A Source of the Backstreaming Ion Beams in the Foreshock Region

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A new source mechanism is proposed for the 'reflected' ion beams observed in the foreshock region of the earth's bow shock. In our model the beams originate in the magnetosheath downstream of the quasi-perpendicular portion of the shock. The quasi-perpendicular shock transition is characterized by two downstream ion populations including high-energy gyrating ions in addition to the directly transmitted anisotropic ions. We show by particle simulations that this highly anisotropic downstream ion distribution (T/Tii ≫ 1) can excite electromagnetic cyclotron waves which, in turn, pitch angle scatter the gyrating ions in a few ion gyroperiods. As a result, some ions acquire large parallel velocities and move fast enough along the convecting downstream magnetic field to escape back across the bow shock into the upstream region. The distribution of escaping ions calculated by using the pitch-angle-scattered ions, as a source, becomes a beam with a large temperature anisotropy T ~ 3-5 Tii and a mean velocity along the magnetic field of about twice that of the solar wind velocity. A significant result is the presence of the maximum angle θe ≈ θs above which no ions can escape, where θs is the angle between the shock normal and the interplanetary magnetic field. A wide peak of constant escaping ion flux is formed below θe whose number density is 1-2% of that of the solar wind. These results are in general agreement with the ISEE observations of the 'reflected' ions.

1. INTRODUCTION

A prominent and well-known feature of the earth's bow shock is the presence of backstreaming ions upstream from the bow shock [Asbridge et al., 1968; Lin et al., 1974]. The morphology of these upstream ions has been clarified in recent years by the accumulation of observational results [Gosling et al., 1978; Bame et al., 1980; Greensstadt et al., 1980; Paschmann et al., 1981; Bonifazi and Moreno, 1981a, b]. The beams are consistently anisotropic with T/Tii ≈ 2-3 [Paschmann et al., 1981], where Tii (Tii) is the temperature parallel (perpendicular) to the magnetic field. The magnitude of Tii is much larger than the solar wind temperature (Tsw, ~ 5 keV); in fact, Tii ~ 0.1-1 keV. For θs > 70°, more energetic beams are observed with speeds a factor of 5 or more larger than Vsw. These beams also have consistently large temperature and thermal anisotropy.

It has been commonly suggested [Greensstadt, 1975; Gosling et al., 1978; Bame et al., 1980; Bonifazi and Moreno, 1981a, b] that these ions are a small part of the solar wind ions which are reflected near the quasi-perpendicular portion of the bow shock; hence the name 'reflected' ions, primarily due to their collimated nature and apparent geometric origin. The major evidence supporting reflection as the source of these ions results from studies of the resulting ion energy gain [Sonnerup, 1969; Greensstadt, 1975; Paschmann et al., 1980]. Sonnerup [1969] has shown using conservation laws that ions can be accelerated by their displacement along the interplanetary motional electric field during their reflection at the shock. His results, which have been extended to a more general shock geometry by Paschmann et al. [1980], relate the energy gain of an ion during reflection to θs and the change in the direction of the ion trajectory. The maximum energy gain is obtained when the ion magnetic moment is conserved during reflection; for large θs the energy gain is proportional to tan² θs and can be quite large. Paschmann et al. [1980] analyzed a number of ISEE upstream ion events and found that the geometric energization formulas assuming μ is conserved generally agree with the observed ion beam streaming energies.
The case for direct reflection as the ion beam source is not compelling, however. Observationally, there is evidence that at least a substantial fraction of the ions are not geometrically accelerated. In a statistical analysis of ion events observed at ISEE 2, Bonifazi and Moreno [1981a, b] found the average ion beam speed to be approximately constant at twice the solar wind speed for $35^\circ \leq \theta_{ab} \leq 70^\circ$. These results are significantly below the beam speeds predicted by the formulas of Sonnerup [1969] and Paschmann et al. [1980], assuming that $\mu$ is conserved, and would appear to contradict the findings of Paschmann et al. [1980]. However, there is considerable scatter in the Paschmann et al. [1980] data for the events with $\theta_{ab} < 70^\circ$ with most of the beam speeds below the predicted values. In fact, for $35^\circ \leq \theta_{ab} \leq 65^\circ$, while they also report significantly faster ion beams, a substantial number of the ion events in the work of Paschmann et al. [1980] are consistent with the Bonifazi and Moreno [1981a, b] results. Further, it is difficult to explain the large temperatures of the upstream ion beam observed in the context of direct reflection, since if $\mu$ is conserved, the beam temperature would be approximately the solar wind temperature. If $\mu$ is assumed to increase during reflection, then the ion perpendicular temperature could be increased significantly. Since this gain comes at the expense of the streaming energy, deviations in $\mu$ could in principle explain the observed scatter in the upstream ion beam speeds. However, in this case, there should be an inverse correlation between beam temperature and speed (at fixed $\theta_{ab}$). The observed beam parallel temperatures cannot be explained in this way, as it is quite insensitive to changes in $\mu$.

