Nonthermal Features of the Auroral Plasma Due to Precipitating Electrons

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In a recent rocket experiment *Reasoner and Chappell* [1973] measured energy spectra and pitch angle distributions in auroral arcs at altitudes of 600–700 km. The most striking feature of the data was the existence of power law continuous electron energy spectra between 40 eV and 2.0 keV. In this energy range they observed a backscatter ratio of 1. At still higher energies they observed an energetic peak (Figure 1).

This letter discusses a physical mechanism that can account for the observed particle spectra and the high backscatter ratio. This mechanism is a consequence of the existence of high-energy electron bursts frequently observed in the auroral arcs [*Reasoner and Chappell*, 1973; *O'Brien and Reasoner*, 1971]. The only requirement of the proposed model is the existence of high-energy beamlike bursts with a drift velocity component along the ambient magnetic field. We do not concern ourselves here with the mechanism that creates such bursts, but we simply assume that they were formed at a much higher altitude.

This letter constitutes a preliminary presentation of the model, with emphasis on the physical processes involved. A more detailed quantitative analysis will be presented elsewhere.

Qualitative Description of the Model

Consider first a high-energy electron burst whose component along the magnetic field has a velocity V_b and a thermal spread ΔV_b (Figure 2). The system of part *a* of Figure 2 is linearly unstable to waves ϵ_{k_1} (Figure 2, part *b*) in the region of phase velocity $V_2 \leq v_{ph} \leq V_b$, the frequency being approximately equal to the background plasma frequency ω_e and the group velocity ($v_{gr} \sim (V_e/V_b)V_e$) being very low. According to quasi-linear theory these waves grow in a fashion such that $W_1/\frac{1}{2}m_b m V_b \Delta V_b \sim 1$, where $W_1 = \Sigma\epsilon_{k_1}$, and will flatten the beam within times of the order of $(n_p/n_b)\omega_e^{-1}$ [*Davidson*, 1972]. However, if one considers nonlinear effects, the picture is radically modified. Consider each of the unstable waves as an external driver wave. One finds that for the parameters relevant to the auroral regions the threshold for excitation of parametric instabilities is exceeded before the beam can stabilize quasi-linearly. Since the frequency of the driver waves is just below the plasma frequency ω_e, the relevant parametric interaction is the oscillating two-stream instability. This instability produces purely growing ion fluctuations and electron waves that travel parallel and antiparallel to the driver wave with phase velocities on the region $V_1 \leq |v_{ph}| \leq V_2$ (Figure 2). In this way, wave energy is transferred out of region 1 into regions 2 and 3 (Figure 2), and the beam can be stabilized against quasi-linear diffusion. These new waves are in resonance with the tail end of the background electron distribution function and can therefore produce energetic tails (Figure 2), whereas the driver wave decays to a much lower level. When the Landau damping due to these tails exceeds the growth of the instability, the system reaches a marginally stable nonthermal quasi-equilibrium that can be altered only by Coulomb collisions. In this way, proper consideration of nonlinear plasma effects results in electron distributions having (1) an energetic component with $\Delta V_b/V_b < 1$; (2) symmetric heavily populated tails, which can give a backscatter ratio of unity; and (3) a number density larger in the tails than in the beam. Such features have frequently been observed in the auroral regions. It is important to note that the physical processes described above have been observed clearly in a particle computer simulation experiment [*Kainer et al.*, 1972; *Thode and Sudan*, 1973].

Quantitative Estimates

A detailed quantitative description in three dimensions will be published elsewhere. In order to be consistent with the purpose of this letter we shall make some quantitative estimates by applying the proposed model to the auroral regions. We concentrate on one-dimensional considerations parallel to the magnetic field, although we will later remark on the accuracy of our results with respect to data for pitch angles $\theta < 60°$.

