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GENERATION OF ELF/ULF WAVES IN THE IONOSPHERE BY DYNAMO PROCESSES

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Abstract. It is shown that amplitude modulated HF heating in the ionospheric E-region, can produce substantial power in the ELF/ULF range even in the absence of ambient ionospheric currents. The signals are associated with spontaneous generation of magnetic fields caused by the coupling of the hot spot temperature gradient to the ambient ionospheric density gradient ($\nabla n \times \nabla T$). The mechanism produces values for PC5 micropulsations consistent with those observed by Stubbe and Kopka (1981).

Introduction

Excitation of VLF/ELF/ULF waves by the interaction of powerful HF waves in the ionosphere has been a very active theoretical and experimental topic. Up to now the most successful excitation techniques have been modulation of the polar and equatorial electrojets. In this technique an HF wave with a frequency of a few MHz is strongly absorbed in the lower ionosphere, resulting in an instantaneous increase in the electron temperature T. The dominant energy loss process, excitation of rotational N_2 levels, is also very fast (i.e. less than 1 msec), resulting in instantaneous temperature relaxation, when the HF power is reduced. Transmission of an amplitude modulated HF wave results in oscillation in the electron temperature followed by corresponding oscillation in the conductivity tensor. If the HF heating occurs in ionospheric regions where ambient d.c. currents flow, an a.c. current is generated in the heated volume which radiates at the modulation frequency [Chang et al. 1981, Stubbe et al. 1981]. In this letter we demonstrate that the presence of a d.c. current is not necessary and a substantial a.c. current at the modulated frequency can be spontaneously generated by a dynamo process. The efficiency of the process is inversely proportional to the low frequency. It is also suggested that this process rather than current modulation is the source of the PC5 pulsations received by Stubbe and Kopka (1981) during ionospheric heating experiments.

Generation of Oscillatory Magnetic Field

Spontaneous generation of magnetic fields from the interaction of electromagnetic (em) waves with plasmas was demonstrated by Stamper et al. (1971) in a laser-plasma interaction experiment. A simple equation was derived in this paper for the spontaneously created magnetic field $\underline{B}$ in the form of

$$\frac{\partial \underline{B}}{\partial t} + \nabla \times \left[\frac{c^2}{4\pi\sigma} (\nabla \times \underline{B}) \right] - B_o c \nabla \times \left[\frac{\hat{e}_z \times (\nabla \times \underline{B})}{4\pi\, ne} \right] - \nabla \times (\underline{u} \times \underline{B}) = \frac{c}{en} (\nabla n \times \nabla T) \qquad (1)$$

In Eq. (1) n is the electron density, $\underline{u}$ is the plasma flow velocity, σ the conductivity, and the ambient magnetic field was taken as $\underline{B}_o = \hat{e}_z B_o$. The source term on the r.h.s. of Eq. (1) represents a dynamo current due to the presence of a density gradient perpendicular to the temperature gradient. It is obvious from Eq. (1) that in the presence of a stationary density gradient (∇n) and an oscillatory temperature T the spontaneously generated magnetic field will have a similar time dependence phase shifted by $\pi/2$; therefore, the heated region will act as a radiating antenna. It is not difficult to solve Eq. (1) including the diffusion as well as the Hall and convection terms. It is, however, obvious that for a low β-plasma (β defined as the ratio of plasma pressure to magnetic pressure) the convective term can be neglected. In addition, since this letter addresses only the feasibility of the process and not the details of the radiation pattern, we consider the simple case where the ambient magnetic field $\underline{B}_o$ is parallel to the density gradient ∇n; this corresponds to the experimental configuration in the auroral zones. The azimuthal (B_θ) component of Eq. (1), is then given by

$$\frac{\partial B_\theta}{\partial t} - K \nabla^2 B_\theta = \frac{c}{en} (\nabla n)_z (\nabla T)_r \qquad (2)$$

where $K \equiv c^2/4\pi\sigma$ is the magnetic diffusivity in the ambient plasma.

