

# ON THE THEORY OF THE TYPE III BURST EXCITER

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**Abstract.** *In situ* satellite observations of type III burst exciters at 1 AU show that the beam does not evolve into a plateau in velocity space, contrary to the prediction quasilinear theory. The observations can be explained by a theory that includes mode coupling effects due to excitation of the parametric oscillating two-stream instability and its saturation by anomalous resistivity. The time evolution of the beam velocity distribution is included in the analysis.

## 1. Introduction

The most fundamental problem in the theory of type III solar radio bursts is that of understanding the propagation of the exciter and its interaction with the background plasma. The quasilinear theory of the beam-plasma interaction predicts that the exciter would be decelerated near the acceleration region, relaxing to a plateau distribution in velocity space. That such relaxation does not occur is shown clearly by particle observations at 1 AU (Lin *et al.*, 1973). This is not, of course, to say that there are no wave-particle effects on the beam propagation; conservation of energy demands that some deceleration occur. Attempts to explain the type III bursts in terms of a locally relaxed exciter, however, are generally contrived and unsuccessful when compared with observations.

In an earlier paper (Papadopoulos *et al.*, 1974, hereinafter referred to as Paper I) it was shown that in the parameter regime characteristic of the type III exciter, the parametric oscillating two-stream instability (OTSI) provides an effective mechanism for the spectral transfer, in  $k$ -space, of plasma waves from the linearly unstable beam-resonant region to lower, nonresonant phase velocities. The OTSI produces symmetric spectra of plasma waves at both positive and negative phase velocities in the nonresonant region, as well as purely growing (i.e., aperiodic) ion density waves.

The object of Paper I was to demonstrate that the OTSI should be important in the interaction of the exciter with the background plasma; to that end, the treatment of Paper I only considered the temporal instability of a uniform beam. The beam-resonant waves were considered as a monochromatic pump wave at  $k=0$ ; this is known as the dipole approximation. Although it may be objected

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that the linearly unstable spectrum is in fact composed of finite wavelengths and has some width  $\Delta k \approx (\Delta V_b/V_b^2) \omega_e$ , it is shown in the appendix to Paper I and in subsequent work by Papadopoulos (1974) that the dipole approximation is valid for sufficiently large energy density in the pump, as long as the pump wavelength is very long compared with the Debye length.

In this paper we report on recent work that extends the theory of Paper I. We consider the dynamic evolution of the velocity distribution function of the beam (at a fixed point in space) and the simultaneous evolution of the electrostatic turbulence spectrum. Knowledge of the shape of the turbulence spectrum is essential for computing the electromagnetic spectrum, for consideration of secondary particle acceleration, and for the question of the observability of the plasma-wave turbulence itself. The present paper is intended as a *precis*; a fuller exposition will be published elsewhere.

## 2. Invalidation of Weak-Turbulence Theory

The necessity for strong-turbulence mechanisms in the beam stabilization is emphasized when we consider that the type III exciter is not a uniform beam pervading all space, but is injected for some finite duration into a half-space. In this situation, 'oscillation pile-up' occurs (Tsytovich, 1970); induced emission of plasma waves by freshly-injected particles is enhanced by the presence of the waves amplified by previously-injected particles. The oscillation pile-up is limited by convection of the waves away from the injection region. The asymptotic turbulence energy density for continuous injection is given by

$$\tilde{W} \approx \left(\frac{V_b}{V_e}\right)^2 n_b m V_b \Delta V_b, \quad (1)$$

where  $V_e$  is the electron thermal velocity of the ambient plasma,  $V_b$  is the nominal beam velocity,  $\Delta V_b$  is the spread in beam velocities,  $n_b$  is the beam density, and  $m$  is the electron mass. The level  $\tilde{W}$  is an enhancement of order  $(V_b/V_e)^2$  of the asymptotic level predicted by quasilinear theory, and is reached over a scale distance from the injection point ( $x = 0$ ) given by

$$\tilde{X} \approx \frac{V_g}{\gamma_b} \approx \left(\frac{\Delta V_b}{V_b}\right)^2 \left(\frac{V_e}{V_b}\right) \frac{\lambda_e}{\eta}, \quad (2)$$

where  $\lambda_e$  is the electron Debye length and  $\eta = n_b/n_e \ll 1$  is the density ratio of the beam to the ambient plasma. In the lower corona,  $\tilde{X}$  is the order of a few tens of meters, much smaller than the dimension of the injection region itself. Similar effects will prevail throughout the propagation region, because  $\tilde{X}$  is everywhere smaller than the inhomogeneity length scale. Furthermore, Equation (1) shows that  $W/N_e T_e$  may be of order  $(V_e/V_b)^2$ , and for such energy densities weak turbulence theory is invalid. Thus, we see that a strong mode-coupling mechanism is essential to stabilize the exciting beam.

