5. Complex Processes in Plasmas and Nonlinear Dynamical Systems, AIP Conf. Proc. 1582, ISBN 978-0-7354-1214-9, Eds., Amita Das and A. Surjalal Sharma, Amer. Inst. Phys., New York,...

CONFERENCE PAPER · NOVEMBER 2012

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Citation: AIP Conference Proceedings **1582**, 35 (2014); doi: 10.1063/1.4865343
View online: http://dx.doi.org/10.1063/1.4865343
View Table of Contents: http://scitation.aip.org/content/aip/proceeding/aipcp/1582?ver=pdfcov
Published by the AIP Publishing

Complexity in Nature and Data-enabled Science: The Earth's Magnetosphere

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Abstract. Understanding complexity in nonequilibrium systems requires multiple approaches and the well established approaches of experiment, theory and numerical simulation have led to the key advances. The data-enabled science, referred to as the fourth paradigm, is an inherently suitable framework for the study of complexity in nature. The data-driven modeling of the Earth's magnetosphere, based on the dynamical systems theory, highlights the achievements of this approach in the study of complexity in natural systems.

Keywords: complexity, data-driven models, nonequilibrium processes, magnetosphere, plasmas
PACS: 52.35.Ra, 05.70.Ln, 89.75.-k, 94.30.cp, 05.45.Tp

Complexity science is widely recognized as studies of organized behavior of systems that are intermediate between perfect order and perfect disorder. Interactions among interdependent components of a system lead to a competition between organized (interaction dominated) and irregular (fluctuation dominated) behavior, and consequently to complexity [1]. Early studies of complex systems were focused on model dynamical systems with a small number of degrees of freedom, e. g., the well-known Lorenz attractor, and have provided deeper understanding of the inherent properties. The understanding achieved with such dynamical systems has enabled a new focus on extended systems with many interdependent components which exhibit regular as well as irregular behavior. In such systems, the behavior of the whole system is more than the sum of their parts. Many natural and anthropogenic phenomena exhibit features readily recognized as typical of complex systems. For example, the plasma in Earth's magnetosphere is inherently nonlinear and exhibit instabilities, nonlinear behavior and nonlinear coupling among the unstable modes. While these processes are complex and lead to irregular behavior, on larger scales they exhibit certain classes of simple behavior that seem insensitive to the details. During a magnetospheric substorm, which is the basic phenomenon responsible for the auroras, a multitude of plasma processes - from the microscopic to the macroscopic - act together to yield large scale coherence. The empirical evidence of the episodic or convulsive nature of the magnetosphere [2] is now understood in terms of its low-dimensional dynamics [3, 4]. However, the magnetospheric dynamics is not limited to low-dimensional behavior and multiscale features are evident. Thus the magnetosphere exhibits both coherence, described by low-dimensionality, and multiscale behavior, arising from the internal dynamics as well as driven by the turbulent solar wind.

Numerical simulation is an essential approach in the study of a wide variety of complex phenomena. For example, plasmas are inherently nonlinear and simultaneous interaction of a large number of degrees of freedom in such nonlinear systems limits the theoretical approach. In the case of plasmas the fundamental laws governing

International Conference on Complex Processes in Plasmas and Nonlinear Dynamical Systems
AIP Conf. Proc. 1582, 35-45 (2014); doi: 10.1063/1.4865343

plasmas are known but their consequences can not be worked out because of the complexity, making numerical simulation a suitable approach [5]. In this approach a numerical model of the system is constructed, e. g., based on the fluid and kinetic plasma descriptions. Then numerical experiments are carried out on a computer, allowing the system to evolve from given initial conditions in accordance with the laws used. This approach or paradigm of numerical simulation, often referred to as the third approach, has provided much of the advances in our understanding of complex systems. Along with the approaches of experiment and theory, numerical simulation is now a distinct intellectual and technological discipline.