In order to provide quantitative estimates of the amplitude B_θ, we assume cylindrical symmetry about the z-direction and prescribe the source term in Eq. (2) as

$$S(r,z,t) \equiv \frac{c}{en} (\nabla n)_z (\nabla T)_r = \frac{S_o}{2} \exp\left[-\frac{r^2}{L_T^2}\right] \exp\left[-\frac{z^2}{L_Z^2}\right] [1-\cos(\omega t)] \qquad (3a)$$

where

$$S_o \equiv \frac{c}{e} \frac{1}{L_N} \frac{\tilde{T}}{L_T} \quad , \qquad (3b)$$

$L_N = [(\nabla n)_z/n]^{-1}$ is the ambient density gradient length, $\tilde{T}/L_T$ is the volume averaged temperature gradient in the HF heated plasma, and L_Z represents the characteristic dimension of the heated region in the vertical direction.

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The cylindrically symmetric version of Eq. (2) with the source term given by Eqs. (3) can be solved as generalized diffusion equations by using a combination of Fourier-Bessel transforms. Following the procedure outlined in the appendix (Eq. A5) we find

$$B_\theta(r,z,t) = \frac{S_o}{2\omega}\int_o^\tau d\tau'$$

$$\frac{(1-\cos\tau')\exp\left[-\frac{(r/L_T)^2}{D(\tau-\tau')}\right]\exp\left[-\frac{(z/L_Z)^2}{F(\tau-\tau')}\right]}{(D(\tau-\tau'))\quad(F(\tau-\tau'))^{\frac{1}{2}}} \tag{4}$$

where $\tau=\omega t$, $\tau'=\omega t'$ and

$$D(\tau-\tau') \equiv \frac{4K}{\omega L_T^2}(\tau-\tau')+1 \tag{5a}$$

$$F(\tau-\tau') \equiv \frac{4K}{\omega L_Z^2}(\tau-\tau')+1 \tag{5b}$$

The amplitude of the spontaneously generated magnetic field B_θ will be controlled by the values of $4K/\omega L_T^2$ and $4K/\omega L_Z^2$. These parameters correspond to the ratio of the radial and axial diffusion time scales L_T^2/K and L_Z^2/K to the ELF/ULF time scale $4/\omega$. They represent the balancing between magnetic field dissipation due to the diffusion term in Eq. (2) and the magnetic field generation due to the source term. Equation (4) has simple solutions in the limiting cases of $4K/\omega(L_T^2, L_Z^2) >> 1$ and $4K/\omega(L_T^2, L_Z^2) << 1$. In the first case the diffusion time through the modified region is short compared to the ELF/ULF time scale $(4/\omega)$ and limits the amplitude to

$$B_\theta(0,0,t) = \frac{S_o}{16K}[\min(L_T^2,L_Z^2)][1-\cos(\omega t)] \tag{6}$$

Eq. (6) was found by approximating the time integration by taking the most significant contribution from the integrand at $\tau'=\tau$ and multiplying by the width of the contributed part $[\min(L_T^2,L_Z^2)]\ \omega/8K$. This simply states the physically obvious conclusion that the field amplitude is $S_o\tau_{diff}$, where τ_{diff} is the minimum axial or radial diffusion time through the modified region.

The second limit corresponds to long diffusion time τ_{diff} relative to the ELF/ULF build up time $4/\omega$. The value of B_θ found by performing the time integration in the limit that $k\to 0$ is

$$B_\theta(0,0,t) = \frac{S_o}{2\omega}[\omega t-\sin(\omega t)] \tag{7}$$

ULF Generation in the Lower Ionosphere

In this letter we will concentrate on the limit of long diffusion times, which for the frequenices under consideration is applicable in the lower ionosphere. The situation described by Eq. (6) as well as intermediate cases will be discussed in a future publication.

