Modulational instability in a plasma with suprathermal electrons

H. P. Freund, Robert A. Smith, and K. Papadopoulos[a)]

Science Applications, Inc., McLean, Virginia 22102

P. Palmadesso

Geophysical and Plasma Dynamics Branch, Plasma Physics Division, Naval Research Laboratory, Washington, D.C. 20375
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The consequences of the presence of suprathermal electrons on the linear stage of modulational instabilities are investigated for a range of parameters appropriate to both laboratory and astrophysical plasmas. Substantial modifications to the growth rate and secondary spectrum are found to occur for instabilities driven by large amplitude Langmuir waves in the dipole limit due to the kinetic effects of the suprathermal electrons. In particular, for reasonable choices of pump amplitude and suprathermal energy density, additional modes become unstable.

One of the major theoretical advances in the study of both laboratory and space plasmas has been the recent progress in the understanding of strong turbulence phenomena associated with the localized behavior of Langmuir wave packets excited by laser- or electron beam-plasma instabilities. Most of the work to date has been restricted to the study of thermal plasmas; however, space and laboratory plasmas are observed with a significant component of suprathermal electrons. Moreover, the suprathermal electron tails are often well correlated with plasma configurations where modulational instabilities and Langmuir collapse are expected to be important. Such situations arise in laser-plasma interactions,[1-4] beam-plasma interactions,[5-8] and solar wind plasmas in areas where type III solar radio emission occurs.[9-13] It is therefore imperative to examine the role of such energetic tails on the behavior of modulational instabilities. The information will be used not only for plasmas with suprathermal tails before the start of Langmuir collapse, but also as an important step in the understanding of the later stages of evolution for laser- and beam-plasma systems where the suprathermal tails were self-consistently produced by the effects of strong turbulence.

The purpose of this work is to consider the modulational instability of large amplitude Langmuir waves in the dipole approximation, sometimes referred to as the oscillating two-stream instability. The plasma is assumed to consist predominantly of thermal electrons and ions, but to also contain a low density energetic electron component in the form of symmetric suprathermal tails. In this process, the fundamental concept is a four-wave interaction in which a large amplitude pump wave with frequency and wave vector $(\omega_0, \mathbf{k}_0)$ excites secondary Langmuir waves $(\omega_1, \mathbf{k}_1)$ and a low-frequency ion-acoustic mode $(\omega_2, \mathbf{k}_2)$, subject to the requirements that $\omega_0 = \omega_1 \pm \omega_2$ and $\mathbf{k}_0 = \mathbf{k}_1 \pm \mathbf{k}_2$. It is found that the essential nature of the linearized parametric instability remains unchanged by the suprathermal electrons for wavelengths $\mathbf{k}_1$ corresponding to phase velocities much greater than the energetic electron thermal speed. In this regime, both the thermal plasma and the suprathermal electrons can be treated as fluids. However, for wavelengths in which the phase velocities of the secondary Langmuir waves are comparable to or less than the energetic electron thermal speed, resonant wave-particle interactions must be included in the analysis, and a fully kinetic treatment is required. In the analysis described herein, a kinetic form of the dispersion equation of the modulational instability is employed to treat the presence of the suprathermal electrons, with the principle result that for sufficiently high levels of suprathermal energy density, new unstable modes appear due to resonant interactions. The electrostatic approximation is made throughout.

The linearized dispersion equation is obtained in the following manner.[14,15] For a pump wave $\mathbf{E}_0 = E_0 \hat{\mathbf{e}}_z \sin\omega_0 t$, the single particle trajectories of the αth species are

$$v_x(t) = v_x, \quad v_y(t) = v_y,$$

$$v_z(t) = v_\parallel - (e_\alpha E_0/m_\alpha \omega_0) \cos\omega_0 t,$$

and

$$x(t) = x + v_x t, \quad y(t) = y + v_y t,$$
$$z(t) = z + v_\parallel t - (e_\alpha E_0/m_\alpha \omega_0^2) \sin\omega_0 t,$$

