Amplification of ion cyclotron waves via high frequency electron plasma wave turbulence

J. D. Huba

Science Applications, Inc., McLean, Virginia 22101

P. K. Chaturvedi[a)]

University of Maryland, College Park, Maryland 20742

K. Papadopoulos[b)]

Naval Research Laboratory, Washington, D.C. 20375
(Received 24 October 1979; accepted 28 March 1980)

It is shown that the presence of high frequency electron plasma wave turbulence can nonlinearly amplify obliquely propagating, electrostatic, ion cyclotron modes.

It is well known that high frequency, short wavelength turbulence can affect low frequency, long wavelength modes either by anomalous transport effects or by direct mode-mode coupling. Substantial work has been done in both of these areas and we address the latter possibility in this note. Vedenov *et al.*[1] have developed a general formalism which describes the interaction between high frequency waves and low frequency turbulence. In fact, it has been shown that high frequency electron plasma wave turbulence can generate and/or modify low frequency drift waves in inhomogeneous plasmas.[2,3] Recently, Pozzoli and Ryutov[4] have investigated the modulational instability produced by broad spectra of Langmuir turbulence in a magnetic field.

The physical processes involved can be described by considering the high frequency turbulence as quasi-particles or plasmons which obey the kinetic wave equation

$$\frac{\partial N_k}{\partial t}+\mathbf{V}_g\cdot\frac{\partial}{\partial \mathbf{r}}N_k-\frac{\partial\omega_k}{\partial \mathbf{r}}\cdot N_k=0\,, \tag{1}$$

where $N_k=|E_k|^2/4\pi\omega_k$ is the plasmon distribution function, $\mathbf{V}_g=\partial\omega_k/\partial k$ is the group velocity of the plasma waves, and ω_k is the wave frequency of the plasma waves. Based upon Eq. (1), we consider the behavior of the plasmons on a slow time scale (i.e., low frequency cyclotron mode) where the dominant coupling is via the ponderomotive force affecting the electron dynamics. Indeed, we demonstrate that the plasma wave turbulence can generate or enchance ion cyclotron modes. This process may be relevant to both tokamak and space plasmas. Although the recent observations of ion cyclotron turbulence in the TFR tokamak[5] and in the auroral region[6] may be attributed to a current-driven ion cyclotron instability, it is possible that this nonlinear effect is important since there is evidence of high frequency turbulence in these plasmas.[7,8] We point out that the mathematical analysis parallels that of Satya and Kaw.[3]

We assume that a stationary, high frequency, short wavelength spectrum of plasma wave turbulence exists in a low β magnetized plasma ($\mathbf{B}=B_0\hat{e}$). The eigenfrequency of the waves is given by

$$\omega_k^2=(k_z^2/k^2)\omega_{pe}^2(1+3k^2\lambda_{De}^2)\,, \tag{2}$$

where $\omega_{pe}=(4\pi ne^2/m_e)^{1/2}$ is the electron plasma frequency and $\lambda_{De}=(T_e/4\pi ne^2)^{1/2}$ is the electron Debye length. We have assumed $\omega_{pe}^2\ll\omega_{ce}^2$ and $k^2\lambda_{De}^2\ll 1$ (where $\omega_{ce}=eB_0/m_ec$ is the electron cyclotron frequency) since this parameter regime is consistent with the TFR tokamak and auroral plasmas. We note that Pozzoli and Ryutov[4] consider the case $\omega_{pe}^2\gg\omega_{ce}^2$. Moreover, we consider low frequency, long wavelength ion cyclotron modes described by

$$\Omega_q^2=\omega_{ci}^2+q^2C_S^2\,, \tag{3}$$

where $\omega_{ci}=eB_0/m_ic$ is the ion cyclotron frequency, $C_S=(T_e/m_i)^{1/2}$ is the ion sound speed, and it is assumed that $q_\parallel\ll q_\perp$. In order that the plasma wave turbulence be accurately described by the wave kinetic equation [Eq. (1)] in the presence of ion cyclotron turbulence, we require $\omega_k\gg\Omega_q$ and $k\gg q$.

