Radio Science, Volume 17, Number 5, pages 1321–1326, September–October 1982

Excitation of the earth-ionosphere waveguide by an ELF source in the ionosphere

V. K. Tripathi,[1] *C. L. Chang, and K. Papadopoulos*[1]

Science Applications, Inc., McLean, Virginia 22102

(Received August 13, 1981; revised May 17, 1982; accepted May 17, 1982.)

It is shown that for polar regions, where the conductivity is predominantly reactive, the wave functions for the transverse electromagnetic mode of the earth-ionosphere waveguide are Hankel functions of the first kind with real argument; therefore they extend up to large heights over the polar ionosphere. A Green's function for the excitation of the waveguide by extended current or field sources in the ionosphere is obtained for an exponential ionosphere. The potential of ELF waveguide excitation by wireless antennas is discussed.

1. INTRODUCTION

There has been considerable interest in the past in the excitation of the earth-ionosphere waveguide at ELF frequencies by satellite-borne current sources located in the ionosphere. *Galejs* [1971], *Einaudi and Wait* [1971], *Pappert* [1973], and *Kelly et al.* [1974] studied this problem in considerable detail and demonstrated that the satellite-based antennas could be radiators at least as efficient as the ground-based dipoles. One important result of these investigations is that the principle of reciprocity holds individually for all modes of the waveguide and hence the radiation fields on the surface of the earth due to a dipole source in the ionosphere could be deduced from the radiation pattern of a ground-based dipole.

More recently, wireless ELF generation in the ionosphere has emerged as an alternative to antenna excitation. There are two main techniques in achieving wireless ELF generation. The first relies on modulating the electrojet current in the lower ionosphere, using amplitude-modulated HF from ground antennas [*Davis and Willis*, 1974; *Stubbe and Kopka*, 1977; *Stubbe et al.*, 1981; *Chang et al.*, 1982]. The HF waves heat the electrons and produce an oscillatory electron temperature at the modulation frequency; this produces a conductivity modulation and hence a current modulation. *Stubbe et al.* [1981] have actually reported extremely encouraging experimental results on ELF generation by modulating the polar electrojet current with a 4.04-MHz RF transmitter. The second technique relies on resonant nonlinear parametric excitation of an Alfven wave, by mixing two HF waves in the ionosphere [*Papadopoulos et al.*, 1982]. In both cases the interaction region over which the ELF power is produced is comparable to the ELF wavelength in the ionosphere, and computations based on the reciprocity principle cannot be applied.

The excitation of the earth-ionosphere waveguide by extended sources requires knowledge of the mode structure of the continuation of the free space transverse electromagnetic (TEM) mode in the ionosphere. *Galejs* [1971] has determined analytic expressions for the mode structure of the TEM mode, assuming a homogeneous ionosphere. *Greifinger and Greifinger* [1974] examined the inhomogeneous ionosphere case and determined the mode structure for the case of purely real conductivity increasing exponentially with height. While this model gives good results for the equatorial zones, it is of very limited validity for higher latitudes; in actual fact, it produces very deceptive results. This is caused by the fact that the ELF index of refraction is mainly real at higher latitudes, corresponding to reactive conductivity. As it will be seen later, this has the very profound effect of allowing the mode to extend to much greater altitude. This can be physically seen as follows: the horizontal component of the TEM propagation wave number β is close to ω/c, which is several orders of magnitude smaller than the propagation constant $|\mathbf{k}|$ in the ionosphere; this implies that $\mathbf{k}$ is predominantly vertical. In the equatorial zones, $\mathbf{k}$ is perpen-

[1] Now at Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742.

Paper number 2S0770.
0048-6604/82/0910-0770$08.00

dicular to the earth's magnetic field **B**, and the refractive index is predominantly imaginary. At higher latitudes, **B** is more aligned with **k**, and the ELF attenuation is substantially reduced.

It is the purpose of this paper to study the mode structure of the TEM mode for ELF propagation at higher latitudes where the index of refraction is predominantly real. The conductivity is modeled as reactive and varying exponentially with height. The problem is solved analytically, and the field structure can be represented by height dependent Hankel functions. A basic difference between our solution and that of *Greifinger and Greifinger* [1974] is that the argument of the Hankel functions is real in our case while it was imaginary in theirs, allowing thereby a much higher altitude penetration. Knowledge of the proper mode structure allows us to determine the Green's function in the presence of a δ function source in the ionosphere and to calculate by simple integration the waveguide excitation for extended wireless ionospheric sources.

