



Brief

Communication:  
Breeding vectors in  
the phase space  
reconstructed

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# Brief Communication: Breeding vectors in the phase space reconstructed from time series data

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Received: 19 February 2015 – Accepted: 15 May 2015 – Published: 5 August 2015

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Published by Copernicus Publications on behalf of the European Geosciences Union & the American Geophysical Union.

**NPGD**

2, 1301–1315, 2015

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## Abstract

Bred vectors characterize the nonlinear instability of dynamical systems and so far have been computed only for systems with known evolution equations. In this article, bred vectors are computed from a single time series data using time-delay embedding, with a new technique, nearest-neighbor breeding. Since the dynamical properties of the standard and nearest-neighbor breeding are shown to be similar, this provides a new and novel way to model and predict sudden transitions in systems represented by time series data alone.

## 1 Introduction

Prediction of sudden regime changes in the evolution of dynamical systems is a challenging problem. For systems with known dynamical models, such as the Earth's atmosphere, simulated trajectories under judiciously chosen, finite-size perturbations can provide useful information regarding regime changes by detecting fast-growing instabilities along the model representation of the system evolution, called the "control." Breeding is a technique to generate an ensemble of such perturbations, developed for operational ensemble forecasting of the numerical weather prediction (Toth and Kalnay, 1993, 1997; Kalnay, 2003), and the resulting perturbations are called "bred vectors." Evans et al. (2004) demonstrated that the growth rate of the bred vectors could be used as a means of predicting the regime changes in the chaotic Lorenz (1963) system (Lorenz, 1963). They found that the appearance of high growth rate typically indicated that a regime change would occur upon completion of the current orbit, and that the longer the duration of the high growth rate, the longer the next regime would last.

Models of most natural systems like the Earth's atmosphere are described by a very large number of dynamical variables and thus are high dimensional. However, the variables or degrees of freedom are nonlinearly coupled, and consequently in dissipative

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systems the dimensionality of the phase space is significantly reduced. This is the basis for the time-delay embedding method in the reconstruction of phase space (Packard et al., 1980; Takens, 1981). The ability of this method to yield the dynamics inherent in observational data, independent of modeling assumptions, has stimulated the modeling of dynamics using time series data, and has led to the development of forecasting tools. For example, reconstruction of the dynamics of the Earth’s magnetosphere using time series data has led to low dimensional models and forecasts of space weather (Sharma et al., 1993; Sharma, 1995). The data-derived models of magnetospheric substorms and geospace storms (Vassiliadis et al., 1995; Ukhorskiy et al., 2002; Valdivia et al., 1996) now provide near real-time forecasts using the solar wind data monitored by the ACE spacecraft at the first Lagrange point (L1).

This paper presents a novel extension of the original breeding technique to the phase space reconstructed from time series data using the time-delay embedding method. Because dynamic instabilities are intrinsically low dimensional (Patil et al., 2001), such an extension is an appealing approach for the systems known to exhibit sudden regime changes in their data. The predictive capabilities of this new breeding technique are tested using data taken from the chaotic Lorenz system (Lorenz, 1963).

## 2 Phase space reconstruction

To extend the breeding technique to a system represented by a time series  $x(t)$  at discrete times  $t = t_1, t_2, \dots, t_N$ , we first give a brief introduction of the time-delay embedding and set the notation used in this study. The state of the system at  $t_i$  in the reconstructed phase space is defined by an  $m$  component vector:

$$\mathbf{x}_i = \{x_1(t_i), x_2(t_i), \dots, x_m(t_i)\}, \quad (1)$$

where  $x_k(t_i) = x(t_i - \tau(k - 1)\Delta t)$  is the time-delay coordinate for  $k = 1, \dots, m$ ,  $\tau$  is the delay time, and  $\Delta t$  is the temporal resolution of the time series. The vectors  $\mathbf{x}_i$  specify the dynamical behavior inherent in the data in the form of a trajectory. The time-delay

embedding (Sauer et al., 1991; Abarbanel et al., 1993; Kantz and Schreiber, 1997) requires only two parameters, viz. the dimension  $m$  and time delay  $\tau$ . Among the several techniques available for obtaining these parameters from the data, the convergence of the correlation among the trajectories (Sauer et al., 1991; Abarbanel et al., 1993; Kantz and Schreiber, 1997) and the minimum in the mean-square prediction error (Chen and Sharma, 2006) yield the optimum values of  $m$ . The time delay  $\tau$  also depends on the correlations, and the minimum of the mutual information function yields reliable values (Sauer et al., 1991; Abarbanel et al., 1993; Kantz and Schreiber, 1997; Chen and Sharma, 2006).

