# Spontaneous emission of radiation from localized Langmuir perturbation

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The radiation at the first and second harmonics of the electron plasma frequency ($\omega_e$) from localized Langmuir oscillations is computed for a field-free plasma. The localized perturbations are assumed to have cylindrical symmetry about the direction of propagation of the localized perturbation. It is shown that when the wavelengths of the excited modes are much greater than the scale lengths of the Langmuir perturbation, a quadrupole radiation pattern is recovered for emission at $2\omega_e$. An application to type III solar radio bursts is discussed briefly.

Studies of strong Langmuir turbulence occurring in beam-plasma interactions[1] or laser-target absorption[2-5] have produced some of the most exciting and unpredictable results in plasma physics. While the dynamics of the collapse and the stability of the predicted solitons are still subjects of active investigation,[6-8] measurements in both space and the laboratory have supported the existence and longevity of localized soliton structures. Such localized solitonlike clumps of Langmuir oscillations have electromagnetic signatures at harmonics of the electron plasma frequency $\omega_e$. In this work, we compute the emissivity for radiation at the fundamental and first harmonic of $\omega_e$. The results of the calculation are of importance both from the point of view of diagnostics in the laboratory as well as for the interpretation of such astrophysical radio emissions as type III solar radio bursts.[9]

The organization of this paper is as follows: We first derive an expression for the radiation emissivity from arbitrary clumps of localized Langmuir oscillations. In order to determine the angular dependence and scaling of the emission with the physical dimensions and amplitude, we then choose a reasonable model for the soliton envelope in two dimensions and discuss the specific character of the radiation. Finally, we discuss the principal features of the emission and some applications.

We assume the localized clump of Langmuir oscillations to be of the form $\mathbf{E}(\mathbf{x},t)=\nabla\phi(r,z)\sin\omega_e t$, and calculate the radiation for frequencies of $\omega=\omega_e, 2\omega_e$, where $\omega_e$ denotes the electron plasma frequency and $\nabla\phi(r,z)$ defines the wave envelope. The interaction between the soliton field and the associated slow time scale cavity oscillations in the plasma density is implicitly included in this treatment, and we use $\delta n(r,z)$ to describe the caviton structure. Both $\phi(r,z)$ and $\delta n(r,z)$ are assumed to possess cylindrical symmetry about the direction of propagation of the soliton (which is taken to define the $z$ axis) and, in addition, $\phi(r,z)$ and $\delta n(r,z)$ are assumed to be odd and even functions of $z$, respectively.

In the case of an unmagnetized plasma, a modified Klein–Gordon equation can be derived for the second-order radiation field $\delta\mathbf{B}$ which is of the form[10]

$$\left[\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\frac{\omega_e^2}{c^2}\left(1+\frac{\delta n}{n_0}\right)\right]\delta\mathbf{B}$$
$$=\frac{\omega_e}{c}\nabla\times\left(\frac{\delta n}{n_0}\nabla\phi\right)\cos\omega_e t-\frac{e}{2m_e\omega_e c}\nabla\times(\nabla^2\phi\nabla\phi)\sin 2\omega_e t\,, \qquad (1)$$

where $n_0$ denotes the ambient density. Equation (1) can be solved directly for radiative solutions, and the emission spectrum follows immediately from consideration of the Poynting flux.

We first consider the emission for $\omega\sim\omega_e$, and Fourier transform (1) in space and time to obtain $(k^2-k_1^2)\delta\mathbf{B}_{\mathbf{k},\omega}=\mathbf{S}_{\mathbf{k}}^{(1)}[\delta(\omega-\omega_e)+\delta(\omega+\omega_e)]$, where $k_1^2=(\omega_e^2/c^2)\,|\delta n/n_0|_{\max}$,

$$\mathbf{S}^{(1)}(\mathbf{x})\equiv\frac{\omega_e}{2c}\left[\frac{\partial}{\partial z}\left(\frac{\delta n}{n_0}\frac{\partial}{\partial r}\phi\right)-\frac{\partial}{\partial r}\left(\frac{\delta n}{n_0}\frac{\partial}{\partial z}\phi\right)\right]$$
$$\times(\sin\varphi\,\hat{\mathbf{e}}_x-\cos\varphi\,\hat{\mathbf{e}}_y)\,,$$

