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TOKAMAK HEATING BY RELATIVISTIC ELECTRON BEAMS

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ABSTRACT. The use of relativistic electron beams for supplementary heating of tokamaks to ignition temperatures is discussed, under the assumption that they can be successfully injected. It is shown that if an electron beam with present-state-of-the-art power, but longer pulse length, can be injected into an Ohmically pre-heated tokamak plasma, it transfers its energy to the plasma on a time-scale short enough for synchrotron radiation to be an unimportant beam energy loss mechanism, while the turbulence generated does not affect the energy confinement. Beam energy and pulse characteristics required for an extension of the ideas to a reactor-size tokamak are presented.

1. INTRODUCTION

With energies as high as 1 MJ available in presently existing relativistic electron beams [1], it has become of considerable interest to utilize them to heat plasmas to temperatures of interest to controlled thermonuclear fusion [2]. The proposed heating applications break down into two categories. The first is the beam-pellet case in which the beam heats an inertially confined, initially solid target [3]. This case, however, requires a very high degree of focussing of a beam having at least one order of magnitude higher power than present beams. The second case, to which this paper is addressed, is the heating of magnetically confined plasma in the density range $\sim 10^{13} - 10^{17}\,cm^{-3}$, which requires the injection of the beam into the confinement geometry and the deposition of the beam energy in the plasma in a length compatible with the geometry. Furthermore, the interaction between the beam and the plasma must not have a significant adverse effect on the equilibrium, stability, and confinement properties of the plasma-magnetic field configuration. The confinement geometry presently receiving the most attention is, of course, the closed, toroidal configuration, the tokamak. We find that a simple extrapolation of present-state-of-the-art electron beams might make a significant contribution to a tokamak fusion feasibility experiment. In particular, we find that although the transfer of beam energy to the plasma is rapid enough so that beam synchrotron radiation is not an important energy loss mechanism, the turbulence levels generated do not appreciably affect the energy confinement in the tokamak.

The utilization of intense electron beams to heat plasmas in a toroidal geometry clearly suffers from a much more severe injection problem than in an open-ended system, and this is, at present, under investigation experimentally [4,5] and theoretically [6] with encouraging preliminary results. It is the purpose of this paper to discuss the beam-plasma interaction process in a tokamak under the assumption that the injection problem can be overcome, and that equilibria, such as those of Mondelli and Ott [7] exist.

In the present work, we find that energy deposition occurs in two stages. The first stage is dominated by the two-stream instability between beam and plasma, which, in the physical situation under consideration, is kinetic in character, owing to the expected large angular spread of the beam [8]. The non-linear mechanism responsible for the saturation is fast spectral transfer of the beam generated waves ($k \sim \omega_e/c$, where k is the wave number, ω_e the plasma frequency and c the speed of light) to larger wave-numbers ($k \gg \omega_e/c$) by parametric instabilities induced by the ponderomotive force [9] of the primary ($k \sim \omega_e/c$) waves, as discussed by Papadopoulos [10-12]. The characteristics of the wave spectrum and the saturation level are such as to have insignificant effect on the plasma diffusion rates. Furthermore, very little energy transfer occurs during the first stage of the interaction for most reasonable beam and tokamak parameters. The second stage begins shortly after saturation, and is a quasi-steady state with the beam instability severely limited by the non-linear effects. During this stage the beam deposits most of its energy by the resistive dissipation of the return current in the presence of the parametrically induced density fluctuations. As will be shown, the energy deposition rate is large enough that intense electron beams may be a viable alternative to neutral beams as a supplementary heat source for tokamak plasmas.

Before discussing the model and theory in detail, let us illustrate how the use of the energy in an electron beam which is nearly present "state-of-the-art" might contribute to a tokamak fusion feasibility experiment. Justification for the details of the example will be discussed in subsequent sections. In considering the minimum requirements on a tokamak ignition experiment using energetic neutral beam injection, Sweetman [13] has found a tokamak having a density of $3\times10^{13}\,cm^{-3}$, major and minor radii of 200 cm and 60 cm, respectively,

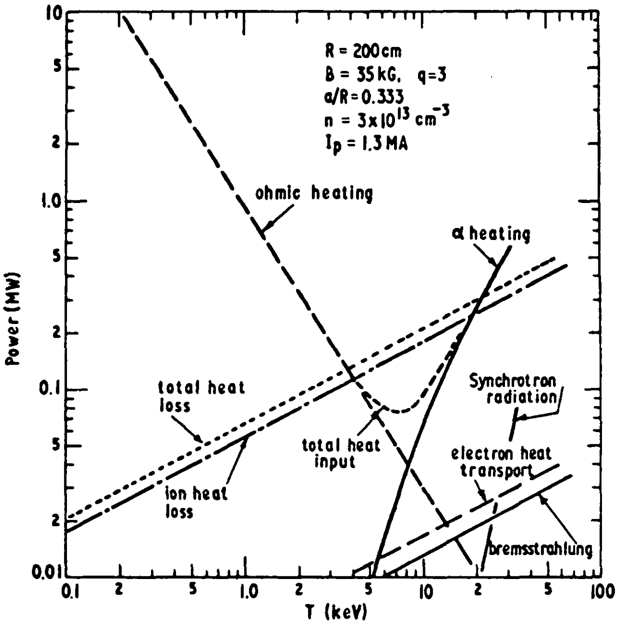

FIG.1. Heating and heat loss rates for a tokamak feasibility experiment (after D.K. Sweetman [13]).