From Eq. (7), we can see that the spontaneously generated magnetic field consists of two parts. A steady magnetic field buildup increasing linearly with time due to zero diffusivity, and an oscillatory piece due to temperature modulation. The amplitude of the oscillatory magnetic field is $S_o/2\omega$, and in practical units, will be given by

$$|\tilde{B}(r,z,t)| = 10^{-1}\left[\frac{Hz}{f}\right]\left[\frac{\tilde{T}}{1000°K}\right]\left[\frac{10km}{L_N}\right]\left[\frac{10km}{L_T}\right]\gamma \tag{8}$$

An important result of Eq. (8) is the scaling of the field amplitude as 1/f. Stubbe and Kopka (1981) detected a B field amplitude of 10.8γ for 10 min. and 5.4γ for 5 min. period micropulsations excited by heating, which is consistent with the 1/f dependence of Eq. (8). Notice that if we take $\tilde{T} \approx 10^3$K, $L_N \approx L_T \approx 10$ km which corresponds to expected experimental values, we find from Eq. (8) $B_\theta \approx 5\gamma$ for f = .02 Hz (T = 5 min.) and $B_\theta \approx 10\gamma$ for f $\approx$ .01 Hz. As mentioned by Stubbe and Kopka (1981) the power expected on the basis of current modulation for the PC5 frequencies is by two orders of magnitude smaller. In comparing the estimates of Eq. (8) with the observations on the ELF-VLF range it is clear that the 1/f dependence does not hold for the range above 200 Hz and that the values expected from Eq. (8) are much lower than the ones expected on the basis of current modulation. For example for f = 2kHz, Eq. (8) predicts $10^{-4}\gamma$ at the source which is much smaller than the value of $10^{-3}\gamma$ measured on the ground. On the other hand the values detected near and below 10 Hz are consistent with the observed. On the basis of the above we expect that the transition from current modulation to dynamo action being the dominant process occurs in the range between 10-200 Hz. We note that inclusion of the Hall term in Eq. (1) does not affect the total magnetic field energy produced by dynamo action; the Hall term simply redistributes the B_θ generated flux to B_r and B_z components. For the E-region and $L_T \approx 10$km the long diffusion time approximation is restricted to frequenices $f \geq .01$Hz. Below this frequency field saturation described by Eq. (6) occurs.

Power Estimates

From Eq. (7) we find that the power radiated in the low frequency wave is

$$P_{LF} = \frac{1}{2}\pi L_T^2 L_N \frac{\partial}{\partial t}\frac{B_\theta^2}{4\pi} = \frac{1}{8}\frac{c^2\tilde{T}^2}{e^2 L_T}\frac{1}{\omega} \tag{9}$$

which in practical units becomes

$$P_{LF} \approx .3\left[\frac{\tilde{T}}{10^3 °K}\right]^2\left[\frac{10km}{L_T}\right]\left[\frac{1Hz}{f}\right] W \tag{10}$$

Since values of $\tilde{T}$ of the order of 4-5 × 10^3°K can easily be obtained, radiating power of the order of several tens of Watts can be generated in the ULF and lower ElF range. Such temperatures require powers of 1-2 MW [Perkins and Roble 1978; Chang et al. 1981]. Optimization of the scheme requires maximizing the electron temperatures (T^2 dependence) while minimizing the heated area ($1/L_N$ dependence). Both can be achieved with high antenna gain and by optimizing the energy deposition altitude. Detailed numerical studies are in progress and will be reported elsewhere.