### 3. Evolution of the Beam Distribution Function

We have developed a semi-empirical model for the beam evolution at a fixed point, based on *in situ* observations of the exciter during type III bursts at 1 AU (Lin, 1972; Lin *et al.*, 1973). Such a model is necessary because the propagation of the exciter in the corona and interplanetary medium is affected by pitch-angle scattering as well as by dynamic friction with self-induced turbulence and by natural dispersive effects associated with the injected distribution. In view of the disparity of length and time scales between macroscopic and microscopic phenomena, any attempt to model the beam evolution must resort to heuristic arguments.

Lin *et al.* observed the differential energy flux of the beam electrons, rather than the velocity distribution itself. Furthermore, their particle detector was pointed southward of the ecliptic plane, and so accepted electrons with pitch angles roughly in the range  $60^\circ$ – $120^\circ$ , because the magnetic field is predominantly in the ecliptic plane. A second detector, with less energy resolution, provided pitch-angle distributions with resolution of about  $30^\circ$  (R. P. Lin, private communication). For purposes of our model calculations, we interpret the differential energy flux distributions observed by Lin as one-dimensional velocity distributions (i.e., integrated over perpendicular velocities). Some implications of this interpretation are discussed below.

The parameters to be determined are  $\eta$ ,  $V_b$ , and  $\Delta V_b$ , all considered to be functions of time. Lin *et al.* showed that the first particles of any given velocity to reach their detector all appeared to have traversed the same distance  $L$  after injection at a common time  $t_0$ . Because of perturbations in the magnetic field,  $L$  is always larger than the nominal distance associated with the underlying Archimedian spiral field. Therefore at time  $t$  the slowest particles at 1 AU have speeds  $\beta_0 c$  given by

$$\beta_0 = \frac{L}{c(t - t_0)}. \quad (3)$$

The peak of the distribution is at  $V_b = \beta_0 c + \Delta V_b$ . Empirically, we find that  $\Delta V_b$  can be well approximated by

$$\Delta V_b = \beta_0 c [\theta(\beta_0) - 1], \quad (4)$$

and so

$$\frac{\Delta V_b}{V_b} = 1 - 1/\theta(\beta_0), \quad (5)$$

where

$$\theta(\beta_0) = 1.4(1 + 0.075\beta_0 + 0.085\beta_0^2)(0.3/\beta_0)^{0.4}. \quad (6)$$

Equations (4) and (6) were found empirically from a sequence of particle spectra of the event of 16 May, 1971. As an independent check, the expressions were

matched against similar spectra for other events for which  $L$  and  $t_0$  were readily available, and agreed well in each case. It should be noted that the only parameter that varies from event to event in this scheme is  $L$  (c.f., Equation (3)). Therefore, these expressions probably contain some implicit description of spatial diffusion as a function of distance.

To model  $\eta(t)$ , we note that the fully-evolved beam is a power law distribution in energy,  $f(E) \sim E^{-\zeta/2}$ . The corresponding one-dimensional velocity distribution is  $f_0(v) \sim v^{-\zeta}$ . For example, the spectral indices  $\zeta$  in the events of 16 May, 1971 and 28 February, 1972, discussed by Lin *et al.*, were  $\zeta = 4.6$  and  $\zeta = 10$ , respectively. (These are approximately the extremes of the range of  $\zeta$  in the many events so far observed [R. P. Lin, private communication].) By integrating the power law distribution for velocities greater than some defined minimum, say,  $\bar{\beta}$ , we obtain the normalization for  $f_0$ ; i.e.,

$$f_0(\beta) = A(\bar{\beta})\beta^{-\zeta},$$

where  $A(\bar{\beta})$  is defined by the condition

$$\int_{\bar{\beta}}^{\infty} d\beta f_0(\beta) = \bar{n}_b$$

Our procedure is to use the distribution function  $f_0$  for  $\beta \geq \beta_b$  and to assume a ramp distribution for  $\beta_0 \leq \beta \leq \beta_b$ . (Note that this implies that the peak of the distribution is at its fully-evolved value.) Thus, we take