The plan of the paper is as follows. In the next section we list the value of the index of refraction as a function of altitude for polar and equatorial regions and calculate the attenuation of an ELF wave propagating downward toward the bottom of the ionosphere. It can be seen that the attenuation is very strong at the equator, forcing us to locate any sources below 70 km, while it is weak in the polar regions, allowing excitation from much higher altitudes. Restricting ourselves to the polar regions, we determine the structure of the waveguide TEM mode inside the ionosphere for an exponential density profile in section 3. On the basis of this structure, we compute the Green's function for a δ function source in the ionosphere in section 4. The last section summarizes and discusses the relevance of our results for wireless communications.

2. ATTENUATION OF ELF WAVES IN THE IONOSPHERE

Consider the propagation of an ELF wave in the TEM mode in the earth-ionosphere waveguide. The propagation constant for this mode has a horizontal component $k_x \simeq \omega/c$ (cf. next section). In the ionosphere the magnitude of propagation constant $k \simeq 10^3\omega/c$; hence

$$k_z = (k^2 - k_x^2)^{1/2} \gg k_x$$

i.e., the ELF waves propagate in the vertical direction (i.e., along the z axis). An ELF current source located at a height z in the ionosphere will produce signals which will be attenuated by a factor A,

$$A = \exp\left[-\int_h^z k_i \, dz\right] \tag{1}$$

before reaching the bottom of the ionosphere at a height h, where k_i is the attenuation constant given by

$$k = k_r + ik_i = n\omega/c$$

It may be mentioned that the concept of an attenuation constant is strictly applicable in media with slow spatial variations. However, in media with moderate spatial variations such as the lower ionosphere it also provides a reasonable estimate of the attenuation rate [*Al'pert and Fligel'*, 1970]. The value of n^2 is given by

$$\begin{aligned} n^2 = {} & \frac{1}{2(\varepsilon_{zz}\cos^2\theta + \varepsilon_{xx}\sin^2\theta)}\{\varepsilon_{xx}\varepsilon_{zz}(1+\cos^2\theta) \\ & + (\varepsilon_{xx}^2 + \varepsilon_{xy}^2)\sin^2\theta \pm [(\varepsilon_{xx}\varepsilon_{zz}(1+\cos^2\theta) \\ & - (\varepsilon_{xx}^2 + \varepsilon_{xy}^2)\sin^2\theta)^2 - 4\varepsilon_{zz}^2(\varepsilon_{xx}^2 + \varepsilon_{xy}^2)\cos^2\theta]^{1/2}\} \end{aligned} \tag{2}$$

Here

$$\varepsilon_{xx} = 1 + \frac{\omega_p^2(\omega + i\nu)}{\omega[(\nu - i\omega)^2 + \omega_c^2]} + \frac{\omega_{pi}^2(\omega + i\nu_{in})}{\omega[(\nu_{in} - i\omega)^2 + \omega_{ci}^2]}$$

$$\varepsilon_{xy} = -i\frac{\omega_p^2\omega_c}{\omega[(\nu - i\omega)^2 + \omega_c^2]} + i\frac{\omega_{pi}^2\omega_{ci}}{\omega[(\nu_{in} - i\omega)^2 + \omega_{ci}^2]}$$

$$\varepsilon_{zz} = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)} - \frac{\omega_{pi}^2}{\omega(\omega + i\nu_{in})}$$

and θ is the angle of the wave vector **k** with the magnetic field; ω_p, ω_c, and ν are the electron plasma, cyclotron, and collision frequencies respectively; ν_{in} is the ion-neutral collision frequency; and the quantities with subscript i refer to ions. For the extraordinary mode (i.e., the whistler mode or the compressional Alfven wave), (2) simplifies to give the following expressions of different heights above the ground.