$\varphi=\tan^{-1}(y/x)$, and the Fourier transform is defined as

$$f_{\mathbf{k},\omega}=(2\pi)^{-4}\int d^3x\int dt\,\exp(i\omega t-i\mathbf{k}\cdot\mathbf{x})f(\mathbf{x},t)\,.$$

It has been implicitly assumed in the derivation that $k_1^{-1}$ is much less than the scale size of the localized Langmuir perturbation. As a result, care must be exercised in the evaluation of the power radiated at $\omega_e$. After inverting the solution and choosing outgoing waves, one finds that in the radiation zone (i.e., $|\mathbf{x}|\gg$ source size)

$$\delta\mathbf{B}(\mathbf{x},t)=i(2\pi|\mathbf{x}|)^{-1}\sin(\omega_e t-k_1|\mathbf{x}|)\int d^3x'$$
$$\times\mathbf{S}^{(1)}(\mathbf{x}')\exp(ik_1\hat{\mathbf{n}}\cdot\mathbf{x}')\,,$$

where $\hat{\mathbf{n}}=\mathbf{x}/|\mathbf{x}|$. Since the system is cylindrically symmetric, we may choose, without loss of generality, $\hat{\mathbf{n}}\cdot\mathbf{x}'=r'\sin\theta\cos\varphi'+z'\cos\theta$ (see Fig. 1) and write

$$\delta\mathbf{B}(\mathbf{x},t)=-\frac{\omega_e}{2c}\frac{\sin(\omega_e t-k_1|\mathbf{x}|)}{|\mathbf{x}|}I_1\hat{\mathbf{e}}_y\,, \qquad (2)$$

where

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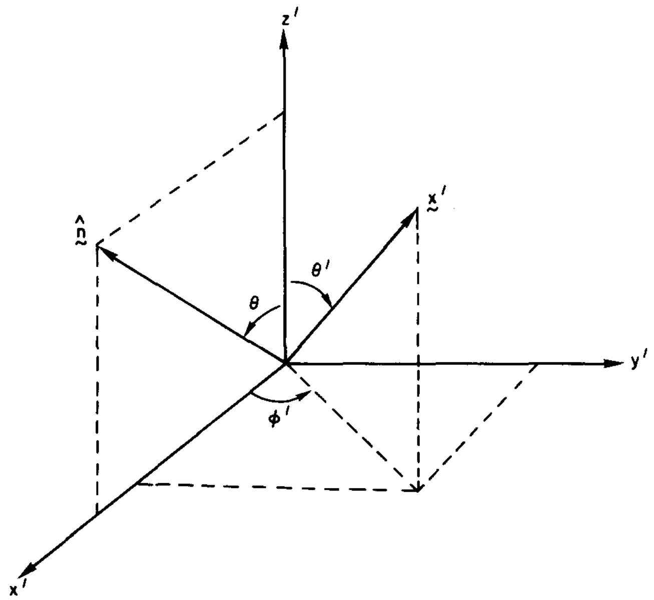


FIG. 1. Schematic representation of the geometrical configuration.

$$I_1 = k_1 \int_{-\infty}^{\infty} dz' \int_0^{\infty} dr' \; r' \frac{\delta n}{n_0}\Big(\cos\theta \sin(k_1 z' \cos\theta) \times J_1(k_1 r' \sin\theta)\frac{\partial}{\partial r'}\phi + \sin\theta\cos(k_1 z'\cos\theta) \times J_0(k_1 r' \sin\theta)\frac{\partial}{\partial z'}\phi\Big). \quad (3)$$

A self-consistent solution for the radiation electric field can be found by means of (2) and the $\nabla \times \mathbf{E}$ Maxwell equation, and

$$\delta\mathbf{E}(\mathbf{x},t) = -\frac{\omega_e^2}{2c^2 k_1}\frac{\sin(\omega_e t - k_1|\mathbf{x}|)}{|\mathbf{x}|} I_1(\cos\theta\hat{\mathbf{e}}_x - \sin\theta\hat{\mathbf{e}}_z). \quad (4)$$