$$f(\beta) = \begin{cases} f_0(\beta) = \bar{n}_b \frac{(\zeta-1)}{\bar{\beta}} (\bar{\beta}/\beta)^{\zeta}, & \beta \geq \beta_b; \\ f_1(\beta) = f_0(\beta_b) \frac{(\beta - \beta_0)}{(\beta_b - \beta_0)}, & \beta_0 \leq \beta \leq \beta_b. \end{cases} \quad (7)$$

The instantaneous beam density in the model is thus given by

$$\begin{aligned} n_b(t) &= \int_{\beta_0(t)}^{\beta_b(t)} d\beta f_1(\beta) + \int_{\beta_b(t)}^{\infty} d\beta f_0(\beta) \\ &\equiv n_b^{(1)} + n_b^{(0)}, \end{aligned} \quad (8)$$

where  $n_b^{(1)}$  and  $n_b^{(0)}$  are the contributions from  $f_1$  and  $f_0$ , respectively. In accord with equation (7), the intensity growth rate  $\gamma_b$  of the linear beam-plasma instability is taken to be constant over the range of phase velocities  $\beta_0 \leq V_{\phi}/c \leq \beta_b$  and equal to zero elsewhere. In the unstable region,  $\gamma_b$  is given by

$$\gamma_b(t) = \pi \eta_1 \left( \frac{V_b}{\Delta V_b} \right)^2 \omega_e, \quad (9)$$

where  $\omega_e$  is the electron plasma frequency and  $\eta_1 \equiv n_b^{(1)}/n_e$ .

We may inquire into the plausibility of interpreting the differential energy flux as a one-dimensional velocity distribution  $f(u)$ , where  $u = v_x$ . The fact that the particles observed by Lin have large pitch angles  $\alpha$  necessarily means that the

one-dimensional  $f$  will contain low velocities, i.e.,  $u = v \cos \alpha$  where  $\cos \alpha$  is small. In principle, the distribution may even fill in to  $u \leq 0$ . Throughout the event, however, there exists anisotropy in the pitch-angle distributions, and this anisotropy is particularly marked early in the event and for a considerable time thereafter. It may easily be shown that if  $dj/dE d\Omega$  is of the form

$$\frac{dj}{dE d\Omega} = AE^{-\zeta/2} G(\mu) H(E - E_b) \tag{10}$$

(where  $\mu = \cos \alpha$  and gyrotropy is assumed, and  $H$  is the Heaviside step function), then the one-dimensional distribution  $f(u)$  is given by

$$f(u) = \text{const.} \int_{\lambda}^{\infty} dv v^{-\zeta-1} G(u/v), \tag{11}$$

where

$$\lambda = \begin{cases} u & (u \geq v_b) \\ v_b & (u < v_b) \end{cases}$$

and  $v_b = (2E_b/m)^{1/2}$ . Our assumption that  $f(u) \sim m(dj/dE)$  amounts to assuming that  $G(\mu)$  is sharply peaked near  $\mu = 1$ . If  $G(\mu)$  is expanded in Legendre polynomials, it may also be shown that the peak of  $f(u)$  is preserved at  $u = V_b$ , while the slope for  $u < V_b$  depends on the relative weights of the various Legendre polynomials in  $G(\mu)$ . (This part of the discussion does not consider the contribution from the particles at  $E < E_b$ .) As an example, we note that if  $G(\mu) = (1 + \mu)/2$ , we have the distribution shown in Figure 1a while if  $G(\mu)$  is dominated by higher-order odd polynomials,  $f(u)$  may be as schematically depicted in Figure 1b. Although the pitch-angle distributions provided to us by Lin