Polar regions

For $z \lesssim 72$ km: $\nu^2 > \omega_c^2$; ion motion negligible

$$n^2 \simeq i\omega_p^2/\omega\nu$$

For $75 \lesssim z < 95$ km: ion motion unimportant

$$n^2 \simeq \frac{\omega_{pi}^2}{\omega\omega_{ci}\cos^2\theta}\left(\cos\theta + i\frac{\nu}{\omega_c}\right)$$

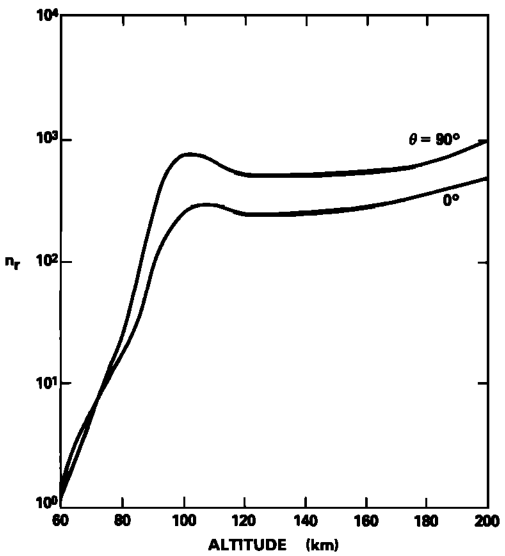

Fig. 1. Refractive index n_r as a function of altitude for two different propagation angles $\theta = 0°$ and $90°$, at a frequency of 100 Hz; $\theta = \pi/2 - \phi$ where ϕ is the geomagnetic latitude.

where

$$\tan^2 \theta < \frac{\omega_c^2}{\nu^2}$$

For $100 < z < 110$ km: ion motion important in ε_{xx}

$$n^2 \simeq \frac{\omega_{pi}^2}{\omega\omega_{ci}\cos^2\theta}\left[\cos\theta + \frac{i}{2}\left(\frac{\nu}{\omega_c}\frac{\omega_{ci}}{\nu_{in} - i\omega}\right)(1 + \cos^2\theta)\right]$$

For $z > 120$ km

$$n^2 \simeq \frac{\omega_{pi}^2 p(1 - p^2)}{2\omega\omega_{ci}\cos^2\theta}\left[\cos^2\theta + 1 - (\sin^4\theta + 4p^2\cos^2\theta)^{1/2}\right]$$

where

$$p = \frac{\omega + i\nu_{in}}{\omega_{ci}}$$

At the equator

$$n^2 = \varepsilon_{xx} + \frac{\varepsilon_{xy}^2}{\varepsilon_{xx}}$$

For $z \lesssim 100$ km

$$n^2 \simeq i\,\frac{\omega_p^2}{\omega\nu}$$

For $z > 120$ km

$$n^2 \simeq \frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1 + i\,\frac{\nu_{in}}{\omega}\right)$$

To appreciate the variation of the refractive index and the attenuation factor, we plotted them as a function of z for different values of θ ($\theta = 0$ corresponds to poles and $\theta = 90°$ corresponds to the equator) in Figures 1 and 2. The results are plotted by using the general expression for n^2 and the ionospheric data as given by *Al'pert and Fligel'* [1970].

The attenuation factor $\exp\left[-\int_h^z k_i\,dz\right]$ is unity at $z = h$ and decreases with height. In the equatorial regions it decreases very rapidly with z, suggesting that the current source should not lie above 90 km in order to have appreciable coupling with the waveguide. In polar regions the attenuation factor decreases very slowly with z, meaning that the current source can be located at any altitude allowed by the mode structure, which will be discussed next.

3. MODAL ANALYSIS

We model the earth-ionosphere waveguide by a perfectly conducting flat earth below $z \leq 0$, vacuum between $0 \leq z \leq h$, and the ionosphere above $z \geq h$. The refractive index of the ionosphere is much greater than unity, and since the horizontal propagation constant of the TEM mode is $\sim\omega/c$, the electromagnetic fields in the ionosphere propagate mainly in the vertical direction. Consequently, the ionosphere may be approximated by an isotropic medium with effective index of refraction corresponding to the compressional Alfven wave and $\theta = \pi/2 - \phi$ in (2) where ϕ is the geomagnetic latitude.