The low frequency density perturbation δn_e produced by the ion cyclotron modes perturbs the plasmon distribution function and linearizing Eq. (1) we obtain

$$\delta N_k=-\frac{\omega_k}{2}\frac{\delta n_e}{n_0}\frac{\mathbf{q}\cdot\partial N_k^0/\partial\mathbf{k}}{\Omega-\mathbf{q}\cdot\mathbf{V}_{gr}}\,, \tag{4}$$

where N_k^0 is the unperturbed plasmon distribution function, and we have used $\partial\omega_k/\partial\mathbf{r}\simeq(iq\omega_k/2)\,\delta n_e/n_0$ which is obtained from Eq. (2). The modification of the plasma turbulence affects the low frequency electron motion through the ponderomotive force term. We find that the parallel electron motion on the slow time scale is described by[3,9]

$$-\frac{T_e}{m_en_0}\frac{\partial}{\partial z}\delta n_e+\frac{e}{m_e}\delta\phi-\frac{\omega_{pe}^2}{2m_en_0}\frac{\partial}{\partial z}\sum_k\frac{\delta N_k}{\omega_k}\simeq 0\,, \tag{5}$$

where $\delta N_k=|\delta E_k|^2/4\pi\omega_k$. Thus, by combining Eqs. (4) and (5), we find that the perturbed electron density is

$$\delta n_e/n_0=(e\,\delta\phi/T_e)(1-A)^{-1}\,, \tag{6}$$

$$A=\frac{W}{4}\omega_{pe}\int\frac{\mathbf{q}\cdot(\partial N_k^0/\partial\mathbf{k})}{\Omega-\mathbf{q}\cdot\mathbf{V}_{gr}}d^3k\left(\int N_k^0\frac{k_z}{k}d^3k\right)^{-1}, \tag{7}$$

$W=(c/n_0T_e)\int N_k^0\omega_k d^3k$ is the normalized energy density of the electron plasma wave turbulence, and c is a normalization constant involved in going from $\sum_k\to\int dk$.

The ion motion is unaffected by the plasma wave tur-

0031-9171/80/071479-02$00.90

bulence and the ion density perturbation is simply given by

$$\frac{\delta n_i}{n_0} = \left(\frac{q^2 C_S^2}{\Omega^2 - \omega_{ci}^2}\right) \frac{e\delta\phi}{T_e} \, . \tag{8}$$

We now make use of quasi-neutrality $\delta n_i \simeq \delta n_e$ to determine the dispersion equation and find from Eqs. (6) and (8) that

$$\Omega^2 = \omega_{ci}^2 + q^2 C_S^2 (1 - A) \, . \tag{9}$$

In the absence of high frequency turbulence ($A = 0$) we recover Eq. (3) from Eq. (9). Note that A is a complicated function of Ω and q. Thus, Eq. (9) describes the modification of low frequency ion cyclotron waves due to high frequency electron plasma wave turbulence.

Certainly for realistic plasmas a two-dimensional plasma wave spectrum should be used. However, such a spectrum complicates the analysis considerably and is not expected to significantly alter the results based upon a one-dimensional wave spectrum.[3] Thus, for simplicity, we assume the plasma wave turbulence to be one dimensional (i.e., $\mathbf{k} = k\hat{e}_z$) and model the unperturbed plasmon distribution by

$$N_k^0 = (\pi \Delta^2)^{-1/2} \exp[-(k - k_0)^2/\Delta^2] \, . \tag{10}$$

Substituting Eq. (10) into Eq. (7), we find that the dispersion equation [Eq. (9)] becomes

$$\Omega^2 = \omega_{ci}^2 + q^2 C_S^2 \{1 - (W/6n_0 T_e)(1/\Delta^2 \lambda_{De}^2)[1 + \zeta Z(\zeta)]\} , \tag{11}$$

where

$$\zeta = \frac{k_0}{\Delta}\left(\frac{\Omega}{q_z V_0} - 1\right), \quad V_0 = V_{gr}\big|_{k=k_0} \simeq 3\frac{\omega_{pe}}{k_0} k_0^2 \lambda_{De}^2 ,$$

and Z is the plasma dispersion function. We now solve Eq. (11) in the "cold" and "warm" limits.

"Cold" limit ($\zeta \gg 1$): We assume $k_0(\Omega/q_z V_0 - 1) \gg \Delta$ and find that the dispersion equation becomes

$$\Omega^2 - (\omega_{ci}^2 + q^2 C_S^2) - q^2 C_S^2 \frac{3W_0}{4n_0 T_e} \frac{1}{k_0^2 \lambda_{De}^2} \frac{q_z^2 V_0^2}{(\Omega - q_z V_0)^2} = 0 \, . \tag{12}$$

