# Enhanced Bremsstrahlung from Plasmas with Relativistic Electron Tails

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The bremsstrahlung emitted from thermal plasmas which coexist with a flux of relativistic electrons is calculated. The emission is found to be greatly enhanced at the fundamental and second harmonic of the electron plasma frequency. Some possible astrophysical applications of the theory are discussed.

## I. INTRODUCTION

In a recent paper Tidman and Dupree[1] calculated the enhanced bremsstrahlung radiation from "thermal" plasmas coexisting with a flux of energetic electrons. They found that the radiation at $\omega = \omega_e$ and $2\omega_e$ can be enhanced by several orders of magnitude compared with the thermal emission from a Maxwellian plasma with no energetic particles. The increase in radiation is due to the suprathermal electrons which drive the wave field of the longitudinal spectrum up to a high amplitude through a process of Cerenkov emission of electron plasma oscillations. These components of the fluctuation spectrum then collide with each other and with low-frequency ion fluctuations and emit electromagnetic radiation. The emission formula used in this calculation was derived nonrelativistically and for this reason is of limited use to astrophysical plasmas like the Sun, the Crab Nebula, some radio sources, etc., which contain electrons with relativistic energies.

In the present paper we derive the relativistically exact emission formulas and we apply them to a "thermal" plasma coexisting with a flux of relativistic electrons. It is found that the previous calculation underestimates the contribution to the radiation of the relativistic particles. In order to demonstrate the importance of the theory for astrophysical plasmas we present at the end two examples.

In deriving the formulas we use a technique developed by Dawson and Nakayama.[2] According to this method the kinetics of a plasma is investigated by expanding the distributions in terms of deflections of the particles from their noninteracting orbits. The first-order expansion is equivalent to Rostoker's[3] dressed particle picture, that is the plasma is viewed as an uncorrelated gas of dressed test particles. The relationship of the expansion to the work of Klimontovich[4] and Dupree[5,6] was clarified by Tidman, Birmingham, Dawson, and Nakayama,[7] who noted that if one wishes to go higher than first order, it is better to expand the fluctuations $\delta f$ and $\delta \mathbf{E}$ directly. Thus, one writes

$$\delta f = f_1 + f_2 + \cdots, \quad \delta \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots,$$

where $\delta f = f - \langle f \rangle$, and $\langle f \rangle$ is the slowly time-dependent ensemble average. Since the propagating transverse field first appears to second order (the first-order transverse field corresponds to a heavily Landau damped near zone field), the second-order equations must be used in computing the emission formula and sources in terms of first-order correlations. The first-order equations are then used to calculate the necessary first-order correlation functions.

The present calculations are valid for any infinite, homogeneous, isotropic plasma with no ambient electric or magnetic field. It is assumed that sufficient time has elapsed so that the plasma particles have no memory of any initial effects. This is achieved mathematically with the so-called adiabatic switching factor,[3] which switches on all the particles at $t = -\infty$.

The plan of the paper is as follows. Section II is devoted to a review of the basic equations and the calculation of the formula for the transverse emission from the second-order equations. The emission formula is given as a function of the first-order sources. There was no attempt at this stage to explicitly calculate the sources in terms of the one-body distribution functions. In Section III we calculate the bremsstrahlung radiation without any multipole expansion. The emission sources are found exactly (correct to all orders of magnitude) from the second-order Dawson–Nakayama equations in terms

[1] D. Tidman and T. Dupree, Phys. Fluids **8**, 1860 (1965).
[2] J. Dawson and T. Nakayama, Phys. Fluids **9**, 252 (1966).
[3] N. Rostoker, Nucl. Fusion **1**, 101 (1961).
[4] Y. L. Klimontovich Zh. Eksp. Teor. Fiz. **34**, 173 (1958) [Sov. Phys.—JETP **7**, 119 (1958)].
[5] T. H. Dupree, Phys. Fluids **6**, 1714 (1963).
[6] T. H. Dupree, Phys. Fluids **7**, 923 (1964).
[7] D. Tidman, T. Birmingham, J. Dawson, and T. Nakayama, Phys. Fluids **9**, 1881 (1966).

of first-order correlations. The first-order correlations are calculated in terms of the average distribution functions of the species involved. In Sec. IV the formulas found above are applied to electron–ion and electron–electron radiation from a plasma with a tail of relativistic electrons. Since only the longitudinal fields contribute significantly to the radiation at $\omega_e$, $2\omega_e$, only these fields are retained. This is done in the interest of computational simplicity and clarity. In Sec. V we compare our results with those of Tidman and Dupree and discuss the physics of the situation. Section VI completes the treatment with the application of the results to a simple model of (a) the Crab Nebula and (b) the solar corona, which indicates the relevance of the theory in astrophysical situations.

## II. RADIATION FORMULA IN TERMS OF SOURCES

As an introduction to the method to be employed in handling the radiation in the ensuing work, in this section we review the Dawson–Nakayama equations to first and second order and then derive the frequency spectrum of the transverse power emission in terms of the sources entering the second-order equations. We consider a fully ionized, multispecies, classical plasma. The particles are distributed uniformly in space with an isotropic (though not necessarily Maxwellian) spread of momentum. The usual assumption that the number of particles in a Debye sphere is large is taken to be valid throughout this work.

### A. Review of Basic Equations

Consider a plasma composed of discrete particles and assume that there are no ambient electric or magnetic fields. Let $f^\alpha(\mathbf{r}, \mathbf{p}, t)$ be the number of particles of the $\alpha$ species per unit volume in $\mathbf{r}$, $\mathbf{p}$ space. These functions are given by

$$f^\alpha(\mathbf{r}, \mathbf{p}, t) = \sum_{i=1}^{N_\alpha} \delta[\mathbf{r} - \mathbf{r}_i(t)]\ \delta[\mathbf{p} - \mathbf{p}_i(t)],$$

where $\mathbf{r}_i(t)$ and $\mathbf{p}_i(t)$ are the position and momentum of the $i$th particle of the $\alpha$ species. They satisfy the Maxwell–Klimontovich equations:

$$\left[\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} + q_\alpha\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\frac{\partial}{\partial \mathbf{p}}\right] f^\alpha(\mathbf{r}, \mathbf{p}, t) = 0,$$

$$\nabla\times\mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0,$$

$$\nabla\times\mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}\sum_\alpha q_\alpha \int d\mathbf{p}\, f^\alpha \mathbf{v},$$

$$\nabla\cdot\mathbf{E} = 4\pi \sum_\alpha q_\alpha \int d\mathbf{p}\, f^\alpha,$$

$$\nabla\cdot\mathbf{B} = 0.$$

According to the scheme in Refs. 2 and 7 we directly expand the singular distribution $f^\alpha$ in terms of the deflections of the particles from their noninteracting orbits. Thus, if

$$f^\alpha = \langle f^\alpha\rangle + \delta f^\alpha,$$

where $\langle f^\alpha\rangle$ is the slowly time varying ensemble average and $\delta f^\alpha$ the fluctuations about $\langle f^\alpha\rangle$, we can directly expand

$$\delta f^\alpha = f_1^\alpha + f_2^\alpha + \cdots$$

and in a similar way

$$\delta\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots,$$

$$\delta\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 + \cdots.$$

Assuming that $\langle\mathbf{E}\rangle = \langle\mathbf{B}\rangle = 0$, we find to first order that