A weakness of the direct reflection model is that no reasonable reflection mechanism has been proposed that conserves, or approximately conserves, $\mu$. In fact, for the quasi-perpendicular bow shock ($\theta_{ab} \gtrsim 45^\circ$), such a mechanism is theoretically difficult to justify. It is well established observationally (J. D. Scudder et al., unpublished manuscript, 1982) and theoretically [Tidman and Krall, 1971; Wu, 1982] that the major magnetic field jump has a scale much shorter than the ion gyroradius. In fact, observational [Paschmann et al., 1982] and numerical simulation [Leroy et al., 1981, 1982] studies of ion reflection at the quasi-perpendicular bow shock show that $\mu$ is far from constant. They find that reflection is instead specular and that reflected ions do not receive the large energy gain assumed in the work of Paschmann et al. [1980]. Moreover, the magnetic mirror force [e.g., Terasawa, 1979], which has been suggested as an ion reflection mechanism and could be expected to conserve $\mu$, cannot reflect solar wind thermal ions. Typically 99.9% of the solar wind ions have pitch angles less than $15^\circ$, whereas the pitch angle must be greater than $\sim 30^\circ$ for mirror reflection.

Thus it is important to consider sources other than direct reflection for the upstream ion beams. One possible source is leakage upstream of the hot magnetosheath ions. Recently, Edmiston et al. [1982] considered this approach in an idealized model in which the ions are heated and thermalized in a thin layer at the shock front. They calculated the ions returning upstream from a hot Maxwellian distribution at this layer. Within their model, which does not address the actual heating mechanism, significant upstream ion fluxes occur only for the quasi-parallel portion ($\theta_{ab} \leq 45^\circ$) of the bow shock. Their results thus appear to be relevant to the diffuse rather than the 'reflected' ions.

In this paper we propose a new and self-consistent source mechanism for the 'reflected' ions based on the observed [Paschmann et al., 1982] and theoretically understood [Leroy et al., 1981, 1982] ion behavior in the quasi-perpendicular shock. As the work of Edmiston et al. [1982], the ion source is in the magnetosheath. The process is summarized in Figure 2. The solar wind, when it encounters the quasi-perpendicular portion of the earth's bow shock, is adiabatically compressed and heated in the perpendicular direction and transmitted into the magnetosheath (called 'core ions' hereafter). At the same time, some of the solar wind ions are reflected at the shock front through a combination of magnetic and electrostatic forces and gain energy due to their Larmor displacement in the direction of the $\mathbf{E} = \mathbf{V}_{SW} \times \mathbf{B}_1$ electric field. These ions, however, are all magnetically deflected back through the shock front (called 'gyrating ions') [Leroy et al., 1981, 1982]. These two distinct types of ions at the quasi-perpendicular shock have recently been observed [Montgomery et al., 1970; Paschmann et al., 1982]. Both kinds of ions thus acquire large perpendicular
energies compared with their parallel energies. The free energy associated with this anisotropy then drives the electromagnetic ion cyclotron (EMIC) instability. As the result of the instability, the gyrating ions are pitch angle scattered, producing some ions with large parallel velocities. These ions then move along the convecting magnetic field lines, cross the bow shock, and escape into the upstream region.

In section 2 of this paper, it will be shown by particle simulations that the gyrating ions created at the quasi-perpendicular bow shock are effectively scattered by the EMIC instability. Using the ion distribution resulting from this process, we calculate the flux and distribution functions of ions escaping upstream in section 3. Finally, section 4 is devoted to a discussion and summary of the results.

2. Ion Scattering Mechanism—Particle Simulations of EMIC Waves

The results of the hybrid simulations [Leroy et al., 1981, 1982] show that there are two types of ions behind the quasi-perpendicular bow shock. Figure 3 shows the velocity distribution of ions (x corresponds to the shock normal direction) perpendicular to the magnetic field just behind the shock front computed in the simulation. (The parameters were chosen to emulate the ISEE shock crossing of November 7, 1977; for a detailed description of the hybrid model and results, see Leroy et al. [1981]). Most of the ions ( ≥ 85%) are slowed and adiabatically heated in the perpendicular direction (their perpendicular temperature becomes roughly 4–8 times greater than their parallel temperature) as they traverse the shock transition layer to form an anisotropic ion ‘core.’ Surrounding these ‘core’ ions is a diffuse ring of more energetic ions. These ions were initially reflected at the shock front and then magnetically deflected downstream. Their gyrating orbits and phase mixing cause them to be spread out roughly in a ring in velocity space. Typically the perpendicular energy of these ‘ring’ ions is 3 to 4 times larger than their bulk flow energy before encountering the bow shock. This configuration contains a large amount of free energy, due to the large differences in the perpendicular and parallel energies of both the ‘ring’ and ‘core’ ions and also to the loss cone nature of the perpendicular velocity distribution of the ring ions, which drive various instabilities, the most important of which is the electromagnetic ion cyclotron instability [Davidson and Ogden, 1975; Tajima et al., 1977; Lee et al., 1981]. Although this instability has been studied previously, the presence of the energetic ring changes its characteristics significantly. In particular, the eventual saturated nonlinear state of the instability is of interest here. In order to investigate the nonlinear behavior, we have performed one-dimensional particle simulations using a finite-size particle magnetostatic code, in which the transverse component of the radiation term is neglected in Maxwell equations [Busnardo-Neto et al., 1977].

