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A Coherent Nonlinear Theory of Auroral Kilometric Radiation 1. Steady State Model

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A theory of auroral kilometric radiation due to the nonlinear interaction between negative energy electromagnetic waves and coherent electrostatic ion cyclotron waves is developed. The theory predicts that such radiation produced must have X mode polarization and must have a frequency that lies in a narrow band between the right-hand cutoff and the Doppler-shifted beam cyclotron frequency for each local point of origin. The basic requirement for free space accessibility is the presence of high-energy beams in the inverted V events and a density-depleted cavity of the type observed by Isis 1 measurements. Beam densities of the order of 10^{-3} of the (depleted) background density appear to be necessary for the instability producing the radiation. Under ideal conditions, paths of wave growth of the order of 100 km may be adequate to produce observed radiation levels from electromagnetic noise at the appropriate frequency in the source region.

1. INTRODUCTION

There have been several observational studies made in recent years of high-intensity kilometric radiation of terrestrial origin. Measurements have been made by satellite from the source region in the auroral zone at $R \sim 2$–$3\ R_E$ by Isis 1 [*Benson and Calvert,* 1979] all of the way out to $R \sim 100\ R_E$ by Voyager 1 [*Kaiser et al.,* 1978].

A number of interesting features of the auroral kilometric radiation (AKR) have been deduced as a result of these satellite measurements. The first measurements were made on Ogo 1 [*Dunckel et al.,* 1970]. More detailed measurements were made on Imp 6 and Imp 8, which were analyzed by *Gurnett* [1974]. Gurnett found the spectrum to lie in 50 kHz $< f <$ 500 kHz, with peak intensity at $f \sim 200$ kHz. The peak intensity power emission was estimated to be about 10^9 W. He noted that the radiation appears to originate at low altitudes ($R \sim 2$–$3\ R_E$) in the auroral region and to be closely associated with the occurrence of discrete auroral arcs, which are believed to be generated by intense inverted V electron precipitation bands. When arcs do not occur, only a small band of diffuse aurora is present, and the radiation disappears. The radiated power was estimated to be close to 1% of the maximum energy dissipated by the auroral charged particle precipitation.

Recent observations in the source region from Isis 1 reveal and establish further properties of the AKR that were not well-established from the far field measurements [*Benson and Calvert,* 1979]. Those observations showed that the radiation was generated in the X mode just above the local cutoff frequency and propagated almost perpendicular to the background magnetic field. It was found to be generated within density-depleted regions with peak density such that $\omega_{pe} < 0.2\omega_{ce}$. (These are probably caused by the large potential drop in those regions associated with the discrete auroral arcs.)

A number of theories have been proposed in the literature to explain the occurrence of the high-intensity radiation. *Oya* [1974] proposed a mechanism for the well-known Jovian decametric radiation (thought to have an origin similar to that of the terrestrial kilometric radiation) that involved the creation of electromagnetic radiation by a mode conversion process. This mechanism was extended to kilometric radiation by *Benson* [1975]. In this model, excited plasma waves were converted into the X mode at the upper hybrid frequency. This mode was then converted to the O mode. The overall efficiency of this mechanism is estimated to be a few percent.

There are two major drawbacks with this theory. First, the polarization is in disagreement with recent polarization measurements, including both far field measurements [*Gurnett and Green,* 1978] and measurements at the source [*Benson and Calvert,* 1979]. Second, the overall efficiency, which is the product of the efficiencies of two inefficient conversion mechanisms, is too small. This does not even take into account the conversion efficiency of precipitating electrons into electrostatic plasma waves in the first place. Thus it is highly unlikely that this indirect mechanism could produce final output radiation that is of the order of 1% of precipitating electron (or electron beam) energy.

