Problem 4 : (15)

A beam of charged particles enter into a region of magnetic field $B$ moving with the same speed $V$ along the $x$-axis and complete half a revolution as shown in the figure. The beam consists of two types of particles with the same charge $q$ but one type of particles is heavier than the other.

a. Are they all positively or negatively charged? Explain. (4)

At the entrance to the $B$ region

$$\vec{v} = \hat{e}_x v$$

$$\vec{B} = -\hat{e}_z B$$

$$\vec{F} = q\vec{v} \times \vec{B} = q v B [\hat{e}_x \times (-\hat{e}_z)] = q v B \hat{e}_y$$

Force in positive y-direction if $q$ positive.

At all other locations force towards the center

Both positively charged particles

b. If the mass of particles with diameter $D_2$ is $m_2$ and of those with diameter $D_1$ is $m_1$, find the mass $m_2$ in terms of $m_1$ and other given variables. Which type particles are heavier? (4)

After they entered the magnetic field region

$$\vec{v} = \hat{e}_\theta v$$

$$\vec{F} = m\vec{a} = q v B [\hat{e}_\theta \times (-\hat{e}_z)] = -q v B \hat{e}_r$$

Central force

$$m v^2 / r = q v B$$

$$r = m v / q B$$

Particle gyroradius proportional to $m$

$$D_2 / D_1 = m_2 / m_1$$

$$m_2 = (D_2 / D_1) m_1$$

$m_2$ is heavier and has a larger gyroradius
c. If possible, find the magnitude (in terms of the given variables) and direction of an electric field that can make the whole beam go through the region in a straight line instead of the shown half-circles. If it is not possible, explain why not. (7)

In order to move along the x-axis the magnetic force should be balanced by the electric force so that

\[
\vec{F} = q[\vec{E} + v\hat{e}_x \times (-\hat{e}_z)B] = 0
\]

\[
\vec{E} = vB(\hat{e}_x \times \hat{e}_z) = -vB\hat{e}_y
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