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) R_1^\alpha(\mathbf{r}, \mathbf{p}, t) + q_\alpha \mathbf{E}_1\cdot\frac{\partial f_0^\alpha}{\partial \mathbf{p}} = 0,$$

$$\nabla\times\mathbf{E}_1 + \frac{1}{c}\frac{\partial \mathbf{B}_1}{\partial t} = 0, \tag{1}$$

$$\nabla\times\mathbf{B}_1 - \frac{1}{c}\frac{\partial \mathbf{E}_1}{\partial t} - \frac{4\pi}{c}\sum_\alpha q_\alpha \int d\mathbf{p}\, R_1^\alpha(\mathbf{r}, \mathbf{p}, t)\mathbf{v} = \frac{4\pi}{c}\mathbf{j}_0(\mathbf{r}, t),$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) \psi_1^\alpha(\mathbf{r}, \mathbf{p}, t) = 0, \tag{2}$$

$$\mathbf{j}_0(\mathbf{r}, t) = \sum_\alpha q_\alpha \int d\mathbf{p}\, \psi_1^\alpha(\mathbf{r}, \mathbf{p}, t)\mathbf{v},$$

where

$$f_1^\alpha(\mathbf{r}, \mathbf{p}, t) = \psi_1^\alpha(\mathbf{r}, \mathbf{p}, t) + R_1^\alpha(\mathbf{r}, \mathbf{p}, t). \tag{3}$$

From the first equation of (2) we get

$$\psi_1(\mathbf{r}, \mathbf{p}, t) = \psi_1(\mathbf{r} - \mathbf{v}t, \mathbf{p}, 0). \tag{4}$$

The initial conditions are[2]

$$\psi_1^\alpha(\mathbf{r}, \mathbf{p}, t=0) = \sum_{i=1}^{N_\alpha} \delta(\mathbf{r} - \mathbf{r}_{i0})\ \delta(\mathbf{p} - \mathbf{p}_{i0}) - \langle f^\alpha\rangle,$$

$$R_1^\alpha(\mathbf{r}, \mathbf{p}, t=0) = 0.$$

Note from above that the fluctuations $f_1$ are split into two parts $\psi_1$ and $R_1$. From Eqs. (1) and (2) we see that $\psi_1$ acts as a source term in Maxwell's equations to drive $R_1$. In this way $R_1$ can be interpreted as the shielding cloud of $\psi_1$ and $\psi_1$ as the fluctuation in the distribution function produced when the particles move in their noninteracting orbits [Eqs. (3) and (4)], thus representing the bare particle fluctuations (here $\psi_1$ is singular and cannot be zero). Therefore, one is presented with the picture of the bare particle with its shielding cloud. The summation over $\alpha$ in (2) is related to Rostoker's superposition principle.

In a similar fashion we find the equations to second order as

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) R_2^\alpha(\mathbf{r}, \mathbf{p}, t) + q_\alpha \mathbf{E}_2 \cdot \frac{\partial f_0^\alpha}{\partial \mathbf{p}} = 0,$$

$$\nabla \times \mathbf{E}_2 + \frac{1}{c}\frac{\partial \mathbf{B}_2}{\partial t} = 0, \qquad (5)$$

$$\nabla \times \mathbf{B}_2 - \frac{1}{c}\frac{\partial \mathbf{E}_2}{\partial t} - \frac{4\pi}{c}\sum_\alpha q_\alpha \int d\mathbf{p}\, R_2^\alpha(\mathbf{r}, \mathbf{p}, t)\mathbf{v} = \frac{4\pi}{c}\mathbf{j}_s(\mathbf{r}, t),$$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}}\right) \psi_2^\alpha(\mathbf{r}, \mathbf{p}, t) = -q_\alpha \left[\left(\mathbf{E}_1 + \frac{\mathbf{v}\times\mathbf{B}_1}{c}\right)\cdot\frac{\partial f_1^\alpha}{\partial \mathbf{p}} - \left\langle\left(\mathbf{E}_1 + \frac{\mathbf{v}\times\mathbf{B}_1}{c}\right)\cdot\frac{\partial f_1^\alpha}{\partial \mathbf{p}}\right\rangle\right], \qquad (6a)$$

$$\mathbf{j}_s(\mathbf{r}, t) = \sum_\alpha q_\alpha \int d\mathbf{p}\; \psi_2^\alpha(\mathbf{r}, \mathbf{p}, t)\mathbf{v}, \qquad (6b)$$

where

$$f_2^\alpha(\mathbf{r}, \mathbf{p}, t) = \psi_2^\alpha(\mathbf{r}, \mathbf{p}, t) + R_2^\alpha(r, p, t).$$

The physical interpretation of $\psi_2$, $R_2$ is similar to the previous one, with $\psi_2$ viewed as bare density fluctuations produced by the direct interaction of the first-order fluctuations with the first-order fields, and with $R_2$ representing the shielding of these fluctuations.

We note that the form of the left-hand side of Eqs. (1) and (5) are identical. They could be written in Dupree's[5] notation as

$$\left(\frac{\partial}{\partial t} + T\right) F_i = J_i, \qquad i = 1, 2, \qquad (7)$$

where the components of $F_i$ are $(f_i, \mathbf{E}_i, \mathbf{B}_i)$, of $J_1$, $(0, \mathbf{j}_0, 0)$ and of $J_2$, $(0, \mathbf{j}_s, 0)$. The solution of (7) is well known as long as we can solve the equations for $\mathbf{j}_0$ and $\mathbf{j}_s$. In the next section we use the second-order equations to find the transverse emission spectrum in terms of $\mathbf{j}_s$.

### B. Radiation Formula

We start from Eq. (5) and try to derive the emission spectrum, for the time being without using

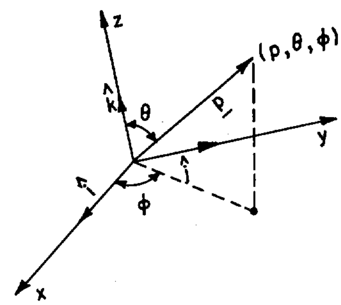


FIG. 1. Coordinate system.

Eq. (6) for the specific form of the sources $\mathbf{j}_s(\mathbf{r}, t)$. In this respect the following calculation is similar to that by Birmingham, Dawson, and Oberman[8] who calculate the emission from an arbitrary current source embedded in a plasma.

