JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 86, NO. A11, PAGES 9215–9218, OCTOBER 1, 1981

Scaling of the Beam-Plasma Discharge

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It is shown that the scaling observed in a recent experiment of the beam-plasma discharge is consistent with the assumption that triggering occurs when an instability threshold condition is exceeded for electron plasma waves with $\omega \approx \omega_e$, the electron plasma frequency.

1. INTRODUCTION

The importance of the beam-plasma discharge (BPD) in the feasibility and interpretation of active experiments using electron beams injected in space is very well known [*Galeev et al.,* 1976; *Bernstein et al.,* 1978, 1979]. This type of discharge can be described qualitatively as follows. When an electron beam propagates through a neutral gas, it collisionally produces a plasma with a density comparable to its own. As the beam interacts with the plasma, a two-stream instability sets in, and the electric fields of the excited waves either heat or accelerate some of the plasma electrons to energies comparable with the ionization energy. Subsequently, the collisions of these electrons with the neutral gas atoms lead to an onset of an avalanche breakdown; the neutral gas is ionized over a time comparable to the mean free time of plasma electrons between inelastic collisions.

Although several experiments have been carried out on the BPD [*Getty and Smullin,* 1963; *Alexeff et al.,* 1966; *Kharchenko et al.,* 1962; *Smullin,* 1968; *Cabral et al.,* 1969], a theoretical understanding has not yet emerged. The more recent results of *Bernstein et al.* [1978, 1979] under steady state conditions and long interaction lengths provide us with sufficient details to test against a theoretical model. It is the purpose of this paper to examine whether the scaling of the critical beam current required for ignition I_c, as well as the narrow band emissions observed for $I < I_c$, can be understood on the basis of a model of the two-stream instability with finite boundaries. The plan of the paper is as follows. In the next section we review the pertinent experimental results. Section 3 deals with the theory of the two-stream interaction in a finite geometry. Section 4 presents a model of the BPD scaling, and section 5 discusses the emissions observed for $I < I_c$.

2. REVIEW OF THE EXPERIMENTAL OBSERVATIONS

The experiments were performed in the large vacuum facility at Plum Brook, Ohio, and at the Johnson Space Center. The electron gun produced a cold, moderately high perveance ($k = 10^{-6}$ AV$^{-3/2}$) energetic beam. The ambient magnetic field varied between 0.8 and 1.5 G, and the neutral density varied from 3×10^{10} to 10^{12} cm^{-3}. The detailed experimental results can be found in *Bernstein et al.* [1978, 1979]. However, the main features were the following:

1. The BPD appeared at a critical current I_c, which scaled as

$$I_c \propto \frac{V^{3/2}}{B^{0.7}PL} \tag{1}$$

where V is the voltage, B is the magnetic field, P is the pressure, and L is the length of the experiment. During BPD the plasma density increased and was accompanied by high-frequency emissions with frequency $f \gg f_{ce}$ (where f_{ce} is the electron cyclotron frequency).

2. For $I < I_c$, radiation was observed with $f \cong nf_{ce}$. At low pressures and currents the level of emissions was seen to depend exponentially on I. As I was increased, this dependence was greatly reduced. On the other hand, if I became greater than I_c, the BPD would take place. However, f_{ce} radiation was not a necessary precursor for the BPD. At higher pressure levels, f_{ce} emissions were not observed prior to BPD.