Melrose [1976] proposed a theory for the Jovian decametric and terrestrial kilometric radiation that involves directly amplified gyroemissions. In this mechanism, radiation is emitted at the electron cyclotron frequency and its harmonics, which have been Doppler-shifted by electron streaming. This radiation is then amplified because of assumed anisotropic velocity distribution functions. It is also assumed that the electron streaming velocity is large enough to allow the radiation to escape to free space.

The frequency for the radiation required in Melrose's theory will turn out to be the most favorable frequency for radiation in the theory proposed in the present paper. However, to

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get growth rates required to produce kilometric radiation at the observed level by anisotropic velocity space instabilities would appear to require very large anisotropies in velocity space. There is at present no experimental evidence for such large anisotropies.

Wu and Lee [1979] have recently proposed a mechanism that involves a velocity space instability, namely, a loss cone type instability. In their model, precipitating electrons stream down the (converging) magnetic field lines toward earth until they encounter an effective magnetic mirror in a region of higher magnetic field strength. They then undergo reflection and travel up diverging magnetic field lines. Some of the upgoing particles are lost through the loss cone, and the resulting velocity distributions lead to a loss cone instability.

Wu and Lee incorporate into their model a long density-depleted cavity in which the unstable wave grows, as found by Benson and Calvert on Isis 1. This cavity was not considered in any of the previous papers but will be assumed in our theory. The most probable explanation is that the cavity is formed by large potential drops known to be present in the auroral region, which also tend to drive large electron (and even ion) beams. For example, if we assume the background density is close to thermodynamic equilibrium, then the density profile is determined by the Boltzmann form

$$n_e(x) = n_0 \exp\left[-e\phi(x)/\kappa T_e\right]$$

where $\phi(x)$ is the local potential and T_e is the electron temperature. Thus the regions of high potential have lower density.

One drawback of the Wu-Lee theory is that to get the large growth lengths required, loss cone angles of typically $\theta > 45°$–$60°$ appear to be required, as, for example, the effective angles in the distribution functions assumed by Wu and Lee. It remains to be seen whether loss cones of this magnitude occur in the precipitating electron bands.

Another recent theory for kilometric radiation was proposed by *Roux and Pellat* [1979]. Their mechanism involved the nonlinear beating of electrostatic waves near the upper hybrid to produce an electromagnetic wave. Their theory predicts that the O model should dominate at $\omega = \omega_{uh}$ (at the upper hybrid frequency) for $\omega_{pe} > \omega_{ce}$ and that the X mode should dominate at $\omega = 2\omega_{uh}$ for $\omega_{pe} < \omega_{ce}$.

Roux and Pellat argue that a coherent nonlinear theory, such as their proposal or the present proposal, should be more efficient in general than an incoherent linear mechanism or even a coherent linear mechanism. However, their particular nonlinear mechanism, which requires the creation of a high phase velocity transverse wave from two low phase velocity longitudinal waves, would be expected to be rather inefficient. Also, their prediction of radiation in the X mode at $\omega = 2\omega_{uh}$ for the density-depleted case observed is in apparent disagreement with the observation that the frequency at the origin is just above the right-hand cutoff frequency. Roux and Pellat also mention the possibility of the nonlinear interaction of an upper hybrid wave and a lower hybrid wave to produce radiation at $\omega = \omega_{uh} + \omega_{lh}$. However, since $\omega_{pe} \ll \omega_{ce}$ in the density cavity, the resulting radiation would be below the right-hand cutoff.

Finally, a theory proposed by *Palmadesso et al.* [1976] assumed that electromagnetic 'noise' gets amplified through its interaction with ion turbulence. The nonlinear beat wave of these two waves is amplified by electron beams, and thus the electromagnetic wave is thereby parametrically amplified. We will employ a similar mechanism in the theory presented in the present paper.

