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Laminar interactions in high Mach number plasma flows

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The laminar interaction of counterstreaming plasmas in the presence of a magnetic field was studied by numerical simulation. The model is one-dimensional and is based on the Vlasov equation with straight orbits for ions ($\Omega_i t \ll 1$). Electrons are treated as a massless, zero-termperature fluid maintaining charge neutrality with the ions, and a finite resistivity is assumed. Magnetic field and density profiles are determined for different Mach numbers, density ratios, and resistivities. Effects of various finite initial density profiles in the streaming plasma are investigated. The distribution function is examined to determine whether microinstabilities would develop in a turbulent model. The relevance of these results to the formation of magnetic pistons is discussed.

I. INTRODUCTION

High Mach number flows exist in various astrophysical situations, laser produced plasma expansions, shock wave experiments, etc. The physical configuration that occurs in such cases is that of two plasmas streaming through each other in the presence of a magnetic field. The interactions which follow are a complex mixture of laminar and turbulent processes. The turbulent processes may be weak, so that they can be represented by anomalous transport coefficients modifying simply the laminar state, or strong in which case the "zeroth-order" laminar structure can change radically.

Recent nonlinear theoretical calculations and computer studies have demonstrated that for scale lengths shorter than an ion gyroradius, electron-ion instabilities are weak, in the sense that they can be represented by the addition of an anomalous resistivity to the laminar equations, while ion–ion instabilities are strong and can change the basic structure of the laminar flow.[1] Whether such strong ion–ion instabilities will occur in a high Mach number flow depends on the *local* magnetic field and electron state conditions.[2]

The purpose of the present study is to examine if and when a flow will result in strong ion–ion momentum coupling. The problem is a time dependent one where effects such as resistivity, initial density profile, Mach number, etc., might be important. This brings it out of the regime of possible analytic solution, and numerical methods seem appropriate.

For this particular study we developed a numerical scheme[3] whereby the ions are followed in their one-dimensional unmagnetized Vlasov orbits, while the electrons are a cold, massless fluid maintaining charge neutrality with the ions. The ions and the electron fluid are coupled through a collision frequency which can represent Coulomb collisions or the effects of an electron-ion instability. In the following we report results concerning the roles of the initial density profile in the leading edge, the Mach number, and finite resistivity in the laminar evolution of the system. The parameter range chosen was such as to fit recent laboratory experiments.

II. MODEL

An infinitely extended, homogeneous background plasma slab is immersed in a uniform magnetic field B_0, and partly penetrated by another plasma with a given density profile. At time $t=0$, the latter plasma is given a streaming velocity u_0, such that $M_{A0} \equiv u_0/V_{A0} \gg 1$, where V_{A0} is the Alfvén speed in the background. A typical phase space representation of such a configuration is given in Fig. 1.

The ions are considered unmagnetized (Lorenz forces are neglected), which restricts the time of applicability of our model to a fraction of an ion gyroperiod. However, this time interval is long enough to include the essential features of the laminar interactions. A similar model was used by Chodura and Finchenstein to study the compression pulse driven by a fast rising magnetic field.[4] The ion velocity distribution function is obtained by means of a hybrid numerical technique[3] in which weighted particles are advanced in phase space as in particle simulations, but where the distribution function is periodically reconstructed from the particles by a local averaging operation as in numerical solutions of the Vlasov equation.

We restrict ourselves to the case of magnetic fields transverse to the streaming, although the method can be trivially extended to arbitrary angle. The relevant ion particle equation is then

$$\ddot{x}_i = -\frac{B}{4\pi n m_i}\frac{\partial B}{\partial x}. \tag{1}$$

The electrons are treated as a cold massless fluid maintaining charge neutrality. This restricts us to low β_e ($\beta_e \ll 1$), laminar situations. However, the effects of electron-ion microturbulence are included through the use of a phenomenological resistivity (Coulomb or anomalous). Thus, the equation for the