3. THEORETICAL CONSIDERATIONS

Consider the interaction of a nonrelativistic cold electron beam, with velocity v_b, density n_b, and radius r_0, with a plasma of density n and radius r_0 in the presence of a magnetic field B_0. The dispersion relation is given by [*Apel,* 1969; *Simpson and Dunn,* 1966; *Bogdankevich and Rukhadze,* 1971]

$$k_z^2\left(1 - \frac{\omega_e^2}{\omega^2}\right) + k_\perp^2\left(1 - \frac{\omega_e^2}{\omega^2 - \Omega_e^2}\right) - \frac{k_z^2\alpha\omega}{(\omega - k_zV_b)^2} - \frac{k_\perp^2\alpha\omega_e^2}{(\omega - k_zV_b)^2 - \Omega_e^2} = 0 \tag{2}$$

where $\omega_e^2 = 4\pi n_e^2/m$, $\alpha = n_b/n$, $\Omega_e = eB/mc$, $k_\perp^2 = \mu_{sl}^2/r_0^2$, and μ_{sl} are the roots of the Bessel function $J_l(\mu_{sl}) = 0$. Neglecting the beam contribution in the first approximation ($\alpha \to 0$), we determine the spectrum of oscillations of a bounded plasma as

$$\omega_{1,2}^2(k_z) = \frac{1}{2}(\omega_e^2 + \Omega_e^2) \pm \left[\frac{1}{4}(\omega_e^2 + \Omega_e^2)^2 - \omega_e^2\Omega_e^2\cos^2\theta\right]^{1/2} \tag{3}$$

where

$$\cos\theta \equiv \left[\frac{k_z^2}{k_z^2 + k_\perp^2}\right]^{1/2}$$

As seen from (2) the contribution of the beam is maximal near the straight lines $\omega = k_zV_b$ and $\omega = k_zV_b \pm \Omega_e$, where the strong interaction occurs.

The instability at the intersections of the lines $\omega = k_zV_b$ with the branches of the eigenoscillations $\omega_{1,2}(k_z)$ is usually called

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Paper number 1A1001.
0148-0227/81/001A-1001$01.00

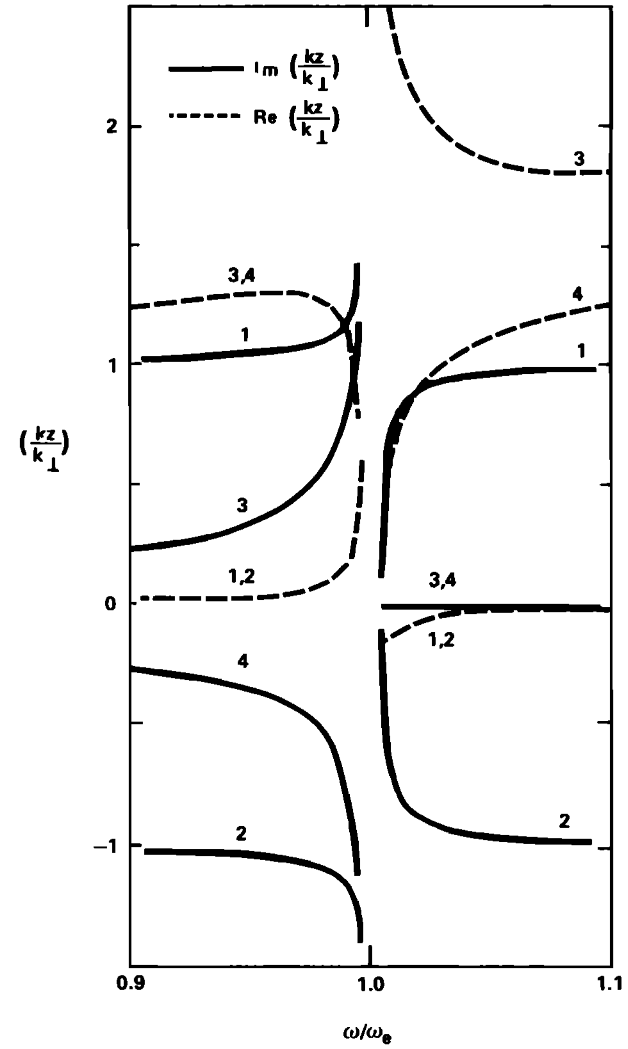

Fig. 1. Four k_z roots of the dispersion equation (8) for real frequency ω near ω_e. Other parameters are $\alpha = 0.01$, $\omega_e/\Omega_e = 6$ and $\omega_e/k_\perp V_b \approx 1.38$.

the two-stream or Cerenkov instability; the instability at the intersections of $\omega = k_z V_b \pm \Omega_e$ and $\omega_{1,2}(k_z)$ is called the cyclotron instability. We focus here on the Cerenkov instabilities, since the cyclotron instabilities are substantially weaker and more susceptible to thermal effects due to their shorter wavelength [*Linson and Papadopoulos*, 1980].