a toroidal magnetic field of 35 kG, and an induced current of 1.3 MA to be the minimum allowing a reasonable ignition experiment. (Ignition here means self-sustaining, i.e. fusion-reaction alpha-particle heating balances total heat losses.) As can be seen from the heating and heat loss rates in Fig.1, ignition in this experiment occurs at 20 keV, and, without supplementary heating, there is a gap between about 3 keV and 20 keV between which total heat loss exceeds the sum of Ohmic heating and alpha-particle heating by $\lesssim 200$ kW. The actual energy required to heat the particles from 3 to 20 keV is 3.2 MJ. Clearly, depositing 200 kW of neutral beam power can make up the difference, assuming no disruptive effects. However, since this power would barely exceed the energy loss rate, it would require 20 s and 4 MJ of deposited energy to achieve ignition. Thus, Sweetman suggests using about 1 MW in order to deposit the energy in a time short compared to the energy loss time, bringing the required energy much closer to the minimum.

We next demonstrate that an electron beam is capable of achieving the same result. We assume that a 600-kA beam of 10-MeV electrons is injected for 800 ns into the tokamak of Fig.1, when the temperature is already up to 3 keV from Ohmic heating. The 40-ns transit time for the beam around the 12-m-circumference torus implies 12 MA of circulating beam current at the end of the 800-ns injection time. Since the 800-ns pulse duration is much longer than the transit time of the beam around the torus, an injection method which is not limited to the transit time, such as the drift injection scheme discussed by Benford et al. [5], is required. Note that the beam has the specifications of one beam from the Aurora generator [14], except for a four-fold increase in pulse duration, which should be a straightforward technological advance at such high electron energy. We further assume that the beam current is injected parallel to the induced plasma current of the tokamak, in accordance with the recent results of Bailey et al. [15] and Swain et al. [16] and that, within a transit or two around the torus, the beam pretty much fills the plasma cross-section. Since the plasma is an excellent conductor with radius large compared to c/ω_e, all necessary conditions are met for complete current neutralization of the beam [17]. Since the 12 MA of plasma current induced by the beam is flowing in the opposite direction to the initial plasma current, 10.7 MA of plasma current is flowing after beam injection. However, the net current is still 1.3 MA, which is still well under the Kruskal-Shafranov limit (note that q = 3, where q is the safety factor). If the beam uniformly fills the plasma cross-section, its density will be $\sim 10^{11}$ cm^{-3}, giving a beam to plasma density ratio of $\sim 3\times 10^{-3}$. As we shall soon show, we expect that only the kinetic electron-electron two-stream instability will be present, and it will saturate due to parametric coupling of the high-frequency plasma waves to low-phase-velocity ion sound waves. Moreover, we shall find that the level of fluctuation resulting from the above saturation mechanism is sufficiently low and of short duration not to cause any significant plasma energy loss. However, it should be sufficient to increase the plasma resistivity by about a factor of 10^3. This will result in the Ohmic dissipation of the return current in a time of < 1 ms. The energy for the return current heating comes inductively from the beam electrons, since the system L/R time is long compared to the energy dissipation time. The 1 ms dissipation time is much shorter than energy loss times, and the energy gap indicated in Fig.1 will be overcome. When the beam energy is dissipated to the point of system stability, the magnetic-field configuration will be maintained by the coasting electron beam plus residual plasma currents, which will be maintained inductively near 1.3 MA total.

In the next section, we shall in detail discuss the model used in the parametric instability calculations which follow in Section 3. In Section 4, we apply the results of Section 3 to an intense beam in a tokamak, as described above. In the final section — Section 5 — we discuss the results, and provide estimates indicating the applicability to a tokamak fusion test reactor if repeatable pulse 10^{13}-W electron beam generators can be developed.

2. THE BEAM-PLASMA CONFIGURATION

We discuss next the beam-plasma configuration. We assume that the beam is injected into the plasma so that it uniformly fills the plasma cross-section, and that it has a mean angle to the total magnetic field $[\overline{\theta}^2]^{\frac{1}{2}} > 1/\gamma$, where $\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}$, v is the electron velocity and c is the velocity of light.