### 3 Bred vectors in reconstructed phase space

Having defined the reconstructed phase space by the time-delayed embedding, the new approach to breeding is in essence a matter of selecting the perturbed trajectories that capture the unstable directions along the control. Over a breeding cycle with window size  $n$ , the control starting from  $\mathbf{x}_i$  evolves to  $\mathbf{x}_{i+n}$  and its neighbor (perturbed)  $\mathbf{x}_j$  to  $\mathbf{x}_{j+n}$ . The corresponding growth rate of the bred vector is given by

$$g_i = \frac{1}{n\Delta t} \ln \left( \frac{\|\delta \mathbf{x}_i^f\|}{\|\delta \mathbf{x}_i^0\|} \right) \quad (2)$$

where  $\delta \mathbf{x}_i^0 = \mathbf{x}_j - \mathbf{x}_i$  and  $\delta \mathbf{x}_i^f = \mathbf{x}_{j+n} - \mathbf{x}_{i+n}$  are the initial and final perturbations of the breeding cycle. To select the perturbed trajectory of the next breeding cycle around the control starting from  $\mathbf{x}_{i+n}$ , we follow the spirit of the standard breeding in which the initial perturbation is given by  $\delta \mathbf{x}_{i+n}^0 = \alpha \delta \mathbf{x}_i^f / \|\delta \mathbf{x}_i^f\|$ , i.e., the final perturbation of the previous cycle is rescaled and bred as the new perturbation. Here the perturbation size  $\alpha$  is constant for all breeding cycles. In the reconstructed phase space, however, the trajectory is defined by discrete points, and the rescaled position  $\mathbf{x}_{i+n} + \alpha \delta \mathbf{x}_i^f / \|\delta \mathbf{x}_i^f\|$  may not be a trajectory point. We thus search and select the nearest point  $\mathbf{x}_{j^*}$  and

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refer to the distance between these two points as the displacement distance. In order to avoid selecting nearest neighbors that are on the control trajectory, points  $\mathbf{x}_{i+j\pm l}$  immediately adjacent to  $\mathbf{x}_{i+n}$  for small  $l$  are excluded from the search for  $\mathbf{x}_{j^*}$ , so as to ensure that  $\delta\mathbf{x}_{i+n}^0 = \mathbf{x}_{j^*} - \mathbf{x}_{i+n}$  captures the instability around the control. We call this technique the “nearest-neighbor breeding”. Like the standard breeding, it involves two parameters, viz., the window size  $n$  and the target perturbation size  $\alpha$ . For successful applications of the nearest-neighbor search, the density of the trajectory points must be high enough that, on average, the displacement distance in the nearest-neighbor search is small with respect to target initial perturbation size  $\alpha$  and the correlation between  $\delta\mathbf{x}_i^f$  and  $\delta\mathbf{x}_{i+n}^0$  is nearly 1.

## 4 Results and discussion

To test whether the nearest-neighbor breeding shares with the standard breeding the ability to predict regime changes in the reconstructed phase space, we use the 3-dimensional Lorenz (1963) system (Lorenz, 1963) and generate a time series data set. Along with its simplicity, this system possesses dynamical properties desirable for this study, namely, high nonlinearity that manifests in a low dimensional attractor and chaotic transitions between two regimes (Fig. 1). The model equations are given by:

$$\begin{aligned}\frac{dx}{dt} &= \sigma(y - x) \\ \frac{dy}{dt} &= -xz + rx - y \\ \frac{dz}{dt} &= xy - bz\end{aligned}\tag{3}$$

with the commonly used parameter values  $r = 28$ ,  $b = 8/3$ , and  $\sigma = 10$ . Forward integration of the model is performed using a fourth order Runge–Kutta with time step 0.01. To reconstruct the phase space from a single time series  $x(t_i)$  with the temporal







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We note that, in addition to large bred vector growth rate, two other methods have been also proposed to predict regime changes in the Lorenz three variable system. In his original paper (Lorenz, 1963), Lorenz pointed out that regime changes were associated with large values of the variable  $z$ . Yadav et al. (2005) showed that large absolute magnitudes of the  $x$  variable are also a good predictor. We have implemented the method used in Yadav et al. (2005) for the time series  $x(t)$  and got equally good results. However, the main objective of this paper is to determine whether bred vectors can predict stability from a single time series data, i. e., in the reconstructed phase space. The reasons for the choice of bred vectors as predictor of fast growth in the dynamical system are two-fold. First, unlike the size of a particular variable, breeding can be tested in any dynamical model. Second, bred vector perturbations and their growth have a clear physical meaning in that they detect instabilities (Hoffman et al., 2009) and are akin to the leading local Lyapunov vector and their corresponding growth (Norwood et al., 2013). Thus while predictions based on threshold values of a single variable work well for the Lorenz model, bred vector growth rate may be suitable for making predictions in a broad range of dynamical systems.