In examining the Cerenkov instabilities, we note that the lower branch is unstable if

$$\min(\omega_e, \Omega_e) > \frac{\mu_{sl} V_b}{r_0} \geq 2.4 \frac{V_b}{r_0} \tag{4}$$

This instability is always convective. The upper branch, in the absence of dissipation, has no threshold. However, for absolute or almost absolute instability, one can compute a length dependent threshold condition. This can be seen by maximizing the inverse of the growth length Im d_z. We derive below the conditions under which this occurs. Equation (2) for Cerenkov excitation can be written as

$$(k_z^2 - K^2)\left(k_z - \frac{\omega}{V_b}\right)^2 = \alpha \frac{\omega_e^2}{V_b^2} \frac{k_z^2}{[1 - (\omega_e^2/\omega^2)]} \tag{5}$$

where

$$k_{z1,2} = K \tag{6}$$

and

$$K^2 \equiv -k_\perp^2 \frac{\{1 - [\omega_e^2/(\omega^2 - \Omega_e^2)]\}}{[1 - (\omega_e^2/\omega^2)]} \tag{7}$$

Notice that the roots $k_{z1,2}$ of (6) correspond to the roots $\omega_{1,2}$ of (2). Examining (8) under the double synchronism condition $k_z \approx \omega/V_b$, $\omega \approx \omega_{1,2}$, we find

$$\left(k_z - \frac{\omega_{1,2}}{V_b}\right)^3 = \frac{\alpha}{2} \frac{\omega_e^2}{V_b^2} \frac{k_{z1,2}}{[1 - (\omega_e^2/\omega_{1,2}^2)]} \tag{8a}$$

or

$$\left(k_z - \frac{\omega_{1,2}}{V_b}\right)^3 = \frac{\alpha}{2} \frac{\omega_e^2}{V_b^2} \frac{ik_\perp\{1 - [\omega_e^2/(\omega_{1,2}^2 - \Omega_e^2)]\}^{1/2}}{[1 - (\omega_e^2/\omega_{1,2}^2)]^{3/2}} \tag{8b}$$

It is obvious from (8) that the Im k_z for real $\omega_{1,2}$, due to the strong dependence of the denominator on the $\omega_{1,2} \approx \omega_e$ resonance, becomes maximum when the frequency ω_1 or ω_2 approaches ω_e. Furthermore, in examining (3) we notice that ω approaches ω_e only when $\cos^2\theta$ approaches unity, or equivalently when

$$\frac{k_\perp^2}{k_z^2} \ll 1 \tag{9}$$

Thus, with $k_z \approx \omega_e/V_b$ and $k_\perp = 2.4/r_0$, (9) yields the following condition:

$$\omega_e > \frac{2.4 V_b}{r_0} \tag{10}$$

To illustrate the existence of large amplification we solved numerically (5), for the specific case where $\alpha = 0.01$, $\omega_e/\Omega_e = 6$ and $\omega_e/k_\perp V_b \approx 1.38$. It is seen from Figure 1 that large amplification is achieved for $\omega_1 \rightarrow \omega_e$, which corresponds to the condition given by (10).

