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STOCHASTIC ACCELERATION OF LARGE M/Q IONS BY HYDROGEN CYCLOTRON WAVES IN THE MAGNETOSPHERE

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Abstract. It is shown that in hydrogen dominated multi-ion plasmas supporting coherent hydrogen cyclotron waves, the minority ion species with large M/Q are preferentially accelerated and the maximum energy achieved scales as $(M/M_{H^+})^{5/3}$. The importance of this scaling to O^+ acceleration in the auroral zones and to other high energy heavy ion observations in the earth's and Jupiter's magnetospheres is discussed.

Introduction

One of the most spectacular and puzzling observations of high latitude magnetospheric physics has been the discovery (Shelley, et al. 1972) of large fluxes of upstreaming O^+ ions with energies of several keV. Further studies (Shelley, et al., 1976; Sharp, et al., 1977; Mozer, et al., 1977; Kintner, et al., 1978) indicate a strong correlation of fluxes of energetic ions with measurements of coherent electrostatic hydrogen cyclotron (EHC) waves. An additional observational fact is the virtual absence of upgoing helium ions in situations where H^+ and O^+ fluxes are of comparable intensity, ranging up to 10^8 cm^{-2} sec^{-1} ster^{-1} keV^{-1} at the peak, which occurs at an energy of ~ 1 keV.

These observations have posed a series of physics questions such as, the excitation mechanism of EHC, the reason for their coherence, the heating processes for H^+, the acceleration processes for the O^+, the source of conic ion distributions, etc. The purpose of this letter is to address one key question concerning the acceleration mechanism of large ion mass to charge ratio $eM/QM_{H^+} \gg 1$ particles. Such particles form the minority constituents of many magnetospheric multi-ion plasmas. In general they do not affect the collective mode structure of the plasma due to their small abundance ratios; their behavior, however, in the presence of plasma turbulence can serve as an important diagnostic measurement of the properties of the excited plasma turbulence. We examine below the acceleration of large M/Q ions in the presence of a coherent large amplitude EHC wave. It should be noted that calculations based on quasilinear or orbit modification theory (Palmadesso, et al., 1974; Dakins, 1976; Lysak, et al., 1980), cannot produce acceleration past the wave phase velocity, and only particles satisfying the resonance conditions can participate in the interaction. In our analysis we show that for wave amplitudes above a certain threshold the orbits of all particles with velocity $v_\perp > \omega/k_\perp$ (ω is the wave frequency and $k_\perp$ is the transverse wave-number) become stochastic and thereby can be accelerated by the wave, without any adiabatic invariance restrictions. The most important conclusion of this study is the strong dependence of the maximum energy achieved by this mechanism on M/Q; this allows for preferencial acceleration of heavy ions.

Physical and Mathematical Considerations

Consider a plasma whose main ionic component is H^+, but includes a variety of minority ions with large M/Q with densities such that they do not significantly affect the dispersive properties of the EHC waves. We examine the acceleration of minority ions with $M/M_{H^+} > 1$, in the presence of a coherent EHC wave with frequency $\omega \approx \Omega_H$ (Ω_H is the H^+ cyclotron frequency) propagating perpendicular to a uniform magnetic field $\vec{B} = B_o\hat{e}_z$. The EHC wave is given by

$$\vec{E} = (-k_\perp\Phi)\ \hat{e}_y \cos\ (k_\perp y-\omega t-\varphi) \tag{1}$$

This field is taken to be imposed, supported by the bulk H^+ ions and the electrons. This is consistent with the auroral observations of Mozer, et al. (1977). The source of the EHC waves and their effect on the H^+ are not considered here. The equations of motion for an ion with $\mu = M e/M_{H^+} Q$, in the frame where the canonical momentum $P_x = 0$, are

$$\ddot{y} + y = \alpha \cos\ (y-\upsilon t-\varphi) \tag{2}$$

$$\dot{x} = y \tag{3}$$

where the length is normalized to $k_\perp^{-1}$ and the time to $\Omega_i^{-1} = Mc/QB_o$, $\upsilon = \mu\ \omega/\Omega_H$, $\alpha = (e\Phi/2T_H)(k_\perp R_H)^2\ \mu$ and T_H, R_H are the hydrogen temperature and gyroradius, respectively.