For the initial state we use three species: isotropic electrons, anisotropic core ions with the perpendicular temperature $T_c^⊥$ greater than the parallel temperature $T_c^∥$, and gyrating ions which have a ringlike velocity distribution and thus have large perpendicular velocities. Then, the average temperature anisotropy $\langle T_c^⊥ \rangle / \langle T_c^∥ \rangle = \frac{1}{2} (v_z^2 + v_x^2)/\langle v_x^2 \rangle \sim \frac{3}{2} \left( \varepsilon_0 \sigma_c^2 + 2 \varepsilon \sigma_c v_x^2 \right) / (\varepsilon \sigma_c^2 + \varepsilon v_x^2)$ is much larger than unity. Here $v_x$ is the gyrating speed of the ring ions; $v_z$ ($v_x$) is the perpendicular (parallel) ion thermal speed. The subscripts $c$ and $r$ denote core and ring, respectively, and $\varepsilon = n_r/(n_r + n_c)$ is the ratio of ring ion density to total ion density, $\varepsilon' = 1 - \varepsilon$. There is an ambient magnetic field pointing in the x direction, and all physical quantities are assumed periodic in this direction within the system of size $L = 16c/\omega_{ce} = 128$ cells, where $c/\omega_{ce}$ is the ion inertia length. Particles (electrons and ions) are allowed to move in only one dimension (x) spatially, but all three velocity components are kept. We use the following parameters for the initial conditions: the ratio of electron cyclotron frequency to electron plasma frequency $\omega_{ce}/\omega_{pe} = 0.5$, the ratio of electron thermal speed to the speed of light $v_e/c = 0.14$, and the mass ratio of ions to electrons $m_i/m_e = 25$. The 6400 ions and 6400 electrons with finite sizes used in the simulation are sufficient to insure that collisions are insignificant on the time scale of the simulation. As usual, the cell size (which corresponds to the particle size) is roughly an electron Debye length.

Results of two runs (A and B) for which the initial perpendicular energy is the same, but distributed between the core and ring ions differently, are presented. For each case the density ratio $\varepsilon = n_r/(n_r + n_c)$ is chosen to be 0.1 (compare hybrid simulations by Leroy et al. [1981, 1982]), and the average temperature anisotropy at $t = 0$ is $\langle T_c^⊥ \rangle / \langle T_c^∥ \rangle \sim 12$. The difference between run A and run B is the different gyration velocity of ring ions. It is represented in terms of the effective temperature $T_{ce}^⊥ = \frac{1}{2} m_r v_x^2 \sim 80 T_{ce}^∥$ and $40 T_{ce}^∥$ for run A and run B, respectively, where $T_{ce}^∥ = \frac{1}{2} m_i v_x^2$ is the parallel temperature of core ions. For run A, two thirds of the ion energy is contained in the ring ions, while for run B, only one third of the perpendicular ion energy is in the ring.

The temporal development of Fourier amplitudes of the perturbed magnetic field $|B_k(k, \omega)|$ for run A is shown in Figure 4. Waves grow in time to $\omega_{ce} t \sim 15$ and saturate ($\omega_{ce} = eB/m_i c$ is ion cyclotron frequency). Mode 4 with $\omega_{ce} t \sim 1.6$ is the most dominant mode during the early stage of the instability. After $\omega_{ce} t > 15$, the longer-wavelength mode $n = 2$ dominates, in accordance with decrease in temperature anisotropy [Davidson and Ogden, 1975]. The oscillations of each Fourier mode are due to the existence of both left and right propagating waves. The frequency of the waves can be determined from these oscillations by assuming that the amplitudes of the right and left traveling waves are equal, yielding

$$B_k(t) = 2B_0 \sin kx \cos \omega t$$

Therefore $|B_k(t)|$ oscillates in time with period $\Delta t = \pi/\omega$, where $\omega$ is the frequency. The growth rate is calculated from the growth of the upper envelope of $B_k(t)$. Linear characteristics of the waves obtained in this manner (to a measurement accuracy of about 20%) for run A (solid circles) and B (open circles) are

![Fig. 3. Velocity distribution of ions in the downstream region of the perpendicular bow shock [Leroy et al., 1982]. The magnetic field is perpendicular to the page.](image-url)
shown in Figure 5. The solid lines in the figure are the theoretical results obtained from the linear dispersion relation of the electromagnetic ion cyclotron instability for the same parameters [Davidson and Ogden, 1975]. This dispersion relation is given by

\[
D^+(\omega, k) = \omega^2 - c^2 k^2 + \omega_{pe}^2 \frac{\omega}{k v_i} Z(\xi_+^2) + \omega_{pi}^2 \frac{\omega}{k v_{pi}} Z(\xi_i^2) - \omega_{pi}^2(1 - T_{i1}/T_{i2})(1 + \xi_i^2 Z(\xi_i^2)) = 0
\]

where \(\xi_+^2 = (\omega + \omega_{pe})/k v_{pe}, \xi_i^2 = (\omega + \omega_{ci})/k v_{ci}, v_+ = (2T_{i1}/m_i)^{1/2}, v_i = (2T_{i1}/m_i)^{1/2}, Z(\xi)\) is the plasma dispersion function, and the ion perpendicular temperature is

\[
T_{i1} = 2\pi \int_0^\infty dv_{i1} f_i (m_i v_{i1}^2/2) G(v_{i1})
\]

where the ion distribution function \(f_i = (m_i v_{i1}^2)^{-1/2} G(v_{i1}) \exp (-v_{i1}^2/m_i v_{i1}^2)\) is assumed. The ion perpendicular temperature appears only once through \((1 - T_{i1}/T_{i2})\) and is independent of the form of \(G\). In the limit of \(T_{i1}/T_{i2} \to \infty\), the frequency and the growth rate are \(\omega = \pm \omega_{ci}, c \leq (\beta_{i1}/2)^{1/2} \omega_{ci}\). The growth rate is an increasing function of the anisotropy \(T_{i1}/T_{i2}\) and the ion beta \(\beta_{i1}\). Because both runs have the same total perpendicular energy, the linear properties of runs A and B are expected to be the same. This is verified in the figure. In addition, the frequency and growth rate of the individual modes (2, 3, 4) agree well with the theory.

