# One-dimensional direct current resistivity due to strong turbulence

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Strong Langmuir turbulence acts to generate localized spiky fields and ion cavities. It is shown that these cavities generate potential barriers that can trap a significant fraction of the electrons. For example, a 20% cavity traps 50% of the electrons. This can seriously change the direct current properties of a collisionless plasma and lead to the appearance of anomalous dc resistivity.

## I. INTRODUCTION

The problem of anomalous plasma dc resistivity in one dimension is one of the oldest basic plasma physics questions. Its resolution has eluded the plasma community despite intensive theoretical and experimental efforts.[1,2] Besides its fundamental nature, the question of anomalous resistivity is of utmost practical importance for laboratory plasma heating and for energy dissipation and particle acceleration in space plasmas.[2] In weak turbulence models, the presence of a field aligned electron current in the plasma with a drift velocity $v_d$, higher than a threshold, excites density fluctuations which then scatter the electrons with a collision frequency $\upsilon^*$ larger than the small angle Coulomb frequency $\upsilon$, thereby producing a higher dissipation rate. This provides a consistent and experimentally satisfactory picture if $\upsilon^* > \Omega_e$, where $\Omega_e$ is the electron cyclotron frequency. However, when $\upsilon^* < \Omega_e$, the electron magnetic moment is conserved and the electrons should be treated as one dimensional. In this case, as clearly shown by Petviashvilli,[3] the formation of a plateau in the electron distribution function precludes any resistivity enhancement beyond a few percent of the classical value. It is the purpose of this paper to demonstrate that inclusion of strong turbulence effects can remove most of the fundamental difficulties encountered in one-dimensional anomalous resistivity and produce a consistent and satisfactory picture. The basic idea is that density cavities in the ion background cause the existence of low frequency potential barriers. When a dc electric field is applied, these barriers can prevent the free streaming acceleration of a significant fraction of the electrons.

An important step forward in understanding when anomalous dc resistivity can appear, was the observation of Papadopoulos and Coffey[4] that the low frequency density fluctuations could be generated by means other than current-driven instabilities. This removes the threshold requirement on $v_d$ of the ambient plasma. In their paper the low frequency fluctuations appeared due to the presence of suprathermal electron beams. Such situations exist both in relativistic beam plasma heating and in auroras due to the presence of the energetic precipitating electrons. It was noted[4,5] that the presence of the beams creates electron plasma oscillations whose ponderomotive force, acting on the plasma, drives low frequency density fluctuations. In going into the one-dimensional situation and within the framework of weak turbulence theory the same difficulties reappear as in the current-driven case. However, under strong turbulence conditions [i.e., $W/nT > (k\lambda_D)^2$, where $W$ is the field energy in the beam resonant waves and $k$ is their wavenumber], the density fluctuations have been shown to form large localized density cavities (cavitons) in which the high frequency field is trapped (solitons).[6-9] Figure 1 shows a typical structure as seen in a computer simulation of such cavities produced by the high frequency ($\omega_{pe}$) waves due to the existence of a beam (see also Figs. 13 and 16 of Ref. 8).

While our original work centered on such beam-generated cavities which occur in many situations of current physical interest, the basic physical effects noted here will appear whenever finite amplitude ion cavities exist in the ambient plasma. A description of the interaction of the background electrons with the cavities requires a local theory going beyond the limit of weak turbulence quasi-linear wave-particle interaction. In view of the analytical complexity of the subject we present below a phenomenological theory supplemented by computer simulations.

## II. THEORY

Assume that there is a density cavity in the ion background with a depth $\delta n_i/n_0$, where $n_0$ is the average ion density. In order to maintain charge neutrality at low frequency, there must exist a low frequency potential internal to the plasma, $\delta\phi$, large enough to exclude a

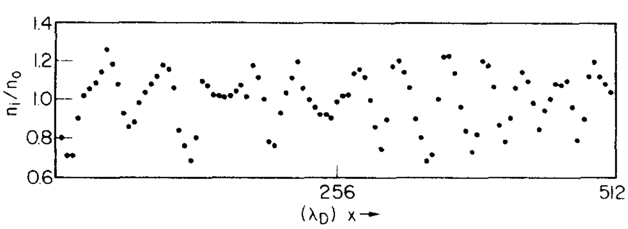


FIG. 1. Typical ion density fluctuations generated by the presence of a high velocity electron beam as seen in a particle simulation.

