n_0T_e, we find that a bump forms at about v_m. Below we present a systematic study of the formation of this bump by varying E_0/E_D, Ω_e/ω_e, α and W_0/n_0T_e. Unless otherwise specified, the following values are used: $\alpha \cong 0.05$, $\Omega_e \cong 3.5\ \omega_e$, $W_0 = 3.53 \times 10^{-3}\ n_0T_e$, $T_e = 1$ keV, $T_i = 200$ eV, $n_0 = 10^{13}$ cm^{-3}, $v_m \approx 8\ v_{th}$, $t \approx 20\ \tau_{ei}$. Figure 1 shows the formation of the bump with different values of E_0. As the E_0/E_D increases, the bump is wiped out by the acceleration of the electrons below the cutoff v_m. Figure 2 is similar to Fig.1 except $\Omega_e/\omega_e \approx 2.5$; this reduces the ratio $v_m/v_{th} = (1/k_z\lambda_D)\,(\Omega_e/\omega_e)$ so that the positive-slope formation is prevented by a smaller electric field.

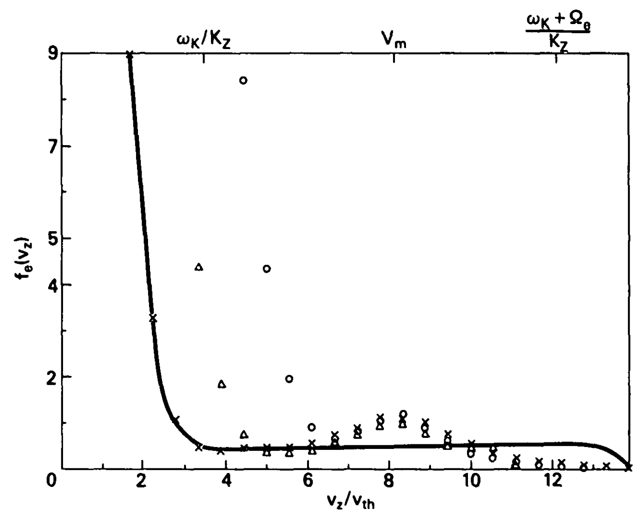

FIG.1. Bump formation for different values of E_0/E_D : E_0/E_D = 0.0 (X), 0.358 (Δ), 0.715 (O) for Ω_e/ω_e = 3.5.

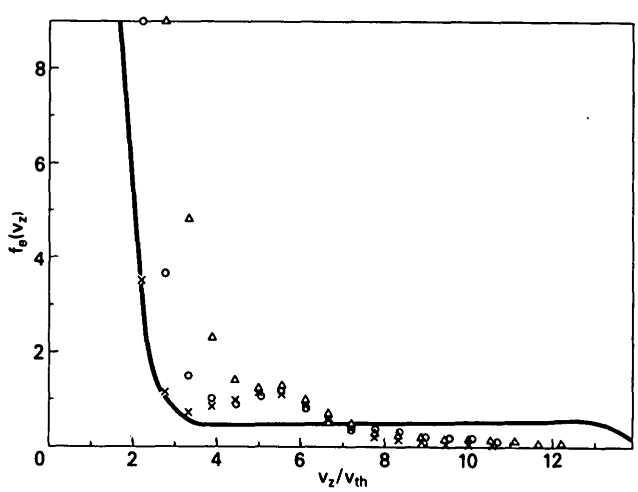

FIG.2. Bump formation for different values of E_0/E_D : E_0/E_D = 0.0 (X), 0.179 (O), 0.358 (Δ) for Ω_e/ω_e = 2.5.

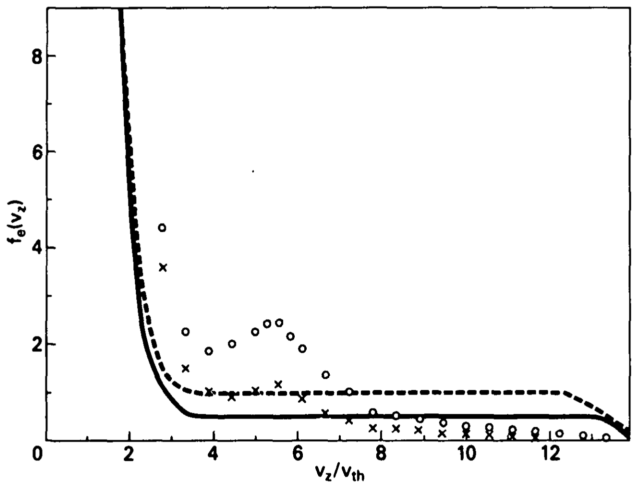

FIG.3. Bump formation for $\alpha = 0.1$ (○), $\alpha = 0.05$ (X) for $\Omega_e/\omega_e = 2.5$. f_T ($t = 0$) is given by the solid curve for $\alpha = 0.05$ and by the broken curve for $\alpha = 0.1$.