The basic equations describing the system of Figure 2 are as follows (the exact set of equations should include explicitly the mode coupling terms, which cause energy to cascade from high- to lower-phase velocity modes; however, for the physical picture that we present we replace them by an equivalent growth rate type description):

1. In the region $V_2 \leq v \leq V_b$, $V_2 \leq v_{ph} \leq V_b$,

$$\frac{\partial}{\partial t} f_b = \frac{8\pi^2 e^2}{m^2} \frac{\partial}{\partial v} \left(\int dk_1 \epsilon_{k_1}(t) \delta(\omega_{k_1} - k_1 v) \frac{\partial f_b}{\partial v} \right) \quad (1)$$

$$\frac{\partial \epsilon_{k_1}(t)}{\partial t} = 2\gamma_{k_1}\epsilon_{k_1}(t) - 2\epsilon_{k_1}(t)\Sigma_{k_2}\alpha_{k_2}(\epsilon_{k_1})\epsilon_{k_2} \quad (2a)$$

$$\alpha_{k_2} \equiv \gamma_{k_2}(\epsilon_{k_1})/\epsilon_{k_1} \quad (2b)$$

2. In the region $V_1 \leq |v| \leq V_2$, $V_1 \leq v_{ph} \leq V_2$,

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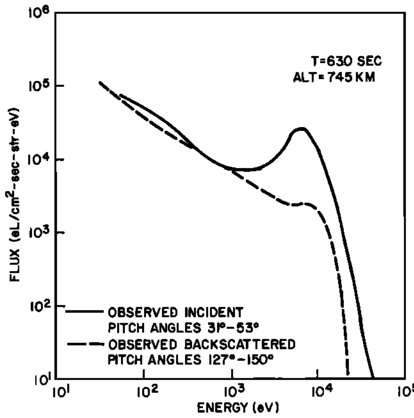

Fig. 1. Incident (solid line) and backscattered (dotted line) electron energy spectra [after *Reasoner and Chappell*, 1973].

$$\frac{\partial}{\partial t}\epsilon_{k_2}(t) = 2\gamma_{k_2}(\epsilon_{k_1})\epsilon_{k_2}(t) - \nu_{eff}\epsilon_{k_2}(t) \tag{3}$$

$$\frac{\partial}{\partial t}f_T(v, t) = \frac{8\pi^2e^2}{m^2}\frac{\partial}{\partial v}\left(\int dk_2\epsilon_{k_2}(t)\,\delta(\omega_{k_2} - k_2v)\frac{\partial f_T}{\partial t}\right) \tag{4}$$

where ν_{eff} is the sum of the collisional and collisionless damping rates.

3. In the region $0 \leq |v| \leq V_1$, $0 \leq |v_{ph}| \leq V_2$, the behavior of waves and particles is assumed to be adiabatic.

In the above equations, f_b is the distribution function of the beam, and $f_T(v, t)$ is the tail end of the ambient Maxwellian distribution of electrons (Figure 2); that is,

$$f_T(v, t) = 0 \qquad |v| \leq V_1$$

$$f_T(v, t) = [n_p/(2\pi)^{1/2}V_e] \cdot \exp -(v^2/2V_e^2) \qquad |v| > V_1 \tag{5}$$

where the value of V_1, to be determined later, represents the phase velocity of the last unstable mode due to the oscillating two-stream instability. The subscript k_1 refers to waves resonant with the beam ($V_2 \leq v_{ph} \leq V_b$), and γ_{k_1} is the growth rate of the beam plasma instability [*Davidson*, 1972]. The subscript k_2 refers to waves nonresonant with the beam ($V_1 < v_{ph} < V_2$), and $\gamma_{k_2}(\epsilon_{k_1})$ is the growth rate of the oscillating two-stream instability [*Nishikawa*, 1968]. Notice that in a numerical solution of these equations the separation of regions 1 and 2 is not necessary. One can replace $f_T(v)$ in (4) by the total background distribution function of the electrons f_e and proceed to solve the equations. However, since here we are interested more in the physical clarity of the interactions, we separate regions 2 and 3 to show that the dominant resonant interaction is with very few particles at the tail, whereas the rest of the distribution interacts nonresonantly and remains essentially adiabatic for the wave energies ϵ_{k_2} under consideration.