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fraction of the electrons equal to $\delta n_i$. This means that inside the cavity

$$\left(1-\frac{\delta n_i}{n_0}\right)n_0=\int_{-\infty}^{\infty} f_{e,\text{in}}(v)\,dv. \qquad (1)$$

From the Vlasov equation, since $f$ is constant along lines of constant energy we can determine $f_{e,\text{in}}$ in terms of $f_{e,\text{out}}$. If $f_{e,\text{out}}$ is a nondrifting Maxwellian then $(1-\delta n_i/n_0)=\exp(-\delta\phi/2)$, where $\delta\phi$ is normalized to the electron thermal energy and $v_{te}\equiv 1$. Defining a trapping velocity $v_{\text{tr}}^2\equiv\delta\phi$, the fraction of trapped electrons is

$$\frac{\delta n_{e,\text{tr}}}{n_0}=\left(\frac{2}{\pi}\right)^{1/2}\int_0^{v_{\text{tr}}}\exp(-\tfrac{1}{2}v^2).$$

This is plotted in Fig. 2 as a function of cavity depth. As can be seen in Fig. 2, the fraction of trapped electrons is greater than the cavity depth. For example, a cavity with $\delta n_i/n_0=0.2$ generates a low frequency potential that traps or blocks half of the electrons. One might assume that in order to maintain quasi-neutrality the plasma would generate a potential just large enough so that $\delta n_{e,\text{tr}}=\delta n_i$. However, as noted earlier, a collisionless plasma will conserve flux along a constant energy surface. As particles pass over the potential barrier they slow down and bunch, increasing the local density. In order to maintain charge neutrality, the plasma must raise the height of the potential barrier so that $\delta n_{e,\text{tr}}>\delta n_i$.

On the basis of this picture we can compute the electron response to an external dc field. In the absence of any cavities, the electrons will free stream with an average velocity $v_{fs}(t)=(q/m)Et$ when an external dc field is applied. The presence of the density cavities divides the electrons into two classes (Fig. 3). The central part of the distribution with $|v|<v_{\text{tr}}$ does not accelerate, since it is reflected by the potential barriers. The tail with initial velocities parallel to $v_{fs}$ being untrapped is displaced by $v_{fs}$; untrapped particles that have an initial velocity opposite to $v_{fs}$ can only slow down to a velocity $|v_{\text{min}}|\simeq(\delta\phi)^{1/2}$. At that point they are scattered for 180° by the potential. This

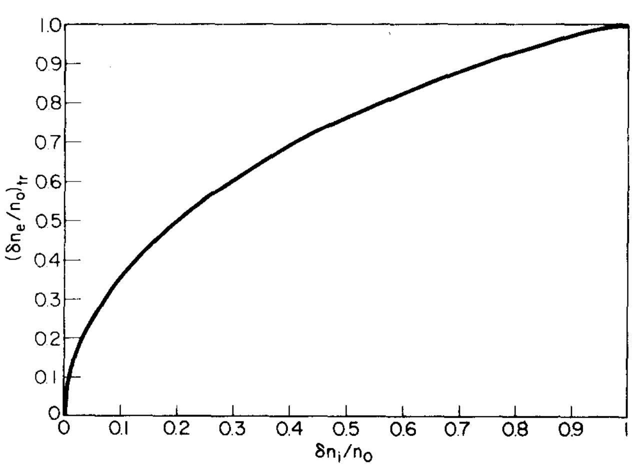


FIG. 2. The fraction of the electron population trapped by an ion cavity of depth $\delta n_i$. Note that a cavity of depth of 0.2 traps approximately half of the electrons.

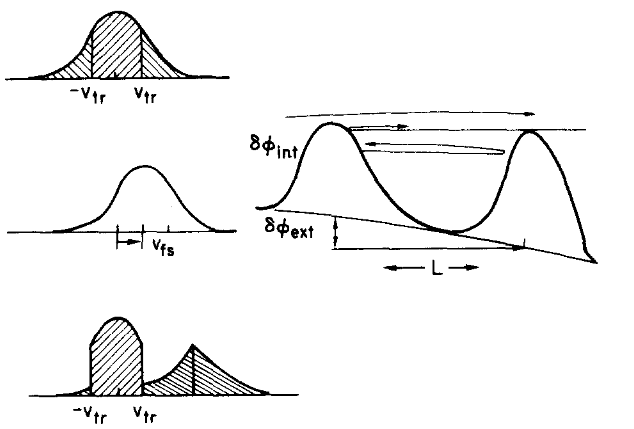


FIG. 3. The presence of a finite amplitude ion cavity causes a colocated low frequency potential, $\delta\phi_{\text{int}}$. When an external potential is applied, $\delta\phi_{\text{ext}}$, a fraction of the electrons are trapped and prevented from accelerating. If we ignore streaming instabilities, the electron velocity distribution will form the shape shown on the left.

