35.51. Model: The earth is a complete absorber of sunlight. An object gains momentum when it absorbs electromagnetic waves.

Solve: The radiation pressure on an object that absorbs all the light is

\[ p_{\text{rad}} = \frac{I}{c} = \frac{1360 \text{ W/m}^2}{3.0 \times 10^8 \text{ m/s}} = 4.53 \times 10^{-6} \text{ Pa} \]

Seen from the sun, the earth is a circle of radius \( R_{\text{earth}} \) and area \( A = \pi R_{\text{earth}}^2 \). The pressure exerts a force on this area

\[ F_{\text{rad}} = p_{\text{rad}} A = p_{\text{rad}} (\pi R_{\text{earth}}^2) = (4.53 \times 10^{-6} \text{ Pa}) \pi (6.37 \times 10^8 \text{ m})^2 = 5.78 \times 10^8 \text{ N} \]

The sun’s gravitational force on the earth is

\[ F_{\text{grav}} = \frac{G M_{\text{sun}} M_{\text{earth}}}{R_{\text{sun-earth}}^2} = \frac{(6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2)(1.99 \times 10^{33} \text{ kg})(5.98 \times 10^{24} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = 3.53 \times 10^{25} \text{ N} \]

\[ \Rightarrow \frac{F_{\text{rad}}}{F_{\text{grav}}} = \frac{5.78 \times 10^8 \text{ N}}{3.53 \times 10^{25} \text{ N}} = 1.64 \times 10^{-17} \]

That is, \( F_{\text{rad}} \) is \( 1.64 \times 10^{-17} \% \) of \( F_{\text{grav}} \).

35.53. Model: Assume that the black paper absorbs the light completely. Use the particle model for the paper.

Visualize:

For the black paper to be suspended, the radiation-pressure force must be equal to the gravitational force on the paper.

Solve: From Equation 35.39, \( F_{\text{rad}} = p_{\text{rad}} A = IA/c \). Hence,

\[ I = \frac{c}{A} F_{\text{rad}} = \frac{c}{A} F_c = \frac{\left(3.0 \times 10^4 \text{ m/s}\right)\left(1.0 \times 10^{-7} \text{ kg}\right)\left(9.8 \text{ m/s}^2\right)}{\left(8.5 \text{ inch} \times 11 \text{ inch}\right)\left(2.54 \times 10^{-2} \text{ m/inch}\right)} = 4.9 \times 10^3 \text{ W/m}^2 \]
35.55. Model: Use the particle model for the astronaut.

Solve: According to Newton’s third law, the force of the radiation on the astronaut is equal to the momentum delivered by the radiation. For this force we have

\[ F = \frac{P}{c} = \frac{1000 \text{ W}}{3.0 \times 10^8 \text{ m/s}} = 3.333 \times 10^{-6} \text{ N} \]

Using Newton’s second law, the acceleration of the astronaut is

\[ a = \frac{3.333 \times 10^{-6} \text{ N}}{80 \text{ kg}} = 4.167 \times 10^{-8} \text{ m/s}^2 \]

Using \( v_f = v_i + at \) and a time equal to the lifetime of the batteries,

\[ v_f = 0 \text{ m/s} + (4.167 \times 10^{-8} \text{ m/s}^2)(3600 \text{ s}) = 1.500 \times 10^{-4} \text{ m/s} \]

The distance traveled in the first hour is calculated as follows:

\[ v_f^2 - v_i^2 = 2a(\Delta s)_{\text{enc/caps}} \]

\[ \Rightarrow (1.500 \times 10^{-4} \text{ m/s})^2 - (0 \text{ m/s})^2 = 2(4.167 \times 10^{-8} \text{ m/s}^2)(\Delta s)_{\text{enc/caps}} \Rightarrow (\Delta s)_{\text{enc/caps}} = 0.270 \text{ m} \]

This means the astronaut must cover a distance of 5.0 m - 0.27 m = 4.73 m in a time of 9 hours. The acceleration is zero during this time. The time it will take the astronaut to reach the space capsule is

\[ \Delta t = \frac{4.73 \text{ m}}{1.500 \times 10^{-4} \text{ m/s}} = 31,533 \text{ s} = 8.76 \text{ hours} \]

Because this time is less than 9 hours, the astronaut is able to make it safely to the space capsule.