Defining Fourier transforms as

$$\mathbf{j}(\mathbf{k}, \omega) = \int d\mathbf{r} \int_{-\infty}^{\infty} dt \exp\left[-i(\mathbf{k}\cdot\mathbf{r} - \omega t)\right]\mathbf{j}(\mathbf{r}, t)$$

Eq. (5) gives for isotropic distributions

$$E_{2\perp}(\mathbf{k}, \omega) = -\frac{4\pi i}{\omega}\frac{j_{s\perp}(\mathbf{k}, \omega)}{\epsilon_t(\mathbf{k}, \omega)}, \qquad (8)$$

where

$$\epsilon_t(\mathbf{k}, \omega) = 1 - \frac{k^2c^2}{\omega^2} - \sum_\alpha L_t^\alpha(\mathbf{k}, \omega)$$

and

$$L_t^\alpha(\mathbf{k}, \omega) = \frac{\omega_\alpha^2 m_\alpha}{\omega}\int d\mathbf{p}\,\frac{(\hat{\mathbf{j}}\cdot\mathbf{v})[\hat{\mathbf{j}}\cdot(\partial f_0^\alpha/\partial\mathbf{p})]}{\mathbf{k}\cdot\mathbf{v} - \omega}.$$

The coordinate system $(\hat{\mathbf{\imath}}, \hat{\mathbf{j}}, \hat{\mathbf{k}})$ used is shown in Fig. 1.

We now use the fact that the rate at which power is radiated into transverse modes is equal to the rate at which work is done by transverse current fluctuations on the transverse field. Thus,

$$W = -\lim_{T\to\infty,\, L\to\infty}\frac{1}{T}\int_{-T/2}^{+T/2} dt\, \frac{1}{L^3}\cdot\int_{-L/2}^{+L/2} d\mathbf{r}\,\langle j_{s\perp}(\mathbf{r}, t)E_{2\perp}(\mathbf{r}, t)\rangle.$$

Using Eq. (8), Parseval's theorem, and the fact that $\mathbf{E}_2^*(\mathbf{k}, \omega) = \mathbf{E}_2(-\mathbf{k}, -\omega)$ it follows that

$$W = \frac{4\pi i}{(2\pi)^4}\lim_{T\to\infty,\, L\to\infty}\frac{1}{TL^3}\int\frac{d\omega}{\omega}\int d\mathbf{k}\,\frac{\langle\, |j_{s\perp}(\mathbf{k}, \omega)|^2\,\rangle}{\epsilon_t^*(\mathbf{k}, \omega)}.$$

The transverse waves propagate at any frequency

[8] T. Birmingham, J. Dawson, and C. Oberman, Phys. Fluids 8, 297 (1965).

greater than $\omega_e$, with a wavenumber $k_0$ given by the roots of

$$\epsilon_t(k_0, \omega) = 0.$$

These waves do not suffer any Landau damping.

Defining a spectrum $W(\omega)$ as

$$W = \int_0^\infty d\omega\, W(\omega)$$

and carrying out the $\mathbf{k}$ integration we obtain the formula for the emission spectrum as

$$W(\omega) = (2\omega k_0/8\pi^5 c^2)P(k_0, \omega), \qquad (9)$$

where

$$P(\mathbf{k}, \omega) = \sum_{\alpha,\beta} \int d\mathbf{k}''\, d\omega'' \cdot \exp\{i[(\mathbf{k} - \mathbf{k}'')\cdot\mathbf{x} - (\omega - \omega'')T]\} \cdot \langle j_{s\perp}^{\alpha}(\mathbf{k}, \omega) j_{s\perp}^{\beta}(\mathbf{k}'', \omega'')\rangle. \qquad (10)$$

In the next part $P(k_0, \omega)$ is calculated from the sources of the second-order equations in terms of the one-body distribution functions.

## III. RADIATION SOURCES AND CORRELATION FUNCTIONS

In this section we consider the radiation emitted from plasmas containing relativistic electrons. For this case, the multipole expansion method used by Birmingham, Dawson, and Kulsrud[9] and by Dupree[6] ceases to be a useful means of calculating radiation. Therefore, we calculate the sources without any expansion.

The calculations are substantially simplified if the particles are assumed to interact only via their longitudinal fields. This does not have any effect on the "wave" part of the emission, since only longitudinal–longitudinal interactions give rise to the resonant peaks at $\omega = \omega_e$, and $2\omega_e$.[10]

### A. Radiation Sources

Consider the Fourier transform of Eq. (6) and keep only the longitudinal fields. Thus,[11]

$$\mathbf{j}_s^{\alpha}(\mathbf{k}, \omega) = q_\alpha \int d\mathbf{p}\, \mathbf{v}\psi_2^{\alpha}(\mathbf{k}, \omega, \mathbf{p}),$$

$$\psi_2(\mathbf{k}, \omega, \mathbf{p}) = -\frac{iq_\alpha}{\omega} \frac{1}{1 - [(\mathbf{k}\cdot\mathbf{v})/\omega]} \cdot \int \frac{d\mathbf{k}'\, d\omega'}{(2\pi)^4} \mathbf{E}_1(\mathbf{k} - \mathbf{k}', \omega - \omega') \cdot \frac{\partial f_1^{\alpha}(\mathbf{k}', \omega', \mathbf{p})}{\partial \mathbf{p}}, \qquad (11)$$

$$\mathbf{E}_1(\mathbf{k} - \mathbf{k}', \omega - \omega') = -\frac{4\pi i(\mathbf{k} - \mathbf{k}')}{|\mathbf{k} - \mathbf{k}'|^2} \sum_\gamma q_\gamma n_1^{\gamma}(\mathbf{k} - \mathbf{k}', \omega - \omega').$$

From Eq. (11) integrating by parts

$$\mathbf{j}_s^{\alpha}(\mathbf{k}, \omega) = \sum_\gamma \frac{\omega_\alpha^2}{\omega} q_\gamma \int \frac{d\mathbf{k}'\, d\omega'}{(2\pi)^4} \frac{1}{|\mathbf{k} - \mathbf{k}'|^2} \cdot n_1^{\gamma}(\mathbf{k} - \mathbf{k}', \omega - \omega') \mathbf{Q}_1^{\alpha}(\mathbf{k}', \omega'), \qquad (12)$$

where

$$\mathbf{Q}_1^{\alpha}(\mathbf{k}', \omega') = m_\alpha \int d\mathbf{p}\, f_1^{\alpha}(\mathbf{k}', \omega', \mathbf{p})\mathbf{g}(\mathbf{k}'; \mathbf{k}, \omega; \mathbf{p}), \qquad (13)$$

$$\mathbf{g}(\mathbf{k}'; \mathbf{k}, \omega; \mathbf{p}) = (\mathbf{k} - \mathbf{k}')\cdot\frac{\partial}{\partial \mathbf{p}} \frac{\mathbf{v}}{1 - [(\mathbf{k}\cdot\mathbf{v})/\omega]}. \qquad (14)$$