Summary and Discussion

We have shown that amplitude modulated ionospheric heating can generate spontaneous magnetic fields by coupling to the density gradients of the ionospheric plasmas. The time dependence of the spontaneous magnetic fields is similar to the dependence of the temperature gradient perpendicular to the ionospheric temperature gradient, shifted by $\pi/2$ in phase. Therefore a periodic heating pulse will generate periodic magnetic pulsation. The amplitude and power of the low frequency signal scales of $1/f$ favoring the lower frequency signals. It is important to note that the mechanism does not require and is not affected by the presence of ambient ionospheric currents. The field generation is due to the presence of a source term producing fields with $\nabla \times \underline{E} \neq 0$ and is not a plasma or parametric instability such as discussed by Kuo and Lee (1983). In this paper we demonstrated the basic physical mechanism along with approximate estimates of the expected amplitude and power. It is easy to show that short scale density fluctuations due to ponderomotive force or density modulation will not affect the zero order field generation. The role of diffusion as well as application and scaling of the process for situations that ∇n is at angle to $\underline{B}_o$, are presently under study and will be reported elsewhere.

Appendix

For cylindrical symmetry, such as expected in the HF heating experiment, Eq. (2) reduces to

$$\left\{\frac{\partial}{\partial t} - K\left[\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial}{\partial r}\right] + \frac{\partial^2}{\partial z^2}\right]\right\} B_\theta(r,z,t) = S(r,z,t) \quad \text{(A1)}$$

where $S(r,z,t)$ is an arbitrary time dependent source given by Eqs. (3). Introducing a Bessel transform in r, and a Fourier transform in z, such as

$$B_\theta(\ell,k,t) = \frac{1}{2\pi}\int_0^\infty dr\, r \int_{-\infty}^\infty dz\, B_\theta(r,z,t) J_o(\ell r) e^{-ikz} \quad \text{(A2)}$$

equation (A1) reduces to

$$\left[\frac{\partial}{\partial t} + K(\ell^2+k^2)\right] B_\theta(\ell,k,t) = S(\ell,k,t) \quad \text{(A3)}$$

This equation is first order in time and yields an exact solution

$$B_\theta(\ell,k,t) = B_\theta(\ell,k,o)e^{-(\ell^2+k^2)Kt} \quad \text{(A4)}$$

$$+ \int_0^t dt' S(\ell,k,t')e^{-(\ell^2+K^2)K(t-t')}$$

where $B_\theta(\ell,k,o)$ is the Fourier-Bessel component of the initial condition $B_\theta(r,z,o)$ at $t = 0$. Final solution of Eq. (A1) is then obtained by the inverse transform

$$B_\theta(r,z,t) = \int_0^\infty d\ell\, \ell \int_{-\infty}^\infty dk\, B_\theta(\ell,k,t)\, J_o(\ell r) e^{ikz} \quad \text{(A5)}$$

Note that this procedure leaves the form of source term $S(r,z,t)$ unspecified, and can be extended to include the azimuthal dependence of the problem by adding a θ transform $\int d\theta\, e^{im\theta}$.

For the source term given by Eqs. (3) and $B_\theta(r,z,o)=0$, Eq. (A5) gives

$$B_\theta(r,z,t) = \frac{S_o}{2\omega}\int_0^\tau d\tau'(1-\cos\tau')$$

$$\frac{\exp\left\{-\left[\dfrac{(r/L_T)^2}{\frac{4K}{\omega L_T^2}(\tau-\tau')+1}\right]\right\}\exp\left\{-\left[\dfrac{(z/L_Z)^2}{\frac{4K}{\omega L_Z^2}(\tau-\tau')+1}\right]\right\}}{\left[\frac{4K}{\omega L_T^2}(\tau-\tau')+1\right]\left[\frac{4K}{\omega L_Z^2}(\tau-\tau')+1\right]^{\frac{1}{2}}} \quad \text{(A6)}$$

where $\tau = \omega t$, $\tau' = \omega t'$ and the integral relations

$$\int_0^\infty dr\, r e^{-(r/L_T)^2} J_o(\ell r) = \frac{L_T^2}{2} e^{-(L_T\ell/2)^2}$$

$$\int_{-\infty}^\infty dz\, e^{-ikz} e^{-(z/L_Z)^2} = \sqrt{\pi}\, L_Z e^{-(L_Z k/2)^2} \quad \text{(A7)}$$

were used to obtain the final result.

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