In the Palmadesso theory, both the RX and LO modes were assumed to be amplified by the above mechanism. However, for the beam to significantly amplify the beat wave, and hence the electromagnetic wave, the beat wave was assumed to be almost a natural mode of the plasma and required to have a slow phase velocity ($\omega/k \ll c$). This required generation at frequencies below the upper hybrid. Thus the RX mode would encounter a cutoff at frequencies between the upper hybrid and the right-hand cutoff and not escape to free space. Thus only the LO mode would be observed, contrary to recent polarization measurements. Also, the ion waves were taken to be incoherent (turbulent) and assumed to be isotropic for convenience of calculation. Recent measurements of electrostatic ion cyclotron (EIC) waves [e.g., *Temerin et al.*, 1978] show strong EIC wave fluctuations that are coherent and primarily perpendicular to the background magnetic field. The coherence of the waves was unanticipated and suggests that a modified application of the mechanism in the work of *Palmadesso et al.* [1976] can be employed.

We will construct the present theory with the same principle underlying mechanism. However, we will construct a new theoretical framework, so the formulation of the theory will be different. We will assume that coherent electrostatic density fluctuations (due to low-frequency EIC waves) propagating perpendicular to the magnetic field are present. We will concentrate on the case that there is a resonant interaction between electromagnetic wave and the coherent density fluctuations, a case not considered by *Palmadesso et al.* [1976]. It will be shown that the X mode is preferentially amplified over the O mode in this model and that the X mode can grow rapidly in a narrow frequency band that is accessible to free space. The basic requirement for this last condition to exist will be shown to be that a density cavity of precisely the density observed is present at the source.

In section 2 we will introduce the basic model for the AKR, derive the wave equation and associated dispersion relation, and discuss solutions. In section 3 we will analyze the wave equation to find the instability criterion and the growth rate for the unstable modes for resonant interaction between the electromagnetic waves and the EIC density fluctuations. In section 4 we discuss in more detail the physics of the wave amplification. Results are summarized in section 5.

2. MODEL AND GEOMETRY

We assume that initially we have electromagnetic radiation propagating at the noise level in the auroral zone, which has been generated by one of a variety of possible mechanisms: Cerenkov, electron cyclotron, bremsstrahlung, etc. We also assume that there are coherent density fluctuations perpendicular to the **B** field due to EIC waves, as was observed by Temerin and co-workers. The background magnetic field $\mathbf{B}_0 = B_0\hat{z}$ is taken along the z direction. Our theory requires a maximum interaction between the electromagnetic wave and the density fluctuations. Thus the electromagnetic wave is assumed to propagate almost perpendicular to the magnetic field, with propagation vector $\mathbf{k} = (k_x, 0, k_z)$, where $k_z^2 \ll k_x^2$, as observed by *Benson and Calvert* [1979] (see Figure 1). A

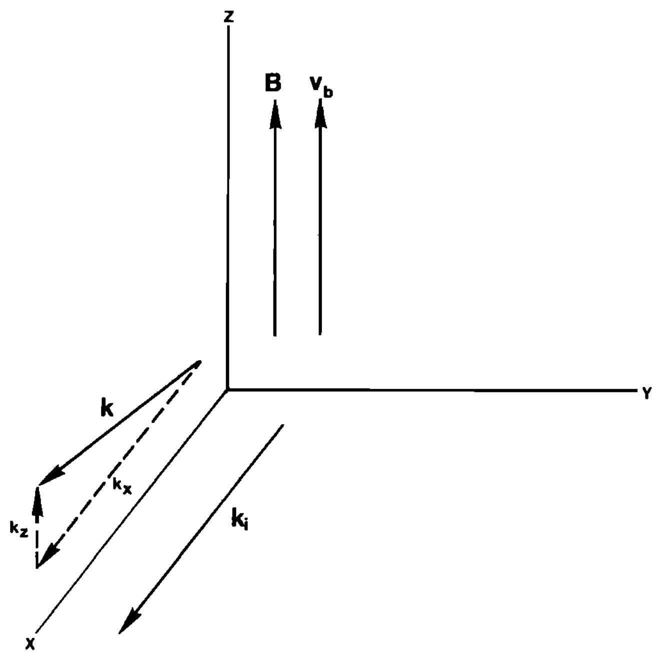


Fig. 1. Geometry of model; **k** is the propagation vector of the electromagnetic wave, and $\mathbf{k}_i$ is the propagation vector for the EIC waves. The latter produce density fluctuations along x.

nonzero k_z is necessary to couple the electron beam into the interaction.