Considering the horizontal space and time variation as $\exp\left[-i(\omega t - \beta x)\right]$, the y and z components

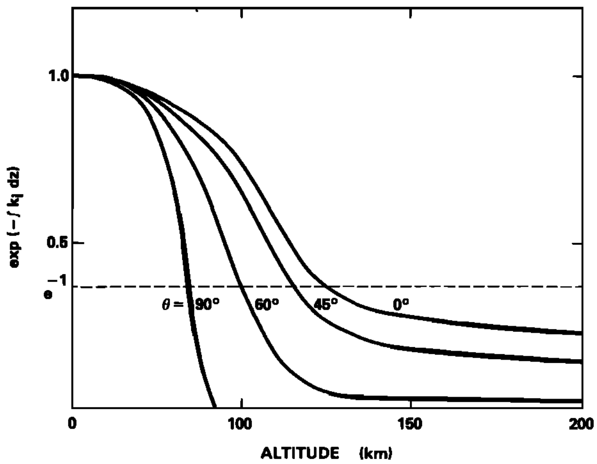

Fig. 2. The attenuation factor as a function of altitude for various angles of ELF wave propagation. The wave frequency is 100 Hz, and the lower boundary of the ionosphere is at 50 km.

of electric and magnetic vectors for TEM modes may be written as

$$E_z = \frac{i\beta}{(\omega^2\varepsilon/c^2 - \beta^2)} \frac{\partial E_x}{\partial z} \tag{3}$$

$$H_y = -\frac{i\omega\varepsilon/c}{(\omega^2\varepsilon/c^2 - \beta^2)} \frac{\partial E_x}{\partial z} \tag{4}$$

where E_x is governed by the wave equation

$$\frac{\partial^2 E_x}{\partial z^2} - \beta^2\left(E_x + \frac{c^2}{\omega^2\varepsilon}\frac{1}{\varepsilon}\frac{\partial\varepsilon}{\partial z}\frac{\partial E_x}{\partial z}\right) + \frac{\omega^2}{c^2}\varepsilon E_x = 0 \tag{5}$$

We take the effective dielectric constant $\varepsilon = n^2$ in the ionosphere to vary with height as

$$\varepsilon = \varepsilon_0 \exp\left[(z-h)/L_n\right] \tag{6}$$

where ε_0 could be real or complex. For ELF waves we are mainly interested in the TEM mode for which $\omega\varepsilon^{1/2}/c \gg \beta$. In this limit, (5) simplifies to

$$\frac{\partial^2 E_x}{\partial z^2} + \frac{\omega^2}{c^2}\varepsilon_0 \exp\left(\frac{z-h}{L_n}\right)E_x = 0 \tag{7}$$

and its solution for $z \geq h$, representing outgoing waves at $z \to \infty$, is

$$\begin{aligned} E_x &= BH_0^{(1)}(\xi) \\ H_y &= -i\varepsilon^{1/2}BH_0^{(1)\prime}(\xi) \end{aligned} \tag{8}$$

where

$$\xi = \xi_0 \exp\left[(z-h)/2L_n\right]$$

$H_0^{(1)}$ is the Hankel function of first kind of argument ξ, $H_0^{(1)\prime} = dH_0^{(1)}/d\xi$, and $\xi_0 = 2\omega L_n \varepsilon_0^{1/2}/c$. The electric and magnetic fields inside the waveguide ($0 \leq z \leq h$) are given by

$$\begin{aligned} E_x &= A\sin(\beta' z) \\ H_y &= -i\omega A\cos(\beta' z)/c\beta' \end{aligned} \tag{9}$$

where

$$\beta' = (\omega^2/c^2 - \beta^2)^{1/2}$$

The continuity conditions at $z = h$ yield the following dispersion relation:

$$\tan\beta' h = \frac{\omega}{c\varepsilon_0^{1/2}\beta'}\frac{H_0^{(1)}(\xi_0)}{H_0^{(1)\prime}(\xi_0)} \tag{10}$$

The right-hand side is less than 1, and the argument of the tangent function is small; hence (10) simplifies to give

$$\beta \simeq \frac{\omega}{c} - \frac{1}{2h\varepsilon_0^{1/2}}\frac{H_0^{(1)}(\xi_0)}{H_0^{(1)\prime}(\xi_0)} \tag{11}$$

For typical ionospheric parameters, $L_n \simeq 5$ km, $\varepsilon_0^{1/2} \simeq 20$, $h \simeq 60$ km, $\omega \simeq 10^2$ Hz, $\xi_0 = 0.4$, $\beta_r \simeq 1.06\omega/c$, and $\beta_i \simeq 2 \times 10^{-4}$ km^{-1}; i.e., the attenuation length is 5000 km. On changing $\varepsilon_0^{1/2}$ to 40 and keeping other parameters fixed we obtain $\beta \simeq 1.06\omega/c$ and $\beta_i \simeq 1.4 \times 10^{-4}$ km^{-1}. For a complex index of refraction the real part of the propagation constant is larger, and the attenuation length is shorter. Well-established values of attenuation of the TEM mode are around 2 dB per megameter.