Although science has always been data-driven, recently there has been dramatic change in the amount of data used in research. There are two origins of large and massive data ('Big Data') in research. The first is large scale numerical simulations. For example, particle-in-cell simulations of magnetic reconnection and laser-plasma interaction use 10^{10} particles on computers with more than 16 petaflop capacity and typically generate exabytes of data. The simulations of medium-range weather forecast models generate many exabytes, and multiple runs are needed for forecasts with the ensemble modeling approach. The second source of massive data is observations and the situation is similar, if not more challenging. For example, in gravitational wave astronomy LIGO acquired 2 petabytes of data in the past decade and the Advanved LIGO instruments (aLIGO) will generate about 1 petabyte of raw data per year, which will be replicated between the geographically distributed observatories and computer centers at the same rate as it is acquired. The Square Kilometre Array, which will be the world's largest and most sensitive radio telescope, will have data rates of many petabits per second, which is more than 100 times the current global internet traffic. The challenges in research using the massive or complex data have stimulated a new approach that is now referred to as data-enabled science [6]. This emerging approach to science is referred to as the fourth paradigm, the first three being experiment, theory and simulation [7]. This new approach is not just data exploration and understanding, but is the use of the data to enable discovery and new understanding. In many nonequilibrium systems including plasmas, significant advances have been achieved with this approach, especially in the modeling of multiscale phenomena [8]. In fact, the earliest forecasting tools for space weather were based on data-enabled models developed from the data of the solar wind-magnetosphere coupling [4].

Although data-enabled science is an emerging paradigm, there has been many important advances already. For example, the phase space reconstruction based on the nonlinear dynamical systems theory yields the dynamical and statistical features of many natural systems, independent of modeling assumptions. For some applications, this approach yields models whose predictive capabilities surpass those of the first principle models, e. g., in space weather forecasting. An important advantage of data-enabled modeling is the capability to provide insights into non-equilibrium systems, in particular extreme events and natural hazards. Extreme events are ubiquitous in natural, social and financial systems, and from a science perspective, they are an emergent property of complex nonlinear systems in which the interaction among interdependent components lead to a competition between organized and irregular behavior. Recent advances in nonlinear dynamics and complexity science provide a new approach to the understanding of extreme events [9]. This paper provides an overview of the data-enabled studies, using the case of the Earth's magnetosphere as an archetypical complex system in nature. Many phenomena in geosciences are inherently nonequilibrium and the data-enabled approach has led to important advances in the modeling of such systems [10].

COMPLEXITY OF THE EARTH'S MAGNETOSPHERE

The Earth's magnetosphere is a multiscale system that exhibits episodes of strong coupling over a wide range space and time scales. The characteristic spatial scales span the range from the electron skin depth (~ a few km) to the global scale (200 R_E $\sim 10^5$ km). Numerical simulation of such multiscale systems is a challenge as they require multiple physics models that cover the relevant scales and the coupling among them. In the case of plasmas, this leads to additional difficulties because of the need to include kinetic processes as the scale sizes in simulations using fluid models approach the microscales. Most large scale simulations of the magnetosphere are driven by the solar wind and have followed the top-down approach. The numerical codes based on the ideal MHD model [11, 12] simulate the solar wind-magnetosphere coupling with a spatial resolution of an ion Larmor radius (~500 km), and are at the limit where the coupling to kinetic codes becomes important. The hierarchy of phenomena in the magnetosphere during a magnetic reconnection event is illustrated in Fig. 1. The global MHD simulations [11] provide the large scale features of the magnetosphere resulting from reconnection in the magnetotail. It should be noted that in the simulation code based on the ideal MHD model, the magnetic field is frozen into the plasma and thus cannot undergo reconnection. The reconnection in the ideal MHD codes is due to an effective resistivity arising from the finite grid of the simulation domain. The characteristic scales in these simulations correspond to those of Alfven waves and provide the connectivity of the different parts of the magnetosphere. In next level of simulations based on the Hall-MHD model the electron and ion fluids are decoupled, thus relaxing the frozen-in condition and allowing magnetic reconnection [13]. A characteristic feature of reconnection is the generation of Hall currents and consequently a quadrupole magnetic field [14] is generated, as shown in the first inset in Fig. 1. The Hall MHD is essentially a limit of a two-fulid (electron and ion**)** model, in which the electron inertia is neglected and the plasma thus evolves on the time scale of the ions and the typical space scale is the ion skin depth. The mesoscale phenomena involve many wave modes and processes and exhibit cross-scale coupling [15]. On the finer scales, the ion dynamics can be neglected and electron fluid is described by the electron-magnetohydrodynamic (EMHD) model [16]. In this model all perturbations are on the electron scale and are essentially whistler waves. The bottom in-set in Fig. 1 shows reconnection at electron scales, with a magnetic field structure consisting of quadrupoles [17]. A new feature of the reconnection at this scale is the nested structure of the quadrupole field, which was observed earlier by Cluster spacecraft [18]. The forthcoming NASA Magnetospheric Multiscale Mission (MMS) is designed to resolve the electron scale processes and will provide data with shortest scale resolution of a few electron electron depths. The EMHD model has the advantage of its simplicity and the code can resolve the finest scale (electron skin depth) processes, and predict the microscale features of the current sheet.