where v_x, v_y, and $v_\parallel$ are constants of the motion. The equilibrium distribution function is assumed to be both homogeneous and isotropic, and to be of the form $F_\alpha = F_\alpha(v^2)$ where $v^2 = v_x^2 + v_y^2 + v_\parallel^2$. Integration of the Vlasov equation then yields the perturbed distribution function, δf_α, due to the excited wave spectrum $\delta\varphi$

$$\delta f_\alpha(\mathbf{x}, \mathbf{v}, t) = \frac{e_\alpha}{m_\alpha} \int_{-\infty}^{t} dt' \, \nabla' \delta\varphi[\mathbf{x}(t'), t'] \cdot \frac{\partial F_\alpha(v^2)}{\partial \mathbf{v}(t')}, \qquad (1)$$

where $\nabla' \equiv \partial/\partial \mathbf{x}(t')$. Defining the Fourier transform as

$$\{\delta f_\alpha(\mathbf{x}, \mathbf{v}, t), \delta\varphi(\mathbf{x}, t)\}$$
$$= \sum_{n=-\infty}^{\infty} \int d^3k \exp[i\mathbf{k}\cdot\mathbf{x} - i(\omega - n\omega_0)t]$$
$$\times \{\delta f_{\alpha n}(\mathbf{k}, \mathbf{v}, \omega), \delta\varphi_n(\mathbf{k}, \omega)\},$$

[a)]Permanent address: Department of Physics and Astronomy, University of Maryland, College Park, Md. 20742.

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after some manipulation we obtain that

$$\delta f_{\alpha n} = -\frac{2e_\alpha}{m_\alpha} \sum_{l,m=-\infty}^{\infty} \delta\varphi_m \frac{J_l(b_\alpha) J_{l+\varepsilon_\alpha(n-m)}(b_\alpha)}{\omega - \mathbf{k}\cdot\mathbf{v} - (m - \varepsilon_\alpha l)\omega_0}, \qquad (2)$$

where $\varepsilon_\alpha \equiv e_\alpha/|e_\alpha|$, $b_\alpha \equiv k_z |e_\alpha E_0/m_\alpha \omega_0^2|$, and $J_n(x)$ is the ordinary Bessel function of order n. The perturbed density $\delta n_{\alpha n}$ of the αth species follows immediately

$$\delta n_{\alpha n} = -\frac{k^2}{4\pi e_\alpha} \sum_{l,m=-\infty}^{\infty} \delta\varphi_m J_l(b_\alpha) J_{l+\varepsilon_\alpha(n-m)}(b_\alpha) \times \chi_\alpha[\mathbf{k}, \omega - (m - \varepsilon_\alpha l)\omega_0], \qquad (3)$$

where

$$\chi_\alpha(\mathbf{k}, \omega - n\omega_0) \equiv \frac{8\pi e_\alpha^2}{m_\alpha k^2} \int d^3v \frac{(\mathbf{k}\cdot\mathbf{v})\partial F_\alpha/\partial v^2}{\omega - \mathbf{k}\cdot\mathbf{v} - n\omega_0}. \qquad (4)$$

The dispersion equation follows from Poisson's equation $k^2\delta\varphi_n = 4\pi e(\delta n_{in} - \delta n_{en})$. In the limit $b_i \ll 1$, we may write

$$[1 + \chi_i(\mathbf{k}, \omega - n\omega_0)]\delta\varphi_n \simeq -\sum_{m,r=-\infty}^{\infty} \delta\varphi_m J_{r-m}(b_e) J_{r+n}(b_e) \chi_e(\mathbf{k}, \omega - r\omega_0). \qquad (5)$$

Under the assumption that $|\omega| \ll \omega_0$, $\chi_i(\mathbf{k}, \omega - n\omega_0) \simeq \chi_i(\mathbf{k}, \omega)\delta_{n,0}$, the dispersion equation becomes

$$\frac{1}{\chi_i(\mathbf{k}, \omega)} = -\sum_{q=-\infty}^{\infty} \frac{J_q^2(b_e)}{1 + \chi_e(\mathbf{k}, \omega - q\omega_0)}, \qquad (6)$$

where we have also assumed that $b_e \ll 1$, and made use of the relation[16] $\sum_{n=-\infty}^{\infty} J_{r-n}(x) J_{q-n}(x) = \delta_{r,q}$. If we expand the Bessel functions and retain terms of order b_e^2, Eq. (6) reduces to the form

$$\frac{1}{\chi_i(\mathbf{k}, \omega)} + \frac{1}{1 + \chi_e(\mathbf{k}, \omega)} = -\frac{1}{4} b_e^2 \left(\frac{1}{1 + \chi_e(\mathbf{k}, \omega + \omega_0)} + \frac{1}{1 + \chi_e(\mathbf{k}, \omega - \omega_0)} \right). \qquad (7)$$