The large-mean-angle assumption is clearly applicable since we must require $\gamma \gtrsim 20$ in order to have reasonably accessible beam parameters. Moreover, the production of a beam with 600 kA even at $\gamma = 20$ precludes that $[\overline{\theta^2}]^{\frac{1}{2}}$ will be less than ~ 0.2 even if we ignore the effects of injection into a torus. (Such a beam has a value of the beam strength parameter $\nu/\gamma \cong 2$, where $\nu = Ne^2/mc^2$, N being the number of electrons per cm of beam length, and e and m being the electron charge and mass, respectively. Therefore, beam production and propagation effects will prevent a smaller mean angle.) Beam uniformity throughout the plasma seems the most reasonable one to assume since, if it is not uniform initially, one might expect multiple small-angle scattering of beam electrons as they travel around the torus to eventually cause it to be uniform.

The growth rate for the electron-electron two-stream instability is [7, 20].

$$\Gamma = \omega_e \frac{n_b}{n} \frac{1}{\gamma} \frac{\overline{\theta}^2 + \frac{1}{\gamma^2}}{\left[\overline{\theta}^2 + \frac{1}{\gamma^2} \frac{\Delta\epsilon_b}{\epsilon_b}\right]^2} \tag{1}$$

where we have assumed a one-dimensional situation along the magnetic field. Since $[\overline{\theta^2}]^{\frac{1}{2}} > 1/\gamma$, the appropriate approximate relation for the growth rate is [20]

$$\Gamma \approx \omega_e \frac{n_b}{n} \frac{1}{\gamma} \frac{1}{\overline{\theta}^2} \tag{2}$$

Note that under the present situation, the instability is in the kinetic limit rather than the hydrodynamic limit characteristic of a "cold" beam examined in detail by Thode and Sudan [18]. This implies that trapping can be eliminated as a probable saturation mechanism for the instability, leaving quasi-linear and parametric saturation as the most likely mechanisms. It will be shown in the next section that the latter mechanism dominates for the parameters of our feasibility experiment, as well as for most other reasonable beam-tokamak experiments.

At this point we have made qualitative statements on $[\overline{\theta^2}]^{\frac{1}{2}}$, but we shall require a numerical value in order to make computations later on. The beam will be injected into the plasma with a certain amount of energy perpendicular to the total magnetic field, and a typical beam electron will gain additional perpendicular energy as a result of multiple small-angle Coulomb scattering. (We expect that neither the beam-plasma interaction fluctuation level nor any ripples in the magnetic field will have any effect on beam electrons.) Synchrotron radiation will reduce the transverse energy of an electron at the approximate rate

$$\frac{d\epsilon_\perp}{dt} \cong \frac{2}{3} \frac{e^2}{c} \gamma^4 \omega_c^2 \theta^2 \beta^2$$

where $\omega_c = eB/\gamma mc$ is the cyclotron frequency of the relativistic electrons and $\theta < 1$ is the angle of a specific electron velocity vector to the total magnetic field direction. In units of MeV/s, this gives

$$\frac{d\epsilon_\perp}{dt} \cong 0.1\ (\gamma^2 - 1)\theta^2 \left(\frac{B}{10\ \mathrm{kG}}\right)^2\ \mathrm{MeV/s}$$

Thus, at $\gamma = 20$ and B = 35 kG, $d\epsilon_\perp/dt = 490\ [\overline{\theta^2}]$ MeV/s is an estimate of the average energy loss rate per electron. For example, if the mean angle $[\overline{\theta^2}]^{\frac{1}{2}} \sim 1/2$, the "typical" electron will require a time of the order of 10 ms to radiate away its transverse energy.

Multiple small-angle scattering increases the mean angle at the rate

$$< \Delta\theta^2 > = \frac{8\pi n v t R_o^2}{\beta^4 \gamma^2} (Z^2 + Z) \ln \left[\frac{a_o \gamma \beta^2}{2\ Z^{4/3} R_o}\right]$$

$$= 4\ \omega_e^2 \frac{e^2}{mc^3} (9 + \ln \gamma\beta^2) \frac{t}{\beta^3 \gamma^2}$$

where R_0 is the classical electron radius, Z is the plasma atomic number and the time after injection is t. In the last formula a hydrogen plasma has been assumed. Therefore, the rate of increase of transverse energy is

$$\frac{d\epsilon_\perp}{dt} \cong \frac{d}{dt}\left(\frac{\gamma m v_\perp^2}{2}\right) \approx \frac{2\ \omega_e^2 e^2}{c} \frac{(9 + \ln \gamma\beta^2)}{\beta\gamma}$$

For $\gamma = 20$, and 3×10^{13} plasma density, this gives a rate of transfer of energy from parallel to the magnetic field lines to perpendicular to them which is about 10^2 times slower than the synchrotron radiation energy loss rate, i.e. 0.8 MeV/s. Even if the effective Z of the plasma is 3 or 4, the synchrotron radiation of transverse energy will dominate multiple scattering so long as $[\overline{\theta^2}]^{\frac{1}{2}} > 0.1$, at which point a balance would occur (for $\gamma = 20$). Thus, if the beam energy will be dissipated in the plasma in $\lesssim 10$ ms, a mean angle of $\sim 0.2 - 0.5$ (i.e. the injection value) is a reasonable estimate, and if the time is longer than 10 ms, ~ 0.1 is reasonable (for $\gamma = 20$).