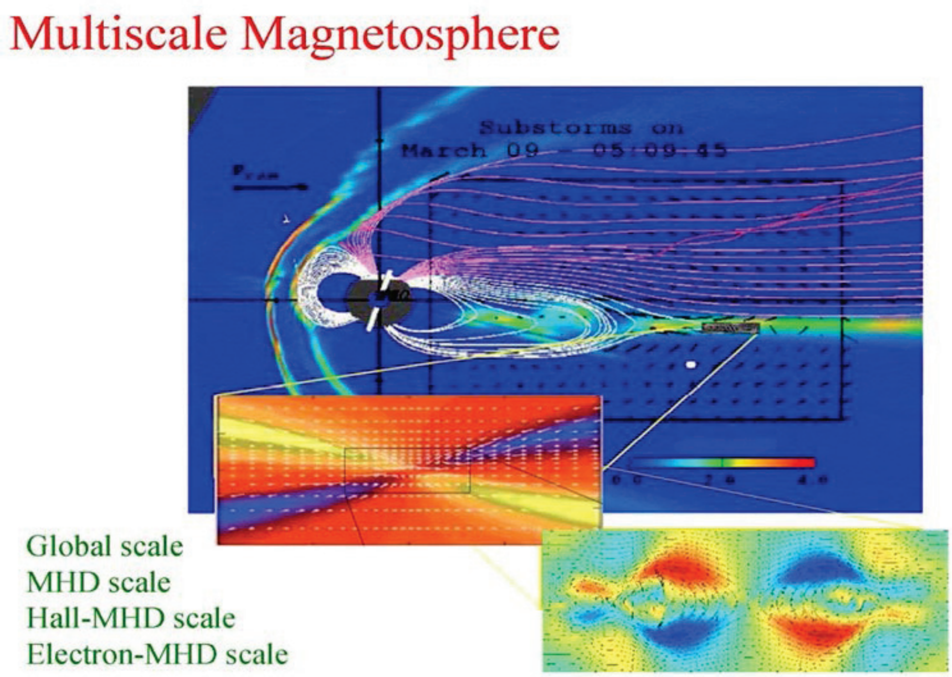

FIGURE 1. The magnetosphere exhibits multiscale behavior ranging from the electron skin depth to the global scale during active periods of magnetic storms and substorms. The global MHD [11], Hall-MHD [13] and electron-MHD [17] codes provide the simulations of the multiscale phenomena.

The multiscale nature of the magnetosphere illustrated in Fig. 1 uses three different types of codes, viz. MHD, Hall-MHD and E-MHD, to highlight the range of scales of the physical phenomena. However, the cross-scale coupling among them is absent in these results from separate codes. A comprehensive simulation that includes all scales simultaneously and their cross-scale coupling requires a two-fluid code in which the dynamics of the electron and ion fluids are fully retained. Such two-fluid codes require time steps that resolve the electron dynamics and stable performance over many ion fluid time scales. This requires extensive computational power and sophisticated numerical algorithms. It should be noted that the present two-fluid codes use time steps that resolve the ion fluid dynamics which are too long to resolve the elctron scale processes, such as the features shown by the EMHD model. Alternatives to such fluid codes for the study of multiscale phenomena are the particle-in-cell, Vlasov and hybrid codes, but at present the capability to simulate the electron and ion scales simultaneously by such codes is limited.