The plasma configuration which is of interest to us can be described by a Maxwellian ion distribution

$$F_i(v^2) = n_0(\pi v_i^2)^{-3/2} \exp(-v^2/v_i^2), \qquad (8)$$

and an electron distribution of the form

$$F_e(v^2) = n_1(\pi v_1^2)^{-3/2} \exp(-v^2/v_1^2) + n_2(\pi v_2^2)^{-3/2} \exp(-v^2/v_2^2), \qquad (9)$$

where $n_0(\equiv n_1 + n_2)$ is the plasma density, $n_2 \ll n_1$, v_i is the ion thermal speed, and v_1 and v_2 $(v_2 \gg v_1)$ are the thermal speeds of the ambient and suprathermal electrons. Throughout, we shall use the subscripts "1" and "2" to describe the thermal and suprathermal electrons, respectively. As a consequence,

$$\chi_i(\mathbf{k}, \omega) = (k\lambda_i)^{-2}[1 + \zeta_i Z(\zeta_i)],$$

$$\chi_e(\mathbf{k}, \omega) = \chi_1(\mathbf{k}, \omega) + \chi_2(\mathbf{k}, \omega),$$

where

$$\chi_1(\mathbf{k}, \omega) = (1 + \mu)^{-1}(k\lambda_1)^{-2}[1 + \zeta_1 Z(\zeta_1)],$$

and

$$\chi_2(\mathbf{k}, \omega) = \mu(1 + \mu)^{-1}(k\lambda_2)^{-2}[1 + \zeta_2 Z(\zeta_2)],$$

$Z(\zeta)$ is the plasma dispersion function,[17]

$$\zeta_\alpha \equiv \omega/kv_\alpha (\alpha = i, 1, 2), \quad \mu \equiv n_2/n_1, \quad \lambda_i^2 \equiv v_i^2/\omega_i^2,$$

$$\lambda_1^2 \equiv v_1^2/\omega_e^2, \quad \lambda_2^2 \equiv v_2^2/\omega_e^2, \quad \omega_i^2 \equiv 4\pi e^2 n_0/m_i,$$

and $\omega_e^2 \equiv 4\pi e^2 n_0/m_e$. Consistent with the dipole approximation, the pump frequency is the local electron plasma frequency, $\omega_0 = \omega_e$. Since the frequency ω characterizes the low frequency ion-acoustic oscillations, we assume that $kv_i \ll |\omega| \ll kv_1 \ll \omega_e$ and write that

$$\chi_i(\mathbf{k}, \omega) \simeq -(\omega_i^2/\omega^2)(1 + 3k^2\lambda_i^2\omega_i^2/\omega^2),$$

and

$$\chi_e(\mathbf{k}, \omega) \simeq (1 + \mu)^{-1}(k\lambda_1)^{-2} \times (1 + \mu T_1/T_2 + i(\pi/2)^{1/2}(|k|\lambda_1)^{-1}\omega/\omega_e),$$

where T_1 and T_2 are the temperatures characterizing the thermal and suprathermal electrons. The dispersion equation, therefore, is