Of equal importance as the characteristics of the beam, are the plasma conditions at the time of injection. In particular, we postulate injection of the beam into the plasma after it has reached 3-keV electron temperature due to Ohmic heating. Therefore, the return current electron drift velocity $v_d \cong (n_b/n)c \cong 3\times10^{-3}$ c is nearly the same as plasma sound speed $c_s = \sqrt{kT_e/m_i}$ and no ion acoustic instability is expected. (Note that even if beam non-uniformity within the plasma caused $v_d > c_s$ locally, the ion acoustic instability would still be very weak since we could wait long enough before injection to have T_i comparable to T_e.)

3. NON-LINEAR THEORY

On the basis of the physical situation described previously, we proceed to describe the coupling of beam energy to the plasma. We begin this section with a brief qualitative picture, and follow it with a quantitative evaluation.

Figure 2 illustrates the beam and plasma distribution functions early in the interaction (i.e. before saturation). It also shows the distribution function of plasma waves in k-space resulting from the kinetic two-stream instability. Notice that our analysis is based on a one-dimensional model, along a strong magnetic field. This is justified, as shown in detail by Fainberg et al. [21], in the case where $[\overline{\theta^2}]^{\frac{1}{2}} > 1/\gamma$, since the modes with large $k_\perp$ are easily stabilized by the transverse velocity spread of the beam. The presence of the strong guide magnetic field acts also as a stabilizing influence for the off-angle modes. A detailed study of such effects can be found in Refs [8, 18-23]. On the basis of this model, and assuming that non-linear effects are negligible, one obtains the quasi-linear theory of the slowing-down of the beam examined by Rudakov [8]. It is described by the following system of equations

$$\frac{\partial F_b}{\partial t} = \frac{\partial}{\partial p} D_p \frac{\partial}{\partial p} F_b \tag{3}$$

$$\frac{\partial W_1}{\partial t} = 2\,\Gamma W_1 \tag{4}$$

where W_1 is the energy in the waves created by the beam with $k_1 \sim \omega_e/c$, $D_p = (\pi/4)(m\omega_e^2/nv)\,W_1(k)$ and Γ is given by Eq.(2). The spectrum width is given by

$$\frac{\Delta k_1}{k_1} \approx \frac{1}{\gamma^2}\frac{\Delta \epsilon_b}{\epsilon_b} + \overline{\theta}^2 \approx \overline{\theta}^2 \tag{5}$$

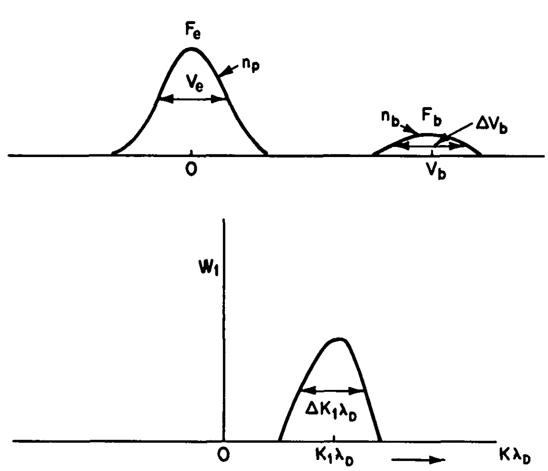

FIG.2. Electron distribution functions and unstable two-stream plasma waves in k-space.

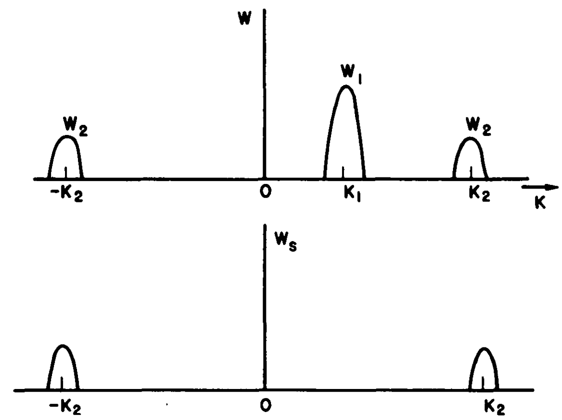

FIG.3. Two-stream (W_1) and parametrically created daughter waves (W_2, W_S).

Based on Eqs (3) and (4) the slowing-down time of the beam is given by

$$t\omega_e \sim \frac{n\gamma}{n_b}\,\overline{\theta}^2 \tag{6}$$

and a large amount of energy (~ 30% of $n_b\epsilon_b$) is in turbulent fields.