The relaxation of the ion temperature anisotropy is shown in Figure 6. After a time \(\omega_{ci} t \gtrsim 8\), when the magnetic field fluctuations \(\langle \delta B^2 \rangle\) become large enough, \(\langle \delta B^2 \rangle/8\pi n\langle T_i \rangle \gtrsim 10^{-3}\), the ion temperature anisotropy starts decreasing. \(\langle T_i \rangle = \langle T_{i1} + 1/2 T_{i2} \rangle\) is the average energy of all the ions. In each run the anisotropy decreases at roughly the same rate to nearly the same final value of \(T_{i1}/T_{i2} \approx 3\). The magnetic field fluctuations shown in Figure 6 as an increasing line in run A stop growing at the same time when the anisotropy reaches its minimum value. The level of the fluctuations of saturation \(\langle \delta B^2 \rangle/8\pi n\langle T_i \rangle\) is about 0.04 in each case. We expect these results to be valid for the bow shock in spite of the unrealistically small mass ratio \(m_i/m_e = 25\) used in the simulations. For \(\omega_{ci}^2/m_i c^2 \ll 1\) and \(\beta_{i1} \approx 1\), the dispersion and quasi-linear equations scale as \(c k/\omega_{pe}, c/\omega_{ci}, \) and \(\omega_{ci}/k v_{ci}\), where \(\omega_{ci} = |E_x|^2/8\pi n\) is the spectral energy density. These scaled equations contain only two parameters \(T_{i1}/T_{i2}\) and \(\beta_{i1}\) and do not include \(m_i/m_e\) [cf. Davidson and Ogden, 1975].

Changes in the ion distribution due to the EMIC instability are clearly manifested in the \((v_{i1}, v_{i2})\) space plots of Figure 7 for run A. The electron distribution remains Maxwellian and is heated very little (consequently not shown here). The core ions are heated in the parallel direction (reducing the anisotropy from 4.6 to 2.6 for run A), but they stay approximately bi-Maxwellian even at \(\omega_{ci} t \sim 36\) of run A. On the other hand, the ring ion distribution changes significantly. Ring ions, initially at \((v_{i1}, v_{i2}) = (0, v_0) = (0, 0.25)\) in the simulation, are pitch angle scattered along paths of constant energy \(v_{i1}^2 + (v_{i2} - v_0)^2 = \) const, where \(v_0 = \pm |c\omega/k|\) is the wave phase velocity; this type of particle scattering was theoretically studied previously [Kennel and Engelmian, 1966]. Thus some of the ring ions eventually gain a parallel velocity \(v_{i1} \sim v_0 - |v_{i2}|\) and the ion distribution function \(f(v_{i1}, v_{i2})\) approaches a constant value along these paths. As shown in Figure 7, such a state is nearly achieved at \(\omega_{ci} t = 24\), when the fluctuating magnetic field energy saturates. It should be noted that these constant density distributions are realized in a finite length of time, say in a few gyroperiods. (Note that the particle density in Figure 7 shows \(f d^3v = 2\pi f_{i1} dv_{i1} dv_{i2}\); therefore there are few particles near the \(v_{i1}\) axis.)

The one-dimensional nature of the simulation restricts wave vectors to being parallel to the ambient magnetic field. Thus instabilities which involve the interaction of the ring ions with either the electrons or the core ions [Kulygin et al., 1971; Lee and Birdsall, 1979a, b] are excluded. The former instability primarily leads to modification of the electrons, and that is not
3. UPSTREAM ION DISTRIBUTIONS

The pitch angle scattering by EMIC waves of the 'ring' ions produces ions with larger $v_p$. These ions can escape upstream by moving along a field line which, owing to the curvature of the bow shock, crosses the shock surface. Due to the convection of the plasma during the finite scattering time $\tau_s$, the isotropization occurs a distance $d = u_2 \tau_s$ downstream of the shock, where $u_2$ is the downstream normal velocity. It was shown in section 2 that $\tau_s \approx 20u_2/\omega_{ci}$, so that $d \approx 20u_2/\omega_{ci}$; for typical downstream parameters $\omega_{ci} \approx 2s^{-1}$, $u_2 \approx 100$ km/s, which implies $d \approx 1000$ km. The parallel velocity of the scattered ions is roughly $v_0$, where $v_0$ is the gyration velocity of the 'ring' ions and is large in comparison with $u_2$. Leroy et al. [1982] have shown $v_0$ to be about twice the incident velocity normal to the shock, independent of Mach number and ion temperature. By following the trajectories of the scattered ions, the properties of resulting upstream ion population can be determined. Since our model is based on the properties of the quasi-perpendicular portion of the bow shock surface (i.e., $\theta_{ab} \gtrsim 45^\circ$), we expect our results to be valid for $\theta_{ab} \geq 45^\circ$ and progressively less accurate for smaller $\theta_{ab}$. For angles $\theta_{ab} \lesssim 45^\circ$, the gyrating ions do not necessarily reenter the shock [Gosling et al., 1982], so our ion source model becomes increasingly doubtful. We further restrict our attention to the ecliptic plane for convenience only; the results generally are valid.