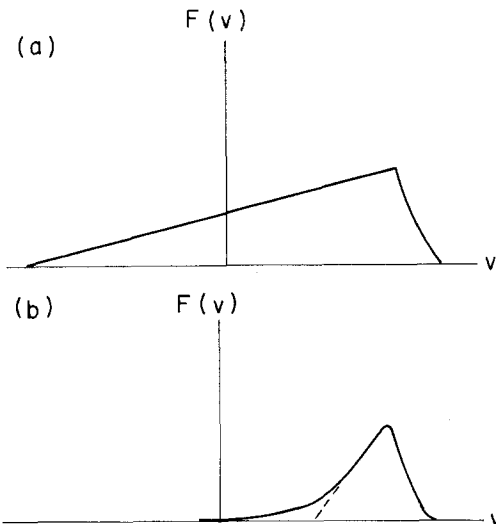


Fig. 1. Schematic of the projected beam distribution function  $f(u)$  for (a) weak anisotropy, (b) strong anisotropy.

are difficult to model analytically owing to lack of resolution, we feel that Figure 1b provides a closer picture of the distribution. Thus, we have simply approximated the steeply rising part by a ramp that goes to zero at  $u = V_0$ , as mentioned above. Although this is clearly a model, we do not feel that the present data justify a more elaborate treatment.

#### 4. Evolution of the Plasma-Wave Spectrum

Our program is to model the temporal development of the turbulence spectrum at a fixed point. Because the distance  $\tilde{X}$  is so short, ranging from less than 1 km in the lower corona to about 3000 km at 1 AU, the effect of oscillation pileup is essentially accounted for by including the temporal evolution of the beam which is assumed injected at  $x = 0$  and evolves at  $x = L$  according to the model described above. The processes affecting the spectral evolution are the linear beam-plasma instability, Landau-damping in the non-resonant region of phase velocities, the OTSI, and the enhanced absorption (scattering) of plasma waves owing to the development of high-frequency anomalous resistivity. We neglect the decay instability, which has a threshold equal to that of the OTSI but a smaller growth rate.

The OTSI has been discussed in Paper I and by Nishikawa (1968a,b), Sanmartin (1970), and Papadopoulos (1973, 1975). In the usual treatment, based on the dipole approximation in which the pump at frequency  $\omega_0$  is spatially uniform ( $k_0 = 0$ ), the driven modes are at finite wave-numbers  $|k| > 0$  and include a purely growing ion density fluctuation and an electron plasma wave, the normal-mode frequency of which is given by the Bohm-Gross dispersion relation  $\omega_{ek} = (\omega_e^2 + 3k^2 V_e^2)^{1/2}$ , but which is frequency-shifted in the OTSI to match the pump frequency. Papadopoulos (1973, 1975; c.f., also the appendix to Paper I) extended the theory to the case of finite wavenumbers in the pump spectrum. The resulting treatment is a generalized one that includes the modified decay and modulational instabilities. In the limit that the dipole approximation may be employed, the modulational and OTS instabilities are equivalent, and we shall use the designation OTSI in a generic sense.

The threshold amplitude of the pump and the growth rate of the OTSI are governed by the frequency shift  $\delta = \omega_0 - \omega_{ek}$  between the pump and daughter modes (note  $\delta < 0$  for the OTSI). In the case of a finite-wavelength pump, the dispersion relation involves a convolution integral over the pump spectrum. In our treatment here we employ a limiting prescription, in which we use the form of the dispersion relation for the dipole pump, but include the effects of finite pump wavenumber  $k$  by using the Bohm-Gross frequencies for both pump and daughter waves:  $\delta = \omega_{ek} - \omega_{ek'}$ .

Neglecting the collision frequencies compared with  $\delta$ , the OTSI threshold is given by

$$W_T \equiv \frac{E_T^2}{8\pi n_e T_e} = \frac{-2\delta}{\omega_e} \left[ (1 + 3k'^2 \lambda_e^2)^{1/2} - \frac{\delta}{\omega_e} \right], \quad (12)$$

while the growth rate for any value of  $W$  is given by

$$\begin{aligned} \gamma_{\text{OTS}}^2(k, k') = & -\frac{1}{2}(\omega_A^2 + \delta^2) \\ & + \frac{1}{2} \left\{ (\omega_A^2 + \delta^2)^2 - 4\delta^2\omega_A^2 \left[ 1 + \frac{W(k)}{2\delta(1+3k^2\lambda_e^2)^{1/2}} \right] \right\}^{1/2}, \end{aligned} \quad (13)$$

where the first argument denotes the driver wavenumber, the second denotes the daughter wavenumber, and

$$\omega_A^2 = \left( \frac{m}{M} \right) \left[ k'^2 \lambda_e^2 \omega_e^2 - \frac{2}{3} \delta \omega_e \right],$$

where  $M$  is the ion mass.