However, as first pointed out by Kainer et al. [23], by Thode and Sudan [18] for the case of a cold relativistic beam, and by Papadopoulos [10] for the warm case, the ponderomotive force [9] exerted on the plasma owing to the plasma waves has the form of negative pressure and can non-linearly transfer energy from the primary waves k_1 to secondary plasma waves k_2 with $|k_2| \gg \omega_e/c$ and non-linear ion waves with the same wave-number $|k_2|$. It was shown in Ref.[12] that this process can lead to a severe limitation in the plasma-wave energy level, a lengthening of the slowing-down time and to an essentially complete suppression of the instability.

As is described in Ref.[12], there are two stages in the development of the beam plasma instability interaction where different non-linear physical mechanisms dominate. In the first stage the physics of the non-linear spectral transfer is similar to the recently discussed [24-29] parametric instabilities. The primary-wave k can be considered equivalent to an externally imposed pump wave whose wavelength with respect to the wavelength of the secondary waves is very large. In the dipole (infinite-wavelength) approximation [25-28], since the frequency of the wave is close to the plasma frequency, one expects the oscillating two-stream instability (O.T.S.) to dominate. This instability produces purely growing ion waves and electron plasma waves with opposite wave-numbers so that momentum is conserved (Fig.3). In this way, energy cascades down to low-phase-velocity electron plasma waves, which create symmetric tails in the electron distribution function. For a single

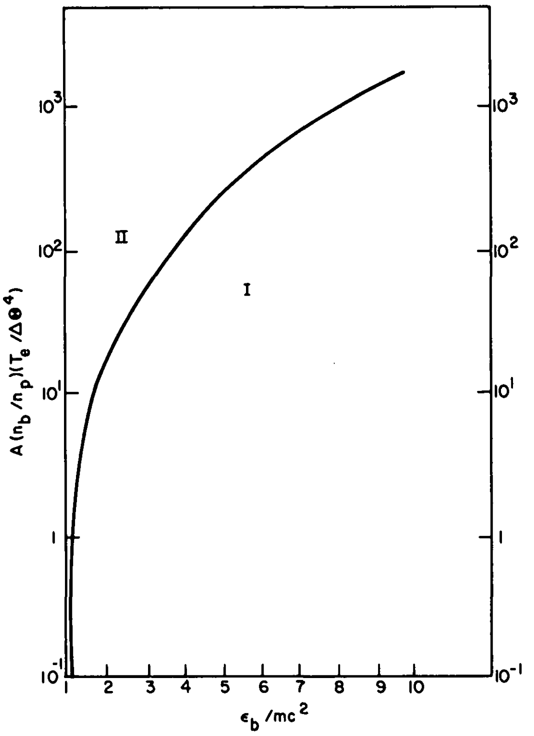

FIG.4. Regions in parameter space where non-linear (I) and quasi-linear (II) stabilization occurs (A is the atomic number of the ionic species) (after K. Papadopoulos [12]).

wave in the dipole approximation, the growth rate of the O.T.S. is [25-28]

$$\Gamma^{NL} \approx \omega_e \sqrt{\frac{m}{M}} \sqrt{\frac{W_1}{nT}} \tag{7}$$

One can show [30] that the wave-packet with wave-number spread Δk will behave as a single wave as long as the frequency spread $\Delta\omega_1 \approx v_g \Delta k_1 \approx V_e/c\, \Delta k_1 < \Gamma^{NL}$, which is easily satisfied in our case. The effect of finite pump-wavelength requires numerical solution. The effects of both finite k_1 and Δk_1 have been examined numerically by Papadopoulos and Haber [30]. According to their results, the use of Γ^{NL} as given by Eq.(7) is justified in our case and the only additional effect introduced by the finite value of k_1 is a small real frequency part in the density fluctuations.

In a simplified fashion the effect of the non-linearity on the growth of W_1 can be seen by adding to the right-hand side of Eq.(4) the term $-2\Gamma^{NL}W_2$ so that

$$\frac{\partial W_1}{\partial t} = 2\,\Gamma W_1 - 2\,\Gamma^{NL}\,W_2 \tag{8}$$

where W_2 is the energy density in the parametrically created waves. The instability is stabilized when $\Gamma W_1 - \Gamma^{NL}(W_1)\,W_2 \leq 0$. An analytic model of these processes was discussed in Ref.[12]. The energy density of the waves W_1, W_2 and the density of fluctuations W_s during this stage are shown in Fig.3.