Solve: For unpolarized light, the electric field vector varies randomly through all possible values of \( \theta \). Because the average value of \( \cos^2 \theta \) is \( \frac{1}{2} \), the intensity transmitted by the first polarizing filter is \( I_1 = \frac{1}{2} I_0 \). For polarized light, \( I_{\text{transmitted}} = I_0 \cos^2 \theta \). For the second filter the transmitted intensity is \( I_2 = I_1 \cos^2 \theta = \frac{1}{2} I_0 \cos^2 \theta \). Similarly, \( I_3 = I_2 \cos^2 \theta = \frac{1}{4} I_0 \cos^2 \theta \), and so on. Thus,

\[ I_3 = I_0 \left( \cos^2 \theta \right)^3 = \left( \frac{1}{4} I_0 \right) \left( \cos^2 \theta \right)^3 = \frac{1}{64} I_0 \left( \cos^2 15^\circ \right)^3 = 0.33I_0 \]
Problem 5.1
Suppose the electric field of a plane electromagnetic wave is given by

$$\mathbf{E}(z,t) = E_0 \cos(kz - \omega t) \mathbf{i}$$

Find the following quantities:

(a) The direction of wave propagation.

(b) The corresponding magnetic field $\mathbf{B}$.

Solution:

(a) By writing the argument of the cosine function as $kz - \omega t = k(z - ct)$ where $\omega = ck$, we see that the wave is traveling in the $+z$ direction.

(b) The direction of propagation of the electromagnetic waves coincides with the direction of the Poynting vector which is given by $\mathbf{S} = \mathbf{E} \times \mathbf{B} / \mu_0$. In addition, $\mathbf{E}$ and $\mathbf{B}$ are perpendicular to each other. Therefore, if $\mathbf{E} = E(z,t) \mathbf{i}$ and $\mathbf{S} = S \hat{k}$, then $\mathbf{B} = B(z,t) \mathbf{j}$. That is, $\mathbf{B}$ points in the $+y$-direction. Since $\mathbf{E}$ and $\mathbf{B}$ are in phase with each other, one may write

$$\mathbf{B}(z,t) = B_0 \cos(kz - \omega t) \mathbf{j} \quad (13.12.2)$$

To find the magnitude of $\mathbf{B}$, we make use of Faraday’s law:

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_s}{dt} \quad (13.12.3)$$

which implies

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} \quad (13.12.4)$$
From the above equations, we obtain

\[-E_0 k \sin(kz - \omega t) = -B_0 \omega \sin(kz - \omega t)\]  
(13.12.5)

or

\[\frac{E_0}{B_0} = \frac{\omega}{k} = c\]  
(13.12.6)

Thus, the magnetic field is given by

\[\mathbf{B}(z, t) = \left(\frac{E_0}{c}\right) \cos(kz - \omega t) \mathbf{j}\]  
(13.12.7)

**Problem 5.2**
Verify that, for \(\omega = kc\),

\[E(x, t) = E_0 \cos(kx - \omega t)\]
\[B(x, t) = B_0 \cos(kx - \omega t)\]

satisfy the one-dimensional wave equation:

\[
\left\{ \frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right\} \begin{bmatrix} E(x, t) \\ B(x, t) \end{bmatrix} = 0
\]

**Solution:**

Differentiating \(E = E_0 \cos(kx - \omega t)\) with respect to \(x\) gives

\[\frac{\partial E}{\partial x} = -kE_0 \sin(kx - \omega t), \quad \frac{\partial^2 E}{\partial x^2} = -k^2 E_0 \cos(kx - \omega t)\]  
(13.12.10)

Similarly, differentiating \(E\) with respect to \(t\) yields

\[\frac{\partial E}{\partial t} = \omega E_0 \sin(kx - \omega t), \quad \frac{\partial^2 E}{\partial t^2} = -\omega^2 E_0 \cos(kx - \omega t)\]  
(13.12.11)
Thus,

\[ \frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = \left( -k^2 + \frac{\omega^2}{c^2} \right) E_0 \cos(kx - \omega t) = 0 \]  

(13.12.12)

where we have made used of the relation \( \omega = kc \). One may follow a similar procedure to verify the magnetic field.

**Problem 5.3**

A parallel-plate capacitor with circular plates of radius \( R \) and separated by a distance \( h \) is charged through a straight wire carrying current \( I \), as shown in the Figure 13.12.1:

![Figure 13.12.1 Parallel plate capacitor](image)

(a) Show that as the capacitor is being charged, the Poynting vector \( \vec{S} \) points radially inward toward the center of the capacitor.

(b) By integrating \( \vec{S} \) over the cylindrical boundary, show that the rate at which energy enters the capacitor is equal to the rate at which electrostatic energy is being stored in the electric field.