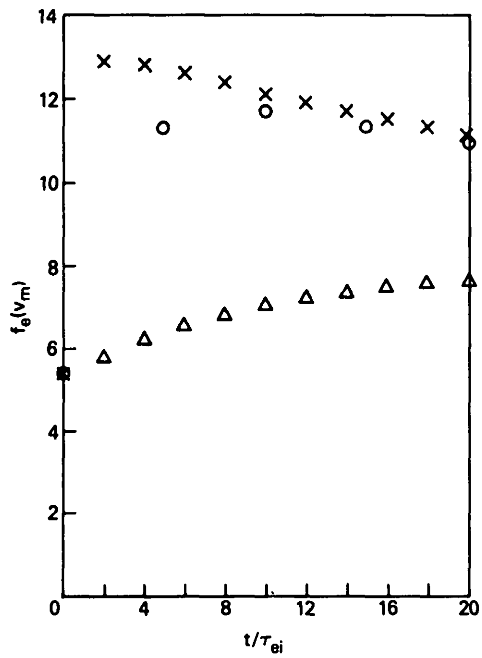

FIG.4. Time history of bump height for different values of W_0 : $W_0/n_0T_e = 3.53 \times 10^{-3}$ (X), 3.53×10^{-5} (○), 3.53×10^{-6} (Δ).

Next we vary the density of the tail in Fig.3; the broken curve and the solid curve show f_T for $\alpha = 0.1$ and 0.05, respectively, at $t = 0$. The circles and crosses are f_T at $t = 20\ \tau_{ei}$ for the same two values of α for $E_0/E_D \sim 0.18$. We see that the bump is more pronounced for larger values of α.

When we change W_0 from $3.53 \times 10^{-3}\ n_0T_e$ to $3.53 \times 10^{-5}\ n_0T_e$ for $E_0/E_D = 0.358$ but keeping all other parameters the same as Fig.1, we find that f_T is almost identical with the data represented by Δ in Fig.1. This implies that the bump forms much faster than $20\ \tau_{ei}$. A time history shown in Fig.4 confirms this fact where we plot the height of the bump versus time. We see that the bump forms in less than a few τ_{ei}, depending on W_0, and then decays away slowly, because of classical collisions.

3. SUMMARY AND CONCLUSIONS

We have presented numerical studies that indicate that monotonically decreasing tails in a tokamak with $(\Omega_e/\omega_e)^2 > 1$ can develop a positive slope near velocities $v_m = (1/k_z\lambda_D)(\Omega_e/\omega_e)v_{th} \approx 3(\Omega_e/\omega_e)v_{th}$. The presence of the constant electric field will not prevent the formation of a positive slope as long as it is only a fraction of the Dreicer field, a condition easily satisfied for tokamak operation after the initial time. The wave energy density W_0/nT does not affect the actual processes but only the time scale. However, even for $W_0/nT \approx 10^{-6}$ the process is fast enough to be of importance. The logarithmic height of the positive slope increases with the ratios α. Note that the above conclusions are subject to the development of an initial distribution function such as the one in Fig.1 before appreciable turbulence develops. In a tokamak this is justified since the initial stabilizing factor are collisions which become small with time, owing to Ohmic heating. In addition, we have assumed two time scales, so that stabilization is reached much faster than the formation of the positive slope.

The purpose of this letter has been to demonstrate that under conditions existing in present-day tokamaks distributions of runaways with positive slope can be produced. The consequences of this transformation can be favourable since they can heat the ambient plasma, but sometimes they may be deleterious since they can scatter energetic electrons in walls leading to liner problems. These important points will be discussed in a future publication.

ACKNOWLEDGEMENTS

The authors wish to acknowledge useful discussions with B. Coppi, C.S. Liu, R. Parker, W. Manheimer and P. Palmadesso.

This work was sponsored in part by USERDA.