Note that $1 - [(\mathbf{k}\cdot\mathbf{v})/\omega]$ can never be zero for the cases under consideration since $\omega/k > c$ for transverse emission.[12]

From Eqs. (10) and (12) it follows that

$$P(\mathbf{k}, \omega) = \sum_{\alpha,\beta,\gamma,\delta} \frac{\omega_\alpha^2 \omega_\beta^2 q_\gamma q_\delta}{\omega^2} \int \frac{d\mathbf{k}'\, d\omega'}{|\mathbf{k} - \mathbf{k}'|^2} \cdot \left(\frac{1}{|\mathbf{k} - \mathbf{k}'|^2} \mathrm{T}^{\alpha\beta}(\mathbf{k}', \omega') S^{\gamma\delta}(\mathbf{k} - \mathbf{k}', \omega - \omega') + \frac{1}{k'^2} \Lambda^{\alpha\delta}(\mathbf{k}', \omega') \Lambda^{*\gamma\beta}(\mathbf{k} - \mathbf{k}', \omega - \omega')\right), \qquad (15)$$

where

$$S^{\alpha\beta}(\mathbf{k}', \omega') = \langle n_1^{\alpha} n_1^{*\beta} \mid \mathbf{k}', \omega'\rangle,$$
$$\Lambda^{\alpha\beta}(\mathbf{k}', \omega') = \langle n_1^{\alpha} Q_{1\perp}^{*\beta} \mid \mathbf{k}', \omega'\rangle, \qquad (16)$$
$$\mathrm{T}^{\alpha\beta}(\mathbf{k}', \omega') = \langle Q_{1\perp}^{\alpha} Q_{1\perp}^{*\beta} \mid \mathbf{k}', \omega'\rangle.$$

In deriving (15) the following prescription was used for the ensemble averages[13]:

$$\langle n_1(\mathbf{k}_1, \omega_1) Q_1(\mathbf{k}_2, \omega_2) n_1(\mathbf{k}_3, \omega_3) Q_1(\mathbf{k}_4, \omega_4)\rangle = (2\pi)^8 [\langle n_1 Q_1 \mid \mathbf{k}_1, \omega_1\rangle\, \delta(\mathbf{k}_1 - \mathbf{k}_2) \cdot \delta(\omega_1 - \omega_2) \langle n_1 Q_1 \mid \mathbf{k}_3, \omega_3\rangle,$$
$$\delta(\mathbf{k}_3 - \mathbf{k}_4)\, \delta(\omega_3 - \omega_4) + \langle n_1 n_1 \mid \mathbf{k}_1, \omega_1\rangle\, \delta(\mathbf{k}_1 - \mathbf{k}_3) \cdot \delta(\omega_1 - \omega_3) \langle Q_1 Q_1 \mid \mathbf{k}_2, \omega_2\rangle\, \delta(\mathbf{k}_2 - \mathbf{k}_4)\, \delta(\omega_2 - \omega_4) + \langle n_1 Q_1 \mid \mathbf{k}_1, \omega_1\rangle\, \delta(\mathbf{k}_1 - \mathbf{k}_4)\, \delta(\omega_1 - \omega_4) \cdot \langle Q_1 n_1 \mid \mathbf{k}_2, \omega_2\rangle\, \delta(\mathbf{k}_2 - \mathbf{k}_3)\, \delta(\omega_2 - \omega_3)].$$

9 T. Birmingham, J. Dawson, and R. Kulsrud, Phys. Fluids 9, 2014 (1966).

10 K. Papadopoulos, Ph.D. thesis, University of Maryland (1968).

11 We neglect the ⟨ ⟩ terms as being of low frequency and thus not contributing to radiation.

12 $\omega$, $k$ are the emitted frequency and wavenumber related via $\epsilon_t(k, \omega) = 0$.

13 L. M. Al'thshul and V. I. Karpman Zh. Eksp. Teor. Fiz. **47**, 15520 (1964) [Sov. Phys.—JETP **20**, 1043 (1965)]; B. B. Kadomtsev, *Plasma Turbulence* (Academic Press Inc., New York, 1965), Chap. II.

Our next task is to calculate the functions given by (16).

### B. First-Order Correlations

We start from the first-order equations (1) and (2) and after Fourier transforming solve for $\mathbf{E}_1(\mathbf{k}, \omega)$ and $R_1^\alpha(\mathbf{k}, \omega, \mathbf{v})$ keeping only the longitudinal fields. This leads to

$$\mathbf{E}_1^l(\mathbf{k}, \omega) = -\frac{4\pi i\mathbf{k}}{k^2\epsilon_l(\mathbf{k}, \omega)} \sum_\beta q_\beta n_0^\beta(\mathbf{k}, \omega),$$

$$R_1^\alpha(\mathbf{k}, \omega, \mathbf{p}) = 4\pi q_\alpha \frac{\mathbf{k}\cdot(\partial f_0^\alpha/\partial \mathbf{p})}{\mathbf{k}\cdot\mathbf{v} - \omega} \sum_\beta q_\beta n_0^\beta(\mathbf{k}, \omega),$$

where

$$n_0^\alpha(\mathbf{k}, \omega) = \int d\mathbf{p}\ \psi_1^\alpha(\mathbf{k}, \omega, \mathbf{p}).$$

Then

$$n_1^\alpha(\mathbf{k}', \omega') = n_0^\alpha(\mathbf{k}', \omega') + \frac{L^\alpha(\mathbf{k}', \omega')}{\epsilon_l(\mathbf{k}', \omega')} \sum_\beta \frac{q_\beta}{q_\alpha} n_0^\beta(\mathbf{k}', \omega'). \quad (17)$$

Similarly,

$$Q_{1\perp}^\alpha(\mathbf{k}, \omega) = Q_0^\alpha(\mathbf{k}', \omega') + \frac{\Delta^\alpha(\mathbf{k}', \omega')}{\epsilon_l(\mathbf{k}', \omega')} \sum_\beta \frac{q_\beta}{q_\alpha} n_0^\beta(\mathbf{k}', \omega'), \quad (18)$$

where

$$L^\alpha(\mathbf{k}', \omega') = \frac{\omega_\alpha^2 m_\alpha}{k'^2} \int d\mathbf{p}\, \frac{\mathbf{k}'\cdot(\partial f_0^\alpha/\partial \mathbf{p})}{\mathbf{k}'\cdot\mathbf{v} - \omega'},$$

$$\Delta^\alpha(\mathbf{k}', \omega') = \frac{\omega_\alpha^2 m_\alpha}{k'^2} \int d\mathbf{p}\, \frac{\mathbf{k}'\cdot(\partial f_0^\alpha/\partial \mathbf{p})}{\mathbf{k}'\cdot\mathbf{v} - \omega'}\, g_\perp(\mathbf{k}'; \mathbf{k}, \omega; \mathbf{p}),$$

$$Q_0^\alpha(\mathbf{k}', \omega') = m_\alpha \int d\mathbf{p}\ \psi_1^\alpha(\mathbf{k}', \omega', \mathbf{p}) g_\perp(\mathbf{k}'; \mathbf{k}, \omega; \mathbf{p}). \quad (19)$$