The appearance of stochasticity for the system of Eqs. (2) and (3), when the value of α exceeds a certain threshold, has been confirmed recently by various authors, who examined the particular cases of ion-acoustic and lower-hybrid heating (Smith and Kaufman, 1975; Fukuyama, et al., 1977; Karney and Bers, 1977). The analysis is based on an examination of the Hamiltonian for the system of Eqs. (2) and (3).

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The Hamiltonian is given by (Karney, 1978)

$$H = \ell_1 + \upsilon\, \ell_2 - \alpha \sin \left[(2\ell_1)^{1/2} \sin q_1 - q_2\right] \quad (4)$$

where (ℓ_1, q_1) and (ℓ_2,q_2) are conjugate action-angle variables of the unperturbed (i.e., $\alpha = 0$) system, $y=(2\ell_1)^{1/2} \sin q_1$ and $x = -\ell_2 - (2\ell_1)^{1/2} \cos q_1$. In Eq. (4) q_2 is the wave phase, so that H describes two coupled harmonic oscillators; (ℓ_1, q_1) describe the ion motion in a magnetic field and (ℓ_2,q_2) describe the wave; and they are coupled by the last term of Eq. (4).

A determination of the general analytic properties of Eqs. (2) through (4) is beyond the scope of the present letter. The key result that we want to emphasize here is the value of α for onset of stochasticity, which can be found by application of the surface of section method (Zasalvskii and Chirikov, 1972). This was performed by Karney (1978) for lower-hybrid waves. The method consists of choosing a cross-section defined by $q_1 = \pi$ and numerically plotting the normalized velocity $r \equiv (2\ell_1)^{1/2}$ against q_2 for each crossing of the $q_1 = \pi$ plane. For $\alpha = 0$ the trajectories lie on horizontal straight lines, since ℓ_1 is conserved. As α is increased, keeping υ constant, the trajectories become more complicated, many lying on chains of "islands". The order of the islands is given by the number of cyclotron orbits it takes for the ion to return to the island it started on. Increasing α further, the orbits become stochastic over a large region of velocity space and therefore the ion can be accelerated by moving from island to island.

The physical picture of the stochastic motion is as follows: the presence of the wave causes a nonlinear change in the cyclotron frequency, $<q_1>$. This in turn causes the ratio of the wave frequency to the cyclotron frequency to become a "simple" rational number in certain regions of velocity space, leading to fixed points. A chain of islands forms around half of these fixed points; the other half are hyperbolic fixed points on the separatrices between islands. It is known from non-linear mathematics (Ford, 1975) that any small perturbation causes the motion in the vicinity of the separatrix to become stochatic. At amplitudes above the "stochasticity threshold", α_{th}, the chains of islands of various orders overlap, leading to diffusion in velocity space (i.e., stochastic acceleration). The overlapping of the islands with increasing α is controlled by the appearance of higher order islands and by their increase in size. The threshold can only be found by a combination of analysis and computer simulation of Eqs. (2) and (3) with various initial conditions. It is given (Karney and Bers, 1977; Karney, 1978) by

$$\alpha > \alpha_{th} = \tfrac{1}{4}\, r \left[\upsilon \left|H_\upsilon^{(1)\prime}(r)\right|\right]^{-1} \text{ for } r \gg \upsilon \quad (5)$$

where $H_\upsilon^{(1)}$ is the Hankel function of the first kind and order υ. For the mathematical details we refer the interested reader to the references given above. Another result of the analysis is the obvious one, that only particles within a trapping distance $\sqrt{\alpha}$ of the resonance velocity can enter the stochastic region; this gives the lower limit of the stochastic region as

$$r = \upsilon - \sqrt{\alpha}\ . \quad (6)$$

Application to the Auroral Regions

The most important result is given by Eq. (5), which can be used to provide a measure of the maximum velocity $r_{max} \equiv k_\perp V_{\perp max}/\Omega_\perp$ achieved by stochastic acceleration for constant υ and α. It is found by taking the limit $r \gg \upsilon$ and is (Karney, 1978)