The second mode-coupling mechanism we include is that of dissipation of long-wavelength waves to shorter-wavelengths owing to high-frequency anomalous resistivity. In the initial stages of the spectral development of the electrostatic turbulence, the linear beam-plasma instability creates a pump spectrum of waves resonant with the beam particles; when the pump amplitude exceeds the OTSI threshold, the OTSI at first becomes the dominant mechanism of spectral energy transfer. The growth *via* OTSI of nonthermal ion-density waves, however, leads to the development of anomalous resistivity (Dawson and Oberman, 1963). In speaking here of ‘dissipation’, we mean only the scattering of waves and not heating of the plasma. The ions are treated as fixed and hence do not constitute a momentum sink; heating effects are of order  $m/M$  compared with scattering. Averaging over the *positions* of the ions in the nonthermal density waves, however, yields some disordering of the motion of the electrons in response to the applied field. Hence there is ‘dissipation’ in the sense that electron energy is conserved but appears in disordered motion which when Fourier-analyzed contributes to the wave spectrum at higher wavenumbers. Denoting by  $S(k)$  the normalized energy density in ion density fluctuations at wavenumber  $k$ , the scattering by these waves is manifested in a nonlinear damping rate  $\gamma_{\text{NL}}(k)$  given by

$$\gamma_{\text{NL}}(k) \approx \frac{S(k)}{k^2 \lambda_e^2} \omega_e. \quad (14)$$

Two other phenomenological damping coefficients are included in the computations. The first, denoted by  $\nu_{\text{eff}}$ , describes Landau damping of non-resonant plasma waves on the high-energy tails in the solar wind electron distribution, and is modelled by

$$\nu_{\text{eff}}(k) = \eta_t \left( \frac{k}{k_t} \right)^\tau, \quad (15)$$

where  $\eta_t$  is the density fraction of the tail population, which is assumed to be given by a power-law distribution  $f(v) \sim v^{-\tau}$  for velocities greater than  $\omega_e/k_t$ . The

role of  $\nu_{\text{eff}}$  is to provide an ultimate sink as the spectrum evolves down to very low phase velocities, but it is not dominant during most of the evolution.

The second, and more important, damping coefficient is that of the ion-density fluctuations, which we designate by  $\nu_s$ . This rate is simply taken to be the trapping width of the density waves, in the Dupree-Weinstock sense, and is given by

$$\nu_s(k) = \left(\frac{m}{M}\right)^{1/2} [k^2 \lambda_e^2 S(k)]^{1/4} \omega_e. \quad (16)$$

The various growth rates  $\gamma_b$ ,  $\gamma_{\text{OTS}}$ ,  $\gamma_{\text{NL}}$ ,  $\nu_{\text{eff}}$ , and  $\nu_s$ , given respectively by Equations (9), (13), (14), (15), and (16), are interpreted here as a set of (phenomenological) transfer coefficients that enable us to describe the evolution of the turbulence spectrum by a set of rate equations. To do so, we re-write the energy densities  $W(k)$  and  $S(k)$  in terms of plasmon and phonon number densities, which are denoted by  $P(k)$  and  $A(k)$  respectively and are defined by

$$\begin{aligned} W(k) &= P(k) \hbar \omega_{ek} / n_e T_e, \\ S(k) &= A(k) \hbar \omega_{Ak} / n_e T_e; \end{aligned} \quad (17)$$

here  $\omega_{AK} \equiv (m/M)^{1/2} k V_e$ . (The definitions (17) are purely conventional; the reason for defining them will become clear below.) Furthermore, it is useful to present the results of the computations in phase-velocity space, rather than  $k$ -space, in order that we may picture the beam and the turbulent spectrum evolving in the same phase-space. There is, of course, a one-to-one mapping between  $k$ -space and phase-velocity space at fixed value of  $\omega_e$ . It is probably clearer, however, to write down the rate equations in  $k$ -space. Using Equations (9), (13)–(17), the rate equations are written separately for plasma waves at positive and negative phase velocities; the ion density waves are non-propagating, however, and so  $A_k = A_{+k} + A_{-k}$ . Thus, defining all  $k$ 's to be positive, we have three rate equations describing the spectra of ion-density fluctuations, forward-going and backward-going electron plasma oscillations, respectively:

$$\frac{dA_k}{dt} = \left\{ \sum_{k' < k} [\gamma_{\text{OTS}}(P_{k'}) + \gamma_{\text{OTS}}(P_{-k'})] - \nu_s(k) \right\} A_k \quad (18)$$

$$\begin{aligned} \frac{dP_k}{dt} &= [\gamma_b(k) - \nu_{\text{eff}}(k)] P_k - \sum_{k' > k} \gamma_{\text{OTS}}(P_{k'}) A_{k'} \\ &\quad - \left[ \sum_{k' > k} \gamma_{\text{NL}}(P_{k'}) \right] P_k + \gamma_{\text{NL}}(k) \sum_{k' < k} P_{k'} \end{aligned} \quad (19)$$

$$+ \frac{1}{2} \left\{ \sum_{k' < k} [\gamma_{\text{OTS}}(P_{k'}) + \gamma_{\text{OTS}}(P_{-k'})] \right\} A_k;$$



$$\begin{aligned}
 \frac{dP_{-k}}{dt} = & \frac{1}{2} \left\{ \sum_{k' < k} [\gamma_{\text{OTS}}(P_{k'}) + \gamma_{\text{OTS}}(P_{-k'})] \right\} P_k - \nu_{\text{eff}}(k) P_{-k} \\
 & + \gamma_{\text{NL}}(-k) \sum_{k' < k} P_{-k'} \\
 & - \sum_{k' > k} \gamma_{\text{OTS}}(P_k) A_{k'} \\
 & - \left[ \sum_{k' > k} \gamma_{\text{NL}}(-k') \right] P_{-k}.
 \end{aligned} \tag{20}$$

In Equations (18)–(20), we have suppressed the wavenumber arguments of  $\gamma_{\text{OTS}}$ , indicating the driving mode by its amplitude. The liberal use of square and curly brackets is to emphasize the structure of the transfer coefficients.

Equations (18)–(20) are written for arbitrary wavenumbers, and it is clear that in general only a few of the various coefficients are nonzero for any particular wavenumber at any instant. Moreover, it may clearly be seen from Equations (19) and (20) why we must write the equations in terms of number densities rather than energy densities. We recall that the OTSI dispersion relation, neglecting collisions, is (cf. Paper I, Equation (A5))

$$(\omega^2 - \omega_A^2) + \frac{1}{n_e M \lambda_e^2} \int dk' W(k') \frac{(9/4)\omega_e(k\lambda_e)^4}{(9/4)\omega_e^2(k\lambda_e)^4 - [\omega - 3\omega_e k k' \lambda_e^2]^2} = 0.$$

The solution  $\omega$  of this dispersion relation is for the low-frequency normal mode and in the dipole approximation yields the purely-growing ion-density wave. Therefore, the OTSI is induced by the level  $A_k$ . Owing to the inclusion of mode coupling by anomalous resistivity (through  $\gamma_{\text{NL}}$ ), however,  $A_k$  and  $P_k$  may differ. Thus, for example, the second and last terms on the RHS (19) must be written in terms of  $A_k$  and  $A_{k'}$ , respectively, in order to balance the equations correctly. Similar considerations apply to Equation (20).

The system of Equations (18)–(20) are solved simultaneously for several modes. In practice, we use finite increments of  $0.01c$  in phase-velocity space, and integrate the number densities over these finite increments. Typical results are shown in the time sequence of Figure 2. We see that once the OTS threshold is reached in the resonant region, there is a rapid transfer of energy to lower phase velocities, resulting in buildup of ion-density fluctuations and the consequent dumping of energy out of the resonant region even below the OTS threshold. This latter effect is of paramount importance, because the OTSI itself cannot reduce the turbulence levels in the resonant region below the threshold values. If such levels are allowed to persist in the resonant region for times of the order of the time scale over which the beam evolves, the relaxation effects on the beam distribution would be significant. Relaxation is not dominant, however, precisely because the dynamic evolution of the beam at any point occurs on a longer time scale than that required for the cascading of the plasma-wave energy to non-resonant phase velocities.