There are two independent modes of the electromagnetic wave, with E field vectors:

X mode

$$\mathbf{E} = [E_x(x)\hat{x} + E_y(x)\hat{y}]e^{i(k_z z - \omega t)} \tag{1a}$$

O mode

$$\mathbf{E} = [E_x(x)\hat{x} + E_z(x)\hat{z}]e^{i(k_z z - \omega t)} \tag{1b}$$

We are dealing with high-frequency waves in this model, so that $\omega \gtrsim \omega_{ce}, \omega_{pe}$. The waves are primarily transverse waves, so $E_x \ll E_y, E_z$.

The EIC waves have associated density fluctuations of both ions and electrons; the electrons are assumed to follow the ions in the fluctuations so that the resulting total plasma fluctuation is quasi-neutral. Then the plasma density is of the form

$$n(x, t) = n_0 + n_1(x, t) \tag{2a}$$

$$n_1(x, t) = \delta n \cos (k_i x - \omega_i t) \tag{2b}$$

where n_0 is the background density, n_1 is the fluctuating density, and ω_i and k_i are the frequency and the wave number of the EIC wave. Since $\omega_i \sim \omega_{ci} \ll \omega_{ce} \sim \omega$, the electromagnetic wave sees the density fluctuation as stationary. Thus we can ignore the $\omega_i t$ term in n_1 in our electromagnetic wave equation:

$$n_1 \approx \delta n \cos k_i x \tag{2c}$$

The wave equation for the electromagnetic wave is

$$\nabla \times (\nabla \times \mathbf{E}) + (\omega/c)^2 \, \mathsf{K} \cdot \mathbf{E} = 0 \tag{3}$$

where K is the plasma dielectric tensor and is of the form

$$\mathsf{K} = \begin{bmatrix} K_\perp & -iK_H & 0 \\ iK_H & K_\perp & 0 \\ 0 & 0 & K_\parallel \end{bmatrix} \tag{4}$$

where $K_\perp$ and $K_\parallel$ are the dielectric constants perpendicular to and along the magnetic field, respectively, and K_H is the Hall term. We will divide the components of K according to

$$K_{ij} = K_{ij0} + \delta K_{ij} \tag{5}$$

where K_{ij0} are the tensor components for the case of $n(x, t) = n_0$ and δK_{ij} are the perturbations introduced by the density fluctuation n_1. In order to take into account the inverted V electron precipitation, we model it as a beam [*Matthews et al.*, 1976]. Including the effect of a beam of velocity $v_b = v_b\hat{z}$ along the **B** field, the dielectric tensor components for the nonfluctuating background plasma for the case of a cold plasma and a warm beam are

$$K_{\perp 0} = 1 - \frac{\omega_{pe0}^2}{\omega^2 - \omega_{ce}^2} - \frac{\omega_b^2}{(\omega - k_z v_b + i\nu)^2 - \omega_{ce}^2} \tag{6a}$$

$$K_{H0} = \frac{\omega_{ce}}{\omega}\left[\frac{\omega_{pe0}^2}{\omega^2 - \omega_{ce}^2} + \frac{\omega_b^2}{(\omega - k_z v_b + i\nu)^2 - \omega_{ce}^2}\right] \tag{6b}$$

$$K_{\parallel 0} = 1 - \frac{\omega_{pe0}^2}{\omega^2} - \frac{\omega_b^2}{(\omega - k_z v_b + i\nu)^2} \tag{6c}$$

with ω_{pe0} the plasma frequency for plasma density n_0. We have neglected the low-frequency ion terms, since they are not of interest for the high-frequency electromagnetic wave. Here ω_b is the beam plasma frequency, and ν enters because of the finite beam width thermal effects for an assumed Lorentzian beam velocity profile. In terms of the beam spread Δv in velocity space,

$$\nu = k_z \Delta v \tag{7}$$

The use of a Lorentzian beam profile for thermal effects is valid provided $v_b \gg \bar{v}_e \sim \Delta v$, where $\bar{v}_e$ is the average electron thermal velocity of the beam. This form is quite consistent with the conclusions of *Matthews et al.* [1976].