$$\omega^4 + i\nu_{ik}\omega^3 - [\tfrac{1}{4}(3k^2\lambda_1^2 - \mu)^2\omega_e^2 + k^2c_s^2]\omega^2 - \tfrac{1}{4}i\nu_{ik}(3k^2\lambda_1^2 - \mu)^2\omega_e^2\omega + \tfrac{1}{4}\frac{m_e}{m_i}k^2\lambda_1^2(3k^2\lambda_1^2 - \mu)\left(3k^2\lambda_1^2 - \mu - \frac{k_\parallel^2}{k^2}\frac{W_0}{(nT)_t}\frac{\lambda_{\text{eff}}^2}{\lambda_1^2}\right)\omega_e^4$$
$$+ \tfrac{1}{2}\omega\omega_e(\omega^2 + i\nu_{ik}\omega - k^2c_s^2)[\chi_2(\mathbf{k}, \omega + \omega_e) - \chi_2(\mathbf{k}, \omega - \omega_e)] + \tfrac{1}{4}\omega_e^2\left((3k^2\lambda_1^2 - \mu)(\omega^2 + i\nu_{ik}\omega - k^2c_s^2) + \tfrac{1}{2}\frac{m_e}{m_i}\frac{k_\parallel^2}{k^2}k^2\lambda_{\text{eff}}^2\frac{W_0}{(nT)_t}\omega_e^2\right)$$
$$\times[\chi_2(\mathbf{k}, \omega + \omega_e) + \chi_2(\mathbf{k}, \omega - \omega_e)] - \tfrac{1}{4}\omega_e^2(\omega^2 + i\nu_{ik}\omega - k^2c_s^2)\chi_2(\mathbf{k}, \omega + \omega_e)\chi_2(\mathbf{k}, \omega - \omega_e) = 0 \qquad (10)$$

where $W_0 \equiv E_0^2/8\pi$ characterizes the pump amplitude, $\nu_{ik} \equiv (\pi m_e/2m_i)^{1/2}|k|\lambda_i\omega_i$ is the well-known ion acoustic damping rate, $c_s^2 \equiv (T_1 + 3T_i)/m_i$, $(nT)_t \equiv n_1T_1 + n_2T_2$ is the total thermal energy density contained by both electron species, and

$$\lambda_{\text{eff}}^2 \equiv \lambda_1^2(1 + \mu)^{-1}(1 + n_2T_2/n_1T_1)$$

is the effective electron Debye length. Equation (10) reduces to the usual oscillating two-stream dispersion equation[18] in the absence of a suprathermal tail.

Before we solve the complete dispersion Eq. (10) numerically, it is instructive to consider limiting cases for which analytic solutions can be obtained. We first treat the limit in which

$$kv_2 \ll \omega_e \ [\text{i.e.}, \ k^2\lambda_{\text{eff}}^2 \ll \mu(1 + n_1T_1/n_2T_2)],$$

and the suprathermal tail may be treated as a fluid. In this case, the dispersion equation becomes biquadratic in ω

$$\omega^4 - (k^2c_s^2 + \frac{9}{4}k^2\lambda_{\text{eff}}^2\omega_e^2)\omega^2 + \frac{3}{4}\omega_e^2k^2\lambda_{\text{eff}}^2\left(3k^2c_s^2 - \frac{m_e}{m_i}\frac{k_\parallel^2}{k^2}\frac{W_0}{(nT)_t}\omega_e^2\right) = 0. \qquad (11)$$

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The threshold condition follows immediately

$$\frac{W_0}{(nT)_t} > 3\frac{k_\parallel^2}{k^2}k^2\lambda_1^2\left(1+3\frac{T_i}{T_1}\right), \tag{12}$$

and shows that the scaling of the threshold with k is not altered by the presence of the suprathermal tail in this regime. Note that since terms of order ζ_2^{-4} have been neglected, the additional restriction that

$$k^2\lambda_{\rm eff}^2 \ll (T_1/5T_2)(1+n_1T_1/n_2T_2)$$

must also be imposed. In the opposite limit where $kv_2 \gg \omega_e$, however, the basic character of the instability is substantially altered due to kinetic wave-particle interactions. Expanding (10) in the small-ζ_2 limit, therefore, we find that the dispersion equation becomes

