Interpretation of soliton formation and parametric instabilities

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It is shown that soliton formation and the resulting plasma heating are nothing more than the description in configuration space of well-known parametric processes and quasi-linear theory.

Parametric instabilities and soliton formation have recently been the subject of extensive studies with regard to laser and electron beam heating of plasmas. A particularly striking effect is the observation in computer simulations of localized large amplitude electric fields accompanied by self-consistent density cavities and electron heating.[1–15] A series of theories has been developed to examine the nature of this spiky turbulence and the associated particle acceleration in localized electric fields.[9,10,12–14] We feel that these interesting and worthwhile studies have not adequately discussed the equivalence of the description of soliton formation and the associated heating in x space, with well understood notions such as the oscillating two-stream instability and quasilinear diffusion in k space. Unfortunately, this inadequacy has resulted in some confusion. It is the purpose of this note to discuss the correspondence of the k and x space description of the above processes.

In a very interesting paper Thompson *et al.*[8] examined the system of equations

$$\frac{\partial^2}{\partial t^2}N_{ek}+\nu_{ek}\frac{\partial N_{ek}}{\partial t}+\omega_{ek}^2 N_{ek}=\frac{ike}{M_e}N_{ik}E_0+\frac{ike}{M_e}\sum_{k'\neq 0}N'_{ik-k}E'_k,$$
$$\frac{\partial^2 N_i}{\partial t^2}+\nu_{ik}\frac{\partial N_{ik}}{\partial t}+\Omega_k^2 N_{ik}=-\frac{ike}{M_i}N_{ek}E_0-\frac{ike}{M_i}\sum_{k\neq k'}\frac{k'}{k-k'}N'_{ek-k}E'_k, \qquad (1)$$

where ω_{ek}^2 is the Bohm–Gross frequency for a plasma oscillation with wavenumber k, Ω_k is the ion acoustic frequency and ν_{ek} and ν_{ik} are the electron and ion Landau damping rates respectively. For pump frequency ω_0 equal to or less than the electron plasma frequency, Eqs. (1) predict the oscillating two-stream instability by neglecting the nonlinear terms (last terms on the right-hand sides). Inclusion of these terms then describes the nonlinear behavior of this instability.

Equations (1) were solved numerically in Ref. 8, coupled to the quasi-linear equation for the electron distribution function f_e. From the instantaneous value of f_e, ν_{ek} can be obtained. The results of this numerical solution were compared with a particle simulation of the instability, showing excellent agreement. For instance, Fig. 3 of Ref. 8 compares the computed values of f_e with the values from the particle simulation. Clearly, they agree extremely well.

However, Eqs. (1) are nothing more than the equations for soliton formation. This can be seen most easily by using Poisson's equation

$$ikE_k=-4\pi eN_e(k) \qquad (2)$$

and writing the equations in the x domain,

$$\frac{\partial^2 E}{\partial t^2}-3\frac{T_e}{M_e}\frac{\partial^2 E}{\partial x^2}+\frac{4\pi N_i e^2}{M_e}E=0,$$
$$\frac{\partial^2 N_i}{\partial t^2}-\frac{T_e}{M_i}\frac{\partial^2}{\partial x^2}N_i-\frac{\partial^2}{\partial x^2}\frac{E^2}{8\pi M_i}=0, \qquad (3)$$

where we have set $\nu_{ek}=\nu_{ik}=0$, and for simplicity assumed quasi-neutrality for the low frequency fluctuation. Equations (3) in the x domain are indeed the standard equations of soliton formation[10–14] and include the trapping of the plasma wave in its own density cavity.

Although the authors of Ref. 8 concentrated on the k domain and the problem of parametric instabilities, they were, in fact, simultaneously studying soliton formation.

Thus, although the stochastic heating by localized field structures may appear to be difficult to understand in the x domain, it actually seems to be quite well understood in the k domain. That is ordinary quasi-linear theory seems to be perfectly adequate to describe the process. Even the qualitative arguments used in Ref. 9 to approximate the soliton width, in the x domain, simply reduce to $V=\omega/k$ (quasi-linear theory) in the k domain. Also, their calculations of energy change by a localized soliton, showing that it is proportional to soliton amplitude squared over a range of five orders of magnitude, is simply further confirmation that the particle acceleration is described by quasi-linear theory. Furthermore, the Born approximation utilized in Ref. 10 is nothing more or less than standard quasi-linear theory, transformed to the x domain. The reason the low velocity particles energy change averages to nearly zero is simply that these particles are not resonant with any wave in the assumed spectrum. In addition, the tail formation is consistent with the quasi-linear calculation used in Ref. 7.

We can, therefore, conclude that the basic nature of caviton formation and their interaction with the electrons are well understood, at least within the approximations of Refs. 9, 10, 12, 13, and 14.

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