means that the electrons in the left-hand tail have undergone a sudden increase in velocity equal to 2 $v_{\text{min}}$. Therefore, the average electron drift velocity is

$$v_d=\left(1-\frac{\delta n_{e,\text{tr}}}{n_0}\right)v_{fs}+2v_{\text{min}}\int_{v_{\text{min}}}^{v_{\text{min}}+v_{fs}}\left(\frac{dv}{(2\pi)^{1/2}}\exp\frac{-v^2}{2}\right). \qquad (2)$$

The maximum contribution from the integral is $(1-\delta n_{e,\text{tr}}/n_0)v_{\text{min}}$. Therefore, on long time scales

$$\dot v_d=(1-\delta n_{e,\text{tr}}/n_0)\,\dot v_{fs}.$$

## III. SIMULATION RESULTS

Here, we present a series of one-dimensional computer simulations showing the effect of ion density cavities on the ability of a collisionless plasma to carry a dc current.

A particle-fluid hybrid code[7] and a Vlasov simulation code[8,10] were used. First, we looked at the acceleration of the plasma by a constant dc electric field (Eq. (2)). To study this case we carried out two groups of runs. We first imposed a fixed ion density fluctuation on the plasma. This allowed us to easily vary $\delta n_i/n_0$ and the shape of the fluctuations. In the next simulations, the ion fluctuations were generated self-consistently by the ponderomotive force of a high frequency long wavelength pump, such as expected in the presence of weak electron beams or a laser. We next looked at the effect of ion cavities on an existing dc current. In these simulations the ion cavities were generated by the collapse of a long wavelength Langmuir wave.

Figure 4 shows the results of the first set of runs. The straight line is the free streaming velocity of the plasma and occurs when $\delta n_i=0$. The curves lines are calculated from Eq. (2) assuming $v_{\text{min}}=v_{\text{tr}}$. Three of the simulations plotted used the particle fluid hybrid code. The electrons were particles. The ions were a fixed fluid background. The small differences in the simulations from the theoretical curves can be at-



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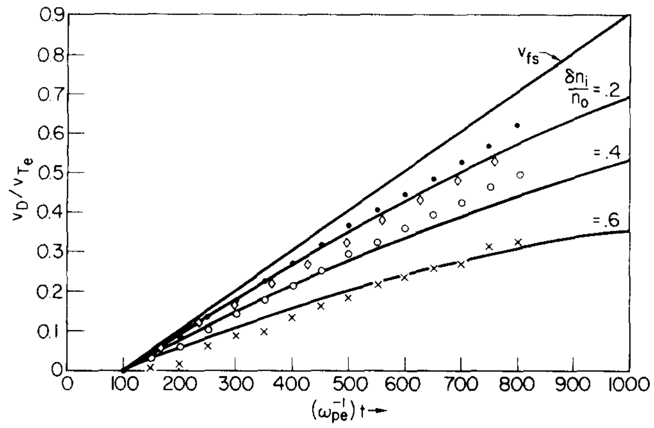


FIG. 4. The electron drift velocity due to a constant dc field. The top line is the free streaming response of the electrons. The three lower lines are the theoretical predictions for different cavity depths as marked, •, ○, × are particle-fluid simulations with $\delta n_i/n_0 = 0.2$, 0.4, and 0.6. ◇ is a Vlasov simulation with $\delta n_i/n_0 = 0.2$.