The monitoring of the coupled solar wind – magnetosphere system using ground-based and space-borne instruments for more than 50 years have provided extensive data suitable for studies of multiscale phenomena. These data are widely used in the study of the plasma processes in geospace and more recently in the data-driven modeling based on dynamical systems theory [3, 19]. Along with the new understanding of geospace phenomena the data-driven studies are early efforts in data-enabled science [6].

DATA-ENABLED SCIENCE AND COMPLEX SYSTEMS

The recent explosion in data has led to the recognition of the potential for scientific advance, and in the same time, to challenges in data management and scientific inference from large volumes of data. Data-enabled science addresses the latter and uses the science to provide the insight that leads to new advances. This is distinct from the practice of data exploration and understanding. An example of this approach is the data-enabled modeling that yields the dynamical and statistical features of many natural systems, without the assumption of specific physical processes.

Most extended systems in nature are described by a large number of variables, while only a small number of these may be monitored for long durations. In such systems, the inherent nonlinearity along with the contraction of its phase space due to dissipation leads to low-dimensionality. The small number of variables describing the low-dimensional dynamics can be reconstructed from the time series data of a single variable, based on the embedding theorem [20, 21]. However, open systems exhibit significant levels of multiscale behavior or high dimensionality, thus limiting the scope of the low-dimensional or global dynamical models. The interplay between the global and multiscale behavior is described below in the case of the Earth’s magnetosphere. These studies require the use many techniques, such as phase space reconstruction [3, 4, 19, 22], mutual information functions [23, 24], and detrended fluctuation analysis [24, 25].

Modeling the Global Magnetospheric Dynamics

The recognition of the large scale coherence in the magnetosphere [2] provided a key motivation for developing low dimensional models and there has been considerable progress in modeling the solar wind-magnetosphere coupling as an input-output system by using techniques of nonlinear dynamics and complexity. The data-derived models are based on the correlated database of the solar wind as the input and the magnetosphere as the output. The solar wind upstream of the Earth is monitored at the Lagrange point L1 by spacecraft such as ACE, and plasma and field are used to derive physical variables that couple effectively to the magnetosphere. Among such variables the solar wind induced electric field, given by the product of the flow velocity **V** and the magnetic field **B**, is widely used. The north-south component of the electric field, VB_z, is the dominant variable that couples the solar wind to the magnetosphere through magnetic reconnection at the magnetopause, its boundary. The magnetospheric response to the solar wind driving is represented by the magnetic field variations monitored by ground-based magnetometers distributed around the globe.

These magnetic field variations are then used to derive geomagnetic indices, such as the auroral electrojet index AL and storm time index Dst, and represent more convenient representations of the magnetospheric dynamics.

The studies using time series data use the phase space reconstruction technique and have shown that the coherent behavior of the magnetosphere during the storms and substorms can be described by low-dimensional models [3, 4, 19, 22]. Further developments in the data-driven modeling have led to a mean field technique of averaging outputs corresponding to similar states of the system in the reconstructed state space [26-28]. The improved models are now used for near real time space weather forecasting. The predictability of the magnetosphere demonstrated by using the data-derived modeling has two implications. First, it shows the global coherence of the magnetosphere in terms of the low-dimensionality of the system [3]. This feature is consistent with the global picture obtained from the theory, modeling, observations, and global MHD simulations. Second, the predictive ability of the data-derived models provide reliable tools for space weather forecasting.

The data-derived low-dimensional model has been used to interprete the magnetospheric dynamics in terms of phase transition-like behavior [29]. The data of the three leading variables of the model can be obtained by projecting the correlated data of the solar wind – magnetosphere system along the leading directions obtained by a singular spectrum analysis [4]. Fig. 2 shows the three variables which correspond to the solar wind input P_i, the magnetospheric response P_o (the color of the manifold) and the third variable P_3 representing the time variability of the response. The two-level structure of the manifold on which the dynamics evolves is interpreted as phase transition-like behavior [29], viz. a transition from the higher state (red) to the lower state (green).