We may substitute (1) into (3) to get the wave equation for each mode. If we now take the values of the dielectric tensor given by (2), (4), (5), and (6), we find for the $\hat{x}$ component of the equation:

X mode

$$E_x = \frac{iK_H E_y}{(K_\perp - n_z^2)} \tag{8a}$$

O mode

$$\frac{\partial E_x}{\partial x} = \left(\frac{\omega}{c}\right)\frac{in_z}{(K_\perp - n_z^2)} \tag{8b}$$

where $n_z = ck_z/\omega$. Substituting these forms into the y component for the X mode wave equation and the z component for the O mode wave equation and expanding to order ϵ, we find the reduced wave equation:

X mode

$$\frac{\partial^2 E_y}{\partial x^2} + (\alpha_1 - \epsilon\alpha_2 \cos k_i x)E_y = 0 \tag{8c}$$

O mode

$$\frac{\partial^2 E_z}{\partial x^2} - (\beta_1 + \epsilon\beta_2 \cos k_i x)E_z = 0 \tag{8d}$$

where

$$\epsilon = \frac{\delta n}{n_0} \tag{9a}$$

$$\alpha_1 \cong \left(\frac{\omega}{c}\right)^2 \left[K_{\perp 0} - \frac{K_{H0}{}^2}{(K_{\perp 0} - n_z{}^2)} - n_z{}^2\right] \tag{9b}$$

$$\alpha_2 \cong \frac{\omega^2 \omega_{pe0}{}^2}{c^2(\omega^2 - \omega_{ce}{}^2)} \left[1 + \frac{2K_{H0}{}^2}{(K_{\perp 0} - n_z{}^2)^2} + \left(\frac{\omega_{ce}}{\omega}\right) \frac{K_{H0}}{(K_{\perp 0} - n_z{}^2)}\right] \tag{9c}$$

$$\beta_1 \cong \frac{(K_{\perp 0} - n_z{}^2)K_{\parallel 0}}{K_{\perp 0}} \tag{9d}$$

$$\beta_2 \cong K_{\parallel 0} \left(\frac{\omega_{pe}{}^2}{\omega^2 - \omega_{ce}{}^2}\right) n_z{}^2 - \frac{\omega_{pe}{}^2}{\omega^2} \left(\frac{K_{\perp 0} - n_z{}^2}{K_{\perp 0}}\right) \tag{9e}$$

The form of (8) is obtained under the assumptions

$$|\omega^2 - \omega_{uh}{}^2| \gg \epsilon \omega_{pe}{}^2 \tag{10a}$$

$$\epsilon^2 \ll 1 \tag{10b}$$

Since measured density fluctuations give $\epsilon < 0.5$, the $\epsilon^2 \ll 1$ assumption appears to be quite good. Also, (10a) is quite valid except for a very narrow range around the upper hybrid frequency:

$$1.01\omega_{uh} \gtrsim \omega \gtrsim 0.99\omega_{uh} \tag{11}$$

The latter frequency range will not be of interest to us because it lies below the right-hand cutoff and involves a wave inaccessible to free space.

Thus the wave equations for both the X mode and the O mode reduce to the Mathieu's equation for the model being used here. From this equation we may find stability conditions and growth rates that arise from resonant spatial interaction between the electromagnetic wave and spatial density fluctuations, as well as the modification of the electromagnetic waves by the density fluctuations.