Non-linear stabilization will dominate quasi-linear stabilization if the rate of parametric depletion of W_1 due to $\Gamma^{NL}(W_1)$ becomes equal to the quasi-linear growth at a lower level than the quasi-linear maximum of W_1. Thus, non-linear stabilization is expected for the beam parameters in region II of Fig.4. From the calculations of Ref.[12] we find that

$$\frac{W_1^{\max}}{nT_e} = \frac{M}{m}\,\frac{\Gamma^2}{\omega_e^2}\,\Lambda^2 \tag{9}$$

where Λ has a weak (logarithmic) dependence on $W_1^{\max}$. It is given by

$$\Lambda = \ell n \left[\left(\frac{M}{m}\right)^{\frac{1}{2}} \frac{\Gamma}{\omega_e} \frac{\left[W_1^{\max}/nT_e\right]^{\frac{1}{2}}}{W_2(0)/nT_e} \right] \tag{10}$$

where $W_2(0)$ is the initial (noise) level for the W_2 waves. The wave-number k_2 of the parametric waves may be obtained from

$$k_2 \approx \frac{1}{2} \left[W_1^{\max}/nT_e\right]^{\frac{1}{2}}/\lambda_D \tag{11}$$

for the case where the dipole approximation is valid. Otherwise, k_2 must be evaluated numerically [30].

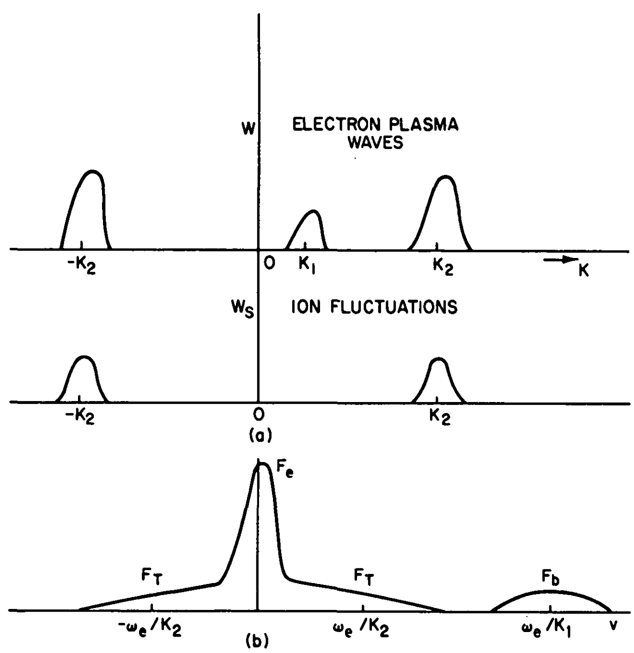

FIG.5. Non-linear quasi-steady state of beam-plasma system. (a) Spectral distribution of electron plasma waves (W) and ion fluctuations (W_S). (b) Particle distributions.

This completes the description of the first stage of the interaction. The second and final stage is again described in Ref.[12] and corresponds to a quasi-steady state. The physics of this non-linear equilibrium can be seen from Fig.5. The waves W_1 generated by the beam with wave-number k_1 are scattered by the density fluctuations W_s with a rate given by the Oberman-Dawson formula $((W_s/nT_e)/k_2^2\lambda_D^2)\omega_e$ [31]) to wave-numbers k_2, thus feeding W_2. Balancing the growth and scattering processes gives

$$2\Gamma = \frac{W_s/nT_e}{k_2^2\lambda_D^2}\omega_e \tag{12}$$

The waves W_2 are Landau-damped on the tails of the electron distribution, with a rate which depends on its slope. If we assume that this damping rate is ν_{eff}, then in order to balance the source term due to scattering of W_1 on W_s we have

$$\frac{(W_s/nT_e)}{k_2^2\lambda_D^2} W_1 = \frac{\nu_{eff}}{\omega_e} W_2 \tag{13}$$

Finally, to balance the pressure gradient of the density fluctuations, a threshold ponderomotive force is necessary. This threshold value W_2^T for comparable T_e and T_i is given by [25-28, 12]

$$\frac{W_2^T}{nT_e} \approx 8\frac{\nu_{eff}}{\omega_e} \tag{14}$$

On the basis of Eqs (12) - (14), we find the average quasi-steady state values (indicated by a superscript zero) to be given by

$$\frac{W_1^0}{nT_e} = 4\frac{\nu_{eff}^2}{\Gamma\omega_e} \tag{15a}$$

$$\frac{W_2^0}{nT_e} = 8\frac{\nu_{eff}}{\omega_e} \tag{15b}$$

$$\frac{W_s^0}{nT_e} = 2k_2^2\lambda_D^2\frac{\Gamma}{\omega_e} \tag{15c}$$

Notice that this is a steady state only in a time-average sense and the actual values will oscillate about the average values given by (15) with time scales of the order Γ^{-1}.