For the binary forecasts, Evans et al. (2004) noted that the next regime tended to be longer-lasting when the duration of the high growth rate is longer using the standard breeding (Fig. 1d). This tendency is also found using the nearest neighbor breeding in the reconstructed phase space in Experiment (c). However, in Fig. 1f, the cases of high growth rate may be separated by slower growth rates (e.g., at time  $t \sim 23$ ) due to the time delay involved in the construction of the embedded time series.

## 5 Conclusions

The ability to predict regime change in a dynamical system using the time series data of just one of its many variables, demonstrated in this paper, has important implications. For most systems in nature and in laboratory, the time series observations of only a limited number of physical variables, often a single variable, are available. In



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many cases even the actual number of variables is not known. This paper presents and demonstrates that the nearest neighbor breeding enables the prediction of regime change in systems for which regime change follows the appearance of instabilities, thus extending the predictive capability beyond the cases whose time evolution equations are known. Further, when regime change is associated with large changes in the dynamical states, this technique can lead to the prediction of large or extreme events in the cases where nonlinear dynamical predictions are made using time series data, e.g., in the Earth’s magnetosphere and space weather (Chen and Sharma, 2006).

*Acknowledgements.* This work is supported by JPL grant 1485739, CICS grants NOAA-CISC-EKEK\_12 and CICS-EKEK\_14, NSF/IIP grant 1338634, and ONR grant N00014-10-1-0557. This paper is in part Contribution No. 3465 of the Virginia Institute of Marine Science, College of William & Mary.

## References

Abarbanel, H. D. I., Brown, R., Sidorowich, J. J., and Tsimring, L. S.: The analysis of observed chaotic data in physical systems, *Rev. Mod. Phys.*, 65, 1331–1392, 1993. 1305, 1307

Chen, J. and Sharma, A. S.: Modeling and prediction of the magnetospheric dynamics during intense geospace storms, *J. Geophys. Res.*, 111, A04209, doi:10.1029/2005JA011359, 2006. 1305, 1307, 1310

Evans, E., Bhatti, N., Kinney, J., Pann, L., Pena, M., Yang, S. C., Kalnay, E., and Hansen, J.: RISE undergraduates find that regime changes in Lorenz’s Model are predictable, *B. Am. Meteorol. Soc.*, 85, 521–524, 2004. 1303, 1307, 1308, 1309

Hoffman, M. J., Kalnay, E., Carton, J. A., and Yang, S.-C.: Use of breeding to detect and explain instabilities in the global ocean, *Geophys. Res. Lett.*, 36, L12608, doi:10.1029/2009GL037729, 2009. 1309

Kalnay, E.: *Atmospheric Modeling, Data Assimilation and Predictability*, Cambridge University Press, Cambridge, UK, 2003. 1303

Kantz, H. and Schreiber, T.: *Nonlinear Time Series Analysis*, Cambridge University Press, Cambridge, 1997. 1305, 1307



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- Lorenz, E. N.: Deterministic Nonperiodic Flow, *J. Atmos. Sci.*, 20, 130–141, 1963. 1303, 1304, 1306, 1309
- Norwood, A., Kalnay, E., Ide, K., Yang, S.-C., and Wolfe, C.: Lyapunov, singular and bred vectors in a multi-scale system: an empirical exploration of vectors related to instabilities, *J. Phys. A-Math. Theor.*, 46, 254021, doi:10.1088/1751-8113/46/25/254021, 2013. 1309
- Packard, N., Crutchfield, J., Farmer, D., and Shaw, R.: Geometry from Time Series, *Phys. Rev. Lett.*, 45, 712–716, 1980. 1304
- Patil, D. J., Hunt, B., Kalnay, E., Yorke, J. A., and Ott, E.: Using bred vectors to establish local low dimensionality of atmospheric dynamics, *Phys. Rev. Lett.*, 86, 5878–5881, 2001. 1304
- Sauer, T., Yorke, J. A., and Casdagli, M.: Embedology, *J. Stat. Phys.*, 65, 579–616, 1991. 1305, 1307
- Sharma, A. S.: Assessing the magnetosphere’s nonlinear behavior: its dimension is low, its predictability, high (US National Report to IUGG, 1991–1994), *Rev. Geophys. Suppl.*, 33, 645–650, 1995. 1304
- Sharma, A. S., Vassiliadis, D., and Papadopoulos, K.: Reconstruction of low dimensional magnetospheric dynamics by singular spectrum analysis, *Geophys. Res. Lett.*, 20, 335–338, 1993. 1304
- Takens, F.: Detecting strange attractors in turbulence, in: *Dynamical Systems and Turbulence, Warwick 1980, Lecture Notes in Mathematics 898*, Springer, Berlin, 1981. 1304
- Toth, Z. and Kalnay, E.: Ensemble forecasting at NMC: the generation of perturbations, *B. Am. Meteorol. Soc.*, 74, 2317–2330, 1993. 1303
- Toth, Z. and Kalnay, E.: Ensemble forecasting at NCEP and the breeding method, *Mon. Weather Rev.*, 125, 3297–3318, 1997. 1303
- Ukhorskiy, A. Y., Sitnov, M. I., Sharma, A. S., and Papadopoulos, K.: Global and multi-scale properties of magnetospheric dynamics in local-linear filter, *J. Geophys. Res. Lett.*, 107, 1369, doi:10.1029/2001JA009160, 2002. 1304
- Valdivia, J. A., Sharma, A. S., and Papadopoulos, K.: Prediction of magnetospheric storms by nonlinear models, *Geophys. Res. Lett.*, 23, 2899–2902, 1996. 1304
- Vassiliadis, D., Klimas, A. J., Baker, D. N., and Roberts, D. A.: A description of the solar wind-magnetosphere coupling based on nonlinear filters, *J. Geophys. Res.*, 100, 3495–3512, doi:10.1029/94JA02725, 1995. 1304
- Wilks, D.: *Statistical Methods in the Atmospheric Sciences: An Introduction*, International Geophysical Series, vol. 59, Academic Press, Oxford, UK, 1995. 1308

