Precipitation of Energetic Protons from the Radiation Belts

using Lower Hybrid Waves

- Lower hybrid waves are quasi-electrostatic whistler mode waves whose wave normal direction is very close to the whistler mode resonance cone.

- The refractive index of ELF/VLF whistler mode waves in the radiation belts propagating along the Earth’s magnetic field $B_0$ generally lies in the range $10^{-100}$.

- The refractive index of ELF/VLF lower hybrid waves in the radiation belts generally lies in the range $10^3 - 10^4$.

- The $E$ field of the lower hybrid waves is much larger than, and the $B$ field of the lower hybrid waves is much smaller than, the corresponding fields of whistler mode waves propagating along $B_0$.

- Lower hybrid waves can be generated by many different means:
  1. ELF/VLF transmitters
  2. Linear mode coupling
  3. Propagation transformation
  4. Plasma instabilities
Power spectral density as a function of wave number $k$. The dipole antenna length is equal to $l$. The antenna preferentially radiates whistler mode waves whose wave length is approximately equal to the antenna length.
An electromagnetic whistler mode wave reflecting from a plasma density irregularity and exciting lower hybrid waves.
$L = 2$

$L = 3$

lat = 30^\circ$

$f = 21.4$ kHz
Proton Resonance Interactions with Lower Hybrid Waves

- We are currently modifying our WIPP (Wave Induced Particle Precipitation) code to include energetic radiation belt protons.

- The WIPP code is a deterministic code which presently uses the Lorentz force law to determine the changes in energetic electron pitch angle and energy which are caused by known whistler mode waves in the radiation belts:

\[
\frac{dv}{dt} = \frac{e}{m} [E + v \times B]
\]

- An example of the WIPP code output for energetic electrons is shown on the next page.
Energy Flux > 100 keV [milli-ergs/cm²-sec] vs. $L$-shell.

- NAA
- NAU
- NPM
- NLK
- NWC

The graph shows the energy flux for different $L$-shells, with the flux values on a logarithmic scale.

(f)
From inspection of the Lorentz force law, \( \frac{dv}{dt} = \frac{e}{m} [E + v \times B] \), it may appear that one only needs to change \( m_e \) to \( m_p \) in order to use the WIPP code to find the proton precipitation. However, this is not the case.

The largest pitch angle changes occur during gyroresonance and Landau resonance interactions:

\[
v_z = \frac{(n\omega_c - \omega)}{k_z}
\]

where \( \omega_c \) is the particle gyrofrequency, \( k_z \) is the component of the wave vector \( k \) parallel to \( B_0 \), where \( n \) is an integer, and where setting \( n = 0 \) yields the parallel Landau resonance.

The fundamental (\( n = 1 \)) gyro-resonance interaction between a 100 keV electron and a 5 kHz whistler mode wave near the magnetic equatorial plane takes place over a distance of \( \approx 600 \) km.

Over this same distance a 100 keV proton in gyro-resonance with the same wave would experience approximately 20 resonances.

This means that the step size in the WIPP code needs to be reduced by a factor of \( \approx 20 \).

Since the gyro-radius of the protons is so large, we must now include radial gradients in \( B_0 \) and \( N_0 \).
The gyro-motion of the protons can be neglected when:

\[ \square \ll r_g \]
If we can assume that the protons are unmagnetized, the Lorentz force law has the simple form:

\[
\frac{dv}{dt} = \frac{e}{m_p} E_o \ e^{i(\omega t - k \cdot r)}
\]

where \( E_o \) is the amplitude of the lower hybrid wave.

A resonance will occur when the phase of \( E \) is constant:

\[
\omega = k \cdot v = k_\perp \cdot v_\perp + k_z v_z \simeq k_\perp \cdot v_\perp
\]

Thus for the case of unmagnetized protons there is only a single resonance, the transverse Landau resonance.
Distribution of k values

If we assume that the k vectors of the lower-hybrid waves are uniformly distributed around $\mathbf{B}_0$, the pitch angle diffusion coefficient can be written: $D_\alpha = D_\perp / v_\perp^2$, where [Retterer et al., 1980]:

$$D_\perp = (e/m)^2 \int \frac{I(\omega) \omega^2 d\omega}{8\pi^2 v_\perp^2} \int_{k_o}^{\infty} \frac{S(k_\perp)dk_\perp}{k_\perp \sqrt{k_\perp^2 v_\perp^2 - \omega^2}}$$

where $I(\omega)$ is the spectral intensity of the LH waves in the frequency domain, $S(k_\perp)$ is the normalized spectral density of the LH waves in the $k$ domain, $v_\perp$ is the proton perpendicular velocity, and $k_o = \omega / v_\perp$.

We assume that the LH wave energy is uniformly distributed over wavelengths from 20 m to 200 m, and that the illumination region extends to $30^\circ$ latitude on both sides of the magnetic equator.

We also assume the average E field amplitude of the LH waves to be 1 mV/m.

In this case we find for a 50 keV proton with an initial pitch angle $\alpha$ of $45^\circ$: $\Delta \alpha_{\text{total}} \simeq \sqrt{D_\alpha T} \simeq 45^\circ$, where $T = 6$ days.
Use of existing VLF Transmitters

- Ray tracing results suggest that $\sim 20$ kHz signals from ground based VLF transmitters can transform into lower hybrid waves as the signals propagate into the conjugate hemisphere.

- However, the transformation is completed at low altitudes where energetic proton pitch angle scattering is very inefficient.

- This situation could be significantly improved if a chemical release took place in the ionosphere above the transmitter, creating plasma density irregularities.

- The transmitter signals propagating upward within these irregularities would transform into lower hybrid waves through linear mode coupling.

- The lower hybrid waves would then propagate upward along $B_0$, filling the plasmasphere with waves which can change the pitch angles of energetic protons at every point along their trajectory.

- Recent DEMETER data suggests that this technique might be very successful.
Energetic Proton Precipitation produced by Lower Hybrid Waves

Stanford University

- We are currently modifying our WIPP (Wave Induced Particle Precipitation) code to include energetic radiation belt protons.
- The WIPP code is a deterministic code which uses the Lorentz force law to determine the changes in energetic electron pitch angle and energy which are caused by known whistler mode waves in the radiation belts.
- We are also developing a new code which determines proton pitch angle and energy changes in a lower hybrid wave field with a wide spectrum of wave vectors $\mathbf{k}$.
- We will treat a number of cases:
  1. Lower hybrid wave fields produced in the radiation belts by a space based ELF/VLF transmitter.
  2. Lower hybrid wave fields produced in the radiation belts by existing ground based VLF transmitters.
  3. Lower hybrid wave fields produced in the radiation belts by a ground based VLF transmitter operating in the 5 - 10 kHz frequency range.