Given the turbulence levels as determined by Eq.(15), we can find the slowing-down time of the beam, and the energy deposition due to the return current dissipation. Since the value of k_2 can be found from relation (11) in terms of the system parameters, the only unknown is the value of ν_{eff}. This value, which is a function of the slope of the distribution function of the tails, F_T, depends on the loss rate of fast particles from the interaction region, which might be to walls, to radiation, collisional relaxation, etc. However, to obtain some estimates we assume that there are negligible losses since we have a toroidal system. In this case, a steady-state distribution of the tails can be established by balancing quasi-linear diffusion with collisions with other particles. We have, therefore, for the tail distribution,

$$\frac{\partial}{\partial t}F_T = \frac{\partial}{\partial v}D_2\frac{\partial}{\partial v}F_T + \frac{\partial}{\partial v}\nu\left(vF_T + \frac{T_e}{m}\frac{\partial F_T}{\partial v}\right) \tag{16}$$

where D_2 is the diffusion coefficient in velocity space, and the collision frequency $\nu \approx \lambda\omega_e^4/nv^3$ (λ is the usual Coulomb logarithm). It is easy to solve Eq.(16) for a steady state. We find [32] that

$$\frac{\partial F_T}{\partial v} = \frac{1}{1 + D_2\frac{m}{T_e\nu}}\frac{\partial F_M}{\partial v} \tag{17}$$

where F_M is the Maxwellian distribution towards which the tail relaxes. Using the usual quasi-linear diffusion coefficient, Eq. (17) becomes

$$\frac{\partial F_T}{\partial v} = \frac{\lambda\omega_e}{N_D}\frac{\Delta k_2}{k_2}\frac{1}{v^3}\left(\frac{T_e}{m}\right)^{3/2}\frac{nT_e}{W_2^0}\frac{\partial F_M}{\partial v} \tag{18}$$

where $N_D \equiv n\lambda_D^3 4\pi/3$. If we take the tail-to-plasma density ratio as α_T, the damping ν_{eff} will be given by

$$\nu_{eff} = \alpha_T\, 1.5\,(k_2\lambda_D)^3\frac{nT_e}{W_2^0}\frac{\lambda}{N_D}\omega_e \tag{19}$$

The value of α_T is of the order $\alpha_T \approx 10^{-3}$ (see Ref.[12]).

We have now in Eqs (2), (9) - (11), (15) and (19) a closed system which determines the turbulence levels in terms of the system parameters. This will be performed in the next section for the case under investigation.

4. APPLICATION

Let us now return to the configuration of a beam in a tokamak as discussed in Section 2. To set the situation again, we heat the tokamak described in Fig.1 by Ohmic heating until the electron temperature has reached ~ 3 keV and the ion temperature is not far behind. We then inject a 10-MeV, 600-kA electron beam into the plasma with the beam current and initial plasma current in the same direction. The injection duration is 800 ns, by which time 12 MA of circulating beam current will be flowing around the torus. It will in turn induce 12 MA of return current in the plasma, in the opposite direction to the initial plasma current. Therefore, the net plasma current will be 10.7 MA, the beam current 12 MA, and the total net current 1.3 MA. The beam uniformly fills the plasma cross-section and has $\overline{\theta}^2 \gg 1/\gamma^2$. In terms of the parameters of Section 3, we have that $n_b/n = 3\times10^{-3}$, $\gamma = 20$, $\overline{\theta}^2 = 1/25$, $\lambda_D = 7.4\times10^{-3}$ cm, $c/\omega_e = 9.7\times10^{-2}$ cm, $N_D = 5.1\times10^7$ and $\lambda = 20$. For these parameters we find that $k_1\lambda_D \approx 0.07$, $\Delta k_1/k_1 \approx 1/25$, which easily justifies single-wave dipole approximation [30]. From Eqs (9) and (10) we find $W_1^{max} \approx 0.09$, and from Eq.(11) $k_2\lambda_D \approx 0.15$. The time to reach this state is $\omega_e t \sim 6\times10^3$ or $t \sim 20$ ns. A comparable time is required to establish the quasi-steady state. The time for this to occur is, therefore, $t < 100$ ns. Thus, during most of the beam injection time, the fluctuation level will be that of the quasi-steady state rather than the first-stage values. Once the first stage is completed the predominant energy transfer is due to return current. The turbulence levels during this state can be found from Eqs (15) and (19) with $k_2\lambda_D \sim 0.15$, $\alpha_T \approx 10^{-3}$ and $\lambda \sim 20$. We find that

$$\frac{W_1^0}{nT_e} \approx 3\times10^{-11}$$

$$\frac{W_2^0}{nT_e} \approx 4\times10^{-6}$$

$$\frac{W_s^0}{nT_e} = 1.7\times10^{-4}$$

We can see that the level of W_1^0 is noise, and the slowing down of the beam due to the scattering by the waves W_1^0 will be classical. As discussed in Ref.[12], the predominant slowing-down and energy transfer mechanism is due to the return current which, in contrast to Lovelace and Sudan [33], although stable with respect to ion sound, sees an enhanced resistivity due to the non-thermal level of W_s, supported by the beam. The value of the anomalous resistivity is given by [34, 35].

$$\eta_a = \alpha\frac{4\pi}{\omega_e}\;\frac{W_s}{nT_e}\;\frac{1}{k_2\lambda_D}$$

where $\alpha \cong 0.25 - 0.4$ depending on the angular spectrum of the fluctuations. Within factors of order unity this gives $\eta_a \approx 10^{-14}$ s. This should be compared with the value of the classical resistivity which is $\eta_{cl} \approx 2\times10^{-18}$ for $T_e \approx 3$ keV. The deposition time t^E for the energy is then given by $\eta_a j^2 t^E = n_b(\gamma - 1)\,mc^2$, with $en_bc = j$. We find the dissipation time t^E of the order of 1 ms.