The average power radiated per unit solid angle subtended by $\hat{\mathbf{n}}$ follows immediately, and we find that

$$\frac{dP(\omega_e)}{d\Omega} = \frac{\omega_e}{2\pi}\int_0^{2\pi/\omega_e} dt \frac{c}{4\pi}|\mathbf{x}|^2 \hat{\mathbf{n}}\cdot(\delta\mathbf{E}\times\delta\mathbf{B}) = \frac{\omega_e^2}{32\pi c^2 k_1} I_1^2. \quad (5)$$

It is important to note that in the evaluation of (5), we are restricted to the regime in which the wavelength of the radiation ($k_1^{-1}$) is much shorter than the scale length of the localized structure.

The treatment proceeds in an analogous manner for emission at $2\omega_e$, and we write $(k^2 - k_2^2)\delta\mathbf{B}_{\mathbf{k},\omega} = (2i)^{-1}\mathbf{S}_{\mathbf{k}}^{(2)}[\delta(\omega + 2\omega_e) - \delta(\omega - 2\omega_e)]$, where $k_2^2 = 3\omega_e^2/c^2$, and

$$\mathbf{S}^{(2)}(\mathbf{x}) \equiv -\frac{e}{2m_e\omega_e c}\left[\frac{\partial}{\partial z}\left(\nabla^2\phi\frac{\partial}{\partial r}\phi\right) - \frac{\partial}{\partial r}\left(\nabla^2\phi\frac{\partial}{\partial z}\phi\right)\right] \times (\sin\varphi\hat{\mathbf{e}}_x - \cos\varphi\hat{\mathbf{e}}_y).$$

It should be pointed out that the term in $\delta n\delta\mathbf{B}$ in Eq. (1) has been neglected here. This is valid for sufficiently small $\delta n$ such that $k^2 \sim k_2^2 \gg \omega_e^2\delta n/c^2 n_0$. The principal consequence of this is that the restriction of the analysis to wavelengths much shorter than the soliton scale length imposed in the treatment of $\omega_e$ radiation does not apply to radiation at $2\omega_e$ when $\delta n/n_0 \ll 1$. The outgoing wave solution in the radiation zone can be shown to be

$$\delta\mathbf{B}(\mathbf{x},t) = (4\pi|\mathbf{x}|)^{-1}\sin(2\omega_e t - k_2|\mathbf{x}|) \times \int d^3x' \, \mathbf{S}^{(2)}(\mathbf{x}')\exp(ik_2\hat{\mathbf{n}}\cdot\mathbf{x}'),$$

where we employ the same geometry as in the case of radiation at $\omega_e$. After some manipulation, this reduces to

$$\delta\mathbf{B}(x,t) = -\frac{e}{4m_e\omega_e c}\frac{\sin(2\omega_e t - k_2|\mathbf{x}|)}{|\mathbf{x}|} I_2\hat{\mathbf{e}}_y, \quad (6)$$

where

$$I_2 = k_2\int_{-\infty}^{\infty} dz'\int_0^{\infty} dr' \, r'\nabla'^2\phi\Big(\sin\theta\sin(k_2 z'\cos\theta) \times J_0(k_2 r'\sin\theta)\frac{\partial}{\partial z'}\phi - \cos\theta\cos(k_2 z'\cos\theta) \times J_1(k_2 r'\sin\theta)\frac{\partial}{\partial r'}\phi\Big). \quad (7)$$

The corresponding self-consistent solution for the radiation electric field is

$$\delta\mathbf{E}(\mathbf{x},t) = -\frac{e}{4m_e\omega_e c^2 k_2}\frac{\sin(2\omega_e t - k_2|\mathbf{x}|)}{|\mathbf{x}|} \times I_2(\cos\theta\hat{\mathbf{e}}_x - \sin\theta\hat{\mathbf{e}}_z), \quad (8)$$

and the radiation emissivity follows immediately

$$\frac{dP(2\omega_e)}{d\Omega} = \frac{e^2}{64\pi m_e^2 c^2 k_2\omega_e} I_2^2. \quad (9)$$