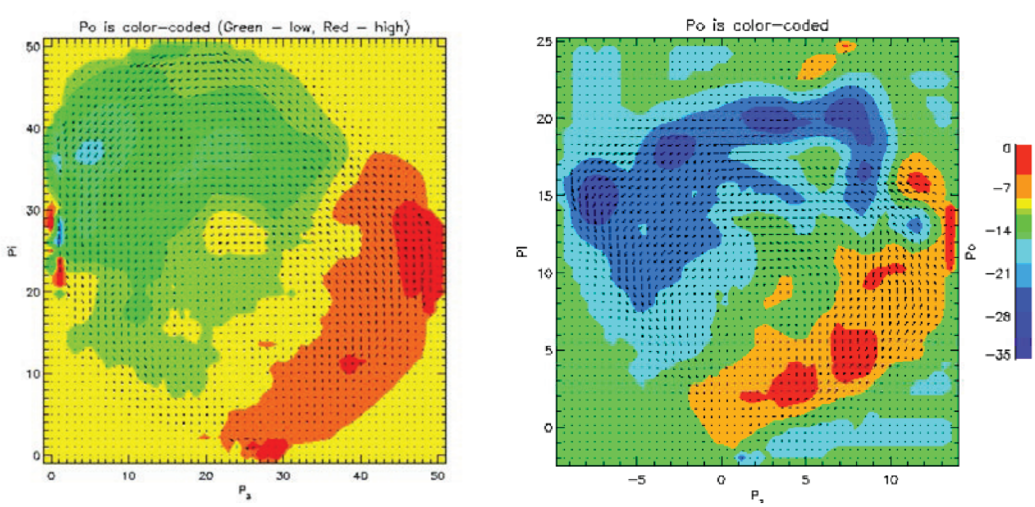

FIGURE 2. The global dynamics of the Earth's magnetosphere obtained from observational data (left panel) [29] and simulations using a global MHD code (right panel) [30]. The two-level structure, with the red/yellow at a higher level than the green/blue, shows a phase transition-like behavior of magnetospheric transitions.

The global MHD simulations provide the large scale features of the magnetosphere and the dynamical features from the simulations have been used to compare with the data-driven model. For this purpose, the global MHD simulations were carried out for the solar wind conditions used in the data-driven modeling. Since the latter uses extensive data of many substorms this required extensive runs on supercomputers. The simulation data was then used to compute the equivalent of the auroral electrojet index AL and the three leading variables obtained in the same manner as the observational data [30]. A plot of the three variables is shown in the right panel of Fig. 2. The two results, one from observational data and the other from simulations, yield similar dynamical manifolds and both show phase transition-like behavior. It should be noted that the simulations require extensive resources, more than 300 hours on supercomputers, and yields similar results as the simple dynamical model derived from data, which run on a typical workstation.

EXTREME EVENTS AND MULTISCALE PHENOMENA

The multiscale nature of nonequilibrium systems is intimately connected to extreme events, which are the largest scale events. The statistical feature of such events in the context of the total distribution is illustrated in Fig. 3 by comparing two types of distributions. The Gaussian distribution includes a negligible number of extreme events compared to the distribution described by a heavy tail with a scaling exponent. Such probability distribution functions (pdf) are common in many natural systems, e. g., earthquakes, floods, river runoffs, geospace storms, etc. The data on the extreme events are usually limited and the nature of their distribution is not known with high accuracy. However, if the exponent of the power law dependence can be computed from the available data it provides a key parameter. With this exponent the likelihood of extreme events can be estimated, assuming a validity of the scaling, and this yields a predictive capability.

The scaling properties of the probability density functions have been studied for many systems in order to characterize extreme events. For example, the studies of the floods of Nile river using the R/S analysis [31] led to the well-known Hurst exponent [32, 33] and is now widely used in characterizing nonequilibrium phenomena. The scaling exponent reflects the presence of correlations and in the presence of non-stationarity and periodicity, such as intermittency exhibited by many systems [34, 35]. Detrending of the data [36-38] is a widely used technique in the computation of the scaling exponents from time series data.

The large database of the auroral electrojet index AL provides a suitable data set for the studies of long-range correlations in the magnetosphere and thus for characterizing the extreme events in space weather. In order to analyze the correlations in AL data sets with 1-hr and 5-min averages were used [24]. The hourly averaged AL for the period 1978-1988, which covers a typical 11-year solar cycle and 5 min averaged data for 1978 were used to compare the scaling properties.