Yadav, R. S., Dwivedi, S., and Mittal, A. K.: Prediction rules for regime changes and length in a new regime for the Lorenz model, *J. Atmos. Sci.*, 62, 2316–2321, 2005. 1309

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**Table 1.** Contingency tables based on the rule that regime change will occur in the orbit following the appearance of high growth rate bred vectors using three different methods. In (b) and (c) using the nearest-neighbor breeding, high growth rate points in orbits with absolute values of extrema above 1 are excluded. OBS and FCST stand for observed and forecast, respectively; (a)–(c) are the same as in Fig. 1.

				OBS	
			Yes	No	Total
(a)	FCST	Yes	374	38	412
		No	80	573	653
		Total	454	611	1065
(b)	FCST	Yes	396	67	463
		No	58	544	602
		Total	454	611	1065
(c)	FCST	Yes	383	77	460
		No	71	534	605
		Total	454	611	1065

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**Table 2.** Measures of forecast accuracy in terms of the Hit Rate (HR), Threat Score (TS), and False Alarm Rate (FAR); (a)–(c) are the same as in Fig. 1. The final row shows the values when the threshold of  $x(t_i)$  rule is used.

	HR (%)	TS (%)	FAR (%)
(a)	82.4	76.0	5.2
(b)	87.2	76.0	11.0
(c)	84.4	72.1	12.6

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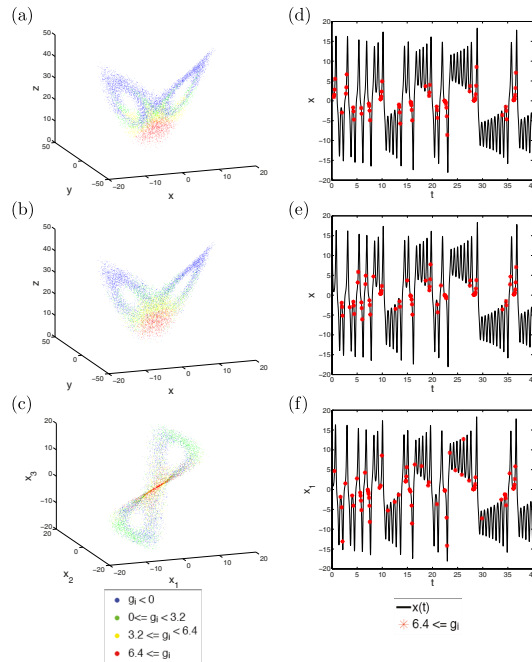
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**Figure 1.** The first column of figures depicts the growth rates of bred vectors in the Lorenz system using three different methods: **(a)** standard breeding in the phase space  $(x, y, z)$ ; **(b)** nearest-neighbor breeding in the phase space  $(x, y, z)$ ; and **(c)** nearest-neighbor breeding in the reconstructed phase space  $(x_1, x_2, x_3)$ . The colored points correspond to negative (blue), low (green), medium (yellow) and high (red) growth. The second column of figures depicts the first coordinate of phase space as a function of time, with red stars indicating the points with high growth rate ( $g_i \geq 6.4$ ) bred vectors for the three different methods: **(d)** standard breeding in the phase space  $(x, y, z)$ ; **(e)** nearest-neighbor breeding in the phase space  $(x, y, z)$ ; and **(f)** nearest-neighbor breeding in the reconstructed phase space  $(x_1, x_2, x_3)$ .