The uni-axial theory yields a higher estimate of the temperature than the theory with a finite magnetic field. There is a close agreement between the estimates obtained by Kuehl's formula[6] and by the graphical method of Muldrew and Gonfalone.[8] Thus, Kuehl's formula can be used directly for estimating the electron temperature.

The author appreciates the pointing out by the referee of recent theoretical work on interference structure near the resonance cone by Burrel,[11] Kuehl,[12] and Chasseriaux.[13] According to these authors the interference structure near the resonance cone is not given just by $\mathrm{Ai}(X)$, but by some products of the Airy functions. This discrepancy between the theoretical work[1,2,9] of the author and that of Burrel, Kuehl, and Chasseriaux remains to be resolved. However, the close agreement between the experimentally measured spacings and those of $\mathrm{Ai}(X)$ shows that so far as the interference spacings are concerned, they are determined by the spacing of $\mathrm{Ai}(X)$. This provides a rigorous method for determining the resonance cone angle once the angular location of the primary and secondary maxima are measured.

The author wishes to thank Dr. R. Sharan for reading the manuscript and suggesting improvements.

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Excitation of lower hybrid waves in a plasma by electron beams

K. Papadopoulos and P. Palmadesso

Naval Research Laboratory, Washington, D.C. 20375
(Received 9 June 1975; final manuscript received 9 October 1975)

The linear theory of excitation of electrostatic lower hybrid waves by electron beams is presented. The importance of such waves in tokamaks with runaway electrons is discussed.

Absorption of lower hybrid waves seems to be a very efficient method for heating ions in a plasma.[1–3] At the same time, excitation of lower hybrid waves can, to a large extent, alter the transport properties of a confined plasma. In this note we demonstrate that such waves can be generated by an energetic unmodulated electron beam streaming through a plasma along the magnetic field. Such situations are of interest with respect to beam-plasma heating experiments and to the concept of a long linear solenoid thermonuclear reactor ignited by an electron beam.[4] Such situations also exist in tokamaks, due to the presence of runaway electron beams. In the latter case excitation of lower hybrid waves can be beneficial by providing ion heating or detrimental if it reduces the confinement time.

Our system is one where a warm electron beam with density n_b, velocity V_b, and thermal spread V_{Tb} streams through a plasma of density n, electron thermal velocity V_e, and ion thermal velocity V_i, along a magnetic field B_0. Contrary to most of the previous work,[5] which examined the excitation of electron plasma waves propagating almost parallel to the magnetic field (i.e., $k_\parallel \gg k_\perp$), here we consider the case of almost perpendicular propagation (i.e., $k_\parallel \ll k_\perp$). The importance of these modes can be very significant especially in view of the fact that for the warm electron beam case the electron plasma waves saturate at a low turbulence level.[5–7] We restrict ourselves to the linear theory for nonrelativistic beams. Generalization to relativistic beams is straightforward and will be presented together with nonlinear theory in a future publication.

For electrostatic waves with $\Omega_i \ll \omega \ll \Omega_e$, ω_e, $k_\perp \gg k_\parallel$, $k_\perp R_e \ll 1$, $\omega/k_\parallel \gg V_e$, $\omega/k_\perp \gg V_i$, the dispersion relation is given by

$$\frac{\omega_i^2}{\omega^2} + \left(\frac{k_\parallel}{k}\right)^2 \frac{\omega_e^2}{\omega^2} = 1 + \frac{\omega_e^2}{\Omega_e^2} - \frac{n_b}{n}\frac{\omega_e^2}{k^2 V_{Tb}^2} Z'\left(\frac{\omega - k_\parallel V_b}{k_\parallel V_{Tb}}\right) . \quad (1)$$

For $\omega = \omega_r + i\gamma$ with $\gamma \ll \omega_r$, we find

$$\omega_r^2 = \frac{1}{1+\alpha^2}\left[\omega_i^2 + \left(\frac{k_\parallel}{k}\right)^2 \omega_e^2\right] \quad (2)$$

and

$$\gamma = -\omega_r \left[\sqrt{\pi}\ \frac{n_b}{n}\ \frac{\omega_e^2}{k^2 V_{Tb}^2 (1+\alpha^2)}\ \rho \exp(-\rho^2)\right] , \quad (3)$$

where we have defined

$$\rho = (\omega_r - k_\parallel V_b)/k_\parallel V_{Tb} \quad (4)$$

and

$$\alpha^2 = \frac{\omega_e^2}{\Omega_e^2} - \frac{n_b}{n}\ \frac{\omega_e^2}{k^2 V_{Tb}^2} \mathrm{Re} Z'(\rho) . \quad (5)$$