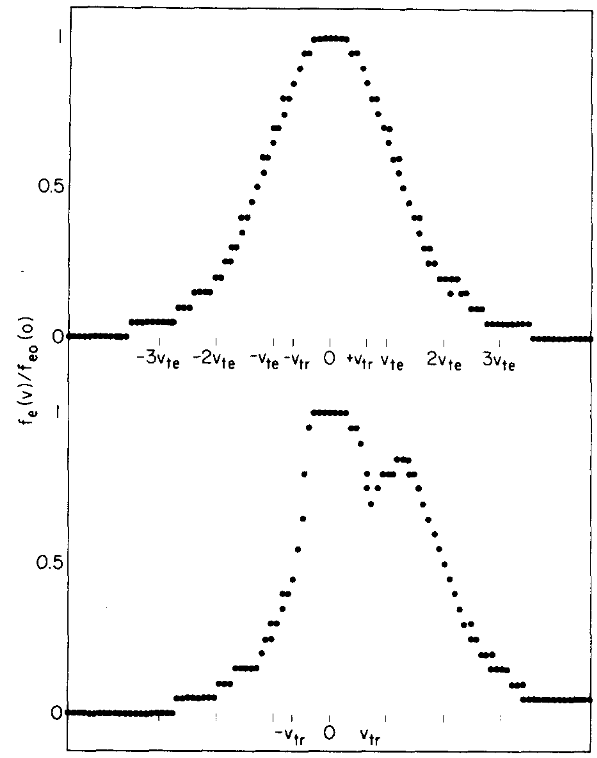


FIG. 5. The electron velocity distribution from the Vlasov simulation shown in Fig. 4, $\delta n_i/n_0 = 0.2$. The top figure shows the initial distribution; the bottom figure shows the distribution at $750\omega_{pe}^{-1}$. The theoretically predicted $v_{tr}$ is marked.

tributed to the electron noise due to the finite number of particles. In the first simulations we ran with what we considered to be a reasonable number of particles per Debye length ($n_0 = 10 \sim 20$); we did not see the enhancement of $\delta n_{e,\mathrm{tr}}$ over $\delta n_i$. As we increased the number of particles per cell the simulation results were in better and better agreement with theory. For the simulations shown in Fig. 4 the number of particles per cell was 400. The appearance of the enhancement is due to the conservation of flux along constant energy surfaces. If the plasma is collisional, this condition is not true. The numerical collision frequency, $\nu$, can be estimated as $(\alpha n_0)^{-1}$, where $\alpha$ depends upon the spatial filtering applied in the code. Based upon test runs with this code, $\alpha \approx 7$. For these particle simulations the width of the cavities is approximately $30\lambda_D$, for $\delta n_i/n_0 = 0.2$, $n_{\mathrm{tr}} \simeq 0.5 v_{te}$; particles with $v \simeq v_{\mathrm{tr}}$ are most slowed by the barrier and contribute most to the enhancement of $\delta n_{e,\mathrm{tr}}$ over $\delta n_i$. From this we can make a rough estimate of the time required for such a particle to pass over the barrier, $t$, and thus we see that $\nu_c t \approx 1$ for $n_0 = 10$. This, of course, is really a lower limit on $\nu_c t$ since the barrier will slow such particles down therefore increasing $t$. As can be seen in Fig. 4 this was actually confirmed by using the noise-free Vlasov code. This code due to its noise free property allowed us to clearly distinguish between trapped and untrapped electrons. Figure 5 shows the electron distribution at $t = 0$ and $\omega_{pe} t = 750$. With $\delta n_i/n_0 = 0.2$, 50% of the particles were trapped in agreement with the theory. Whitfield and Skarsgard[11] observed this splitting of the electron distribution function in particle simulations in which they modeled the effect of a bumpy magnetic field with a fixed sinusoidal density variation.

In order to determine the effects of the cavity width and spacing on the interactions, we initialized the ions in the Vlasov simulations four different ways as shown in Fig. 6. It was found that the interaction was independent of both the width and the spacing and dependent only on the value of the density minimum. These results are consistent with the simplified theory presented here.

We next examined three situations where the ion cavities are self consistently generated due to strong turbulence.

Figure 7 shows the results of a hybrid simulation with $m_i = 128 m_e$ where a kinetic beam plasma instability with growth rate $\gamma = 0.006\omega_{pe}$ and phase velocity $v_{ph} = 81 v_{te}$ was driven. The mode was nonlinearly stabilized[5-9] by creating a set of 10 density cavities (see Fig. 1). The rms value of the density fluctuations was 15%, while the localized cavity depth was 0.2–0.3. An electric field was turned-on at $\omega_{pe} t = 700$. It can be seen that the value of the acceleration was substantially smaller than free streaming.