Using Eqs. (17)–(19) the quantities given by Eq. (16) can be calculated. We are interested in the case of an isothermal electron–ion plasma. Thus, neglecting the ion motion, we find

$$S^{ee}(\mathbf{k}', \omega') = \frac{\langle n_0^e n_0^e \mid \mathbf{k}', \omega'\rangle}{|\epsilon_l(\mathbf{k}', \omega')|^2}, \quad (20)$$

$$\Lambda^{ee}(\mathbf{k}', \omega') = \frac{1}{|\epsilon_l(\mathbf{k}', \omega')|^2} \cdot \{[1 - L^e(\mathbf{k}', \omega')]^* \langle Q_0^e n_0^e \mid \mathbf{k}', \omega'\rangle + \Delta^{e*}(\mathbf{k}', \omega')\langle n_0^e n_0^e \mid \mathbf{k}', \omega'\rangle\}, \quad (21)$$

$$\mathrm{T}^{ee}(\mathbf{k}', \omega') = \frac{1}{|\epsilon_p(\mathbf{k}', \omega')|^2} \{ |1 - L^e(\mathbf{k}', \omega')|^2 \cdot\langle Q_0^e Q_0^e \mid \mathbf{k}', \omega'\rangle + \{[1 - L^e(\mathbf{k}', \omega')]\, \Delta^{e*}(\mathbf{k}', \omega') + [1 - L^e(\mathbf{k}', \omega')]^*\, \Delta^e(\mathbf{k}', \omega')\}\langle Q_0^e n_0^e \mid \mathbf{k}', \omega'\rangle + |\Delta^e(\mathbf{k}', \omega')|^2 \langle n_0^e n_0^e \mid \mathbf{k}', \omega'\rangle\}. \quad (22)$$

The spectral densities, $\langle n_0^e n_0^e \mid \mathbf{k}', \omega'\rangle$, etc., can be found simply from the equations for the bare particle fluctuations $\psi_1$. We have that

$$\langle n_1^\alpha(\mathbf{r}_1, t_1) n_1^\beta(\mathbf{r}_2, t_2)\rangle = \delta(\alpha, \beta) \int d\mathbf{v}\, f_0^\alpha(\mathbf{v})\ \delta(\mathbf{r} - \mathbf{v}t),$$

where $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $t = t_1 - t_2$. Thus,

$$\langle n_0^\alpha n_0^\beta \mid \mathbf{k}', \omega'\rangle = 2\pi\ \delta(\alpha, \beta) \int d\mathbf{p}\, f_0^\alpha(\mathbf{p})\ \delta(\omega' - \mathbf{k}'\cdot\mathbf{v}). \quad (23)$$

In a similar fashion

$$\langle n_0^e Q_0^e \mid \mathbf{k}', \omega'\rangle = 2\pi m_e \int d\mathbf{p}\, f_0^e(\mathbf{p})\, g_\perp(\mathbf{k}', \mathbf{k}, \omega; \mathbf{p})\ \delta(\omega' - \mathbf{k}'\cdot\mathbf{v}), \quad (24)$$

$$\langle Q_0^e Q_0^e \mid \mathbf{k}', \omega'\rangle = 2\pi m_e^2 \int d\mathbf{p}\, f_0^e(\mathbf{p})\, g_\perp^2(\mathbf{k}'; \mathbf{k}, \omega; \mathbf{p})\ \delta(\omega' - \mathbf{k}'\cdot\mathbf{v}). \quad (25)$$

Equations (9), (15), and (20)–(25) give the relativistically correct bremsstrahlung radiation due to longitudinal fluctuations in terms of the particle distribution functions. For nonrelativistic plasmas we can approximate $g_\perp(\mathbf{k}'; \mathbf{k}, \omega; \mathbf{p})$ by $(\mathbf{k} - \mathbf{k}')_\perp/m_e$. Thus, the nonrelativistic emission formula derived by Dupree[6] is recovered.

In the next section we apply the emission formulas to the particular situation of enhanced bremsstrahlung from a plasma with relativistic electron tails.

## IV. ENHANCED RADIATION FROM PLASMAS WITH RELATIVISTIC ELECTRON TAILS

We are ready to apply the results derived in the previous section to any plasma whose distribution function is known.

Assume an ion–electron plasma in which

$$f_0^i(\mathbf{v}) = (2\pi)^{-3/2} V_i^{-3} \exp(-v^2/2V_i^2),$$
$$f_0^e(\mathbf{p}) = \beta(2\pi)^{-3/2} V_e^{-3} \exp(-v^2/2V_e^2) + (1 - \beta) f_r(\mathbf{p}), \quad (26)$$

where

$$V_{i,e}^2 = T_{i,e}/m_{i,e},\ (1 - \beta) \ll 1,\ T_e \sim T_i,$$

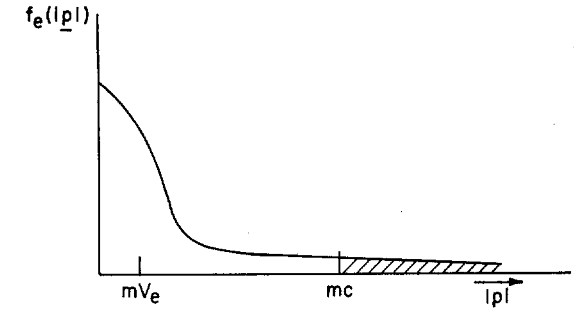


FIG. 2. Electron distribution function.

and

$$f_r(p) = \frac{1}{4\pi m_e^3 c^3}\left(\frac{m_e c^2}{T_0}\right) K_2^{-1}\left(\frac{m_e c^2}{T_0}\right) \cdot \exp\left[-\frac{m_e c^2}{T_0}\left(1+\frac{p^2}{m_e^2 c^2}\right)^{1/2}\right]$$

represents a Maxwellian relativistic tail of electrons ($T_0 > m_e c^2$) (Fig. 2). $K_2$ is the second-order Bessel function.