We write $\Omega = \Omega_r + \delta\Omega$ and let $\Omega_r \approx \pm(\omega_{ci}^2 + q^2 C_S^2)^{1/2}$ and $\Omega_r \approx q_z V_0$. Assuming $\delta\Omega \ll \Omega$, we find that

$$\delta\Omega^3 = \frac{3W}{8n_0 T_e} \frac{1}{k_0^2 \lambda_{De}^2} \frac{q_z^2 V_0^2 q^2 C_S^2}{\Omega_r} \tag{13}$$

and obtain an instability for $\Omega_r < 0$. The growth rate is given by

$$\gamma \approx \frac{3}{2}\left(\frac{W}{24 n_0 T_e} \frac{1}{k_0^4 \lambda_{De}^4} \frac{q^2}{q_z^2} \frac{m_e}{m_i}\right)^{1/3} \Omega_r \, . \tag{14}$$

Note that substantial growth can occur in the weak turbulence regime, $W/n_0 T_e \sim m_e/m_i$ for $q_z/q \sim (m_e/m_i)^{1/2}$ and $k_0 \lambda_{De} \lesssim 1$.

"Warm" limit ($\zeta \ll 1$): In this limit we assume $k_0(\Omega/q_z V_0 - 1) \ll \Delta$ and obtain the following dispersion equation:

$$\Omega^2 - \left[\omega_{ci}^2 + q^2 C_S^2 \left(1 - \frac{W}{6n_0 T_e} \frac{1}{\Delta^2 \lambda_{De}^2}\right)\right] + i\sqrt{\pi} q^2 C_s^2 \times \frac{W}{6n_0 T_e} \frac{1}{\Delta^2 \lambda_{De}^2} \frac{k_0}{\Delta} \left(\frac{\Omega_r}{q_z V_0} - 1\right) = 0 , \tag{15}$$

which has the solution (for $\gamma \ll \Omega_r$)

$$\Omega_r = \pm \left[\omega_{ci}^2 + q^2 C_S^2 \left(1 - \frac{W}{6n_0 T_e} \frac{1}{\Delta^2 \lambda_{De}^2}\right)\right]^{1/2} , \tag{16}$$

$$\gamma = -\sqrt{\pi} \left[\frac{q^2 C_S^2}{\Omega_r^2} \frac{W}{12 n_0 T_e} \frac{1}{\Delta^2 \lambda_{De}^2} \frac{k_0}{\Delta} \left(\frac{\Omega_r}{q_z V_0} - 1\right)\right] \Omega_r \, . \tag{17}$$

Thus, we require $\Omega_r < q_z V_0$ and that the phase velocity of the ion cyclotron wave be in the same direction as the group velocity of the Langmuir turbulence for instability.

Thus, we have shown that for rather modest levels of turbulence, nonlinear coupling of high frequency electron plasma wave turbulence can excite ion cyclotron oscillations. This interaction, in fact, may be important in both tokamak and space plasmas. However, due to the limited experimental data, it is not possible to make a quantitative comparison between this theory and observational results. Several improvements of the theory are presently being considered and will be reported in a future publication. First, the ion density perturbation can be determined using kinetic theory. This will allow higher-order cyclotron harmonics to be investigated since several harmonics have been observed[6] and will include finite ion Larmor radius corrections. Second, a two-dimensional wave spectrum of electron plasma wave turbulence will be considered which is more realistic than a one-dimensional spectrum. Finally, the dispersion equation will be solved numerically for parameters typical of relevant laboratory and space plasmas.

One of us (P.K.C.) thanks Y. S. Satya and M. Sinha for helpful discussions.

This work has been supported by the Office of Naval Research.

[a]Present address: Berkeley Research Associates, Arlington, Va. 22116.
[b]Present address: University of Maryland, College Park, Md. 20742.

[1]A. A. Vedenov, A. V. Gordeev, and I. I. Rudakov, Plasma Phys. **9**, 719 (1967).
[2]E. N. Kricorutsky, V. G. Makhanakev, and V. N. Tystovich, Nucl. Fusion **9**, 97 (1969).
[3]Y. S. Satya and P. K. Kaw, Phys. Rev. Lett. **31**, 1453 (1973).
[4]R. Pozzoli and D. D. Ryutov, Phys. Fluids **22**, 1782 (1978).
[5]TFR Group, Phys. Rev. Lett. **41**, 113 (1978).
[6]P. M. Kinter, M. C. Kelley, and F. S. Mozer, Geophys. Res. Lett. **5**, 139 (1978).
[7]C. S. Liu, Y. C. Mok, K. Papadopoulos, F. Engelmann, and M. Bornaticci, Phys. Rev. Lett. **39**, 701 (1977).
[8]C. S. Lin and R. A. Hoffman, J. Geophys. Res. **84**, 6547 (1979).
[9]R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, New York, 1969), p. 32.