In another run the Vlasov code was used with a constant dipole pump of energy $E_0^2/8\pi n_p T_{e0} = 0.1$. Such a run simulates the possibility of a microwave or laser pump

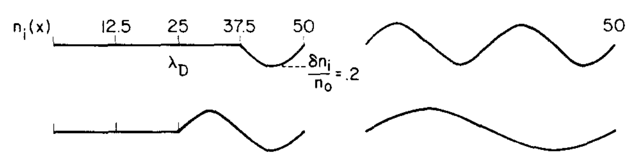


FIG. 6. The different ion densities for the Vlasov simulation shown on Fig. 4. $\delta n_i/n_0 = 0.2$. All four simulations gave the same response.



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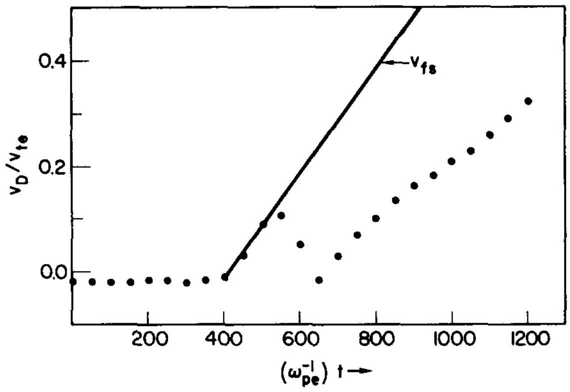


FIG. 7. The response of the electrons to a dc electric field with an electron beam present. The electric field was turned on at $400\omega_{pe}^{-1}$. The ion density at $400\omega_{pe}^{-1}$ is shown in Fig. 1. Note that the ion cavities in this simulation are generated self-consistently.

at $\omega_{pe}$ across the current carrying system. A large single cavity was formed in this case. The plasma response was again consistent with the simple theoretical picture.

Next, we considered the effects of ion cavities on an existing dc current. Figure 8 shows the results from a Vlasov simulation where the electrons have an initial drift, $v_d$, equal to $-0.5v_{te}$; $T_e = T_i$ and, hence, the current is stable. Between 0 and 200 $\omega_{pe}$, a long wavelength Langmuir wave is brought up in the plasma with $E_0^2/8\pi T_{e0} = 1$ and $\omega_{pe}/k_0 = 20\, v_{te}$ ($k_0$ is the wavenumber of the Langmuir wave). The system length is 128 $\lambda_D$ and the ion-electron mass ratio is 100. Since $E_0^2/8\pi n T_{e0} \gg \frac{3}{2}(k_0\lambda_0)^2$, this wave is nonlinearly unstable and we see the transfer of the field energy to localized spikey fields where it can be resonantly absorbed by the electrons. Simultaneously, this leads to the formation of ion cavities inside of which the spikey fields are trapped [Fig. 8(a)]. A series of such simulations without a dc are reported in Ref. 8. In particular, Figs. 12, 13, and 14 of Ref. 8 provide a dynamical picture of the formation of the spikey turbulence and ion cavities. In Fig. 8(b) we show the electron and ion momentum normalized to $m_e v_{te}$. Initially, the electrons drift freely through the stationary ions. As the strong turbulence begins to drive up the ion cavities the electrons and ions start to couple. At the time the ion cavities reach this maximum depth ($\delta n_i/n_0 = 0.28$), we see the strong coupling of the electrons and ions. The electrons lose 30% of their initial momentum to the ions. Figure 9 shows the electron distribution at $t = 0$ and 2000 $\omega_{pe}^{-1}$. One can clearly see that the low energy electrons have been blocked and shifted to smaller velocities.

As can be seen in Figs. 4 and 9, the presence of the ion cavities can lead to the formation of a double peaked electron velocity distribution. If a dc electric field is continuously applied to the plasma, one would see the formation of a runaway beam of electrons. The behavior of the plasma on these longer time scales depends upon how these beams are thermalized. One