We want the wave contribution to the radiation. This will give the usual peaks at $\omega = 2\omega_e$ for electron–electron interactions and at $\omega = \omega_e$ for electron–ion interactions. Noticing that $m_i \gg m_e$, $T_e \sim T_i$, we can neglect ion dynamics. For the distributions (26) one finds that

$$\epsilon_l(\mathbf{k}', \omega') = 1 - \frac{\omega_i^2}{k'^2 V_i^2}\left[\frac{\omega'}{k' V_i} R\left(\frac{\omega'}{k' V_i}\right) - 1\right] - \frac{i\omega_i^2}{k'^2 V_i^2} I\left(\frac{\omega'}{k' V_i}\right) - \frac{\beta\omega_e^2}{k'^2 V_e^2}\left[\frac{\omega'}{k' V_e} R\left(\frac{\omega'}{k' V_e}\right) - 1\right] - \frac{(1-\beta)\omega_e^2}{k'^2 c^2}\left(\frac{m_e c^2}{T_0}\right)\left[\frac{\omega'}{2k'c}\ln\frac{\omega' + k'c}{|\omega' - k'c|} - 1\right] - \frac{i\beta\omega_e^2}{k'^2 V_e^2} I\left(\frac{\omega'}{k' V_e}\right) - \frac{i(1-\beta)\omega_e^2}{k'^2 c^2}\left(\frac{m_e c^2}{T_0}\right)\frac{\pi}{4}\frac{\omega'}{k'c'} \cdot \left(\frac{\omega' - k'c}{|\omega' - k'c|} - 1\right),$$

where

$$I(x) = (\pi/2)^{1/2}\exp(-x^2/2),$$

$$R(x) = \frac{1}{(2\pi)^{1/2}} P\int_{-\infty}^{\infty} dy\, \frac{\exp(-y^2/2)}{y+x}.$$

Using the asymptotic expansion

$$x\,R(x) = 1 + 1/x^2 + \cdots \text{ for large } x$$

it is found that in the range

$$k' < \frac{k_D}{\alpha},$$

$$\alpha^2 = 2\ln\left(\frac{2}{\pi}\right)^{1/2}\left(\frac{c}{V_e}\right)^3\left(\frac{m_e c^2}{T_0}\right)^{-2}\frac{K_2(m_e c^2/T_0)}{(1-\beta)} \qquad (27)$$

Eqs. (20)–(22) with (23)–(26) become (Appendix A)

$$\begin{bmatrix} S^{ee}(\mathbf{k}', \omega' \sim \omega_e) \\ \Lambda^{ee}(\mathbf{k}', \omega' \sim \omega_e) \\ T^{ee}(\mathbf{k}', \omega' \sim \omega_e)\end{bmatrix} = 2\pi\left(\frac{T_0}{m_e c^2}\right)^2 \frac{k'^2 c^2}{\omega_e^2}\,\theta(k'c - |\omega_e|)\begin{bmatrix} 1 \\ \Delta^{e*}(\mathbf{k}', \omega') \\ |\Delta^e(\mathbf{k}', \omega')|^2\end{bmatrix}[\delta(\omega' - \omega_e) + \delta(\omega' + \omega_e)]. \qquad (28)$$

We calculate next the radiation at $\omega_e$ and $2\omega_e$.

### A. Electron–Ion Interaction

From Eq. (15) with $S^{ii}(\omega) \approx \pi\delta(\omega)$ Eq. (9) reduces to

$$W^{ei}(\omega) = \frac{2\,|\omega|\,k_0}{8\pi^5 c^2}\frac{\omega_e^4 q_e^2}{\omega^2}\,\pi\int_{\omega_e/c}^{kD/\alpha} d\mathbf{k}'\,\frac{T^{ee}(\mathbf{k}', \omega)}{|\mathbf{k}_0 - \mathbf{k}'|^4}.$$

Using Eq. (28) for $T^{ee}$ and integrating over $\omega$ for $\omega > 0$ we find

$$W^{ei}(\omega_e) = 3 \times 10^{-24}\, n_0^{5/2} T_e^{-3/2}\left(\frac{T_0}{m_e c^2}\right)\alpha^{-4} \quad \text{erg/sec cm}^3. \qquad (29)$$

### B. Electron–Electron Interaction

In a similar way as above

$$W^{ee}(\omega) = \frac{2\,|\omega|\,(\omega^2 - \omega_e^2)^{1/2}}{8\pi^5 c^3}\frac{\omega_e^4 q_e^2}{\omega^2} \cdot \int_{\omega_e/c}^{kD/\alpha} d\mathbf{k}' \int d\omega'\, \frac{1}{|\mathbf{k} - \mathbf{k}'|^2} \cdot \left(\frac{T^{ee}(\mathbf{k}', \omega') S^{ee}(\mathbf{k} - \mathbf{k}', \omega - \omega')}{|\mathbf{k} - \mathbf{k}'|^2} + \frac{\Lambda^{ee}(\mathbf{k}', \omega')\Lambda^{*ee}(\mathbf{k} - \mathbf{k}', \omega - \omega')}{k'^2}\right).$$

Using Eq. (28) and integrating over $\omega$ for $\omega > 0$ we find

$$W^{ee}(2\omega_e) = 1 \times 10^{-23}\, n_0^{5/2} T_e^{-3/2}\left(\frac{T_0}{m_e c^2}\right)^2 \alpha^{-3} \quad \text{erg/sec cm}^3. \qquad (30)$$

## V. DISCUSSION

The formulas derived in Secs. II and III are the relativistically correct emission formulas, in terms of the one-body distribution functions. If we approximate

$$\frac{\partial}{\partial \mathbf{p}} \frac{\mathbf{v}}{1 - [(\mathbf{k}\cdot\mathbf{v})/\omega]}$$

by $1/m_e$ the nonrelativistic formula is recovered. We note here that such an assumption might lead to erroneous results if particles with relativistic velocities are involved. For example the substitution $\partial/\partial\mathbf{p}$ by $(1/m_e)(\partial/\partial\mathbf{v})$ will result in a difference of the order of the Jacobian,

$$J(|\mathbf{v}|/|\mathbf{p}|),$$

which can be substantial. Also, for generation of transverse waves with phase velocities $\omega/k > c$ but $\omega/k \sim c$ the approximation $1 - (\mathbf{k}\cdot\mathbf{v})/\omega \sim 1$ may be rather poor and care must be exercised with regard to the number of terms that must be kept in expanding $\{1 - [(\mathbf{k}\cdot\mathbf{v})/\omega]\}^{-1}$ as a power series in $[(\mathbf{k}\cdot\mathbf{v})/\omega]$.

We compare now the results derived in Sec. IV for enhanced emission with the results derived by Tidman and Dupree. From Tidman[14]

$$I\omega_e = 6 \times 10^{-25}\, n_0^{5/2} T_e^{-3/2} (V_E/c)^2 \alpha^{-4},$$

$$I_{2\omega_e} = 5 \times 10^{-25}\, n_0^{5/2} T_e^{-3/2} (V_E/c)^4 \alpha^{-3}.$$

The maximum amount of radiation that one can get from the above under constant $n_0$, $T_e$ is

$$I_{\omega_e}^{\max} = 6 \times 10^{-25}\, n_0^{5/2} T_e^{-3/2} \alpha^{-4},$$

$$I_{2\omega_e}^{\max} = 5 \times 10^{-25}\, n_0^{5/2} T_e^{-3/2} \alpha^{-3}.$$

Comparing these with our results and using as a typical temperature of the tail $T_0 \approx 5$ Mev we get

$$\frac{W^{ei}}{I_{\omega_e}^{\max}} \approx 50, \quad \frac{W^{ee}}{I_{2\omega_e}^{\max}} \approx 2 \times 10^3.$$

These results show a substantial increase in the radiation, over the nonrelativistic case.