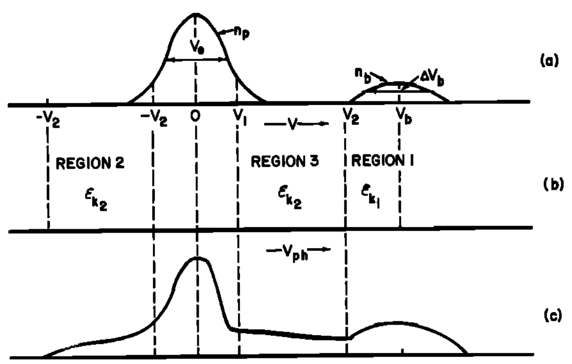

Fig. 2. Velocity and field energy distributions: *a*, Initial velocity distribution functions of the beam and the plasma electrons; *b*, wave spectrum in phase velocity space; region 1 is the region of the growing waves due to the beam plasma interaction; regions 2 and 3 are the regions of spectral energy transfer due to the oscillating two-stream instability; and *c*, final marginal stability state of the beam and the plasma electron distribution functions.

A numerical solution of (1)–(5) will be the subject of a forthcoming publication. In order to find some numerical estimates relevant to the aurora we proceed and introduce the following simplifications.

We replace (2) and (3) with

$$\partial/\partial t\ W_1 = 2\gamma_1W_1 - 2\gamma_2(W_1)W_2 \tag{6}$$

$$\partial/\partial t\ W_2 = 2\gamma_2(W_1)W_2 - \nu_{eff}W_2 \tag{7}$$

where $W_1 = \Sigma_{k_1}\epsilon_{k_1}$, $W_2 = \Sigma_{k_2}\epsilon_{k_2}$, and the $\gamma_{1,2}$ are the average growth rates of the instabilities. For the weak beam plasma interaction,

$$\gamma_1 \approx (\pi/2)\omega_e(n_b/n_p)(V_b/\Delta V_b)^2 \tag{8}$$

The value of γ_2 is given by the theory of the oscillating two-stream instability [*Nishikawa*, 1968; *Sanmartin*, 1970]. Thus $\gamma_2 = 0$ for wave energies W_1 below the instability threshold given by

$$W_{th}/\tfrac{1}{2}n_pmv_e^2 = 4\nu_{eff}/\omega_e \tag{9}$$

For values of W_1 much larger than W_1 the value of γ_2 is given by

$$\gamma_2 = (m/M)^{1/3}\omega_e(k_2^2\lambda_d^2)^{1/3}(W_1/\tfrac{1}{2}n_pmV_e^2)^{1/3} \tag{10a}$$

whereas for W_1 very near threshold

$$\gamma_2 = \pi(k^2e^2/4mM)(W_1 - W_{th}/2\omega_e\omega_a^2) \tag{10b}$$

where ω_a is the ion acoustic frequency and M is the ion mass.

The value of V_1 can be determined from (9) by replacing ν_{eff} by the collisionless damping due to the ambient Maxwellian electrons. Thus

$$4(\pi/2)^{1/2}(V_1^3/V_e^3)\exp -(V_1^2/2V_e^2) = (W_1/\tfrac{1}{2}n_pmV_e^2) \tag{11}$$

The basic features of the physical behavior of the system can be found by inspection of the above equations supplemented by the fact that the wave energy W_1 is bracketed by $W_1 < n_bmV_b\Delta V_b$. Since we would like to compare these estimates with observations, we consider as typical auroral values at 750 km the following: For the auroral plasma we take the electron density to be $n_p = 2 \times 10^3$ cm^{-3}, the electron thermal velocity to be $V_e = 3 \times 10^7$ cm/s, and the electron-ion mass ratio to be $m/M \sim 10^{-4}$. For the electron burst we take $n_b = 2 \times 10^{-2}$ cm^3, the component of streaming velocity along the magnetic field to be $V_b = 6 \times 10^9$ cm/s, and the thermal spread to be $\Delta V_b \sim \frac{1}{3}V_b$. These values correspond to a peak of 10 keV and a particle flux of the order of 10^8 el/cm^2 s. We examine next the various predictions of the model.