In summarizing the results of our linear analysis, we find that (1) waves with frequency ω_e can be excited if the condition given by (10) is satisfied, independently of the ω_e/Ω_e ratio, and (2) if in addition to (10), $\omega_e > \Omega_e$, then the ω_e instability gives very large amplification. A nonlinear analysis of the energy dissipation and the ionization avalanche will be presented elsewhere. In this preliminary report, we attempt to test the hypothesis that the threshold scaling is consistent with the excitation of an almost absolute instability with frequency near ω_e, i.e., (10). The reasoning for the hypothesis is that [*Papadopoulos*, 1981; *Papadopoulos and Rowland*, 1977; *Linson and Papadopoulos*, 1980] in the case that the unstable waves are near ω_e and the wave energy density $W/nT = E^2/8\pi nT > k^2\lambda_D^2$, the energy deposition is completely nonlinear (i.e., soliton collapse), and results in the formation of very energetic (i.e., 100 eV like) suprathermal tails. These tails act as the source of the avalanche ionization and in addition provide axial confinement by building several tens of volts potential sheaths. A more detailed description of this process will be published elsewhere [*Papadopoulos*, 1981].

4. BPD SCALING

The purpose of this section is to test qualitatively the implications of the assumption that the BPD is consistent with the

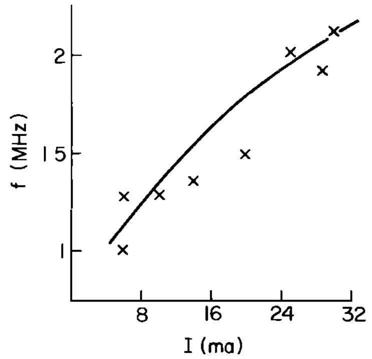

Fig. 2. Experimentally observed frequencies as a function of I for $I < I_c$. The solid line is $f = \omega_1/2\pi = (\Omega_e^2 + \omega_e^2 \cos^2 \theta)^{1/2}/2\pi$. ω_e is determined from the collisional ionization model (equation (12)).

linear ω_e instability trigger condition given by (10). The time evolution of the electron density n_e prior to BPD will be given by

$$\frac{\partial n_e}{\partial t} = A n_b N \langle \sigma V_b \rangle L - \frac{\partial n_e}{\tau} \qquad (11)$$

where N is the density of neutral particles, $\langle \sigma V_b \rangle$ the ionization rate of the beam electrons, is a length normalization factor for ionization with mean free path longer than L and τ the particle confinement time. The maximum density n_e expected from (11) is

$$\frac{n_e}{n_b} = AN\langle \sigma V_b \rangle L\tau \qquad (12)$$

Combining (10) and (12), we find that

$$I_c = C\frac{V_b^3}{\langle \sigma V_b \rangle L\tau} \approx C\frac{V^{3/2}}{PL\tau}$$

where C is a proportionality constant. The observed scaling as shown in (1) is reproduced by considering that $\tau \sim B$, i.e., a Bohm like diffusion. This type of loss is consistent with our ω_e model, since it produces axial plugging. We should note that there is experimental evidence for $1/B$ dependence of the diffusion coefficient [*Szuszczewicz,* 1979] and an independent check of the ω_e/Ω_e ratio resulting from (10) tests very well against the experimental values [*Szuszczewicz et al.,* 1981].

5. Wave Observations

In this section we examine the wave emissions created for currents lower than I_c. In this case also the modes excited are in the upper branch. In a simplified form,

$$\omega^2 \cong \Omega_e^2 + \omega_e^2$$

since $\theta \approx \pi/2$.

In Figure 2 are plotted the experimentally measured frequencies for $E_b = 0.6$ keV as a function of current. The line is the frequency of the upper branch with ω_e determined from (10), with $\theta = 88°$, and results from assuming the smallest $k_\perp$ that satisfied the radial boundary conditions and that $k_z = \Omega_e/V_b$.

We can easily understand the disappearance of the Ω_e radiation when the distance for the first node of the beam became longer than the length of the cylinder. The first node occurs at $\Delta L = V_b/\Omega_e$. When ΔL becomes greater than L, the length of the cylinder, $k_r < k_{z,\min}$ and the instability disappears. This effect was seen by *Maxum and Trivelpiece* [1965].