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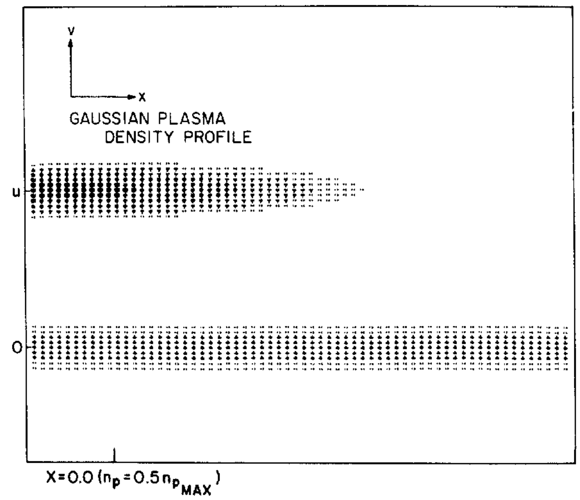

FIG. 1. Initial ion-phase space configuration for a typical counterstreaming plasma situation. A semi-infinite plasma with a Gaussian density profile is streaming at velocity u through a stationary background plasma.

magnetic field is

$$\dot{B}=\frac{\partial}{\partial x}\left(-uB+(\nu_{ei}c^2/\omega_{pe}^2)\,\frac{\partial B}{\partial x}\right), \qquad (2)$$

where ν_{ei} is the effective collision frequency and ω_{pe} is the local electron plasma frequency.

III. NUMERICAL RESULTS

Our system involves the following parameters which can be varied: the streaming to background plasma density ratio n_p/n_B, the Mach number $M_A\equiv u/V_A$, the value of finite resistivity defined by the ratio $\eta\equiv\nu_{ei}/\Omega_e$ and the respective ion thermal velocities expressed by the ratios u/V_i $(i=p, B)$ In addition, the density profile of the streaming plasma can be varied.

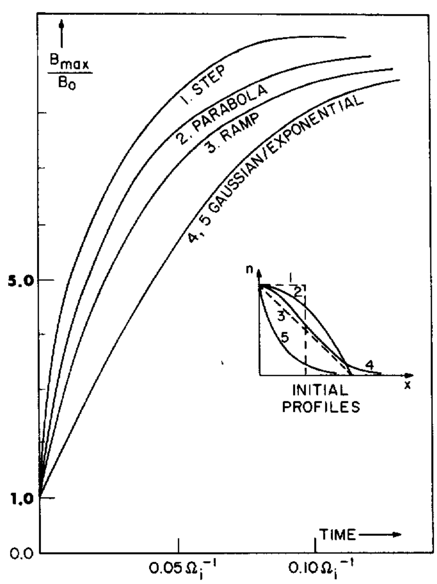

FIG. 2. Peak magnetic compression versus time for various initial density profiles.

A. Effect of Initial Density Profile

We first investigated whether the long-time laminar evolution of the plasma depends on the initial plasma profile. In most analytic calculations a square initial profile is assumed, in addition to zero ion temperature. Keeping n_p/n_B, u/V_i, M_A, and ν_{ei}/Ω_e constant, we experimented with square, linear, parabolic, exponential and Gaussian density contours, each of thickness $\sim 0.02r_i$ where r_i is the ion gyroradius, see insert in Fig. 2.

The runs were performed in a parameter space regime where $n_p/n_B\gg 1$, $u/V_i\gg 1$, $M_A\gg 1$ and $\nu_{ei}/\Omega_e\simeq 1$. Typical results are shown in Figs. 2 and 3. For this particular case, $n_p/n_B=80$, $M_A=10$ and $\nu_{ei}/\Omega_e=0.25$. In Fig. 2 we show the peak value of magnetic field compression $B_{\max}/B_0$ as a function of time. The values converge after a time of the order $0.1\Omega_i^{-1}$. For simulations with less abrupt initial density profiles, the compressions take correspondingly longer to converge.