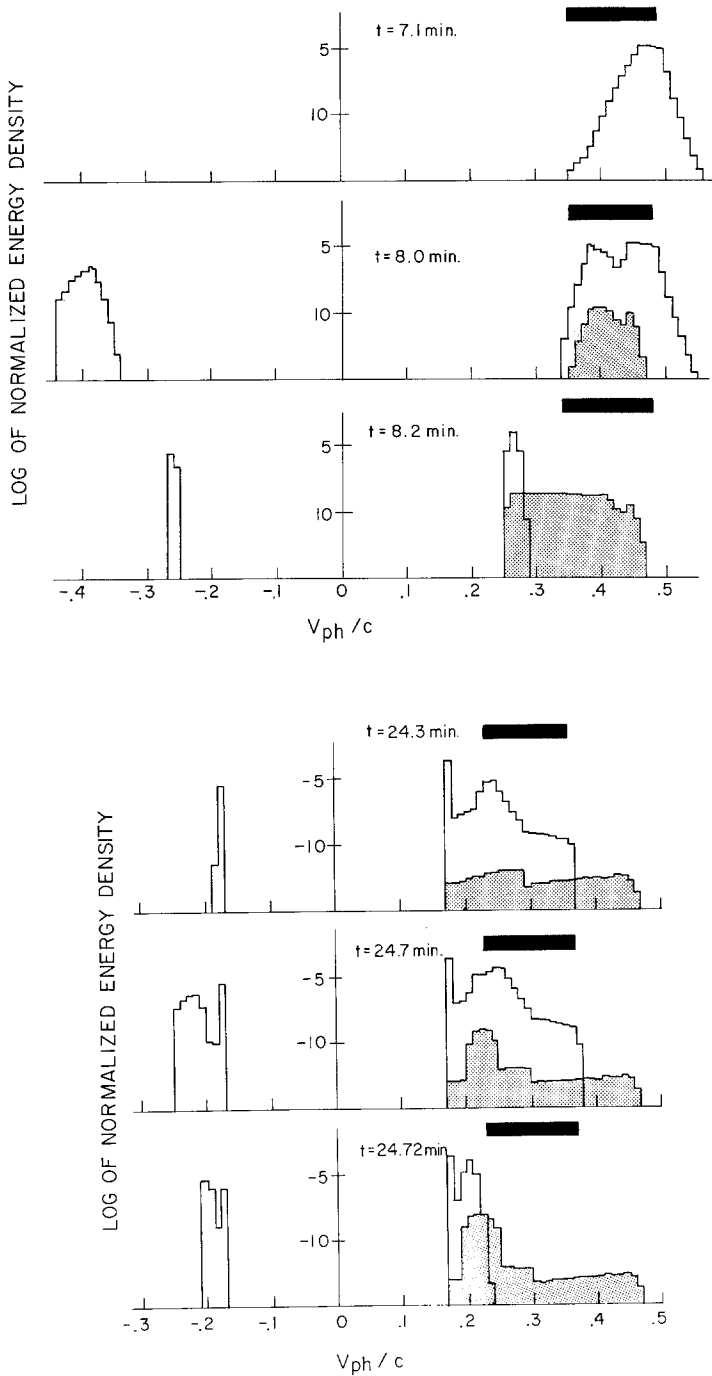


Fig. 2. Time sequence of turbulent energy densities during evolution of the beam at 1 AU. The unshaded histograms correspond to electron plasma oscillations; the shaded histograms represent the ion density waves. The beam parameters are  $\eta(E \geq 1 \text{ keV}) = 2 \times 10^{-6}$ ,  $\zeta = 4.6$ , corresponding to the event of 16 May, 1971. The solid bar denotes the unstable region where  $\partial f / \partial u > 0$ .

## 5. Summary and Discussion

We have extended the theory of Paper I to formulate detailed calculations of the simultaneous temporal development of the type III exciter and its accompanying spectrum of electrostatic turbulence at a given point. Our numerical computations have been done for parameters appropriate at 1 AU; calculations for coronal plasma levels will be presented elsewhere. We have used a limiting form of the dipole approximation; if the growth rate  $\gamma_{\text{OTS}}$  is greater than the frequency spread  $\Delta\omega$  of the pump spectrum, the theory may validly be taken to this limit, in which the OTSI and the modulational instability become identical. From the general discussion of the model of the beam evolution, and in particular from Equation (9), it is seen that we do not treat in detail the interaction of neighboring modes in the resonant region. Therefore although the rate Equations (18)–(20) are integrated on the fastest time scales in the problem, namely  $\gamma_{\text{OTS}}^{-1}$  and  $\gamma_{\text{NL}}^{-1}$ , the results are expected to be valid in detail only when averaged over the slow time scale  $\gamma_b^{-1}$ .