Given the initial injection spread, this instability may be kinetic at the start as discussed in section 3. In that case, convective effects could determine the saturation level of the instability. The maximum wave energy would be

$$W_{\max} = W_0 \exp(\gamma L/v_g)$$

If the amount of energy lost by the beam is small, $\Delta V_b/V_b$ will be determined by the initial injection spread. Using the model presented earlier, this field energy dependence upon current and beam energy is of the form

$$\ln(W_{\max}/W_0) \sim I^{3/2}/E_b^{2\,9}$$

When $(\Delta E_b/E_b) > 2(\Delta V_b/V_b)_0$, the energy loss of the beam will determine the velocity spread and hence the growth rate. In this case, $W_{\max} \sim I^{3/4}/E_b^{1/2}$. Therefore, as the current is increased one would first see an exponential increase in the radiation. However, at higher radiation and currents levels one should see a weaker current dependence. Of course if $I > I_c$, the BPD will take place.

In Figure 3 is plotted the observed beam distribution function where $I < I_c$ and Ω_e radiation is observed. The initial average beam velocity, V_{b0}, is marked. The predominant modification of the beam distribution function is for electrons with $v \leq V_{b0}$. The slope of the distribution $(\partial f/\partial v)$ is also flatter for $v \leq V_{b0}$. These effects are characteristic of a kinetic not cold plasma instability.

6. Summary and Conclusions

In the present paper we have demonstrated that the scaling observed in BPD is consistent with the assumption that triggering occurs when an instability near ω_e is excited. The threshold condition is then determined by the injected beam. In the subthreshold range the excitation and nonlinear amplitude of the f_{ce} waves is consistent with excitation of the kinetic instability of the upper branch (Figure 2) and convective saturation. A more detailed investigation of the nonlinear dynamics, including electron heating and ionization, is presently under way and will be presented elsewhere.

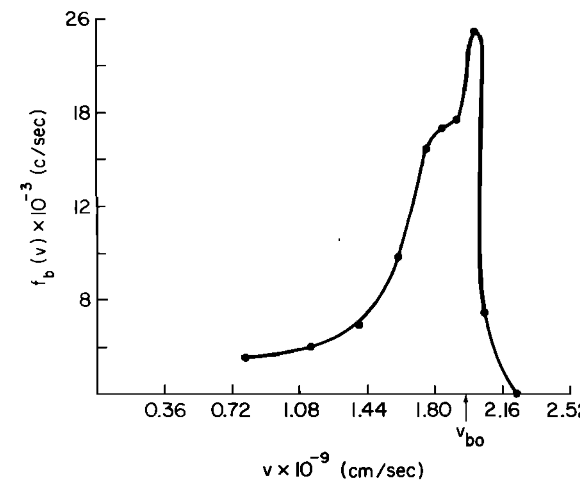

Fig. 3. Experimentally observed beam velocity distribution for $I < I_c$. V_{b0} is the average initial beam velocity. The modification of the beam velocity distribution is consistent with a kinetic interaction of the beam with the upper hybrid mode (ω_1) for $I < I_c$.

Acknowledgments. It is a pleasure to acknowlege the contribution of W. Bernstein in steering our interest in the experiment and helping us with long discussions of the results and their interpretation. Several important suggestions in our work due to L. Linson and P. Kellogg are greatly appreciated. This work was supported under Rice University subcontract NAS8-33777-1.

The Editor thanks P. J. Kellogg for his assistance in evaluating this paper.

REFERENCES

Alexeff, I. W., W. D. Jones, R. V. Neidigh, W. F. Peed, and W. L. Stirling, Plasma heating and burnout in the beam plasma interaction, in *Plasma Physics and Controlled Nuclear Fusion Research,* vol. II, International Atomic Energy Agency, Vienna, Austria, 1966.

Apel, J. R., Experimental studies of linear beam-plasma instabilities in a magnetic field, *Phys. Fluids, 12,* 291, 1969.