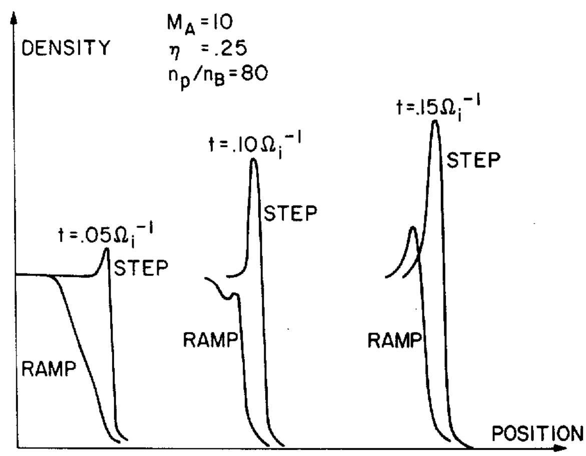

FIG. 3. Time-evolution of "sharp" and "ramped" initial density profiles.

In Fig. 3, density profiles are shown at different times for two extreme cases (linear and square profiles). One can see that the leading edge of the density evolves in similar fashion except for some time lag. This convergence is due to two effects which are not usually included in the analytic theories, namely, finite ion temperature and effects of induced electric fields on the ion orbits.

For studies of ion momentum coupling, knowledge of the exact location of the shell of magnetic compression is important, as is the magnitude of the compression. Figure 4 shows the ion phase space, the density profile, and the magnetic field profile at an

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intermediate stage ($\Omega_i t = 0.075$) for the case of a linear profile. A most interesting feature is the fact that the magnetic spike forms ahead of the region where n_p becomes large.

Consider a plasma impulsively started with uniform velocity u_0 into a stationary background plasma of density n_B in the presence of a transverse magnetic field B_0. At $t=0$, let the moving plasma have a linear density profile $n_p(x)$ between the limits x_1 and x_2:

$$n_p(x) = n_{p0}[(x_2-x)/(x_2-x_1)], \qquad x_1 \leq x \leq x_2.$$

If resistive effects are neglected, the time rate of change of B will be approximately

$$\frac{\partial B}{\partial t} \approx -\frac{\partial}{\partial x}\left(\frac{B u_0 n_p}{n_B+n_p}\right).$$

After a time δt, sufficiently short so that the field is only slightly altered, B will have changed by an amount

$$\delta B \approx \frac{B_0 u_0 \delta t}{(x_2-x_1)} \frac{n_B}{n_{p0}} \left(\frac{n_B}{n_{p0}} + \frac{x_2-x}{x_2-x_1}\right)^{-2}, \qquad x_1 \leq x \leq x_2. \quad (3)$$

In the interval (x_1, x_2), δB will be greatest at $x=x_2$, where it takes the value

$$\delta B \approx [B_0 u_0/(x_2-x_1)](n_{p0}/n_B)\ \delta t.$$

This situation is quite general, and similar results are found for various density profiles, geometries, and velocity fields—the peak magnetic compression initially occurs where n_p is comparable to, and in some cases much smaller than n_B. Even a profile which is initially sharp quickly assumes a finite thickness due to thermal and electrodynamic effects, and the magnetic pulse forms on the resulting density "foot" of the streaming plasma. However, energy to compress the magnetic field is supplied by the kinetic energy of the streaming ions in the vicinity of the shell, and the magnetic peak must, in all such cases, gradually fall back toward regions of increasing n_p.

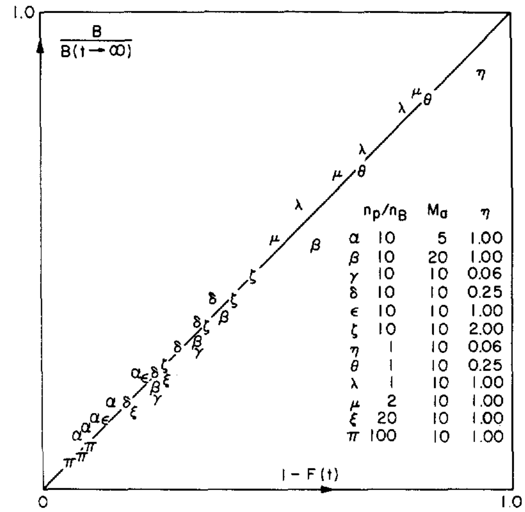

FIG. 5. Computed peak field compressions compared with the predictions of the modified Lampe–Hernandez model (Ref. 5).