It is clear that in the long wavelength limit (i.e., when $k_2 z' \ll 1$ and $k_2 r' \ll 1$) $dP(2\omega_e)/d\Omega \propto \sin^2 2\theta$, so that when $|\partial\phi/\partial r| \ll |\partial\phi/\partial z|$ a quadrupole radiation pattern is indicated with

$$\frac{dP(2\omega_e)}{d\Omega} \cong \frac{e^2 k_2^3\sin^2 2\theta}{256\pi m_e^2 c^2\omega_e}\left(\int_{-\infty}^{\infty} dz' z'\int_0^{\infty} dr' \, r'\nabla'^2\phi\frac{\partial}{\partial z'}\phi\right)^2. \quad (10)$$

In order to illustrate specific properties of the radiation we assume that $|\partial\phi/\partial r| \ll |\partial\phi/\partial z|$ and neglect the radial component of the soliton field. Following Degtyarev *et al.*,[3] we choose $\partial\phi/\partial z = E(r)\,\mathrm{sech}[\kappa(r)z]$, and $\delta n/n_0 = -6\kappa^2(r)\lambda_e^2\,\mathrm{sech}^2[\kappa(r)z]$, where $\lambda_e$ is the Debye length, $\kappa$ denotes the soliton scale length in the direction of propagation, and $E$ denotes the amplitude. The soliton amplitude and scale length are related via

$$E^2/8\pi n_0 T_e = 12(\kappa\lambda_e)^2[1 + \gamma_i(T_i/T_e)], \quad (11)$$

where $T_i$ and $\gamma_i$ are the ion temperature and ratio of specific heats, and $T_e$ is the electron temperature. After some manipulation, we find that (where $v_e^2 \equiv T_e/m_e$)

$$\frac{dP(\omega_e)}{d\Omega} = \frac{3\pi^2}{16}\left(\frac{v_e}{c}\right)^2\omega_e k_1 n_0 T_e\left(1 + \gamma_i\frac{T_i}{T_e}\right)^{-1}\sin^2\theta \times\left[\int_0^{\infty} dr' \, r' J_0(k_1 r'\sin\theta)\frac{W}{n_0 T_e} \times\left(1 + \frac{k_1^2\cos^2\theta}{\kappa^2}\right)\mathrm{sech}\left(\frac{\pi k_1\cos\theta}{2\kappa}\right)\right]^2 \quad (12)$$

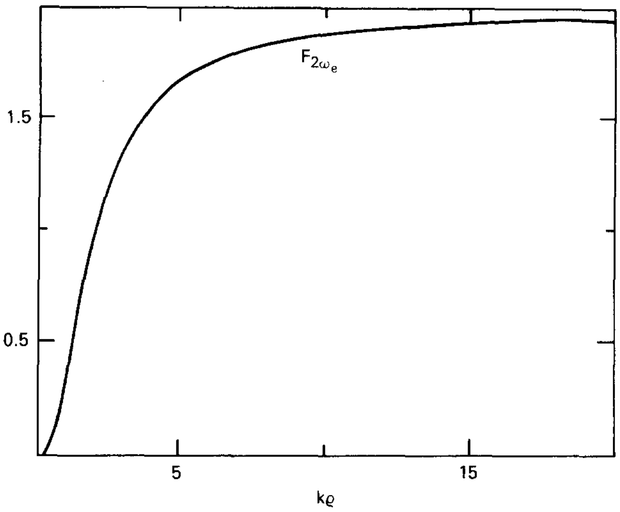


FIG. 2. Graph of $F_{2\omega_e}$ as a function of $k\rho$.