$$(\omega^2 + i\nu_{ik}\omega - k^2c_s^2)\left[\omega^2 + 2i\nu_{2k}\omega - \nu_{2k}^2 + \frac{1}{4}\left(3k^2\lambda_1^2 - \mu - \frac{\mu}{k^2\lambda_2^2}\right)^2\omega_e^2\right]$$
$$= \frac{1}{4}\frac{k_\parallel^2}{k^2}\frac{m_e}{m_i}\frac{W_0}{(nT)_t}k^2\lambda_{eff}^2\omega_e^4\left(3k^2\lambda_1^2 - \mu - \frac{\mu}{k^2\lambda_2^2}\right), \tag{13}$$

where

$$\nu_{2k} \equiv (\pi/8)^{1/2}\mu(k\lambda_2)^{-3}\omega_e\exp(-2/k^2\lambda_2^2)$$

describes the damping due to the suprathermal tail. It is evident from (13) that in the kinetic regime the contribution due to the energetic electrons scales as k^{-2}, which is fundamentally different from the fluid behavior. The threshold condition is obtained by setting $\omega = 0$ in (13)

$$\frac{W_0}{(nT)_t} > \frac{k_\parallel^2}{k^2}\frac{\lambda_1^2}{\lambda_{eff}^2}\left(1+3\frac{T_i}{T_i}\right) \times \frac{[4\nu_{2k}^2/\omega_e^2 + (3k^2\lambda_1^2 - \mu - \mu/k^2\lambda_2^2)^2]}{(3k^2\lambda_1^2 - \mu - \mu/k^2\lambda_2^2)}, \tag{14}$$

valid for $k\lambda_1 > (T_1/2T_2)^{1/2}$. There are two analytically accessible regimes in this case (subject to the restriction on k) which correspond to (1) $k\lambda_1 < (k\lambda_1)_\infty$, where

$$(k\lambda_1)_\infty \equiv (\mu/6)[1 + (1+12n_1T_1/n_2T_2)]^{1/2},$$

and (2) $k\lambda_1 > (k\lambda_1)_\infty$. These regimes are distinguished primarily by the fact that the threshold is singular for $k\lambda_1 = (k\lambda_1)_\infty$, but vanishes for $k\lambda_1 < (k\lambda_1)_\infty$. This corresponds to a radical alteration in the character of the solutions. Specifically, examination of (11) and (13) shows that the instability is aperiodic everywhere except in the small-ζ_2 regime where $k\lambda_1 < (k\lambda_1)_\infty$. The difficulty in dealing with this regime is that for physically reasonable values of the parameters, $(k\lambda_1)_\infty$ can be of the order of $(T_1/2T_2)^{1/2}$, in which case the small-ζ_2 expansion of $\chi_2(\mathbf{k}, \omega \pm \omega_e)$ becomes invalid. As a result, we present the full numerical solution of the complete dispersion Eq. (10) which makes no approximation *a priori* of the suprathermal electron susceptibility χ_2, and includes the ion-acoustic damping terms.

Our primary aim is to demonstrate the effect of the energetic tail on the instability, and we seek solutions to the dispersion equation for different values of the parameters which characterize the tail, while holding