These details in the buildup of magnetic compression can have important consequences. Since the ion–ion instabilities have very short wave-lengths and are expected to occur in regions of high magnetic field,[2] the instability criterion should be based on local conditions in the vicinity of the spike.

Finally, from the ion phase space plot of Fig. 4, we see that the two ion streams are basically uncoupled. This is a general result, and implies that no laminar coupling is to be expected for such a parameter range. We will later discuss the relevance of this fact to experimental situations.

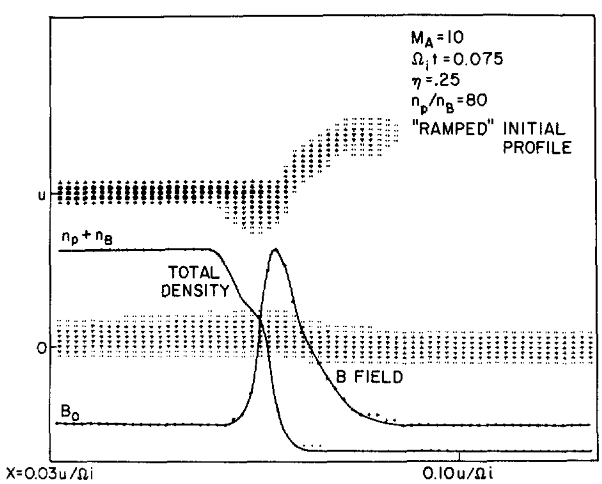

FIG. 4. Ion phase space, with superposed density and magnetic field profiles for a ramped initial profile ($\Omega_i t = 0.075$).

B. Time Evolution of Maximum Field Compression

Lampe and Hernandez[5] considered a simplified version of the counterstreaming ion problem. Assuming the ions to be at zero temperature, moving in straight orbits at constant velocity, they have studied the effect of resistivity with a square streaming plasma density profile and found that the maximum field compression at the leading edge can be given by

$$B(t)/B_0 = [(n_p+n_B)/n_B][1-F(t/\tau)], \qquad (4)$$

where

$$F(t/\tau) = \pi^{-1}\int_{-\infty}^{\infty} \frac{y^2 dy}{(y^2+\frac{1}{4})^2} \exp[-(y^2+\tfrac{1}{4})t/\tau], \qquad (5)$$

and

$$\tau = \Omega_i^{-1}\{(\eta/M_A^2)[(n_p+n_B)/n_B]\}. \qquad (6)$$

Note that in the limit $t\rightarrow\infty$, the magnetic field approaches the value given by the infinite conductivity approximation (i.e., the magnetic flux proportional to the local electron number density).

