Anomalous Resistivity in the Auroral Plasma

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It is shown that the low-frequency ion density fluctuations produced nonlinearly by the interaction of fast precipitating electrons with the auroral plasma can enhance the resistivity of the auroral plasma by several orders of magnitude over its collisional value.

One of the key problems to the understanding of auroral arcs is the resistivity that they present to the flow of field-aligned electric currents. Since collisional resistivity cannot account for the observations, one is faced with the question of what kind of collisionless process can account for the enhanced resistance. Previous theories [*Kindel and Kennel*, 1971] attempted to associate the anomalous resistivity with electron-ion instabilities driven by the electron current.

Such instabilities are excited only when the current exceeds a certain threshold. Field-aligned measurements of currents [*Reasoner and Chappel*, 1973] show that the thresholds predicted by *Kindel and Kennel* [1971] occur only at altitudes greater than 1000 km. It is the purpose of this note to demonstrate that one can have large values of anomalous resistivity even when the auroral plasma is stable with respect to current driven instabilities. The underlying physical notion of the proposed model is that during times of high-energy electron precipitation the auroral plasma is in a state of stable but highly nonthermal collisionless equilibrium. The details of the processes involved and the possible stationary state can be found in *Papadopoulos and Coffey* [1974]. The model proposed there for the explanation of various nonthermal features of the auroral plasma has the following aspects:

1. One can view the high-energy precipitating electrons ($\sim$10 keV) as a spread out beam in velocity space $[(\Delta V_b/V_b) \sim 1/3]$. Unless the beam is completely flat in velocity $[(\Delta V_b/V_b) \sim 1]$, it will act as a source of high-frequency ($\omega_0 \sim \omega_e$) very long wavelength $[k_0\lambda_D \sim (V_e/V_b)^2]$ plasma waves. Although the spectrum of these waves is broad in k space $[(\Delta k/k_0) \sim (\Delta V_b/V_b)]$, it will be seen by the ambient plasma as having a very narrow bandwidth $\Delta\omega$ due to the low group velocity of these waves

$$\frac{\Delta\omega}{\omega_e} \sim \frac{V_{gr}\Delta k}{\omega_e} \sim \left(\frac{V_e}{V_b}\right)^2 \frac{\Delta V_b}{V_b}$$

2. Since the oscillations created by the fast electrons have frequency very near the plasma frequency, they can act as pump waves and drive parametric instabilities [*Nishikawa*, 1968] similar to the ones observed in the absorption of laser light by a plasma. What actually happens is that above a certain threshold the high-frequency wave ω_0 itself becomes nonlinearly unstable and drives ion density fluctuations and other electron plasma oscillations. One can distinguish between two cases. The first one occurs when the modes that grow at the expense of the pump are linear, normal modes of the system, namely, an ion sound wave and an electron plasma wave. This is called the decay instability, and the plasma wave in this case has phase velocity larger than that of the pump wave. The second case occurs when the low-frequency mode is not a normal mode of the linear system but is a mode nonlinearly created by the pump [*DuBois and Goldman*, 1972]. In this case the instability is called the oscillating two-stream instability (OTS) and is an aperiodic instability for the ions (therefore independent of ion Landau damping), and the plasma wave has smaller phase velocity than that of the pump wave (typically, $k\lambda_D \sim 0.1$ in our case). It has been shown in *Papadopoulos and Coffey* [1974] that it is the second interaction that is dominant for the case under consideration (the auroral plasma).

3. The interaction of the fast electrons with the ambient plasma can be stabilized even in the presence of the nonthermal features $[(\Delta V_b/V_b) \sim 1/3]$ because of the fact that wave energy is transferred by the OTS to phase velocities nonresonant with the fast electrons at a rate faster than it is generated.

4. As was shown quantitatively in *Papadopoulos and Coffey* [1974], one can establish a quasi stationary state with $(\Delta V/V_b) \leq 1$ that is characterized by symmetric superthermal electron tails drawn by resonant diffusion of the low phase velocity plasma waves and a spectrum of enhanced low-frequency ion density fluctuations.

We demonstrate in this note that for any nonbeam current, wave particle scattering due to the low phase velocity ion fluctuations can substantially enhance the dc resistivity of the auroral zones [*Nishihara and Hasekawa*, 1972]. The plan of this note is as follows: In the next section we derive the resistivity of a plasma in the presence of fluctuating electric fields on the basis of weak turbulence kinetic theory. In the section after that we discuss the resistivity in the presence of stable electron drifts. The next to the last section provides quantitative estimates of the enhanced auroral resistivity. The last section provides a brief summary of the results.