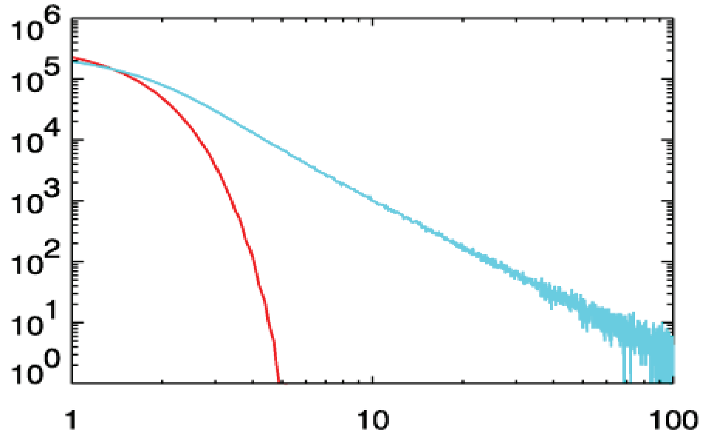

FIGURE 3. Heavy-tailed (power-law) distribution of frequency vs. event size (blue), compared to the light-tailed (Gaussian) case (red). The likelihood for extreme events is much larger for the heavy tailed case.

The detrended fluctuation analysis of time series data is accomplished in four steps [24, 36-38]. The first step computes the profile of the time series data x_i data as:

$$Y(i) = \sum_{k=1}^{i} (x_k - <\mathrm{x}>)$$

The subtraction of the global mean <*x*> of the dataset however is not essential as the third step, described below, usually removes this and other trends. In the second step, the profile *Y(i)* is divided into $N_L = N/L$ non-overlapping segments of length *L*. In order to avoid a loss of data in the case N is not a multiple of *L*, the same process is repeated starting from the other end of the data set, yielding 2 N_L segments. The third step is where the trends in the data are removed by defining a local trend $q_j(i)$ for each segment *j* by a fitting procedure, e.g., a least-squares fit. The detrended time series for the segment duration *L* is then defined as:

$$Y_L(i) = Y(i) - q_j(i)$$

In most cases the local trend is usually represented by a polynomial and a quadratic function is used. In the fourth step, the variance of each segment $Y_L(i)$ is computed:

$$F_L^2(j) = < Y_i^2(i) > = \frac{1}{L} \sum_{i=1}^{L} Y_L^2\,[(j-1)L + i]$$

This leads to the detrended fluctuation function *F(L)*, which is defined by

$$F^2(L) = \frac{1}{N} \sum_{i=1}^{2N_L} F_L^2$$

In the presence of long-range correlations, the fluctuation function scales as

$$F(L) \approx L^{\alpha}$$

For uncorrelated or short-range correlated data, the exponent $\alpha = 0.5$ and larger values show the presence of long-range correlations.

The detrended fluctuation analysis of the 5-min averaged AL data yields a scaling function *F(L)* shown in Fig. 4 [24]. Also shown in this figure is the function using the fluctuation analysis (FA) [36]. The DFA function *F(L)* yields an exponent ≃

0.9, thus showing long-range correlations. The hourly averaged data covering 11 years yields an exponent of 0.87, and thus the two data sets exhibit similar scaling.