$$r_{max} = (4\alpha\upsilon)^{2/3} \left(\frac{2}{\pi}\right)^{1/3} \quad (7)$$

Using Eq. (7) to compute the maximum energy $\epsilon_{max} = \frac{1}{2} M V_\perp{}^2{}_{max} \sim r^2{}_{max}$ and returning to our physical variables we find

$$\frac{\epsilon_{max}}{T_H} = 4\mu^{5/3} \left(\frac{Q}{e}\right) \left[\frac{k_\perp R_H}{\pi} \left(\frac{e\Phi}{T_H}\right)^2 \left(\frac{\omega}{\Omega_H}\right)^2\right]^{2/3}. \quad (8)$$

For ECH with $e\Phi/T_H \approx \frac{1}{2}$ to 1/3, as is measured in the aurora, and $\omega/\Omega_H = 1.2$, $k_\perp R_H \approx 1.7$ (Drummond and Rosenbluth, 1962) we find that

$$\epsilon_{max}/T_H \approx (M/M_{H^+})^{5/3} (e/Q)^{2/3}. \quad (9)$$

The paremeter $\alpha = \frac{1}{2} (k_\perp R_H)^2\, \mu\, e\Phi/T_H = 8$ to 12 for 0^+ and the minimum stochasticity threshold (Karney, 1978) $\alpha_{th} = \frac{1}{4}\, \upsilon^{2/3} = 1.79$, so that the 0^+ ions are in the stochastic regime by almost an order of magnitude. Therefore, 0^+ can be accelerated to energies of over 100 times the ambient hydrogen temperatures. If $T_H \approx$ 10-20eV then 0^+ will have ϵ_{max} of several keV. More importantly, lower M/Q ions such as He^+ will only reach energies of the order of 100 eV. The time required to accelerate the ions to the maximum energy is considerably shorter than the time in which the ions are accelerated or drift out of the acceleration region. Thus, the ions will be able to gain the maximum energy calculated above before escaping. This conclusion is consistent with the available data (Shelley, et al., 1976; Sharp, et al., 1977; Mozer, et al., 1972; Kintner, et al., 1978).

In order to determine the flux one has to know the number of particles that satisfy Eq. (6), so that they enter the stochastic acceleration region. This in turn depends on the plasma composition in the acceleration region. Questions connected with the distribution function of the accelerated 0^+ (i.e. conics or beams) depend on assumptions about the presence of parallel electric fields and the force due the magnetic field gradient $(MV_\perp{}^2/2B)\,(\partial B/\partial r)$ which will further increase the energy of the particles. Such calculations coupled with numerical solutions of Eqs. (1) through (3) for various initial distribution and values of μ are presently under way and will be published elsewhere.

Summary and Conclusions

The purpose of the present letter is to note that for EHC waves above a certain threshold amplitude, minority ion species with large M/Q can be accelerated to substantial energies, contrary to predictions of quasi-linear and orbit modification theories. The most important result is the scaling of the maximum energy ϵ_{max} with M/Q which appears in Eq. (9) as $(M/M_H)^{5/3}$. This observation may shed light on the puzzle of the selective O^+ acceleration in the auroral regions. Moreover, the consequences of the scaling may be more important when dealing with observations of very energetic large M/Q ions in Jupiter's magnetosphere (Krimigis, et al., 1980), other magnetospheric regions (Cornwall and Schultz) or active plasma injection experiments. Accelerations of 10^3 - 10^4 are expected for the heavier elements. It should be noted that the above conclusions are not limited to acceleration by EHC but may be extended directly to other low frequency electrostatic waves. We will discuss these results in future publications.

The importance of the physics of heavy ions in the magnetosphere has been emphasized in an excellent review by Cornwall and Schultz(1979). We feel that our preliminary letter indicating the preferential acceleration of large M/Q ions in the presence of the electrostatic turbulence, will open new avenues of investigation involving plasma and stochastic collective acceleration mechanisms. We are presently examining the possibility of utilizing the difference equations resulting from Eqs. (1) and (2) to derive a diffusion equation which will describe the acceleration of ions above the stochasticity threshold as a function of M/Q and will determine the spectral shape of their distributions.

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