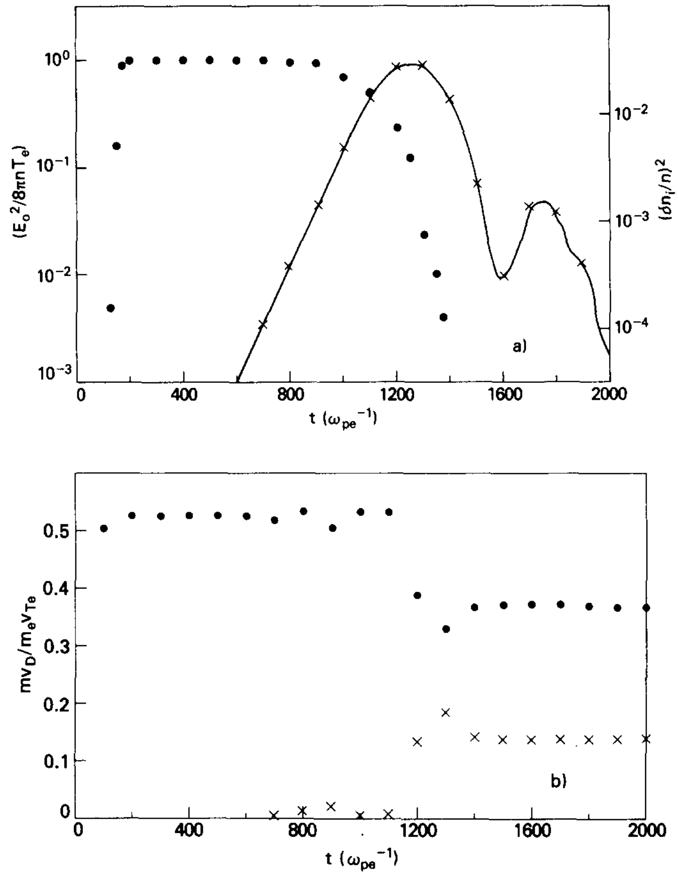


FIG. 8. The effect of strong turbulence collapse on a electron current. (a) This shows the time evolution of the total field energy (•) and $(\delta n_i/n_0)^2$(×). For this simulation a long wavelength Langmuir wave with a phase velocity equal to 20 $v_{te}$ is brought up in the system between 0 and $200\omega_{pe}^{-1}$. At $200\omega_{pe}^{-1}$ all of the field energy was in the long wavelength mode. Strong turbulence acts to transfer the energy to shorter wavelength modes which can resonantly interact with the electrons, damping the high frequency fields. At the same time the strong turbulence drives up ion cavities. (b) This shows the time history of the electron (•) and ion (×) momentum. Initially, the electrons have a drift of 0.5 $v_{te}$ and the ions are stationary. $T_e = T_i$ and therefore the drift is stable. As the ion cavities are driven up, the ions and the electrons are coupled. The electrons are slowed down with the momentum being gained by the ions.

possibility is a two stream interaction between the trapped and untrapped electrons. How this process evolves depends upon the ratio of trapped to untrapped electrons and the spacing between the ion cavities. Also, on these longer time scales, the evolution of the ion cavities becomes important and this depends upon how the ion cavities are generated. As discussed in Ref. 8, if the cavities are generated via strong turbulence due to the beam plasma instability, one can see different behavior depending upon the strength of the beam and the electron ion temperature ratio. For example, when $W/nT_e > 0.1$ and $T_e \simeq T_i$, the ion cavities will begin to damp on longer time scales following the initial stabilization of the beam. Figure 1 of Ref. 8 shows this behavior as seen with the hybrid code and with the



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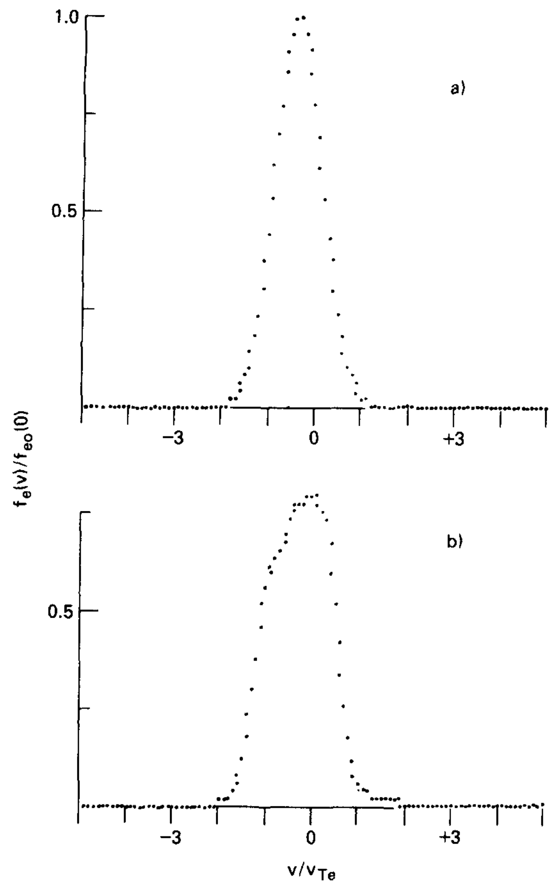


FIG. 9. The electron distribution for the simulation shown in Fig. 8(a), $t=0$. (b) $t=2000\omega_{pe}^{-1}$. The low velocity electrons have been trapped by the finite ion cavities.