The physical model used in our calculations, which are carried out in the shock frame, is illustrated in Figure 8. The solar wind flow is assumed to be radial (the earth is at the origin). The interplanetary field at the bow shock $B_\perp$, unless otherwise specified, is taken to be along the average Parker spiral, that is, in the ecliptic plane inclined by 45° from the sun-earth line. The bow shock surface is taken to be a circular arc (solid arc in Figure 8) of radius $R = 25 R_e$. The downstream flow velocity $u_2$ and density $n_2$ are determined locally using the Rankine-Hugoniot relations. We assume an isotropized ion source located a distance $d = u_2 \tau_s$ behind the shock surface (dashed arc). The scattered ion source extends along the entire shock front, but the density of the scattered ions is assumed to be a maximum at $\theta_{ab} = 90^\circ$ and decreases as $\sin^2 \theta_{ab}$ (sensitivity to this assumption is discussed later in this section). Consistent with the results of section 2, the velocity space density of scattered ions is constant on surfaces of $v_1^2 + (v_1^2 - c^2/k^2) = \text{const} = v^2_0 + c^2/k^2$, where $v_1$ is the parallel downstream bulk velocity and $c/k$ is the EMIC wave phase speed. ($X_0, X_1,$ and $X_2$ in Figure 8 will be explained later.)

Since the scattered ions basically move along the field lines, the global orientation of the downstream field is important to the model. As limiting cases we have first considered the field to be determined locally in accordance with the Rankine-Hugoniot relations (model A) and second to be parallel to the upstream field (model B), which is valid for $\theta_{ab} = 90^\circ$ and (over) emphasizes the global nature of the field. However, we will discuss primarily the results for an intermediate case (model C) in which the field is taken to be the average of the above extremes.

In determining the passage of ions across the shock surface itself, it is necessary to consider the structure of the transition layer, in particular, the electrostatic potential. We assume an asymptotic potential jump $\phi_2$ of $1/4 m_1 v_{sw}^2$; the upstream potential is $\phi_1$, which is taken to be zero. Within the transition layer, however, we assume significant potential overshoot $\phi_{max} > \phi_2$ [Leroy et al., 1981]. The value of $\phi_{max}$ is roughly $\frac{1}{2} \phi_2$ for $\theta_{ab} = 90^\circ$, and its overshoot is assumed to decrease as $\sin^2 \theta_{ab}$. The overshoot blocks the escape of some of the ions, particularly for moderate $\theta_{ab}$. It is also assumed that the magnetic moment of the ions is constant in crossing the layer. The upstream parameters used in this calculation correspond to the ISEE shock crossing of November 7, 1977 (J. D. Scudder et al., unpublished manuscript, 1982). The Alfvén Mach number is $M_A = 8$ at $\theta_{ab} = 90^\circ$, ion and electron beta are $\beta_i = 0.9$ and $\beta_e = 22$, plasma density is $n_i = 12$ cm$^{-3}$, and the magnetic field is $5 \gamma$. The corresponding solar wind velocity is about 360 km/s, and density $n_1$ and the magnetic field $B_1$ just behind the perpendicular shock are $n_1 = 40.4$ cm$^{-3}$ and $B_1 = 16.8 \gamma$.

The velocity space distributions of ions which escape into the foreshock region, calculated using model C, are shown in...
The flux of escaping ions calculated by the above model is shown in Figure 10. For model A, no ions escape above \( \theta_{\text{max}} = 55^\circ \); this model, which uses the Rankine-Hugoniot relations, overestimates the bending of the field line at the shock surface because the global connection of the field lines is not taken into account. For model B with parallel field lines, the escaping ions first appear at \( \theta_{\text{max}} = 70^\circ \), while for model C, which is a hybrid of models A and B, the ions first appear at \( \theta_{\text{max}} = 65^\circ \). Thus the angle \( \theta_{\text{max}} \) at which the escaping ions first appear depends on the orientation of the field lines in the magnetosheath. The existence of this critical angle, \( \theta_{\text{max}} \), is due to the fact that the pitch-angle-scattered ions move with the convecting magnetic field and also that the ion source region is detached from the shock surface. Our calculated ion flux has a peak around \( \theta_{\text{max}} = 40^\circ - 65^\circ \) for model C, and its density is about 1.5% of the solar wind density (the flux density is about 2.5% of the solar wind flux), where we assumed that 10% of the incident solar wind ions become gyrating ions. For each case, the flux density decreases below the angle \( \theta_{\text{max}} 
approx 45^\circ \); however, this is due to the assumed decrease in the source ions as \( \sin^2 \theta_B \). If instead we use an ion source whose density is independent of the angle \( \theta_B \), no qualitative change occurs in the ion density contours in the \( (v_{yi}, v_{z}) \) space. However, a fairly wide plateau of constant escaping flux density appears below \( \theta_{\text{max}} \lesssim 60^\circ \), leading to a significant change in the flux density below \( 50^\circ \) in Figure 10. If the pitch-angle-scattered ion source is switched off below a certain angle \( \theta_r \), then a plateau is formed for \( \theta_r \lesssim \theta_{\text{max}} \lesssim 65^\circ \), and the escaping flux density decreases rapidly below \( \theta_r \). At present, we have little knowledge about the ion behavior at the shock with relatively small \( \theta_B \) angle (say, \( \theta_B \lesssim 60^\circ \)), and our results are increasingly unreliable for \( \theta_B \) smaller than \( 45^\circ \).