Theory of Anomalous Resistivity

Assume that the plasma is subjected to a homogeneous steady state spectrum of turbulence given by [*Davidson*, 1972]

$$\mathbf{S}(\mathbf{k}, \omega) = \int d\mathbf{x} \int dt \exp\left[-i(\mathbf{k}\cdot\mathbf{x} - \omega t)\right] \cdot \delta\mathbf{E}(\mathbf{r}, t)\, \delta\mathbf{E}(\mathbf{r} + \mathbf{x}, t + \tau) \qquad (1)$$

whose energy density $W = \delta\mathbf{E}\cdot\delta E/8\pi$ is small in comparison with the thermal energy of the plasma nk_BT, i.e. $W/nk_BT \ll 1$. Then following weak turbulence plasma theory we find that the ensemble average of the electron distribution function f_e is given by [*Montgomery and Tidman*, 1964; *Davidson*, 1972]

$$\frac{\partial}{\partial t} f_e + \mathbf{v}\cdot\frac{\partial}{\partial \mathbf{r}} f_e - \frac{e}{m}\left(\mathbf{E}_0 + \frac{\mathbf{v}\times B_0}{c}\right)\cdot\frac{\partial f_e}{\partial \mathbf{v}} = \frac{\partial}{\partial \mathbf{v}}\cdot\left(\mathsf{D}\cdot\frac{\partial f_e}{\partial \mathbf{v}}\right) \quad (2)$$

where the diffusion tensor D is given by

$$\mathsf{D} = \frac{e^2}{m^2}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{2\pi}\,\mathsf{S}(\mathbf{k},\omega) \int_{-\infty}^{t} dt'\exp\left[i\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r}) - i\omega(t'-t)\right] \quad (3)$$

where $\mathbf{r}' = \mathbf{r}(t')$ is the zero-order particle orbit.

An effective collision frequency ν_{eff} for the rate of change of the average electron momentum due to turbulent scattering can be found from (2) by multiplying (2) by $\mathbf{v}$ and integrating over velocity. One finds

$$\nu_{eff} = \frac{1}{n V_D^2}\int d\mathbf{v}\mathbf{v}_D\cdot\mathsf{D}\cdot\frac{\partial f_e(\mathbf{v})}{\partial \mathbf{v}} \quad (4)$$

where $\mathbf{v}_D = (1/n_e)\int \mathbf{v} f_e(\mathbf{v})\, d\mathbf{v}$. For electrostatic turbulence, which is what concerns us here, one finds that

$$\mathsf{S}(\mathbf{k},\omega) = (\mathbf{k}\mathbf{k}/k^2)S(\mathbf{k},\omega) \quad (5)$$

Therefore

$$\nu_{eff} = \frac{1}{n v_D^2}\frac{e^2}{m^2}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{(2\pi)}\frac{S(k,\omega)}{k^2}(\mathbf{V}_D\cdot\mathbf{k}) \int d\mathbf{v}\mathbf{k}\cdot\frac{\partial f_e}{\partial \mathbf{v}}\int_{-\infty}^{t} dt'\exp\left[i\mathbf{k}\cdot(\mathbf{r}'-\mathbf{r}^0) - i\omega(t'-t)\right] \quad (6)$$

or more concisely

$$\nu_{eff} = \frac{-e^2}{m^2 v_D^2 \omega_e^2}\int\frac{d\mathbf{k}}{(2\pi)^3}\int\frac{d\omega}{(2\pi)}(\mathbf{k}\cdot\mathbf{v}_D)S(\mathbf{k},\omega)\,\mathrm{Im}\,\epsilon(\mathbf{k},\omega) \quad (7)$$

where $\epsilon(\mathbf{k},\omega)$ is the electron dielectric function. These results can also be found through the use of the fluctuation dissipation theorem for nonequilibrium plasmas [*Rostoker*, 1961]. In deriving (7) we implicitly assumed that $f_e(\mathbf{v})$ was frozen in time during the collisional time scale [*Dupree*, 1964]. Various forms of (7) have been used in calculation of anomalous resistivity by *Sagdeev* [1966], *Krall and Book* [1969], and others.

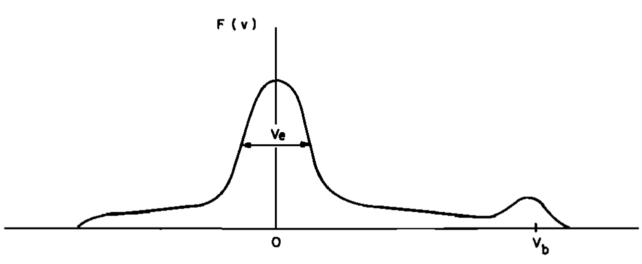

Fig. 1. Marginally stable electron distribution function with a weak fast electron beam.