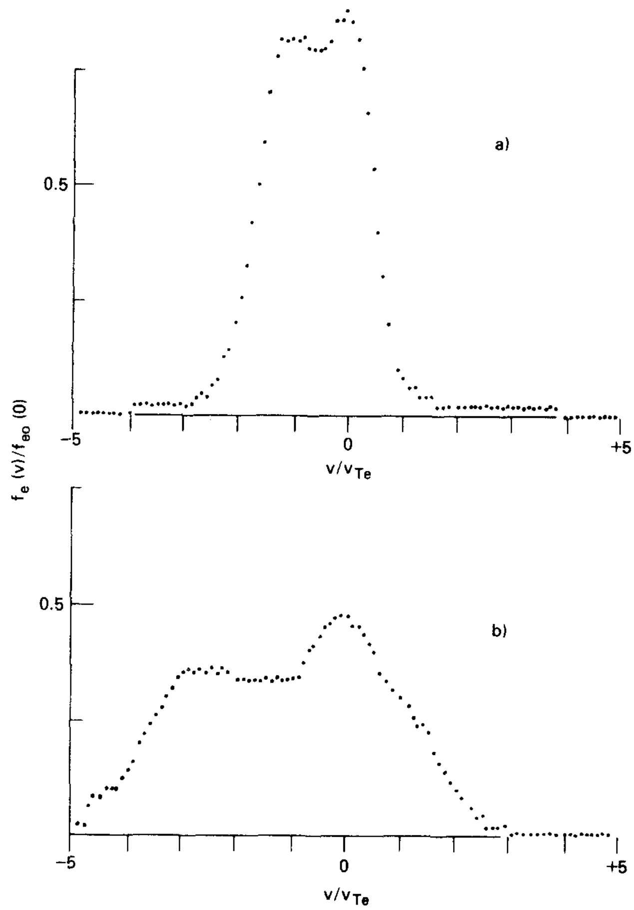


FIG. 10. On longer time scales the acceleration of the untrapped electrons can lead to the appearance of the two-stream instability. The ion cavities were generated by the collapse of a long wavelength Langmuir wave and then, in order to study only the electron dynamics, they were fixed. The maximum $\delta n_i/n_0$ was 0.21. A constant dc field was then applied to the electrons. (a) This shows the initial splitting of the trapped and untrapped electrons. (b) The distribution $1600\omega_{pe}^{-1}$ later. When the separation between the peaks of the two distributions became greater than $1.5\,v_{te}$, a two-stream instability was triggered. This lead to the growth of electrostatic waves with phase velocities midway between the two peaks and a flattening of the runaway distribution.

Vlasov code. Except for the fact that there was no external dc field applied in those simulations, they are the same as the simulation shown in Fig. 7. As the ion cavities are damped away by Landau damping on the ions, fewer particles are trapped and the bulk acceleration starts to increase. This is what we begin to see at the end of the simulation shown in Fig. 7. However, as the level of ion turbulence drops, the beam plasma instability starts to grow again. This acts to drive the ion cavities back up. This long term behavior can be seen in Fig. 4 of Ref. 8. Also note that the damping of the ion cavities as seen in the simulations takes place on a faster time scale because of the artifical mass ratio. This periodic reappearance of the beam-plasma instability and strong turbulence could lead to periodic quenching of the current. However, in other parameter regimes the ion cavities are much more stable in time. Because of this large number of parameters, the following simulation should be viewed as only an example of how these runaway beams could be thermalized. It is most interesting however because of its connection with some earlier work on runaway electrons.[12] To study this interaction we ran a simulation similar to the one shown in Fig. 8 where the ion cavities were created by the strong turbulence collapse of a high phase velocity Langmuir wave. The simulation parameters are the same as those of the simulation shown in Fig. 8 except that there was no initial electron drift. As before, the collapse of the wave led to the formation of ion cavities with most of the high frequency field energy going into the production of superthermal electron tails. At this point we applied a dc electric field and in order to look at just the electron thermalization, we froze the ions. The ions had three large cavities and two smaller ones. The maximum depth was $\delta n_i/n_0 \simeq 0.21$. We again saw the initial splitting of the electron distribution [Fig. 10(a)]. As we continued to accelerate the electrons, the velocity difference between the trapped and untrapped particles increased and the relative minimum in the distribution function between the two peaks increased. When the separation between the peaks became greater than approximately 1.5 to 2 times the electron thermal velocity, we began to see the growth of electrostatic waves and a filling in of the valley between the two peaks. Since $\delta n_i = 0.21\,n_0$, the