The largest angle \( \theta_{\text{max}} \) for which ions, initially incident at the perpendicular shock (\( \theta_B = 90^\circ \)), can escape upstream can be determined in the following manner. As illustrated in Figure 8, we assume that the front one fourth of the bow shock may be approximated by a circle. We fix the \( x \) axis parallel to the interplanetary magnetic field and the \( y \) axis such that the coordinate of the perpendicular shock is \( x_0 = (0, R) \), where \( R \) is the radius of the magnetosheath. In this coordinate system the solar wind velocity is \( \mathbf{V}_{\text{sw}} = (-V_{\text{sw}}/\sqrt{2}, -V_{\text{sw}}/\sqrt{2}) \). After the ions crossed the point \( x_0 \), they move at the downstream flow

![Figure 9](image-url)

Fig. 9. Equal density contours of escaping ions in the \( (v_{yi}, v_{z}) \) velocity space in the shock rest frame for \( \theta_B \) in the range \( 50^\circ - 55^\circ \) and \( 60^\circ - 65^\circ \). The solar wind and the interplanetary magnetic field are assumed to form an angle of (a) \( 45^\circ \) and (b) \( 67^\circ \), and the solar wind velocity is \( 360 \text{ km/s} \). The upstream bulk speed \( V_{\text{sw}} \) is obtained by the vector sum of \( v_{yi} \) and \( V_{yi} \).

The largest angle \( \theta_{\text{max}} \) at which the ions can escape upstream, the distribution peaks at \( v_{yi} \approx 500 \text{ km/s} \) with a parallel velocity spread \( (v_{yi}^2 - v_0^2)^{1/2} \) of about 40 km/s when the angle between the solar wind and the interplanetary magnetic field \( \phi_{\text{sw}} \) is \( 45^\circ \) (Figure 9a). The perpendicular thermal spread for this case is about 95 km/s, that is, \( T_{yi} \approx 50 \text{ eV} \). Thus its perpendicular temperature is 5.5 times larger than the parallel temperature. As the angle \( \phi_{\text{sw}} \) decreases, the distribution changes; the distribution, which is beamlike for \( \phi_{\text{sw}} \approx 65^\circ \), becomes oblate shaped for \( \phi_{\text{sw}} \approx 55^\circ \) with a perpendicular thermal spread of 160 km/s (\( T_z \approx 130 \text{ eV} \)) and a maximum pitch angle of about \( 30^\circ \). The oblateness comes from the fact that the ions originate from two different source regions. One is close to the exit point, and the other is close to the perpendicular shock. The former forms a peak off the \( v_{yi} \) axis in the distribution. When the angle \( \phi_{\text{sw}} \) is changed from \( 45^\circ \) to \( 67^\circ \), the parallel bulk speed increases as much as \( 15-30\% \). At the same time, the perpendicular temperature increases as much, but the parallel temperature does not change.

![Figure 10](image-url)

Fig. 10. Flux of escaping ions versus \( \theta_B \) for the case of \( B \cdot V_{\text{sw}} = \phi_{\text{sw}} = 45^\circ \), where \( \phi_{\text{sw}} \) is the angle between the shock normal and the interplanetary magnetic field \( B \). Solid, dot-dash, and dashed lines correspond to calculations assuming model A, model B, and model C, respectively. Full scale in ordinate is about \( 3\% \) of \( n_i V_{\text{sw}} \).
The angle $\theta_{\text{max}}$, is obtained by putting $x/\sqrt{2}$, yielding $\theta_{\text{max}}$, ion beams in the foreshock region are a natural consequence of $60^\circ$. Ions with $v/\gamma < x/2\gamma$ reach the shock surface with a important and well-documented [Montgomery et al., 1970; Pas- chmann et al., 1982; Leroy et al., 1982] macroscopic feature of the microscale and macroscale structure of the bow shock. An important and well-documented [Montgomery et al., 1970; Paschmann et al., 1982; Leroy et al., 1982] macroscopic feature of the quasiperpendicular portion of the bow shock is the reflec-
tion and energization of some of the incident ions at the mag-
netic ramp. These ions are magnetically deflected into the mag-
netsheath and phase mix to form a high-energy ring in veloci-
ity space. While convecting downstream, the ions are iso-
tropized by microturbulent scattering due to the electro-
magnetic ion cyclotron instability. By this process some ions acquire large enough velocities parallel to the magnetic field to escape along field lines, which are connected to the upstream region due to the macroscale curvature of the shock surface.