Stabilization of the beam against quasi-linear diffusion. From (1) and (2*a*) or (6) we notice that the beam will be stabilized against quasi-linear diffusion when more energy is transferred out of the resonance region than is generated by the beam. This happens when $\epsilon = \gamma_1/\gamma_2 \ll 1$. A more exact condition is [*Papadopoulos et al.*, 1974]

$$\epsilon \ln \epsilon[W_1/W_2(t = 0)] < 1 \qquad (12)$$

Using the fact that $W_1 \leq m_b m V_b \Delta V_b$ and equations 6 and 8 and taking thermal noise for $W_2(t = 0)$, we find that the beam will be stable against quasi-linear diffusion if

$$(n_b/n_p)^{2/3}(V_b/\Delta V_b)^{7/3}(V_e/V_b)^{2/3}(M/m)^{1/3} < 10^{-2}(k_m^2\lambda_d^2)^{1/3} \qquad (13)$$

where k_m is the wave number with the maximum growth rate for the parametric instability ($k_m\lambda_d \sim 0.1 - 0.2$). For the typical values quoted, (13) becomes

$$\Delta V_b/V_b \geq 5(n_b/n_p)^{2/7} \qquad (14)$$

and for $\Delta V_b/V_b \sim 1/3$ and $n_b \sim 2 \times 10^{-2}$ we find that the beam will keep its structure for $n_p \geq 2.6 \times 10^2$ cm^{-3}, which is certainly satisfied at 750 km or below.

Time for tail formation. For cases where (14) is satisfied, the transfer of energy in the region k_2 will cause the resonant particles in f_T to spread according to (4). The time scale for this to occur is given by

$$t \sim v^2/[(8\pi e^2/m^2)(W_2/\Delta\omega)] \qquad (15)$$

Using $\Delta\omega \sim \omega_i$, $W_2 \sim n_b m V_b \Delta V_b$, and $v \sim 10V_e$, we find that $t \lesssim 10^{-4}$ s. Even if W_2 is 2 orders of magnitude smaller, the time for tail formation is very fast with respect to the observation times. One can therefore expect that tails will be formed almost instantaneously.

Lower velocity bound of the power law tails. From (11) using the fact that $W_1 < n_b m V_b \Delta V_b$ and the typical values quoted above, we find that the low-energy bound is given by $V_1/V_e > 3$. This value corresponds to deviations from Maxwellian for energies much higher than 5 eV.

Tail number density. The tail number density can be found directly by integrating the ambient Maxwellian from V_1 to infinity; namely,

$$\frac{n_T}{n_p} = 2\frac{1}{(2\pi)^{1/2}}\frac{\exp -(V_1^2/2V_e^2)}{(V_1/V_e)}$$

For $V_1/V_e \sim 3$ and $n_p \sim 1 \times 10^3$ this equation gives a value of $n_T \sim 1$.

Tail power law. Since all the time scales discussed above are much smaller than the burst duration and the observation time, we expect that a quasi-steady state will be established, so that in an average sense the beam plasma and the parametric instability will be marginally stable. The existence of the tail can stabilize the parametric instability by increasing the Landau damping in the region (V_1, V_2) to balance the growth at each unstable wave number (see equation 7). The Landau damping, due to an arbitrary tail distribution f_T, is proportional to $\nu_{eff}\, \alpha v_{ph}^2 f_T'(v_{ph})$, whereas the growth rate as given by (10*a*) and (10*b*) is either proportional to $1/v_{ph}^{2/3}$ or independent of the phase velocity, depending on whether the fields are much larger than threshold or not. Since in order to establish a marginal stability state, $[\gamma_2(v_{ph})/\nu_{eff}(v_{ph})] \sim 1$, we can determine the velocity dependence of f_T. Depending on whether we use (10*a*) or (10*b*) for the growth rate we find either that $f_T(v_{ph}) \sim 1/v_{ph}^{-5/3}$ or that $f_T(v) \sim 1/v$. More generally, we can say that

$$f_T(v) = A(1/v^n) \qquad 1 \leq n \leq \tfrac{5}{3}, \qquad 3V_e \leq v \leq V_b \qquad (16)$$
$$f_T(v) = 0 \qquad \text{elsewhere}$$

where A is a normalization factor.

Particle flux. From (16) we find that the energy dependence of the directional differential flux is given by $F \sim 1/E^{n/2}$, which for $1 \leq n \leq 5/3$ is in excellent agreement with the data. Most striking is the fact that use of the value of $n_T \sim 1$ for the number of particles in the tails gives excellent agreement with the measured flux magnitude.