**Hint:** The Electric field \( E \) of a capacitor is given by

\[ E = \frac{Q}{\varepsilon_0 \pi R^2} \]
(a) Let the axis of the circular plates be the \( z \)-axis, with current flowing in the \( +z \)-direction. Suppose at some instant the amount of charge accumulated on the positive plate is \( +Q \). The electric field is

\[
\mathbf{E} = \frac{\sigma}{\varepsilon_0} \mathbf{k} = \frac{Q}{\pi R^2 \varepsilon_0} \mathbf{k}
\]

(13.12.13)

According to the Ampere-Maxwell’s equation, a magnetic field is induced by changing electric flux:

\[
\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I_{\text{enc}} + \mu_0 \varepsilon_0 \frac{d}{dt} \oint \mathbf{E} \cdot d\mathbf{A}
\]

From the cylindrical symmetry of the system, we see that the magnetic field will be circular, centered on the \( z \)-axis, i.e., \( \mathbf{B} = B \hat{\phi} \) (see Figure 13.12.2.)

Consider a circular path of radius \( r < R \) between the plates. Using the above formula, we obtain

\[
B(2\pi r) = 0 + \mu_0 \varepsilon_0 \frac{d}{dt} \left( \frac{Q}{\pi R^2 \varepsilon_0} \pi r^2 \right) = \frac{\mu_0 r^2}{R^2} \frac{dQ}{dt}
\]

(13.12.14)

or

\[
\mathbf{B} = \frac{\mu_0 r}{2\pi R^2} \frac{dQ}{dt} \hat{\phi}
\]

(13.12.15)
The Poynting \( \vec{S} \) vector can then be written as

\[
\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \frac{1}{\mu_0} \left( \frac{Q}{\pi R^2 e_0} \hat{k} \right) \times \left( \frac{\mu_\sigma r}{2\pi R^3} \frac{dQ}{dt} \hat{\phi} \right)
\]

\[
= -\left( \frac{Qr}{2\pi^2 R^4 e_0} \right) \left( \frac{dQ}{dt} \right) \hat{r}
\]

(13.12.16)

Note that for \( dQ / dt > 0 \) \( \vec{S} \) points in the \(-\hat{r}\) direction, or radially inward toward the center of the capacitor.

(b) The energy per unit volume carried by the electric field is \( u_\epsilon = e_0 E^2 / 2 \). The total energy stored in the electric field then becomes

\[
U_\epsilon = u_\epsilon V = \frac{e_0}{2} E^2 (\pi R^2 h) = \frac{1}{2} e_0 \left( \frac{Q}{\pi R^2 e_0} \right)^2 \pi R^2 h = \frac{Q^2 h}{2\pi R^2 e_0}
\]

(13.12.17)

Differentiating the above expression with respect to \( t \), we obtain the rate at which this energy is being stored:

\[
\frac{dU_\epsilon}{dt} = \frac{d}{dt} \left( \frac{Q^2 h}{2\pi R^2 e_0} \right) = \frac{Q h}{\pi R^2 e_0} \left( \frac{dQ}{dt} \right)
\]

(13.12.18)

\[
\oint \vec{S} \cdot d\vec{A} = SA_R = \left( \frac{Q r}{2\pi^2 e_0 R^4} \right) \left( 2\pi R h \right) = \frac{Q h}{e_0 \pi R^2} \left( \frac{dQ}{dt} \right)
\]

(13.12.19)

which is equal to the rate at which energy stored in the electric field is changing.
**Problem 5.4:**
Can parallel electric and magnetic fields make up an electromagnetic wave in vacuum?

**Problem 5.5:**
4. Explain why the reception for cellular phones often becomes poor when used inside a steel-framed building.

\[
\oint E \cdot d\mathbf{s} = -\frac{d}{dt} \iint \mathbf{B} \cdot d\mathbf{A}
\]

\[
E(z + \Delta z)\Delta x - E(z)\Delta x = \frac{E(z + \Delta z) - E(z)}{\Delta z} \Delta z \Delta x = \frac{\partial E}{\partial z} \Delta z \Delta x =
\]

\[
= -\frac{d}{dt} \iint BA(e_x \cdot e_y) = 0
\]

\[
\frac{\partial E}{\partial z} = 0
\]

No propagation - no em wave if B is in x-direction

**Problem 5.5**
EM wave reflected as well as absorbed by the metal in the wall reaches room with reduced amplitude