4. GREEN'S FUNCTION

The excitation of ELF waves in the earth-ionosphere waveguide due to an extended current source in the ionosphere is best described by a Green's function. We consider a horizontal current source $\mathbf{J}_{\parallel}\hat{x}$ in the ionosphere. The wave equation governing E_x may be Fourier transformed (or Bessel function transformed) in x and y to obtain

$$\frac{d^2}{dz^2}E_{x\beta} - \frac{\beta^2c^2}{\omega^2\varepsilon}\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\frac{dE_{x\beta}}{dz} + \left(\frac{\omega^2}{c^2}\varepsilon - \beta^2\right)E_{x\beta} = -\frac{4\pi i\omega}{c^2}J_\beta(z) \tag{12}$$

where $E_{x\beta}$ and J_β are the Fourier transforms of E_x and J respectively. The Green's function for (12) satisfies the following equation:

$$\frac{d^2}{dz^2}G(z, z_0) - \frac{\beta^2c^2}{\omega^2\varepsilon}\frac{1}{\varepsilon}\frac{d\varepsilon}{dz}\frac{dG(z, z_0)}{dz} + \left(\frac{\omega^2}{c^2}\varepsilon - \beta^2\right)G(z, z_0) = -\frac{4\pi i\omega}{c^2}\delta(z - z_0) \tag{13}$$

To solve (13), we consider three regions: (I) $0 \leq z \leq h$, (II) $h \leq z \leq z_0$, and (III) $z \geq z_0$. The fields in region I are given by (9), and those in region III by (8). In region II, the fields may be written as

$$\begin{aligned} E_x &= B_1H_0^{(1)}(\xi) + B_2H_0^{(2)}(\xi) \\ H_y &= i\varepsilon^{1/2}[B_1H_0^{(1)\prime}(\xi) + B_2H_0^{(2)\prime}(\varepsilon)] \end{aligned} \tag{14}$$

These solutions must satisfy the continuity conditions at $z = h$ and jump conditions at $z = z_0$,

$$H_y|_{\mathrm{II}} = H_y|_{\mathrm{III}} + \frac{4\pi}{c} \qquad E_x|_{\mathrm{II}} = E_x|_{\mathrm{III}} \tag{15}$$

Employing these conditions, we obtain the Green's function,

$$G(z, z_0) = -i \sin \beta' z \cdot \left\{ \cos \beta' h \left[\tan \beta' h - \frac{\omega}{c\beta' \varepsilon_h^{1/2}} \frac{H_0^{(1)\prime}(\xi_0)}{H_0^{(1)}(\xi_0)} \right] \right\}^{-1} \cdot \frac{4\pi^{1/2}}{c\varepsilon_{z0}^{1/2}} \frac{H_0^{(1)}(\xi_{z0})}{H_0^{(1)\prime}(\xi_0)} \frac{\xi_{z0}}{\xi_0} \tag{16}$$

where $\xi_0 = \xi_0 \exp[(z_0 - h)/2L_n]$ and z lies inside the waveguide. Equation (16) has poles corresponding to different modes of propagation (cf. equation (10)).

Using (16) and taking the inverse Fourier transform in x, y of the resulting $E_{x\beta}$ gives the electric field

$$E_x = \iint dz_0 \, \boldsymbol{\beta} \, G(z, z_0) J_\beta(z_0) \exp[i\boldsymbol{\beta} \cdot \mathbf{X}_\perp] \tag{17}$$

The β integral can be evaluated easily by the residue theorem. At large values of $x_\perp$, only the pole corresponding to the TEM mode contributes; others damp out very rapidly.