REFERENCES

[1] BOXMAN, G.J., et al., in Controlled Fusion Plasma Physics (Proc. 7th Europ. Conf. Lausanne, 1975) 2, Lausanne (1975) 14.
[2] VLASENKOV, V.S., LEONOV, V.M., MEPEZHKIN, V.G., MUKHOVATOV, V.S., Nucl. Fusion 13 (1973) 509.
[3] EQUIPE TFR, in Plasma Physics and Controlled Nuclear Fusion Research (Proc. 5th Int. Conf. Tokyo, 1974) 1, IAEA, Vienna (1975) 135.
[4] ALIKAEV, V.V., et al., Zh. Tekh. Fiz. 45 (1975) 515 (Sov. Phys.–Tech. Phys. 20 (1975) 327).
[5] ALIKAEV, V.V., RAZUMOVA, K.A., SOKOLOV, Yu.A., Fiz. Plazmy 1 (1975) 546 (Sov. Phys. – Plasma Phys. 1 (1975) 303.
[6] KULSRUD, R.M., SUN, Y.C., WINSOR, N.S., FALLON, H.A., Phys. Rev. Lett. 31 (1973) 690.
[7] COSTLEY, A.E., HASTIE, R.J., PAUL, J.W.M., CHAMBERLAIN, J., Phys. Rev. Lett. 33 (1974) 758.
[8] BOYD, D.A., STAUFFER, F.J., TRIVELPIECE, A.W., Phys. Rev. Lett. 37 (1976) 98.
[9] HUI, B., WINSOR, N., COPPI, B., Phys. Fluids (in press).
[10] KADOMTSEV, B.B., POGUTSE, O.P., Zh. Ehksp. Teor. Fiz. 53 (1967) 2025 (Sov. Phys. – JETP 26 (1968) 1146).
[11] TEKULA, M.S., BERS, A., R.L.E. Progress Rept. No.117 (1976) 215.
[12] SHAPIRO, V.D., SHEVCHENKO, V.I., Zh. Ehksp. Teor. Fiz. 54 (1968) 1187 (Sov. Phys. – JETP 27 (1968) 635).
[13] PAPADOPOULOS, K., HABER, I., PALMADESSO, P., COPPI, B., Phys. Rev. Lett. (submitted).
[14] LIU, C.S., MOK, Y., Phys. Rev. Lett. 38 (1977) 162.

(Manuscript received 1 April 1977
Final version received 20 June 1977)

OBSERVATION OF PROPAGATING m = 1 WAVES IN SCYLLAC*

E.L. CANTRELL, G. MILLER, W.E. QUINN, W. RIESENFELD (Los Alamos Scientific Laboratory, Los Alamos, New Mexico, United States of America)

The plasma confinement in a high-beta stellarator is ended by an m = 1 (gross sideways) motion of the plasma column to the discharge tube wall. Detailed understanding of the dynamics of m = 1 motion is thus of great importance for this concept whether feedback or wall stabilization is used. In the Scyllac sector experiments the following specific question is of interest: can the propagation of transverse displacements from the ends, where the beta-dependent equilibrium is probably greatly perturbed, limit the useful experimental time? This time is otherwise limited by the loss of plasma from the ends which reduces beta and destroys the equilibrium. One would generally expect the propagation of m = 1 disturbances along a theta-pinch column to occur at the Alfvén velocity, however the situation in Scyllac is somewhat different due to the existence of the m = 1 instability produced by the helical magnetic fields. In addition, the role of damping in possibly suppressing the propagation of disturbances is not clear. The capability in the Scyllac sector feedback experiment [1] of applying external forces to the plasma in a controlled way and measuring the plasma displacement with feedback position detectors offered a convenient opportunity to study experimentally the propagation of m = 1 disturbances in the presence of the m = 1 instability.

The layout of the Scyllac sector is shown in Fig. 1. The plasma parameters are as given in Ref. [1]. Diamagnetic loop and plasma luminosity measurements show the plasma beta in the central region of the sector to be roughly constant for about 20 μs and similarly Thomson scattering measurements at 20–30 μs show no significant decrease of the electron temperature. The free-streaming end-loss time based on theta pinches with similar parameters is about 100 μs and the classical electron end conduction cooling time is several hundreds of μs. Energy loss due to radiation, assuming oxygen is the principal impurity, is calculated to be small. The most stringent constraint on the time of observation is the time available before a wall hit. By careful adjustment of equilibrium trimming fields this time could be extended to about 20 μs.

To investigate the dynamics of m = 1 motion, a disturbance was launched in the central region of the sector by locally accelerating the column in the vertical direction (perpendicular to the plane of the torus). The vertical direction was chosen to eliminate the additional complication due to imperfect horizontal

* Work performed under the auspices of USERDA, contract number W-7405-ENG.36.