This mechanism provides a natural explanation for the reflected' ion beams in the foreshock region with $\theta_{\text{in}} \lesssim 65^\circ$, roughly 95% of the observed cases [Bonifazi and Moreno, 1981a, b]. Without an additional acceleration mechanism, ions in our model cannot escape upstream for $\theta_{\text{in}} \gtrsim 70^\circ$ and thus cannot straightforwardly explain the remaining 5% of fore-
shock ion beams. There is likely to be some such acceleration, however, so that our model may also explain the ion beams with large $\theta_{\text{in}}$. This possibility is discussed in more detail later. We expect the results of our model to be quantitatively correct for $70^\circ \geq \theta_{\text{in}} \geq 45^\circ$. For smaller $\theta_{\text{in}}$, the results are likely to be progressively more inaccurate, since the ion scattering process assumed is strictly valid only for the quasiperpendicular bow shock. For oblique shocks, there are additional processes which can also lead to ions diverted back upstream. Some of the ions reflected at the shock front, which at the perpendicular shock must all eventually reach the downstream region, can at the oblique shock return directly into the upstream region [Gosling et al., 1982]. These ions may in fact be related to the diffuse population. In addition, scattering of ions by the turbulent fields characteristic of oblique shocks can also increase the flux and the parallel temperature of the reflected ions. However, our model is likely to be qualitatively correct even for the foreshock beams with $\theta_{\text{in}} \leq 45^\circ$.

The characteristics of the ion beams produced in our model agree well with the observations, especially for $65^\circ \geq \theta_{\text{in}} \geq 45^\circ$, for which the model is most accurate [Gosling et al., 1978; Bame et al., 1980; Greenstadt et al., 1980; Paschmann et al., 1981; Bonifazi and Moreno, 1981a, b]. The calculated density of the escaping ions $n_{\text{es}}$ is about 1–2% of the solar wind density $n_{\text{sw}}$ for the parameters given in section 3. This value is in excellent agreement with the average observed relative density of about 1% [Bonifazi and Moreno, 1981a, b]. In our model the relative density of ions able to escape is proportional to the relative density of ions reflected at the magnetic ramp of the quasiperpendicular shock which in turn is roughly proportional to the Alfvén Mach number $M_A$ [Leroy et al., 1981]. Paschmann et al. [1981] appear to have failed to find significant correlation between the density of reflected ions and $M_A$ in an analysis of a small number of cases (20). This correlation could be difficult to measure experimentally. The relevant Mach number $M_A$ is based on the solar wind velocity component normal to the shock where the ions initially cross the shock. Observationally, only the local conditions at the spacecraft are known. Using the observed ion beam velocity, the point where the beam leaves the shock can be inferred. In our model, ions after being scattered re-cross the shock and flow upstream at significantly smaller $\theta_{\text{in}}$. It is further necessary to average the local $M_A$ at the various sections that contribute to the ions observed at some point upstream, which would smear out the correlation.

The calculated ion density decreases below $\theta_{\text{in}} \sim 45^\circ$ (Figure 10); however, the decrease is based on the assumption that ions are mainly supplied by the quasi-perpendicular shock (source density $\propto \sin^2 \theta_{\text{in}}$) which is increasingly questionable for small $\theta_{\text{in}}$. The assumption of a constant source density gives a wide plateau below $\theta_{\text{in}} \sim 60^\circ$, as mentioned in section 3.

The parallel velocity of the calculated escaping ions in the shock frame ranges from $v_{\parallel}/V_{\text{sw}} \sim 1.4 (1.5)$ at $\theta_{\text{in}} = 65^\circ$ to $v_{\parallel}/V_{\text{sw}} \sim 1.7 (1.8)$ at $\theta_{\text{in}} = 45^\circ$ for the case where the angle between the solar wind and the interplanetary magnetic field is $45^\circ$ (67°). Then the total bulk velocity in the foreshock region $V_{\text{es}}/V_{\text{sw}}$ becomes $1.5 (1.9)$ at $\theta_{\text{in}} = 65^\circ$ and $1.8 (2.0)$ at $\theta_{\text{in}} = 45^\circ$. These results are rather independent of $M_{\text{sh}}$, since the ratio of the gyrating ion velocity to solar wind velocity $V_{\text{sw}}$ is roughly constant [Leroy et al., 1982]. Our results are consistent with the observed average value of the backstreaming bulk velocity of the reflected ions, about $2.0V_{\text{sw}}$, which is only very weakly dependent on $\theta_{\text{in}}$ for $\theta_{\text{in}} < 65^\circ$ [Bonifazi and Moreno, 1981a, b]. The calculated upstream velocity, however, depends on the angle between the solar wind and the interplanetary magnetic field. The present calculations assumed this angle is $45^\circ$ and $67^\circ$. When this angle is increased, so that there is less incident parallel velocity, the scattered ions expend less energy in turning around to come back upstream and thus escape with larger energies.

The values of $V_{\text{es}}/V_{\text{sw}}$ calculated from our model appear also to be consistent with the upstream ion events discussed in the work of Paschmann et al. [1980]. We note, however, that our results are significantly smaller than those predicted by the geometric energization formulas of Sonnerup [1969] and Paschmann et al. [1980], assuming $\mu$ to be conserved. Although the data of Paschmann et al. [1980] generally agree with these predictions, there is considerable scatter. For $45^\circ < \theta_{\text{in}} < 65^\circ$ (which, using our model parameters, corresponds to $5 \lesssim (E_{\parallel}E_{\text{in}})^{1/2} \lesssim 15$ in Figure 2 of Paschmann et al. [1980]), there are several events for which $(E_{\parallel}E_{\text{in}})^{1/2}/V_{\text{sw}}$ is approximately 4. These observations agree both with our results and with those of Bonifazi and Moreno [1981a, b].