We proceed next to examine whether the low-frequency density fluctuations will affect the plasma confinement time, during their life-time, which is of the order of < 1 ms, as computed above. Their frequency [29, 30] is of the order of $\omega \sim \frac{1}{2}k_1v_g \approx \omega_e \times (v_e/c)^2 > \Omega_i$ for the case under consideration. We can therefore apply for the diffusion the Yoshikawa-Rose [36] diffusion coefficient $D_\perp \approx (\pi/4)(\delta n/n)^2 \times 10^8\,(T_e)_{eV}/B$. The confinement time $\tau = a^2/4D_\perp$, where a is the plasma radius. Using the values of $k_2\lambda_D \sim 0.15$ and $W_s/nT \sim 1.7\times10^{-4}$ we find that $\tau > 1$ ms if the plasma radius $a > 8$ cm. We can therefore conclude that the turbulence induced by the beam will not affect the confinement time of the tokamak.

5. DISCUSSION

We have presented above an analysis which indicates that, if injection of intense relativistic electron beams into a tokamak geometry becomes feasible, they represent an attractive alternative to neutral-beam injection towards achieving ignition in a feasibility experiment. A clear advantage of the relativistic-beam approach neutral injection is the fact that the technological requirements for the electron beams are very close to current state of the art. For neutral beams, further engineering milestones must be achieved, although injection should be a more straightforward problem. Since the effects of the neutral-beam injection on the plasma transport properties are not clearly known, we feel that development of a parallel programme exploiting ignition schemes using relativistic beams should be strongly encouraged.

We should note that the observed behaviour of runaways in present-day tokamaks is closely related to the theory developed above. Preliminary reports of the MIT Tokamak indicate that the 20-25-keV runaways produced excite density fluctuations of the type described previously, which are absorbed by the ions [37]. Additional preliminary observations of 1-5 MeV runaways in the Oak Ridge Tokamak indicate that they are almost free-accelerating, in accordance with non-linear stabilization processes, rather than the quasi-linear relation [38]. However, in view of the inadequacy of the observations to date, and the as yet unresolved details of their production, we shall defer an examination of the problem until a later time.

A final point to consider is the fact that the energy is absorbed via Landau damping on the tails of the electron distribution function. For the energy to be communicated to the ions a few electron-ion collisions must take place, a process which takes a few seconds in the feasibility experiment discussed in detail above. If neutral-beam injection were to take place during this time, very little energy would be "wasted" by heating electrons. Thus, one can consider combining the two energy injection methods, using the electron beam energy primarily to "pre-heat" electrons (and requiring only half as much electron beam energy) and the neutral beam energy to ignite the ions.

The extension of these ideas for a tokamak fusion feasibility experiment to a reactor-size tokamak is much more speculative than the previous discussion since, at least, a factor of 10 more electron beam energy would be required to heat the plasma to ignition temperature [13]. If the required beam energy could be produced and injected into the tokamak, beam-plasma conditions would be such that the physics of the coupling would be similar to the feasibility experiment case discussed in detail. One might obtain such an increased beam energy in several ways: (1) More energy in the pulse can be obtained by having a longer pulse duration, higher voltage or higher current; (2) several modules, each injecting an appropriate fraction of the energy, can be used; and (3) one module can be pulsed several times in a total time period short compared to characteristic energy loss times. Since energy loss times in a reactor-size system "pre-heated" to the keV-range by Ohmic heating would be much longer than a second, one might expect the third solution to be feasible while at the same time requiring less injection area and less capital expense than the second solution. (Once ignition is achieved, no further beam pulses would be required until a new cycle is begun, perhaps hours later. Thus, large investment in the reactor "match" is not justifiable if it can be avoided.) The first solution may not be possible technologically since even 20-30-cm cathode-anode gaps can close in 10 μs, and a ten times higher current without a smaller gap would require a ten times larger cathode area, a possibility more reasonably achieved with several modules. Significantly higher voltage would be difficult to produce by existing technology and would lead to more substantial synchrotron radiation losses during the interaction in the tokamak. Therefore, it appears that achievement of multi-pulsing capability for intense beam generators would be the best approach for their application to a reactor-size tokamak.

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