The physics of the enhanced emission is the same as the one described by Tidman–Dupree. The relativistic particles drive the wave-field part of the longitudinal fluctuation spectrum up to a high amplitude through a process of Cerenkov emission of plasma oscillations. These components of the fluctuation spectrum, then collide with each other and with low-frequency ion density fluctuations and emit electromagnetic radiation. However one should not fail to notice that the plasma oscillations excited by the relativistic electrons have phase velocities close to the velocity of light. Thus if the transverse wave resulting from the "collision" of two longitudinal waves has also phase velocity close to $c$, a further enhancement is achieved, since the wave–wave interaction is very close to resonance (although *never* exactly resonant).

From (29) and (30) one notices the same density dependence $n_0^{5/2}$ as in the nonrelativistic case. This is expected, since the density enters in the formulation via the plasma frequency $\omega_e$, which in the present case is controlled completely by the nonrelativistic part of the distribution function.

We note here that recently Lerche,[15] starting from the nonrelativistic formulas of Tidman and Dupree and substituting into them spectral densities for distribution functions with relativistic tails, concluded that there is a substantial enhancement of radiation at $2\omega_e$. However, although we agree with this qualitative conclusion, we believe that it is, in general, inconsistent to try to find effects due to relativistic particles from nonrelativistic formulas.[16]

Before considering the examples we mention the conditions of applicability of the previous formulas in physical situations. They are exactly the same as in the nonrelativistic case.[1] That is:

(i) There must be an equilibrium between the emission and reabsorption (collisionless damping) of longitudinal waves for the small wavenumber part of the spectral density in the radiating volume of the plasma. This is the case when the linear dimensions of the plasma $L$ are such that

$$L \gg \omega/k\gamma_L^{-1}(k).$$

(ii) The Landau damping must be larger than that the collisional damping in the wavenumber range considered. That is,

$$\gamma_L > \nu_c.$$

In the present case the Landau damping is given by

$$\gamma_L(k') = \frac{\pi}{4} \frac{(1-\beta)\omega_e^4}{k'^3 c^3} \left(\frac{m_e c^2}{T_0}\right).$$

The collisional damping is $\nu_c \approx O(k_D^3/n_0)$.

## VI. TWO EXAMPLES

In order to show the relevance of the theory to astrophysical situations, we briefly examine two examples. A more detailed account of similar situations will be published elsewhere.[17]

### A. Crab Nebula

Consider as typical parameters[18] $n_0 \sim 10^3$ cm$^{-3}$

[14] D. Tidman, *Dynamics of Fluids and Plasmas* (Academic Press Inc., New York, 1966), p. 399.

[15] I. Lerche, Phys. Fluids **11**, 2459 (1968).

[16] K. Papadopoulos and I. Lerche, Phys. Fluids (to be published).

[17] K. Papadopoulos and I. Lerche (unpublished).

[18] I. S. Shklovsky, *Cosmic Radio Waves* (Harvard University Press, Cambridge, Massachusetts, 1960), p. 282.

$1-\beta \sim 10^{-5}$, $T_e \sim 10^4$ °K, $10^7 \leq T_0 \leq 10^8$. We find then for the radiation at $2\omega_e$ which corresponds in the megacycle band that

$$W^{ee}(2\omega_e) \sim 10^{-20} \text{ erg/sec cm}^3.$$

Although this is a gross estimate and several factors such as reabsorption should be considered, it shows that such a process might be of interest in astrophysical plasmas, in agreement with Lerche.[15] A similar calculation might also explain the unusually high radiation measured by Hewish *et al.*[19] from a small compact part of the Crab Nebula.

### B. Solar Corona

For the solar corona we use the model considered by Tidman,[20] according to which the source of radiation is assumed to be a collisionless plasma shock wave rising through the solar corona. Considering as typical parameters of the corona $n_0 = 2.8 \times 10^8$ cm$^{-3}$, $T_e = 10^6$ °K, $T_0 \sim m_e c^2$, and $1-\beta \sim 10^{-4}$. The fundamental frequency of the emitted radiation is then $f = 150$ Mc/sec. Applying these values to (29) and (30) we find that

$$W^{ei}(\omega_e) \sim W^{ee}(2\omega_e) \sim 10^{-12} \text{ erg/sec cm}^3.$$

For a radiating volume of the order $10^{30}$ cm$^3$ this gives a total power of

$$\mathcal{E} \sim 10^{18} \text{ erg/sec}$$

which is of the same order of magnitude with the experimental observations of the type II solar outbursts.

Note that for the above examples one can easily verify the conditions of applicability of the theory stated previously.

### ACKNOWLEDGMENTS

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### APPENDIX A

Here we present the calculation of $S^{ee}(\mathbf{k}', \omega')$ given by Eq. (20) for the distribution function (26). The values of $\Lambda^{ee}(\mathbf{k}', \omega')$ and $T^{ee}(\mathbf{k}', \omega')$ can be found in a similar way.[10]

From Eqs. (20) and (23) we have that

$$S^{ee}(\mathbf{k}', \omega') = \frac{2\pi F^e(\mathbf{k}', \omega')}{|\text{Re}\,\epsilon_l(\mathbf{k}', \omega')|^2 + |\text{Im}\,\epsilon_l(\mathbf{k}', \omega')|^2}, \qquad \text{(A1)}$$

where

$$F^e(\mathbf{k}', \omega') = \int d\mathbf{p}\, f_0^e(p^2)\, \delta(\omega' - \mathbf{k}'\cdot\mathbf{v}). \qquad \text{(A2)}$$

For the distribution function (26) with $(1-\beta)\cdot T_0 < mV_e^2\beta$, the dominant term is due to the nonrelativistic part of the distribution function. Thus for $k' < k_D$,

$$\text{Re}\,\epsilon_l(\mathbf{k}', \omega') = 1 \pm \left(\frac{\omega_e^2}{\omega'^2} + \frac{3k'^2V_e^2}{\omega'^2}\right)^{1/2}. \qquad \text{(A3)}$$

From (A1) and (A3)

$$S^{ee}(k' < k_D, \omega' \sim \pm\omega_e) = \pi^2\omega_e \frac{F^e(\mathbf{k}', \omega_e)}{|\text{Im}\,\epsilon_l(k', \omega_p)|} \cdot \delta[\omega' \pm (\omega_e + 3k'^2V_e^2)^{1/2}]. \qquad \text{(A4)}$$