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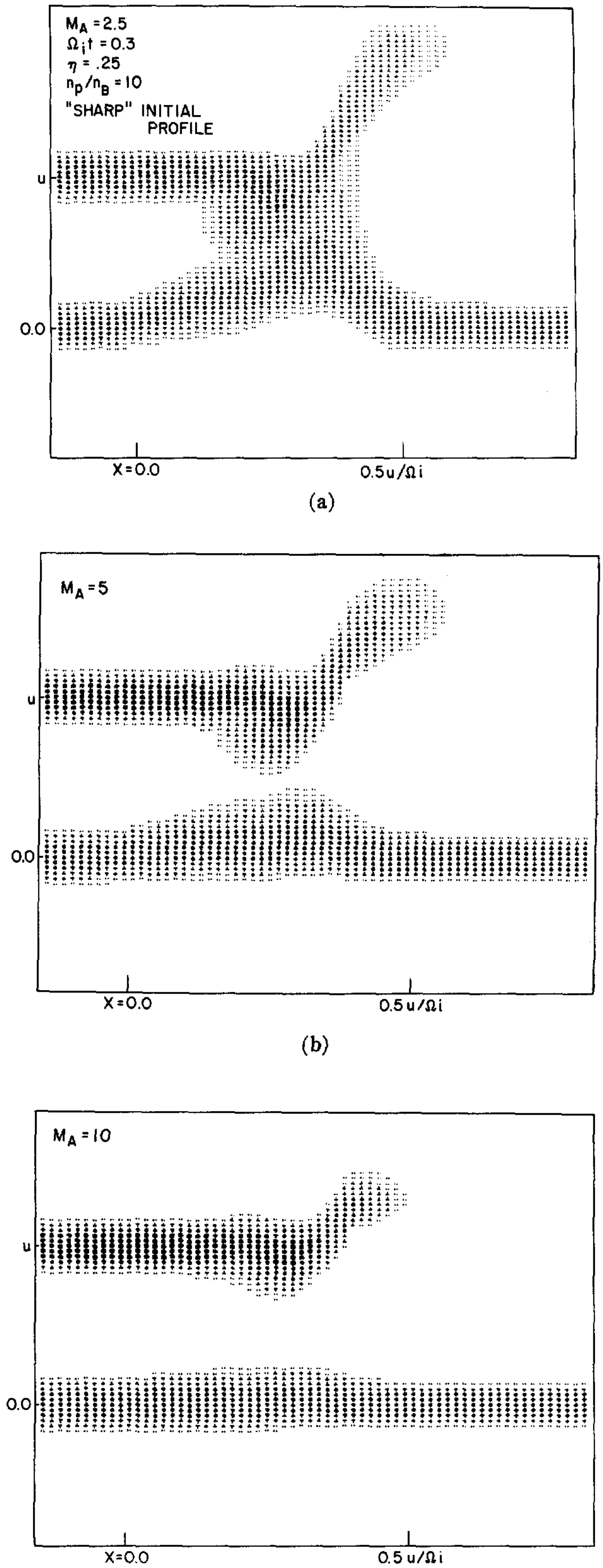

FIG. 6. Dependence of laminar coupling on Mach number: ion phase space configurations at $\Omega_i t=0.30$. (a) $M_A=2.5$, (b) $M_A=5.0$, (c) $M_A=10$.

We tried to find whether this result could be modified so that it would describe the numerical results. Agreement was obtained over a wide range of parameters if the time τ in Eq. (6) was redefined

$$\tau_c=\Omega_i^{-1}\{[3(\eta)^{1/2}/M_A^2][(n_B+n_p)/n_B]^{3/2}\}. \quad (7)$$

Figure (5) shows the agreement of Eqs. (4), (5), and (7) with numerical simulations at different Mach numbers, density ratios and resistivities. The characteristic time described by Eq. (7) takes into account effects not considered in the analytic model, including the influence of induced electric fields on the density profile of the leading edge of the streaming plasma.

C. Turbulent Coupling

The most important question in such high Mach number flows is whether the two plasmas will be coupled, producing either a reflecting or snow-plowing piston. In the case of laminar coupling, one would expect the moving electrostatic potential which accompanies the magnetic pulse to reflect the background ions, which would then move ahead without mixing with the streaming plasma. All of our numerical results indicate that for large M_A, no such laminar coupling occurs.

In Fig. 6, ion phase space plots at $\Omega_i t=0.3$ are presented for different values of M_A in the case of an initially sharp profile, with the other parameters fixed. Absence of laminar coupling on such a time scale is evident for $M_A \gtrsim 5$, leaving only the possibility of a microinstability accomplishing coupling in such a regime. Furthermore, it will be shown that at least one "fast" turbulent process can be expected to eclipse such purely laminar interactions for $M_A \lesssim 5$.