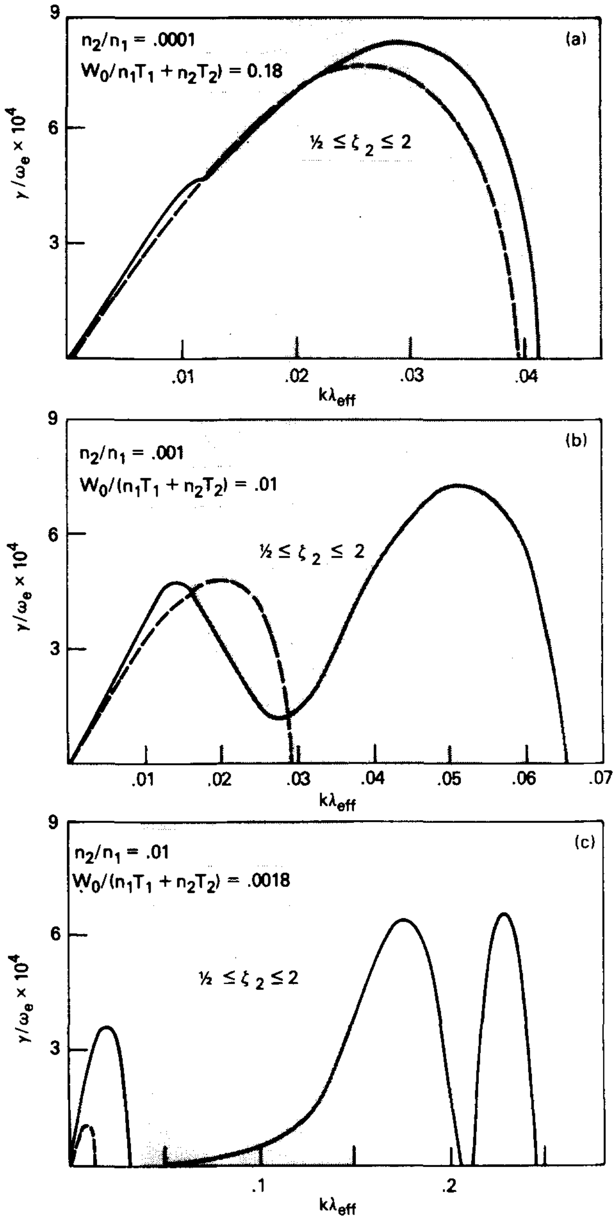

FIG. 1. Graph of the growth rate versus $k\lambda_{eff}$ for $T_1/T_2 = 0.001$ and (a) $\mu = 0.0001$, (b) $\mu = 0.001$, and (c) $\mu = 0.01$.

all other quantities constant. Thus, we assume that $W_0/n_1T_1 = 0.02$, $T_i/T_1 = 1$, and $\theta = 0°$ and plot the results of a numerical solution of (10) for γ ($\equiv$Imω) versus $k\lambda_{eff}$ in Fig. 1 for $T_1/T_2 = 0.001$ and (a) $\mu = 0.0001$, (b) $\mu = 0.001$, and (c) $\mu = 0.01$. The dotted line in each case represents the tail-free limit (i.e., $n_2 = 0$) for the corresponding value of $W_0/(nT)_t$ and the shaded portion represents the region in which ζ_2 is of order unity. In Fig. 1(a), the energy density of the suprathermal species is 10% that of the ambient electrons, and it is clear that only a small modification in the behavior of the instability occurs. As the suprathermal energy density increases, however, the effects of the energetic tail become more pronounced. The curve shown in Fig. 1(b) corresponds to the case in which the ambient and suprathermal electron energy densities are equal, and the growth rate has become double-peaked due to strong damping of the secondary Langmuir waves by the sup-

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rathermal tail. The instability is aperiodic (i.e., $\mathrm{Re}\,\omega = 0$) in each of these two cases. When the suprathermal energy density increases further, new structure appears due to kinetic effects in the small-ζ_2 regime. In Fig. 1(c), we display the growth rate corresponding to a suprathermal energy density which is an order of magnitude greater than that of the ambient electrons. Here, the instability contains additional modes in the kinetic-tail regime. The modes for $k\lambda_1 > (k\lambda_1)_\infty \simeq 0.064$ (i.e., for $k\lambda_{eff} \simeq 0.21$) and for $k\lambda_{eff} < 0.038$ are aperiodic. However, in the intermediate regime, two unstable modes exist having identical growth rates and real frequencies of equal magnitude but opposite sign. The real frequency is relatively insensitive to k, and we find that $0.98 \lesssim |\mathrm{Re}\,\omega/\omega_e \times 10^3| \lesssim 2.6$.

It should also be pointed out that an additional consideration in obtaining the new mode structure in the kinetic regime is that the region where $0.5 < \zeta_2 < 2$ (in which strong Langmuir damping overwhelms the parametric instability) is narrow. Since $\zeta_2 \sim T_2^{-1/2}$, this requirement is equivalent to the condition that $T_2 \gg T_1$. Even when this condition is not satisfied, however, we find that the modulational instability can still manifest a shift to longer wavelength modes due to this process when $n_2 T_2 \gtrsim n_1 T_1$.