From (A2)

$$F^e(\mathbf{k}', \omega_e) = \beta(2\pi)^{-1/2}V_e^{-1} \exp\left(-\frac{\omega_e^2}{2k'^2V_e^2}\right) + (1-\beta)\int d\mathbf{p}\, f_r^e(\mathbf{p})\, \delta(\omega' - \mathbf{k}'\cdot\mathbf{v}). \qquad \text{(A5)}$$

Using polar coordinates[21] (Fig. 1)

$$\begin{aligned} I &\equiv \int d\mathbf{p}\, f_r^e(p^2)\, \delta(\omega' - \mathbf{k}'\cdot\mathbf{v}) \\ &= 2\pi \int_{-1}^{+1} dx \int_0^\infty dp\, p^2\, \delta(\omega' - k'vx) f_r^e(p') \\ &= \frac{2\pi m_e}{k'} \int_0^\infty dp\, \gamma p f_r^e(p^2) \int_{-1}^{+1} dx\, \delta\left(\frac{\omega}{kv} - x\right) \\ &= \frac{2\pi m_e^3 c^2}{k'^2}\, \theta(k'c - |\omega_e|) \int_{\gamma_0}^\infty d\gamma\, \gamma^2 f_r^e(\gamma), \end{aligned} \qquad \text{(A6)}$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}, \gamma_0 = \left(1 - \frac{\omega^2}{k'^2c^2}\right)^{-1/2}, \qquad \text{(A7)}$$

and $x = \cos\theta$.

Define

$$\psi(\gamma) = \int_\gamma^\infty d\gamma_1 \int_{\gamma_1}^\infty d\gamma_2 \int_{\gamma_2}^\infty d\gamma_3\, f_r^e(\gamma_3).$$

This is a solution of the differential equation

$$\frac{d^3\psi(\gamma)}{d\gamma^3} = -f_r^e(\gamma)$$

subject to $\psi(\gamma), \psi'(\gamma), \psi''(\gamma) \to 0\ \gamma \to \infty$.

[19] A. Hewish and S. E. Okoye, Nature **207**, 59 (1965).
[20] D. Tidman, J. Planet. Space Sci. **13**, 781 (1965).
[21] Here $\theta$ is the Heaviside function.

Thus,

$$I = \frac{2\pi}{k'} m_e^3 c^2 [\gamma_0^2 \psi''(\gamma_0) - 2\gamma_0 \psi'(\gamma_0) + 2\psi(\gamma_0)] O(k'c - |\omega_p|). \quad \text{(A8)}$$

From (A5) and (A8) we find for the distribution given by Eq. (26)

$$F^e(\mathbf{k}', \omega_e) = \beta(2\pi)^{-1/2} V_e^{-1} \exp\left(-\frac{\omega_e^2}{2k'^2 V_e^2}\right) + \frac{(1-\beta)\gamma_1^2}{2k'^2 c^2} \frac{\exp(-\gamma_1 m_e c^2/T_0)}{K_2(m_e c^2/T_0)} \cdot \left[1 + \frac{2}{\gamma_1}\left(\frac{T_0}{m_e c^2}\right) + \frac{2}{\gamma_1^2}\left(\frac{T_0}{m_e c^2}\right)^2\right], \quad \text{(A9)}$$

where

$$\gamma_1 = \left(1 - \frac{\omega_e^2}{k'^2 c^2}\right)^{-1/2}.$$

In a similar fashion[10]

$$\operatorname{Im} \epsilon_p(\mathbf{k}', \omega_e) = \left(\frac{\pi}{2}\right)^{1/2} \frac{\beta \omega_e^2}{k'^2 V_e^2} \exp\left(-\frac{\omega_e^2}{2k'^2 V_e^2}\right) + \frac{(1-\beta)\omega_e^3 \gamma_1^2 \pi}{k'^3 c^3} \frac{\exp(-\gamma_1^2 m_e c^2/T_0)}{K_2(m_e c^2/T_0)} \cdot \left[\frac{m_e c^2}{T_0} + \frac{2}{\gamma_1} + \frac{2}{\gamma_1^2}\left(\frac{T_0}{m_e c^2}\right)\right]. \quad \text{(A10)}$$

From (A4), (A5), and (A6) we find that for $\omega_e/c \le k' \le k_D/\alpha$ with

$$\alpha^2 = 2 \ln \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{c}{V_e}\right)^3 \left(\frac{T_0}{m_e c^2}\right)^2 \frac{K_2(mc^2/T_0)}{(1-\beta)}, \quad \text{(A11)}$$

$$S^{ee}(k', \pm\omega_e) = 2\pi \left(\frac{k'^2 c^2}{\omega_e^2}\right) \left(\frac{T_0}{m_e c^2}\right) \cdot \theta(k'c - |\omega_e|)\, \delta(\omega' \pm \omega_e). \quad \text{(A12)}$$

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# Space, Time, and Energy Distributions of Neutrons and X Rays from a Focused Plasma Discharge

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The energy spectrum and spatial distribution of neutrons emitted by a plasma focus device were measured with nuclear emulsions and scintillation detectors, and the results are reported. The average energy of the D–D neutrons emitted along the axis was shifted $\sim$500 keV corresponding to an axial center-of-mass velocity of $2 \times 10^8$ cm/sec for the reacting deuterons. An 11% anisotropy in the neutron fluxes measured in the forward and radial directions correlates with an axially streaming plasma which is not isotropic; the reacting deuterons collide predominantly in the radial direction. Time-resolved collimation measurements of the emitted neutrons showed an axial translation of the neutron source corresponding to velocities up to $2 \times 10^8$ cm/sec; this correlates with the nonsimultaneous pinching of the noncylindrical plasma along the axis. Plasma densities of $\sim 2 \times 10^{20}$ cm$^{-3}$ and ion temperatures of 2 keV were consistent with observed neutron yields of $10^{10}$ and pulse widths of 60 nsec. X-ray intensity measurements were made for x-ray energies of $7 < E_p < 30$ keV and showed an $E_p^{-2}$ dependence which does not agree with plasma bremsstrahlung, but appears to result from anode bombardment of axially accelerated electrons with energies $> 200$ keV. The line radiation corresponded to that of fairly cold ions.

## I. INTRODUCTION

Different mechanisms have been proposed to explain the intense production of neutrons and x rays generated by the dense deuterium plasmas formed in plasma focus devices, for which neutron yields of $10^9$ to $10^{10}$, pulse widths of $\sim$100 nsec, and plasma densities of $2 \times 10^{19}$ to $3 \times 10^{20}$ cm$^{-3}$ have been reported.[1–4] We present the results of measuring the energy spectrum of axially emitted neutrons, the neutron-flux anisotropy, and the time-resolved spatial distribution of the neutron

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