In the case of a transverse magnetic field the obvious candidate is the counterstreaming ion–ion instability discussed recently by Papadopoulos *et al.*[2] Such an instability occurs in one dimension only if the streaming

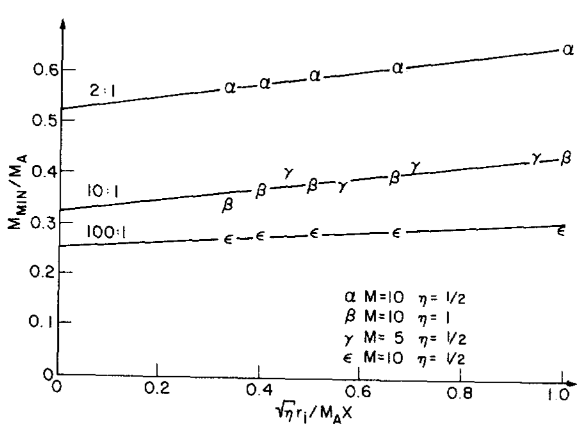

FIG. 7. Minimum local Mach number as a function of coupling length for a sharp initial profile, with $n_p/n_B=2$, 10, and 100.

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velocity u is such that

$$u \leq 2V_A(1+\beta_e)^{1/2},$$

where V_A is the *local* Alfvén speed. The one-dimensional assumption restricts the unstable modes to the direction of the streaming. However, in any experimental situation one has unstable modes over the whole plane perpendicular to the ambient field. If these modes are included in the theory, the condition for an instability able to produce coupling is modified to

$$u(x) \leq 2.5V_A(x)(1+\beta_e)^{1/2}. \quad (8)$$

As we have seen, the magnetic pulse initially occurs where $n_p \approx n_B$. This suggests that the maximum local Alfvén speed can be approximated by

$$V_A(x)\,|_{\max} \approx V_{A0}\frac{B(x)\,|_{\max}}{B_0\sqrt{2}}$$

$$\approx \frac{V_{A0}}{\sqrt{2}}\left(\frac{n_p+n_B}{n_B}\right)[1-F(x/x_0)], \quad (9)$$

where F is defined in Eq. (5), with the corresponding criterion for turbulent coupling given by

$$M_A \leq 2/5[(1+\beta)/2]^{1/2}[(n_p+n_B)/n_B][1-F(x/x_0)], \quad (10)$$

with $x_0 = r_i(\eta)^{1/2}/M_A{}^2(1+n_P/n_B)^{3/2}$ for $x/r_i<1$.

This yields a lower bound for the coupling length x, since the position of the magnetic peak will move into regions of higher n_p with time. The rate at which this occurs is a function of density ratio and Mach number, and the exact coupling criterion must be determined numerically.

The results of several simulations are shown in Fig. 7. By plotting (M/M_A) vs $(\eta^{1/2}/M_A)(r_i/x)$, substantial similarity is obtained with respect to resistivity and Mach number for a given density ratio. In all cases, computation was terminated at $x/r_i=0.3$. Based on results such as these, we can estimate that for the selected density ratios of 2, 10, and 100, conditions for turbulent coupling will be met (i.e., the local Mach number ahead of the magnetic shell will at some point drop below 2.5) within a length $0.3r_i$ for Mach numbers not exceeding about 4, 7, and 9, respectively.

Before closing this part, we would like to note that for laser plasma expansions in an ambient plasma where the geometry effects might be important, one can carry out a similar analysis with added factors (r/r_0) multiplying the right-hand side of Eq. (10). Here, r is the distance from the focal spot to the pulse, and r_0 is the radius where the expansion first becomes collisionless with respect to background ion Coulomb collisions. This factor is required by magnetic flux conservation in spherical geometry.

SUMMARY AND CONCLUSIONS

We believe that we have presented results which are of interest in experiments involving high Mach number flows. In particular:

(a) It was shown that the long-time laminar profiles produced in high M_A flows are essentially independent of the exact initial density contour.

(b) However, the importance of the location of the magnetic pulse with respect to the value $n_p(x)/n_B$ within the framework of a finite density profile was pointed out.

(c) It was demonstrated that, if coupling is observed in a high Mach number flow for times $t\Omega_i<1$, it is turbulent rather than laminar.

(d) A simple formula was presented for the value of the maximum field compression at the front.

(e) Criteria under which a transverse high Mach number flow evolves into a snowplowing piston were derived.

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