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trapped and untrapped electron distributions have close to the same density. This threshold for instability is in good agreement with the Penrose stability criterion for two equal density, warm beams. Figure 10(b) shows the electron distribution 1600 $\omega_{pe}^{-1}$ later. For the constant electric field used the electron should have increased their velocity by 1.28 $v_{te}$. The leading edge of the untrapped electrons has been accelerated to the free streaming velocity. However, the low velocity part of the distribution is still trapped by the ion cavities and the free electrons have formed a long flat runaway distribution. This distribution [Fig. 10(b)] should be compared with the distribution shown in Fig. 1 of Ref. 12. It was shown in this paper that an electron distribution with such a long flat tail can be driven unstable by the resonant transfer of tail particle energy to $\omega_e \cos\theta$ waves. This leads to the formation of a positive slope on the tail distribution which is unstable. The overall effect is a large pitch angle diffusion of the tail particles and a reduction in the velocity of the leading edge of the runaway distribution. As discussed here the effect of the ion cavities is to split the electron distribution into two parts. The part trapped by the ion cavities cannot respond to the dc electric field. The untrapped part gets pulled out to higher velocities and can form a long flat runaway tail. While the response of the plasma electrons to the dc field has been greatly reduced, the leading edge of the distribution can still run away. If we now include the pitch angle diffusion of the leading edge of the beam as shown in Ref. 11, it may be possible to prevent the untrapped electrons from running away. We are presently carrying out simulations to study, simultaneously, the effects of finite ion cavities and pitch angle scattering.

Preliminary results from simulations with fixed ion cavities and the same parameters as those in Ref. 11 show that the pitch angle scattering does strongly limit the acceleration of the runaways while the ion cavities prevent the acceleration of the main body of the electron distribution.

The simulation results reported here are all one dimensional. Recent simulation results[13] of the study of the turbulent collapse of Langmuir waves in two dimensions in a magnetic field indicate that, for an initial spectrum of waves parallel to the magnetic field with a small transverse spread such as would be generated by a warm electron beam, the ion cavities that form are much wider across the field than in the parallel direction. Most importantly for our work, the transverse scale is much greater than the electron gyroradius so that the low velocity electrons will see a one-diemensional barrier due to the cavities.

## IV. SUMMARY AND CONCLUSIONS

The results reported here should be considered as a preliminary but major step in understanding and modeling the long time anomalous resistivity in one dimension. Referring first to our self-consistent results we demonstrated that:

(a) In agreement with previous studies the presence of electron beams[5-9,14] or externally imposed high frequency ($\omega_{pe}$) electromagnetic fields,[15] large local density cavities ($\delta n/n_0 \simeq 0.2-0.6$) can be formed.

(b) The potentials associated with these cavities are large enough to trap a significant fraction of the thermal electrons, so that the entire current will be carried by a small fraction of untrapped electrons. This effect will appear experimentally as an enhanced resistance and the appearance of localized runaway beams. The long time scale behavior of the system will depend on how these beams are thermalized. A possible mechanism is further two-stream interaction of the runaways with the trapped electrons. This process will depend critically on the spacing among cavities and is presently under study. The preliminary simulation results indicate that this can lead to the formation of a flat runaway distribution. This type of distribution has been shown to be unstable to pitch angle scattering in a strong magnetic field.[12] This acts to reduce the velocity of the high velocity edge of the runaway beam. By combining these two effects, it may be possible to produce a quasi-steady current in the presence of a constant dc electric field.

Furthermore, the results of the first simulations demonstrated that similar resistivity will appear any time an instability can create finite amplitude density fluctuations. The ion cyclotron instability seems to be a good candidate and is currently under study.

In concluding we mention that these concepts can help in understanding return current heating in beam-plasma interactions and extend the Papadopoulos-Coffey anomalous resistivity in the auroral zones to regions where $\omega_{pe}/\Omega_e \simeq 1$. The results of the constant pump runs suggest the possibility that a laser created corona with large cavities can produce short electron beam deposition lengths for $e$-beam-pellet fusion.

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