From Eqs. (1)–(4) we see that, physically, the instability is generated by Landau growth of the waves given by Eq. (2), when their phase velocity parallel to the magnetic field falls in the negative slope of the beam (i.e., $\omega_r/k_\parallel \approx V_b$). It is interesting to notice that for $k_\parallel/k_\perp \approx \sqrt{m/M}$ (m and M are the electron and ion masses), the waves given by Eq. (2) are the lower hybrid waves. We next proceed to examine when the instability can be excited under experimental conditions. The first and obvious condition is that the injection time of the beam (or the beam duration time) τ is long, that is,

$$\gamma\tau \gg 1 \quad . \tag{6}$$

The second comes out of the fact that we are dealing with a convective instability. Therefore, since most beams have finite radius R_0, the growth length in the perpendicular direction $L_\perp$ must be much smaller than R_0. The group velocity of the unstable wave is

$$\mathbf{v}_g = \frac{\partial\omega}{\partial\mathbf{k}} = \frac{\omega_r}{k}\;\frac{k_\parallel/\epsilon k}{1+k_\parallel^2/\epsilon k^2}\left(\hat{\mathbf{e}}_\parallel + \hat{\mathbf{e}}_\perp\frac{k_\parallel}{k_\perp}\right) \quad , \tag{7}$$

where $\epsilon = m/M$, the electron to ion mass ratio. (We assume the ions to be singly charged.) From Eq. (3) we find that the growth rate γ is given by

$$\gamma = \omega_r\,\frac{k_\parallel^2}{k^2}\,\frac{\Lambda}{\epsilon} \quad , \tag{8}$$

where

$$\Lambda = \sqrt{\pi}\;\frac{n_b}{n}\;\frac{\omega_i^2}{k_\parallel^2 V_{Tb}^2(1+\alpha^2)}\,|\rho|\,\exp(-\rho^2).$$

From Eqs. (7) and (8) the vector growth length $\mathbf{L}$ will be

$$\mathbf{L} = \frac{\mathbf{v}_g}{\gamma} = \frac{\lambda}{2\pi}\;\frac{\hat{\mathbf{e}}_\parallel + \hat{\mathbf{e}}_\perp\,(k_\parallel/k)}{\Lambda(k_\parallel/k)\,(1+k_\parallel^2/\epsilon k^2)} \quad , \tag{9}$$

where $\lambda = 2\pi/k$, the instability wavelength, therefore the second condition is

$$\frac{L_\perp}{R_0} = \frac{\lambda}{2\pi R_0}\;\frac{1}{\Lambda}\left(1+\frac{k_\parallel^2}{\epsilon k_\perp^2}\right)^{-1} \ll 1 \quad . \tag{10}$$

Notice that for a homogeneous plasma, the conditions on the parallel growth length are almost trivially satisfied.

Detailed analysis concerning the various angles of propagation will appear in a future publication. In this short note we concentrate on the lower hybrid waves which seem to be important with respect to ion heating.[1,2] Such waves are produced when $k_\parallel/k \approx \sqrt{\epsilon}$. For these waves and from Eqs. (9) and (2), we find

$$L_\parallel \approx \frac{\lambda}{4\pi}\frac{\epsilon^{-1/2}}{\Lambda}\;,\quad L_\perp = \epsilon^{1/2}L_\parallel,\quad \omega_r \approx \sqrt{2}\,\omega_i\left(1+\frac{\omega_e^2}{\Omega_e^2}\right)^{-1/2}$$

if $n_b \ll n$.

We also note that, since $k_\parallel^2 \approx \omega_r^2/V_b^2 \approx 2\omega_i^2\,(1+\omega_e^2/\Omega_e^2)^{-1}/V_b^2$, one has

$$k \approx \epsilon^{-1/2}k_\parallel \approx \sqrt{2}\,\omega_e\left(1+\frac{\omega_e^2}{\Omega_e^2}\right)^{-1/2}\Big/V_b \quad\text{and}\quad \Lambda \approx \frac{\sqrt{\pi}}{4}\,\frac{n_b}{n}\,\frac{V_b^2}{V_{Tb}^2}\,.$$

On the basis of these estimates the conditions given by Eqs. (6) and (10) become

$$\tau \gg \sqrt{\frac{8}{\pi}}\;\frac{n}{n_b}\;\frac{(1+\omega_e^2/\Omega_e^2)^{1/2}}{\omega_i}\left(\frac{V_{Tb}}{V_b}\right)^2 \tag{11}$$

and

$$\sqrt{\frac{2}{\pi}}\;\frac{V_b}{R_0\omega_e}\left(1+\frac{\omega_e^2}{\Omega_e^2}\right)^{1/2}\frac{n}{n_b}\left(\frac{V_{Tb}}{V_b}\right)^2 \ll 1. \tag{12}$$

For most linear beam plasma experiments due to the short pulses and the small beam radius, the criteria of Eqs. (11) or (12) are not satisfied. However, the situation is different in tokamaks where one has runaway beams of electrons with densities a few percent of the ambient and energies varying from 25 keV for the M.I.T. tokamak to 10 MeV for the Oak Ridge machine. We should note that such low frequency oscillations have actually been observed at M.I.T.,[8] while for the Oak Ridge machine there are not yet any measurements. We finally examine the possibility of lower hybrid waves in the auroral zones. Since in this case $V_b \approx 3\times 10^9$ cm/sec, $n_b/n \gtrsim 10^{-4}$ $R_0 \gtrsim 100$ km, and the pulse duration τ is very long, Eqs. (11) and (12) appear to be well satisfied.

Before closing we should note that the dispersion relation given by Eq. (1) does not include electromagnetic effects. This is justified as long as $\omega_e^2/k^2c^2 \ll 1$. Inclusion of electromagnetic effects results modifying the definition of α^2 given by Eq. (5) into[9]

$$\alpha^{2\,em} = \frac{\omega_e^2}{\Omega_e^2}\left(1+\frac{\omega_e^2}{k^2c^2}\right) - \frac{n_b}{n}\,\frac{\omega_e^2}{k^2V_{Tb}^2}\,\mathrm{Re}Z'(\rho) \quad .$$

The rest of the theory remains unchanged.

This work was supported by the Naval Air Systems Command.

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