Bernstein, W., H. Leinbach, P. Kellogg, S. J. Monson, T. Hallinan, O. K. Garriott, A. Konradi, J. McCoy, P. Daley, B. Baker, and H. R. Anderson, Electron beam experiments: The beam-plasma discharge at low pressures and magnetic field strengths, *Geophys. Res. Lett., 5,* 127–130, 1978.

Bernstein, W., H. Leinbach, P. J. Kellogg, S. J. Monson, and T. Hallinan, Further laboratory measurements of the beam-plasma discharge, *J. Geophys. Res., 84,* 7271–7278, 1979.

Bogdankevich, L. S., and A. A. Rukhadze, Excitation of longitudinal electromagnetic waves in a restricted plasma by injection of REB, *Usp. Fiz. Nauk, 103,* 609, 1971. (Also *Sov. Phys. JETP Engl. Trans., 35,* 126, 1972.)

Briggs, R. J., *Electron-Stream Interactions With Plasmas,* MIT Press, Cambridge, Mass., 1964.

Cabral, J. H., J. Hopman, F. G. Insinger, and W. Ott, Time resolved beam distribution functions in a beam-plasma experiment, in *Plasma Physics and Controlled Nuclear Fusion Research,* vol. II, International Atomic Energy Agency, Vienna, Austria, 1969.

Galeev, A. A., E. V. Mishen, R. Z. Sagdeev, V. D. Shapiro, and I. V. Shevchenko, Discharge in the region around a rocket following injection of electron beams into the ionosphere, *Sov. Phys. Dokl. Engl. Transl., 21,* 641–643, 1976.

Getty, W. D., and L. D. Smullin, Beam-plasma discharge: Buildup of oscillations, *J. Appl. Phys., 34,* 3421–3429, 1963.

Kharchenko, I. F., Ya. B. Fainberg, R. M. Nikoloev, E. A. Kornilov, E. I. Lutsenko, and N. S. Pedenko, The interaction of an electron beam with a plasma in a magnetic field, *Sov. Phys. Tech. Phys. Engl. Transl., 6,* 551–553, 1962.

Linson, L. M., and K. Papadopoulos, Review of the status of theory and experiment for injection of energetic electron beam in space, *Tech. Rep. LAPS 69,* Science Applications, Inc., McLean, Va., 1980.

Maxum, B. J., and A. W. Trivelpiece, Two-stream cyclotron and plasma interactions, *J. Appl. Phys., 36,* 481, 1965.

Ottinger, P. F., Equilibrium and nonlocal properties of bounded relativistic electron beam-plasma systems, Ph.D. thesis, *Tech. Rep. 78-030,* Univ. of Md., College Park, 1977.

Papadopoulos, K., Theory of the beam plasma discharge, in *Proceedings of NATO Symposium Geilo, Norway,* Brussels, in press, 1981.

Papadopoulos, K., and H. L. Rowland, Collisionless effects on the spectrum of secondary auroral electrons at low altitudes, *J. Geophys. Res., 83,* 5768–5772, 1978.

Simpson, J. E., and D. A. Dunn, Cold beam-plasma interaction theory for finite transverse dimensions and finite magnetic fields, *J. Appl. Phys., 37,* 4201, 1966.

Smullin, L. D., Recent results on the beam plasma discharge, in *A Survey of Phenomena in Ionized Gases,* 129 pp., International Atomic Energy Agency, Vienna, Austria, 1968.

Szuszczewicz, E. P., Plasma diffusion in a space-simulation beam-plasma-discharge, *Geophys. Res. Lett., 6,* 201, 1979.

Szuszczewicz, E. P., C. S. Lin, D. N. Walker, W. Bernstein, and K. Papadopoulos, Ignition of the beam-plasma-discharge and its dependence on electron density, submitted to *J. Geophys. Res.,* 1981.

(Received August 28, 1980;
revised June 10, 1981;
accepted June 16, 1981.)