High-frequency electric fields. In the marginal stability state (Figure 2) the electron tails stabilize the parametric instability. Thereafter, the beam plasma instability should resume unless the electric field energy in the region of the tails is large enough to make the right-hand side of (2) zero or negative. Using (2) and (10*b*), we find that the suprathermal level of fluctuations at ω_e must satisfy

$$W_2/\tfrac{1}{2}nmV_e^2 \geq 4(n_b/n_p)(V_b/\Delta V_b)^2 \qquad (17)$$

Actually, the value of W_2 will be given by (17) or the equilibrium suprathermal level due to the tails, which is of the order of $(V_b/V_e)^2$ above the thermal noise. These high-frequency electric fields can be measured either directly by rockets or indirectly by enhanced radar scattering.

Extension to more dimensions. The beam plasma interaction is strongest along the direction of the beam, whereas the parametric instability grows mostly in the direction of the pump waves. However, waves at an angle can grow. The growth rates fall off as $\cos^2\theta$, where θ is the angle between the pump field and the wave. These waves will produce heating in the transverse direction. A detailed analysis in more dimensions will be presented soon. Here we remark that for fluxes at an angle θ the basic features of the results will be like those in our one-dimensional analysis, whereas the numerical estimates will be correct to within a factor of $\cos^2\theta$, which is less than 4 for $\theta < 60°$. This finding has been verified numerically by *Kruer and Dawson* [1972].

Anomalous resistivity. Since the parametric instability, besides producing the electron plasma waves, also produces purely growing ion density fluctuations, we expect the resistivity of the auroral plasma to be modified. In a simpleminded way one would expect that $|\delta n_i|^2/|\delta n|_{th}^2 \sim W_2/W_{2th}$, which is given by (17). We would therefore expect an anomalous resistivity larger than the collisional one by a factor of $(V_b/V_e)^2$ or $4(n_b/n_p)(V_b/\Delta V_b)^2$, whichever is larger. This gives an enhancement by at least 10^4. Of course, this is valid only on the assumption that the ion density fluctuations do not damp. A detailed examination of the question of anomalous resistivity is presently under study.

CONCLUSION

It has been proposed that the nonlinear plasma interactions that stabilize the instability between ambient and precipitating plasma electrons can account for several of the observed nonthermal features of the auroral plasma.

Comparison of the measurements with the theory shows very good qualitative and quantitative agreement.

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REFERENCES

Davidson, R. C., *Methods of Non-Linear Plasma Theory,* chap. 8, pp. 174–182, Academic, New York, 1972.

Kainer, S., J. M. Dawson, and T. P. Coffey, Alternating current instability produced by the two-stream instability, *Phys. Fluids, 15,* 2419, 1972.

Kruer, W. L., and J. M. Dawson, Anomalous high-frequency resistivity of a plasma, *Phys. Fluids, 15,* 446, 1972.

Nishikawa, K., Parametric excitation of coupled waves, 1, General formulation, *J. Physiol. Soc. Jap., 24,* 916, 1968.

Nishikawa, K., Parametric excitation of coupled waves, 2, Parametric plasmon-photon interaction, *J. Physiol. Soc. Jap., 24,* 1152, 1968.

O'Brien, P. S., and D. L. Reasoner, Measurements of highly collimated short duration bursts of auroral electrons and comparison with existing auroral models, *J. Geophys. Res., 76,* 8258, 1971.

Papadopoulos, K., M. Goldstein, and R. Smith, Stabilization of electron streams in type 3 solar bursts, *Astrophys. J.,* in press, 1974.

Reasoner, D. L., and C. R. Chappell, Twin pay load observations of incident and backscattered auroral electrons, *J. Geophys. Res., 78,* 2176, 1973.

Sanmartin, J. R., Electrostatic plasma instabilities excited by a high-frequency electric field, *Phys. Fluids, 13,* 1533, 1970.

Thode, L. E., and R. N. Sudan, Two-stream instability heating of plasmas by relativistic electron beams, *Phys. Rev. Lett., 30,* 732, 1973.

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