Resistivity for Stable Electron Drifts

We proceed now to investigate the resistivity due to a stable field-aligned current in the presence of electrostatic fluctuations. We assume that $\mathbf{E}_0$, $\mathbf{B}_0$, and the electron drift velocity $\mathbf{v}_D$ are collinear and that $v_D \ll V_e$ (V_e is the thermal velocity of the electrons). In this case and for modes with wave numbers predominately along the direction of the magnetic field the Im $\epsilon(\mathbf{k},\omega)$ is given by

$$\mathrm{Im}\,\epsilon(\mathbf{k},\omega) = \frac{1}{k^2\lambda_D^2}\frac{\omega - \mathbf{k}\cdot\mathbf{v}_D}{2^{1/2}kV_e}\exp\left[-\left(\frac{\omega-\mathbf{k}\cdot\mathbf{v}_D}{2kV_e}\right)^2\right] \quad (8)$$

where $\lambda_D = (4\pi ne^2/T_e)^{1/2}$ is the electron Debye length. From (8) we see that the dominant contribution of Im $\epsilon(\mathbf{k},\omega)$ comes from fluctuations with $\omega \ll \mathbf{k}\cdot\mathbf{v}_d$. This is consistent with the results of *Tidman and Eviatar* [1965] on scattering of test particles by electrostatic fluctuations. They concluded that the scattering is dominated by waves having phase velocity smaller than the velocity of the test particle.

The integral over $d\mathbf{k}$ includes both the collisional ($k\lambda_D > 1$) and the wave ($k\lambda_D < 1$) part of ν_{eff}. Using the low-frequency approximation for the value of $S(\mathbf{k},\omega)$ in thermal equilibrium [*Klimontovich*, 1967] and (8) with $\omega \ll \mathbf{k}\cdot\mathbf{v}_d$, we find from (7) that ν_{eff} has the value of the collision frequency given by *Spitzer* [1956] for binary electron-ion collisions. The dominant contribution comes from the $k\lambda_D > 1$ region.

Experimental measurements indicate that the auroral plasma is far from thermal equilibrium [*Reasoner and Chappel*, 1973; *Pongratz*, 1972; *Frank and Ackerson*, 1971]. The theory developed in *Papadopoulos and Coffey* [1974], which accounts for several of the observed nonthermal features, predicts a nonlinear quasi steady state for the auroral plasma in the presence of fast precipitating electron streams, which includes enhanced electrostatic field fluctuations at the plasma frequency ω_e and ion waves near zero frequency in the wave number region $3 < (k\lambda_D)^{-1} < (V_b/V_e)$ (Figure 1), where V_b is the field-aligned velocity of the fast electrons. The marginally stable model predicts electron distribution functions as composed of an ambient thermal plasma, symmetric suprathermal electron tails, and a weak fast electron bump (Figure 1). Such distributions are in accordance with the experimental rocket measurements [*Reasoner and Chappel*, 1973]. The low-frequency electric fields due to the enhanced ion density fluctuations can give a large enhancement in resistivity. This is true even though the current driven by an applied electric field is itself stable.

Quantitative Estimates of Auroral Resistivity

Using (7) and (8) and integrating over frequency in the low-frequency regime, we find that for the wave part of the resistivity

$$\nu_{eff} = \frac{e^2}{m^2\omega_e^2\lambda_D^2 V_e}\left(\frac{\pi}{2}\right)^{1/2}\frac{2\pi}{(2\pi)^4} \cdot\int_0^{1/\lambda_D} dkk\int_0^{\theta_0} d\theta\sin\theta\cos^2\theta S(\mathbf{k}) \quad (9)$$

where θ_0 is the range of angles over which the magnetic field dependence of Im $\epsilon(\mathbf{k},\omega)$ (8) can be neglected. Two-dimensional computer simulation results [*Kruer*, 1972], indicate that $S(\mathbf{k})$ is strongly anisotropic and strongest in the direction of the drift. We represent the behavior of $S(\mathbf{k})$ as

$S(\mathbf{k}) = S(k)R(\theta)$, with $R(\theta)$ at decreasing function of θ and $R(\theta_0) > 0$. We can then write (9) as

$$\nu_{eff} = \frac{e^2}{mK_BT_eV_e}\left(\frac{\pi}{2}\right)^{1/2}\frac{A}{(2\pi)^4}\int_\theta^{1/\lambda_D} dkkS(k) \quad (10)$$

where

$$A = 2\pi\int_0^{\theta_0} d\theta \sin\theta\cos^2\theta R(\theta) \quad (11)$$

Hence

$$\nu_{eff} = \omega_e\frac{\pi^{1/2}}{4\pi}\frac{1}{nk_BT_e}\frac{A}{(2\pi)^4}\int_0^{1/\lambda_D} dkk^2S(k)(k\lambda_D)^{-1} \quad (12)$$