The physical process responsible for the modifications to the modulational instability can be illustrated by consideration of the dispersion characteristics of Langmuir waves in the presence of a suprathermal tail. The parametric instability arises from competition between linear dispersion and the nonlinear frequency shift due to the ponderomotive force (in the fluid limit this is equivalent to competition between electron pressure and the ponderomotive force), and the instability threshold occurs when these forces are in balance. Since the nonlinear frequency shift acts to reduce the wave frequency (i.e., the ponderomotive force creates density depletions which reduce the local value of ω_e) and is proportional to W_0, the instability threshold is roughly proportional to the dispersive frequency shift of the Langmuir waves. We plot $\mathrm{Re}(\omega_k - \omega_e)/\omega_e$ and the associated instability threshold versus $k\lambda_{eff}$ in Fig. 2 for $T_1/T_2 = 0.001$ and $n_2/n_1 = 0.01$, where ω_k denotes the linear dispersion relation of the Langmuir waves. It is clear from Fig. 2(a) that the presence of the energetic tail results in a reduction in the frequency of the mode. When this reduction is to a frequency below ω_e, a propagating instability results. When the frequency of the mode drops below the Bohm-Gross frequency for the ambient electrons, the instability threshold is correspondingly reduced as well. This is shown in Fig. 2(b), in which the dotted line represents the threshold in the tail-free limit.

We conclude from this study that even a relatively low density suprathermal electron component can substantially alter the character of Langmuir wave modulational instabilities when the suprathermal energy density is comparable to that of the ambient plasma. In such cases, the instability is shifted to longer wavelength modes and a new mode structure appears due to resonant kinetic interactions between the energetic electrons and the secondary Langmuir waves.

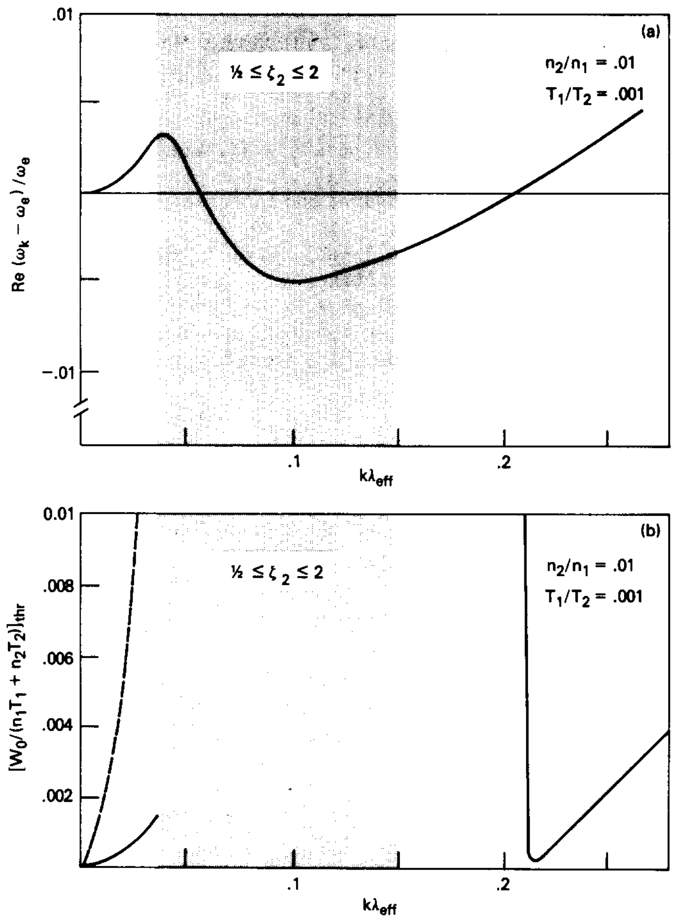

FIG. 2. Graph of the linear dispersion equation for Langmuir waves in the presence of a suprathermal tail, and the associated instability threshold.

Before closing, we should remark that a nonlinear theory of the collapse in the presence of suprathermal tails is under way. Based on the results of this study, however, the existence of stable wavenumber regimes in k space [i.e., zero growth regions shown in Fig. 1(c)] seems to indicate that stable three-dimensional solitons could exist for these scale lengths even in the absence of a magnetic field. We expect to report on this in the near future.

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