for $k_1 \gg \kappa$, $W \equiv E^2/8\pi$, and

$$\frac{dP(2\omega_e)}{d\Omega} = \frac{\pi^2}{16}\left(\frac{v_e}{c}\right)^2 \omega_e k_2 n_0 T_e \sin^2 2\theta \times \left[\int_0^\infty dr'\, r' J_0(k_2 r' \sin\theta)\, \frac{W}{n_0 T_e} \times \frac{k_2^2 \cos\theta}{\kappa^2} \operatorname{csch}\left(\frac{\pi k_2 \cos\theta}{2\kappa}\right)\right]^2 . \quad (13)$$

It is clear that the only case which may be investigated analytically is that of radiation at $2\omega_e$ for which $k_2 \ll \kappa$. The regimes in which $k_{1,2} \gg \kappa$ must be studied numerically for the parameters appropriate to a given problem, and will not be discussed further here. Instead, we consider (13) in the limit in which $k_2 \ll \kappa$, and obtain

$$\frac{dP(2\omega_e)}{d\Omega} \simeq 9\left(\frac{v_e}{c}\right)^4 \omega_e k_2 n_0 T_e \left(1 + \gamma_i \frac{T_i}{T_e}\right) \sin^2 2\theta \times \left[\int_0^\infty dr'\, r' J_0(k_2 r' \sin\theta) \left(\frac{W}{n_0 T_e}\right)^{1/2}\right]^2 . \quad (14)$$

If it is assumed that $\kappa(r) = \kappa_0 \exp(-r/\rho)$ for simplicity, then we obtain

$$\frac{dP(2\omega_e)}{d\Omega} = 9\sqrt{3}\left(\frac{v_e}{c}\right)^4 \left(1 + \gamma_i \frac{T_i}{T_e}\right) \frac{c^3 W_0}{\omega_e^2} \frac{(\omega_e^2 \rho^2/c^2)^2 \sin^2 2\theta}{(1 + k_2^2 \rho^2 \sin^2\theta)^3} , \quad (15)$$

where $W_0[\equiv E^2(r=0)/8\pi]$ denotes the peak amplitude of the soliton field. It is clear from examination of Eq. (15) that a simple quadrupole radiation pattern is possible, as we noted previously, only in the limit in which the wavelengths of the radiated waves are much greater than the typical soliton dimensions.

The total radiated power at $2\omega_e$ is determined by integration of (15) over $d\Omega(\equiv 2\pi \sin\theta\, d\theta)$. We find that

$$P(2\omega_e) = 2\sqrt{3}\pi\left(\frac{v_e}{c}\right)^4 \frac{c^3 W_0}{\omega_e^2}\left(1 + \gamma_i \frac{T_i}{T_e}\right) F_{2\omega_e}(k_2^2 \rho^2) , \quad (16)$$

where $F_{2\omega_e}$ is given by

$$F_{2\omega_e}(k_2\rho) = \frac{1}{\mu_2^2}\left[3 + 2k_2^2\rho^2 - \frac{1}{2\mu_2} \frac{(3 + 4k_2^2\rho^2)}{k_2\rho} \ln\left(\frac{\mu_2 + k_2\rho}{\mu_2 - k_2\rho}\right)\right]. \quad (17)$$

In Eq. (17), $\mu_2^2 \equiv 1 + k_2^2\rho^2$, and a graph of $F_{2\omega_e}$ versus $k\rho$ is given in Fig. 2. Approximate expressions for the radiated power in either the long or short wavelength limits can be obtained from (16) and (17). We obtain

$$P(2\omega_e) \cong \begin{cases} \dfrac{32\sqrt{3}\pi}{15}\left(\dfrac{v_e}{c}\right)^4 \dfrac{c^3 W_0}{\omega_e^2}\left(1 + \gamma_i \dfrac{T_i}{T_e}\right)(k_2^2\rho^2)^2; & k_2^2\rho^2 \ll 1 , \\ 4\sqrt{3}\pi\left(\dfrac{v_e}{c}\right)^4 \dfrac{c^3 W_0}{\omega_e^2}\left(1 + \gamma_i \dfrac{T_i}{T_e}\right)\left(1 - \dfrac{1}{k_2^2\rho^2} \ln k_2^2\rho^2\right); & k_2^2\rho^2 \gg 1 . \end{cases} \quad (18)$$

It is clear from examination of Eq. (18) that in the short wavelength region the emission per soliton at $2\omega_e$ is *independent* of its transverse size. As a result, the emission in this regime does not depend on the details of the mechanism which determines collapse in the transverse direction.