Under the assumption that $R(\theta_0) \ll R(0)$, (12) becomes

$$\nu_{eff} = \left(\frac{\pi}{2}\right)^{1/2}\omega_e\frac{1}{\bar{k}\lambda_D}\frac{W}{nk_BT_e} \quad (13)$$

where we have defined

$$\bar{k}\lambda_D = \frac{1}{W}\frac{A}{(2\pi)^5}\int_0^{1/\lambda_D} dkk^2S(k)(k\lambda_D)^{-1} \quad (14)$$

In (13) $\bar{k}\lambda_D$ represents the predominant wave number of the spectrum, and W the energy density of the low-frequency turbulence. Knowledge of $\bar{k}\lambda_D$ and W will allow us to calculate the anomalous resistivity. These values have to be determined either experimentally or by numerically solving the set of equations of *Papadopoulos and Coffey* [1974]. We are presently pursuing the numerical approach. We can, however, find some order of magnitude estimates in line with the results of Papadopoulos and Coffey and guided by the results of various computer simulations of similar problems [*Kainer et al.*, 1972; *Kruer and Dawson*, 1972; *Kruer*, 1972].

For $\bar{k}\lambda_D$, one can take the wave number value for maximum growth of the oscillating two-stream instability, which is of the order $\bar{k}\lambda_D \sim 0.15$.

The value of W has to be determined by the stabilization mechanism of the parametric interaction. Since this depends predominantly on the interaction of the high-frequency (ω_e) oscillations with the electron tails, it is convenient to relate W to the energy W_0 of these oscillations. Assuming that both grow over the same time starting from noise, we have $W \geq (\bar{k}\lambda_D)^2W_0$. From this it follows that

$$\nu_{eff} \sim \omega_e\left(\frac{\pi}{2}\right)^{1/2}\bar{k}\lambda_D\frac{W_0}{nk_BT} \quad (15)$$

From *Papadopoulos and Coffey* [1974] and various computer simulation results [*Kainer et al.*, 1972; *Kruer and Dawson*, 1972] the value of W_0 can be estimated

$$W_0 \sim \alpha n_b m V_b \Delta V_b \quad (16)$$

where α is a factor of order unity (with a maximum value 1). From (15) and (16),

$$\nu_{eff} = \alpha\left(\frac{\pi}{2}\right)^{1/2}\omega_e\bar{k}\lambda_D\frac{n_b m V_b\,\Delta V_b}{nk_BT}$$

$$\sim 2\alpha\left(\frac{\pi}{2}\right)^{1/2}\omega_e\bar{k}\lambda_D\left(\frac{n_b}{n}\right)\left(\frac{V_b}{V_e}\right)^2\frac{\Delta V_b}{V_b} \quad (17)$$

For typical values $(V_b/\Delta V_b) \sim 1/3$, $(V_b/V_e) \sim [(6\times10^9)/(3\times10^7)] \sim (2\times10^2)$, $\bar{k}\lambda_D \sim 0.15$ we find

$$\nu_{eff} = 5\times10^4\alpha(n_b/n)\omega_e \quad (18)$$

From (17) and (18) we notice the following parametric dependence. (1) The anomalous resistivity is proportional to the energy of the precipitating stream. (2) For the same precipitating energy the resistivity scales as $n^{-1/2}$ and will thus be a decreasing function of altitude. (3) The Coulomb collision frequency is $\nu_c = 80n(T_e^3/2)^{-1}$ (T_e is in °K); hence taking $T_e \sim 3000$°K and $n_b \sim 2\times10^{-2}$, we find that $(\nu_{eff}/\nu_{coll}) \sim 1.2\times10^9\alpha n^{-3/2}$. This expression is valid for altitudes above which $\nu_{ei} > \nu_{en}$ (i.e., above about 200 km). This expression, depending on the altitude, gives resistivity enhancement between 10^2 and 10^4 if one takes α to be of order unity. Computer simulations performed to date indicate that α is of order unity.

Before closing we should point out that the actual problem of anomalous resistivity in the presence of high-frequency and low-frequency electric fields is a very complicated one. Even when the mode coupling terms are neglected, the static electric conductivity has in addition to the term considered here a term due to the adiabatic interaction between the particles and the high-frequency field and a dynamic friction type term for the low-frequency part of the spectrum [*Ichikawa and Nishikawa*, 1971]. Inclusion of these effects requires a substantially more general theoretical formulation to the problem of weak turbulent plasma theory and is presently under consideration. It is expected that these effects will further increase the resistivity above that predicted here.

SUMMARY AND CONCLUSIONS

We have demonstrated that the existence of enhanced low-frequency turbulence due to instabilities produced by field-aligned streams of precipitating electrons in the auroral zones can significantly increase the value of the plasma resistivity even when the plasma is stable with respect to current driven instabilities, and the value of the resistivity will be independent of the drift velocity.

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