Paschmann et al. [1980] also found ion events with beam speed well above the speeds we can straightforwardly produce in our model for $\theta_{\text{in}} \geq 45^\circ$. This subset of their data agrees very well with the geometric energization formulas of Sonnerup [1969] and Paschmann et al. [1980]. These high-energy beams may, in fact, be a separate population of ions that are directly reflected upstream at the shock.

Higher-energy beams can also be produced by a plausible extension of our model. We can assume that in addition to
TABLE 1. Summary of Model Calculations Presented in This Paper
With Observations of ISEE Satellites

| Detection region, θ<sub>ab</sub> | ≤65° | ≤70°± |
| Density, n<sub>esc</sub>/n<sub>sw</sub> | 1-2% | 1%± |
| Bulk speed, V<sub>pe</sub>/V<sub>sw</sub> | 1.5-2.0 | 2.0± |
| Temperature anisotropy, T<sub>e</sub>/T<sub>i</sub> | 3-2| 2-3* |

θ<sub>ab</sub> is the angle between the shock normal and the upstream magnetic field at which ions first escape.

*Paschmann et al. [1981].
†Bonifazi and Moreno [1981a,b].
‡Warm ring with ±20% spread.

being pitch angle scattered, the gyrating ions downstream of the shock are thermalized by electrostatic turbulence. This thermalization, though not the primary agent, is evident observationally [Paschmann et al., 1982]. The gyrating ions would be heated by the turbulence producing some more energetic ions. The calculations of Kulygin et al. [1971] clearly show that the interaction of a cold ring with energy e<sub>0</sub> and the ambient plasma generates a thermal spread around e<sub>0</sub> of the order Δe = 0.5e<sub>0</sub>, with ions reaching energies up to 2.3e<sub>0</sub> (see Figure 1 of Kulygin et al. [1971]). The net effect would probably be to produce higher-energy beams for large θ<sub>ab</sub> (≥70°) rather than increase the beam speed for moderate θ<sub>ab</sub>. We can invert the argument used in section 3 to determine the exit angle of an ion as a function of velocity to obtain

\[ \frac{V_{pe}}{V_{sw}} = \left( \frac{E_i}{E_j} \right)^{1/2} = \frac{1}{\sqrt{2}} \left[ \frac{1 + 2 \frac{B_1}{B_2} \tan \theta_{ab}}{1} \right]^{1/2} \]

Assuming this relation to hold for θ<sub>ab</sub> > 70°, we find that V<sub>pe</sub>/V<sub>sw</sub> is 2.4 for θ<sub>ab</sub> = 75° and 3.2 for θ<sub>ab</sub> = 80°. These results agree very well with findings of Bonifazi and Moreno [1981a, b]. The values calculated for the two events with θ<sub>ab</sub> slightly greater than 70° by Paschmann et al. [1980] are significantly higher, ~5, which are consistent with Sonnerup [1969].

The distributions of escaping ions in section 3 show the large temperature anisotropy inherent in our model. This is an important result as this anisotropy is a universal characteristic of the "reflected" ion beams [Paschmann et al., 1981], which is difficult to explain in the context of the direct reflection model. The model anisotropy is somewhat larger than observed T<sub>e</sub>/T<sub>i</sub> ~ 6 for θ<sub>ab</sub> ~ 65° (φ<sub>ab</sub> = 45°). Furthermore, some of the distributions shown in Figure 9 are peaked at v<sub>i</sub> ≠ 0 and hence are loss-cone-like instead of monotonic in velocity, as is usually observed. These distributions should, however, be considered as upper limits, since we have neglected effects that would tend to make them more isotropic. We have also assumed that the ring ions had negligible thermal spread. The anisotropy is relaxed by introducing a reasonable thermal spread to the source ions. When the thermal spread Δv<sub>e</sub> is included such that

| n<sub>esc</sub>/n<sub>sw</sub> | 1-2% | 1%± |
| V<sub>pe</sub>/V<sub>sw</sub> | 1.5-2.0 | 2.0± |
| Temperature anisotropy, T<sub>e</sub>/T<sub>i</sub> | 3-2| 2-3* |

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\[ \frac{V_{pe}}{V_{sw}} = \left( \frac{E_i}{E_j} \right)^{1/2} = \frac{1}{\sqrt{2}} \left[ \frac{1 + 2 \frac{B_1}{B_2} \tan \theta_{ab}}{1} \right]^{1/2} \]

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angle scattering postulated in our model. Our model cannot easily explain the more energetic subset of the ion beams studied by Paschmann et al. [1980], which may indicate that direct reflection also produces some upstream ions.

Acknowledgments. We are grateful to C. S. Wu and M. M. Leroy for valuable comments and discussions, especially about the results of the hybrid simulations. We are also grateful to J. D. Scudder and A. Mangeney regarding ISEE data prior to publication. This work was supported by the NASA Solar Terrestrial Theory Program NAGW-81 and partially by ONR N00014-79-C-0665.

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