Equation (18) describes the emission per soliton. A more appropriate quantity from the standpoint of observations in the laboratory or space, however, is the emissivity per unit volume. In order to determine this quantity, a knowledge of the total volume occupied by the various solitons as well as the spacing between solitons is required. Unfortunately, a sufficiently detailed knowledge (whether experimental or theoretical) of the dynamics of soliton formation to enable determination of such quantities does not currently exist. In view of this, we shall assume that the solitons are "close-packed," for the purpose of illustration, with volume $V \sim 2\pi\rho^2\kappa^{-1}$ per soliton. In this case the power radiated per unit volume $J \sim P/V$, is given by

$$J(2\omega_e) \simeq \begin{cases} \dfrac{3}{5}\left(\dfrac{v_e}{c}\right)^3 \left(\dfrac{W_0}{n_0 T_e}\right)^{1/2} \omega_e W_0 k_2^2 \rho^2 \left(1 + \gamma_i \dfrac{T_i}{T_e}\right)^{1/2}, & k_2^2\rho^2 \ll 1 , \\ \dfrac{3}{4}\left(\dfrac{v_e}{c}\right)^3 \left(\dfrac{W_0}{n_0 T_e}\right)^{1/2} \omega_e W_0 \dfrac{1}{k_2^2\rho^2}\left(1 + \gamma_i \dfrac{T_i}{T_e}\right)^{1/2}, & k_2^2\rho^2 \gg 1 , \end{cases} \quad (19)$$

for radiation at $2\omega_e$.

The present theory of radio emission from solitonlike structures has been applied to explain the characteristics of the $2\omega_e$ radiation observed during type III solar radio bursts.[9] The fundamental property of this phenomenon is the presence of energetic electron streams associated with the type III bursts which are observed to propagate over distances (~$10^8$ km) which are too large to be accounted for in terms of quasi-linear processes, and which are explained by a strong turbulent stabilization mechanism[1] which proceeds via the excitation of the oscillating two-stream instability that is effective when $W_k/n_0 T_e > (k\lambda_e)^2$. Since $k\lambda_e \sim v_e/v_b \ll 1$ for typical beam-plasma interactions (where $v_b$ is the average beam velocity), the parametric stabilization mechanism is appropriate even for relatively low levels of wave turbulence. Since the strong turbulence dynamical equations for this process are equivalent to the equations defining Langmuir solitons,[11] it is expected that such localized Langmuir perturbations would be formed by such a process and that electromagnetic radiation at harmonics of $\omega_e$ would result. Since $v_e/v_b \sim 10^{-2}$ for these streams, the strong turbulence theory is required even for comparatively low levels of wave turbulence $W_0/n_0 T_e \gtrsim 10^{-4}$. Details of the application of these concepts to the type III bursts can be found in Papadopoulos *et al.*,[12] Smith *et al.*,[13] and Goldstein *et al.*[14] A corollary of this work is the prediction that the electrostatic fields associated with the energetic electron streams should display the bursty character of localized solitonlike structures, and observations consistent with these conclusions have, in fact, recently been reported by Gurnett and Anderson.[15] As a result, it is expected that the strong turbulence radiation mechanism described herein could account for the observed properties of the $2\omega_e$ radiation associated with such bursts, and provide an additional test of the theory. It should be noted that while this strong turbulence radiation mechanism does not provide a significant enhancement in radiation efficiencies over that of weakly turbulent processes, radiation scaling is substantially different and helps to explain basic observational features of the emissions.[16] The interested reader is referred to Papadopoulos and Freund,[9] Rowland and Papadopoulos,[17] and Freund *et al.*[18] for more details of the radiation scaling.

## ACKNOWLEDGMENTS

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