Equation (17) can be expressed in terms of cylindrical polar coordinates r, θ, z,

$$E_x = \sum_n e^{in\theta} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \beta \, d\beta \, dz_0 \, G(z, z_0) J_{\beta n}(z_0) H_n^{(2)}(\beta r) \tag{18}$$

where

$$J_{\beta n} = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} r \, dr \, d\theta e^{-in\theta} J_x J_n(\beta r)$$

J_n is the Bessel function, J_x is the current density in the ionosphere, and $H_n^{(2)}$ is the Hankel function of second kind. For an azimuthally symmetric current distribution, $J_{\beta n} = 0$ for $n \neq 0$; hence only the $n = 0$ term in (18) survives. We consider next two cases.

Dipole current source

$$J_x = Idl\delta(r)\delta(z - z_0)/2\pi r$$

In this case,

$$J_{\beta 0} = \frac{Idl}{2\pi} \delta(z - z_0)$$
$$E_x = \frac{Idl}{2\pi} \int_{-\infty}^{\infty} \beta \, d\beta \, H_0^{(2)}(\beta r) G(z, z_0) \tag{19}$$

Equation (17) can be solved by contour integration, closing the contour in the lower half plane and considering only the TEM mode at large distances,

$$E_x \simeq i \frac{Idl}{\pi^{1/2}} \frac{\beta'}{ch\varepsilon_0^{1/2}} \frac{H_0^{(1)}(\xi_{z0})}{H_0^{(1)\prime}(\xi_0)} \sin \beta' z H_0^{(2)}(\beta r) \tag{20}$$

Here β and β' correspond to the TEM mode.

Gaussian current distribution

$$J_x = J_{00} \exp(-r^2/r_0^2)\delta(z - z_0) \tag{21}$$

For this distribution,

$$J_{\beta 0} = J_{00} \frac{r_0^2}{2} \exp(-\beta^2 r_0^2/4)\delta(z - z_0)$$

$$E_x = J_{00} \frac{r_0^2}{2} \int_{-\infty}^{\infty} \beta \, d\beta \, G(z, z_0) H_0^{(2)}(\beta r) \exp(-\beta^2 \gamma_0^2/4) \tag{22}$$

To carry out the β integration in (22), we approximate the Gaussian by a Lorentzian,

$$\exp(-\beta^2 r_0^2/4) \simeq \frac{0.845}{(\beta^2 \gamma_0^2/4 + 0.408)^2 + 0.678} \tag{23}$$

Using (23), (22) can be solved by the method of contour integration, by closing the contour in the upper half plane. The poles at the Lorentzian occur at $\beta \simeq 2/r_0 \, (1 \pm i)$ and represent highly damped solutions. Consequently, the main contribution to the integral, at long distances, comes from the pole corresponding to the TEM mode,

$$E_x \simeq iJ_{00} r_0^2 \pi^{1/2} \frac{\beta'}{ch\varepsilon_0^{1/2}} \frac{H_0^{(1)}(\xi_{z0})}{H_0^{(1)\prime}(\xi_0)} \sin \beta' z H_0^{(2)}(\beta r) \tag{24}$$

For typical ionospheric parameters in polar regions, $h \simeq 60$ km, $h_0 \simeq 75$ km, a horizontal electric dipole with $Idl \simeq 10^6$ A m and $\omega = 10^2$ Hz produces a magnetic field strength $H_y \simeq 10^{-7}$ A/m at a distance of 1 Mm from the source. At larger distances the field strength goes as $r^{-1/2}$. In addition to this the TEM mode is attenuated at a rate $\simeq 2$ dB/Mm. It must be mentioned here that the H field spectral level of noise at 100 Hz is ~ -120 dB with respect to 1 A/m(Hz)$^{1/2}$ [*Evans and Griffiths*, 1974].

5. DISCUSSION

ELF waves propagate in the earth-ionosphere waveguide in the TEM mode with a phase velocity slightly less than the velocity of light in vacuum; the effective index of refraction of the waveguide is $\beta c/\omega \approx 1.2$. Because of the continuity conditions at the vacuum-ionosphere interface, the horizontal component of the propagation vector for ELF waves in the ionosphere is approximately ω/c. Since the effective index of refraction kc/ω of the ionosphere is $\sim 10^3$, the ELF waves in the ionosphere propagate mainly in the vertical direction. Thus we could model the ionosphere as an isotropic medium having a complex index of refraction of the extraordinary mode. The refractive index is sensitive to the geomagnetic latitude ϕ; $\pi/2 - \phi$ is the angle between $\mathbf{k}$ (vertical) and the earth's magnetic field.