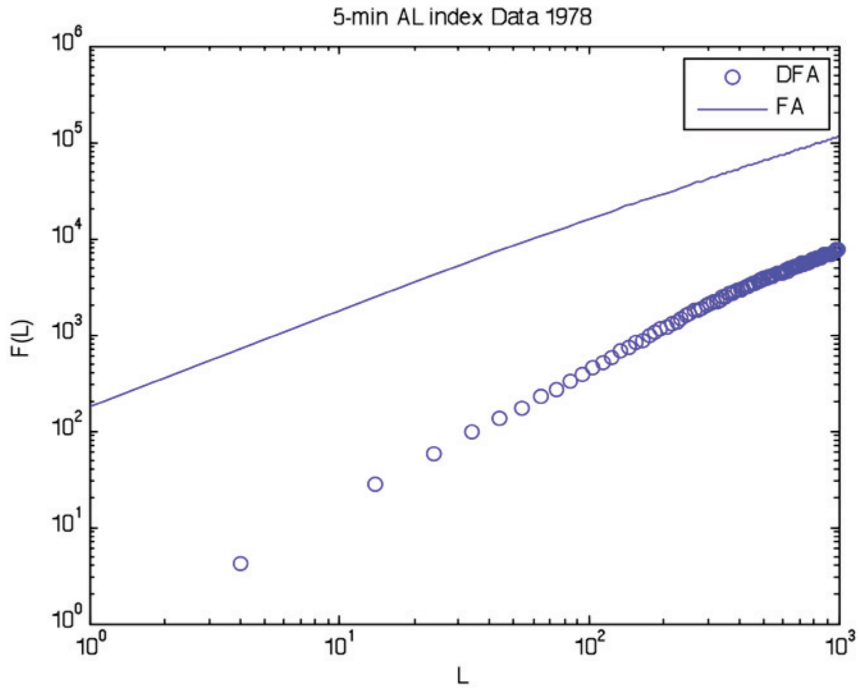

FIGURE 4. Detrended fluctuation analysis of 5-min averaged data of AL for 1978. The function F(L) has a scaling exponent of 0.9 and shows the presence of long-range correlations. The case of Gaussian distributed (random) has an exponent of 0.5.

The scaling exponent α provides a means for estimating the likelihood of extreme events and is widely used in forecasting, e. g., earthquakes, river run-offs, and finance [35]. It also provides an insight into the inherent nature of the system. For example, α=0.5 implies randomness and forecasts in such cases are not meaningful. On the other hand, $\alpha > 1$ implies long-range correlations and thus the system has predictable behavior.

CONCLUSION

The emerging data-enabled science is based on effective use extensive observational data to yield dynamical and statistical properties. Complexity science, which yields data-driven models with predictive capability in cases where the first principle models are not available, provides such a framework. The Earth's magnetosphere is an example of a large scale open system in nature and the studies of its properties with data-enabled modeling have provided highlights of the new approach to understanding complexity.

An important advantage of data-enabled modeling is the capability to provide insights into extreme events and their predictability. Extreme events are an emergent property of many complex, non-linear systems in which various interdependent components and their interaction lead to a competition between organized and irregular behavior. The devastating consequences of extreme events in natural, social and financial systems are well-known, viz. natural hazards and economic downturn. Considering the complexity of these nonequilibrium systems, it is not clear at present whether predictive models based on first principles can be developed. On the other hand, the data-driven approach has the potential of providing models with improved predictability. Many natural hazards and similar phenomena have a heavy-tailed distribution of magnitudes, such as a Pareto or lognormal, and characterizing them is an important step in improving the forecasting skill. In the damages due to natural hazards, about 80% of losses arising from claims come from ~20% of events ("the 80-20 rule") [39], thus underscoring the importance of targeting improved forecasts of extreme events.

The recent explosion of data from observations and simulations requires development of data-enabled approaches. From modeling dynamical behavior to characterization of key features, many requirements of 'Big Data' are beyond the current techniques and approaches [6, 7]. The complexity science provides an attractive perspective for developing ways to make effective use of large volumes of data.

Although the data-enabled modeling has emerged recently, there have been attempts at the use of data-driven techniques as a fundamental approach. Information theory [40], which has advanced our understanding of a variety of systems and is the backbone of many technological systems, is essentially a data-driven approach. In physics the earliest approach to a general theory of nonequilibrium statistical mechanics is based on information theory [41] and this framework can be used for implementing data-enabled science. Such approaches, however, have been the target of criticism, on the premise that modeling be based on first principles alone. With the advances of complexity science in physical, biological and social sciences there is growing recognition that modeling complexity requires going beyond the dependence on fundamental laws that are true for all times and all places [42]. Insights achieved in a complex system, using a consistent framework, can be used to understand similar systems, even in the absence of the fundamental laws.

ACKNOWLEDGMENTS

The author has benefitted greatly from many discussions with Armin Bunde, Predhiman Kaw, Abhijit Sen, and Chippy Thyagaraja. Much of the paper was prepared during a visit to the Institute for Plasma Research, Gandhinagar, and the author is grateful for the hospitality. The research at the University of Maryland is supported by NASA and NSF grants.

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