We close with some remarks on the relation of the theory to observational results from IMP-6 during times when the satellite is clearly within the region of type III bursts at 1 AU.

The first result, reported elsewhere in this issue by Kellogg (1976) and Gurnett and Frank (1976), is that plasma-turbulence levels such as seem to be requisite for the theory (viz.,  $W \sim 10^{-5}$ ) are not observed; the highest levels observed are of order  $W \sim 10^{-9}$ . Because all the instabilities involved in the theory (including the linear instability) are phase-coherent, however, it is plausible to suggest that the collapse of the turbulent spectrum to high wavenumbers, of spectral width  $\Delta k$ , is equivalent to a spatial collapse to a characteristic dimension  $\Delta X \sim (\Delta k)^{-1}$ . The initial coherence length of the linear beam-plasma instability is the order of the wavelength  $\lambda \sim 2\pi V_b/\omega_e \geq 1$  km. Our calculations indicate that  $\Delta X \sim 10^2 m$ ; thus  $\Delta X/\lambda \leq 0.1$ . For any instrument such as those of Kellogg and Gurnett, which sample the medium for some finite integration time  $\tau$  and which make direct measurements of power (and therefore energy density), the ratio of measured energy density to the actual energy density in collapsed wave packets is of the order of the ratio of the volume of the wave packets to the sampled volume\*, and so

$$\frac{W_{\text{meas}}}{W_{\text{packet}}} \approx \left(\frac{\Delta X}{\lambda_0}\right)^3 \leq 10^{-3}.$$

Such collapsed wave packets have recently been observed by Wong and Quon (1975). Theoretical treatments of such wave packets have recently appeared in

\* An alternative suggestion has been offered by M. C. Kelley (private communication), who noted that photoelectrons from the satellite may stream along magnetic field lines in the solar direction and increase the electron density so that the local plasma frequency upstream on the flux tube of the satellite is enhanced over the daughter-wave frequencies, reflecting the waves away from the satellite. We do not yet know if this effect is quantitatively adequate.

the literature; where they are referred to as "spikons" or solitons. As was shown by Manheimer and Papadopoulos (1975) these are the configuration-space manifestation of the OTSI. Morales and Lee (1974) investigated the acceleration of electrons by such localized electric fields in the Born approximation; this is the quasilinear limit of the OTSI, corresponding to the asymptotic regime discussed in Paper I and by Papadopoulos and Coffey (1974), in which tails are drawn out of the thermal-plasma velocity distribution. Particle acceleration has also been treated by Bezzerides and DuBois (1975), who used quasilinear theory to describe the particle acceleration.

The second observational result we shall discuss is the relation between the brightness temperature  $T_b$  and the particle flux  $j$  (Lin, 1975; Evans *et al.*, 1975). The rate of change of  $T_b$  with  $j$  increases from  $T_b \sim j$  below a certain critical flux, to  $T_b \sim j^{2.7}$  above the critical flux; the break is fairly sharply defined. We suggest that the break occurs when the OTSI threshold is exceeded. The electromagnetic radiation from type III bursts observed *in situ* is well known to be at the second harmonic,  $\omega = 2\omega_e$ . Such waves are produced by the coalescence of two plasma waves with nearly oppositely-directed phase velocities. Before the onset of the OTSI, backwards-propagating waves must be scattered from density fluctuations, and this is relatively inefficient. The OTSI, however, creates symmetric spectra of forwards- and backwards-propagating waves, and so a small efficiency factor in the emissivity is replaced by a factor of unity. To investigate this assertion, we have run our program for the beam parameters in the two cases in which a break in the  $T_b$  vs  $j$  curve is clearly evident, and we find that the OTSI is first triggered when the particle fluxes are approximately equal to the observed critical fluxes in each case. As Thoreau (1850) said, "some circumstantial evidence is very strong, as when you find a trout in the milk."

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### Discussion

*D. Smith:* The interaction involves two ES pump waves, a resultant plasma wave and a zero frequency ion sound fluctuation.

*Arons:* Do you neglect convective effects?

*R. Smith:* Yes.

*Hudson:* Are you saying that the zero frequency ion mode forms a soliton which traps and localizes Langmuir oscillations?

*R. Smith:* Yes.