For a current source located inside the ionosphere, the electromagnetic waves penetrating into the earth-ionosphere waveguide are attenuated by a factor exp $[-\int_h^{h_0} k_i \, dz]$, where h_0 is the height of the source and h is the bottom of the ionosphere. Attenuation is very strong for current sources located in the equatorial zone, and one is forced to locate the current sources below 90 km above the ground. In the polar zones, attenuation is much weaker, and electromagnetic energy from current sources at the height of 150 km could effectively tunnel into the waveguide. Since the current sources are expected to be produced by exploiting some nonlinear processes in the ionosphere and the efficiency of the process goes directly as the electron density, one would like to go to heights in excess of 100 km. For this reason, polar regions must be used to excite nonlinear currents.

An ionosphere with an exponential density profile is found to give a mathematically simple modal dispersion relation. The eigenfunctions are expressed in terms of Hankel functions of the first kind. In the polar zones (greater than 40° geomagnetic latitude) the mode extends to large heights in the ionosphere. From the viewpoint of excitation of the earth-ionosphere waveguide by a current source in the ionosphere, this is an important result. This model might not give a very accurate attenuation constant of the mode as it propagates in the horizontal direction, but that is not crucial for waveguide excitation. The decisive factor is the attenuation of an ELF wave as it propagates vertically downward from the source to the bottom of the ionosphere. For a dipole current source, an antenna strength $Idl \simeq 10^6$ A m seems to be large enough for global communications. However, with an extended current source having directivity in the downward direction, the requirements on the current source may be relaxed considerably.

The determination of the TEM mode eigenfunctions in the inhomogeneous ionosphere allows us to examine the possibility of ELF communication by directly exciting them, using the ponderomotive force due to an HF wave, along the lines of force [*Papadopoulos et al.*, 1982]. This will be reported in a future publication.

Acknowledgments. Discussions with S. Ossakow and P. Palmadesso are gratefully acknowledged. This work was supported by NRL contract N00173-80-C-0194.

REFERENCES

Al'pert, Ya. L., and D. S. Fligel' (1970), *Propagation of ELF and VLF Waves Near the Earth*, p. 171, Consultants Bureau (Plenum), New York.

Chang, C. L., V. K. Tripathi, K. Ko, K. Papadopoulos, J. Fedder, P. Palmadesso, and S. Ossakow (1982), ELF communication by electrojet current modulation, submitted to *Radio Sci.*

Davis, J. R., and J. W. Willis (1974), A quest for a controllable ULV wave source, *IEEE Trans. Commun.*, *COM-22*, 578–586.

Einaudi, F., and J. R. Wait (1971), Analysis of the excitation of the earth-ionosphere waveguide by a satellite-borne antenna, *Can. J. Phys.*, *49*, 447–457.

Evans, J. E., and A. S. Griffiths (1974), Design of a Sanguine noise processor based upon world wide extremely low frequency (ELF) recordings, *IEEE Trans. Commun. COM-22*, 528–540.

Galejs, J. (1971), Excitation of the terrestrial waveguide by sources in the lower ionosphere, *Radio Sci.*, *6*(1), 41–53.

Greifinger, C., and P. Greifinger (1974), Generation of ULF by a horizontal electric dipole, *Radio Sci.*, *9*(5), 533–539.

Kelly, F. J., D. J. Baker, and G. A. Chavt (1974), On the spreading of waves launched by an ELF/VLF satellite, *NRL Memo. Rep. 7814*, Nav. Res. Lab., Washington, D. C.

Papadopoulos, K., R. R. Sharma, and V. K. Tripathi (1982), Parametric excitation of Alfvén waves in the ionosphere, *J. Geophys. Res.*, *87*, 1491–1494.

Pappert, R. A. (1973), Excitation of the earth-ionosphere waveguide by point dipoles at satellite heights, *Radio Sci.*, *8*, 535–545.

Stubbe, P., and H. Kopka (1977), Modulation of the polar electrojet by powerful HF waves, *J. Geophys. Res.*, *82*, 2319–2325.

Stubbe, P., H. Kopka, and R. L. Dowden (1981), Generation of ELF and VLF waves by polar electrojet modulation: Experimental results, *J. Geophys. Res.*, *86*, 9073–9078.