For purposes of illustration of the solution, we will solve the wave equation for the X mode. The O mode solution is quite similar. By using (8a) for the relationship between E_x and E_y we may use the standard form of the solution of a Mathieu equation from Floquet's theorem [*Whittaker and Watson,* 1927] to write the solution for **E** in the form

$$\mathbf{E} = A^{\pm}\left[\hat{y} - \frac{K_{H0}}{(K_{\perp 0} - n_z{}^2)}\hat{x}\right] e^{i(k_z z - \omega t)} \sum_n^{\pm} a_n{}^{\pm} e^{\pm i(q + nk_i)x} \tag{12}$$

where $a_0{}^{\pm} = 1$, and for no density fluctuations,

$$a_n{}^{\pm}(\epsilon = 0) = 0 \qquad (n \neq 0) \tag{13}$$

By substituting this form into our wave equation, (3), we find the dispersion relation to order ϵ^2:

$$q^6 - q^4(2k_i{}^2 + 3\alpha_1) - q^2\left[k_i{}^4 + 3\alpha_1{}^2 - \frac{\epsilon^2\alpha_2{}^2}{2}\right] + \left[-\alpha_1(\alpha_1 - k_i{}^2) + \frac{\epsilon^2\alpha_2{}^2}{2}(\alpha_1 + k_i{}^2)\right] + O(\epsilon^4) = 0 \tag{14}$$

(See Appendix A for the method of solution of this equation.) Also, solution of the coefficients to order ϵ yields

$$a_1{}^{\pm} = \frac{-\epsilon\alpha_2}{2[\alpha_1 - (q + k_i)^2]} \tag{15a}$$

$$a_{-1}{}^{\pm} = \frac{-\epsilon\alpha_2}{2[\alpha_1 + (q - k_i{}^2)^2]} \tag{15b}$$

These equations show the interaction between the electromagnetic wave and the first harmonic of the density fluctuation. If we included higher orders of ϵ, we would get the higher harmonics. In general, for the inclusion of harmonics up to n we get a dispersion relation or order $2n + 1$ in q^2 and coefficients $a_n{}^{\pm} \sim \epsilon^n$. It should be noted that in the limit $\epsilon \to 0$ the dispersion relation in (14) has roots

$$q_{1,2} = \pm(\alpha_1)^{1/2} \tag{16a}$$

$$q_{3,4} = \pm[(\alpha_1)^{1/2} - k_i] \tag{16b}$$

$$q_{5,6} = \pm[(\alpha_1)^{1/2} + k_i] \tag{16c}$$

The first two roots are just the fundamental mode, and the last four are simple first harmonics. However, for $\epsilon \to 0$ the amplitudes $a_{\pm 1}{}^{\pm}$ of these harmonics are zero.

3. Stability Criterion and Growth Rates

When the electromagnetic wave is propagating (i.e., for $\alpha_1 > 0$), then the q's in (16) for the unmodulated wave are real. However, the presence of density fluctuations (so $\epsilon \neq 0$) causes the roots for q in (14) to be complex for certain conditions, so that the propagating wave has growing or decaying solutions. For example, in (14) the dispersion relation of the X mode for the fundamental and first harmonic interactions ($n = 0, \pm 1$ in (12), we have such solutions if $(\alpha_1)^{1/2} \approx k_i$ (see Appendix B). In general, the requirement for growing and decaying solutions to exist is

X mode

$$(\alpha_1)^{1/2} \cong pk_i \tag{17a}$$

O mode

$$(\beta_1)^{1/2} \cong pk_i \tag{17b}$$

where p is some positive integer. The larger the relative fluctuation amplitude ϵ, the less strictly this criterion has to be satisfied. Basically, (17) is just the condition for resonant interaction between the electromagnetic wave and a spatial harmonic of the EIC wave density fluctuation. A stability diagram is shown in Figure 2. It should be noted that whether the unstable roots that can exist in the shaded regions in the dia-

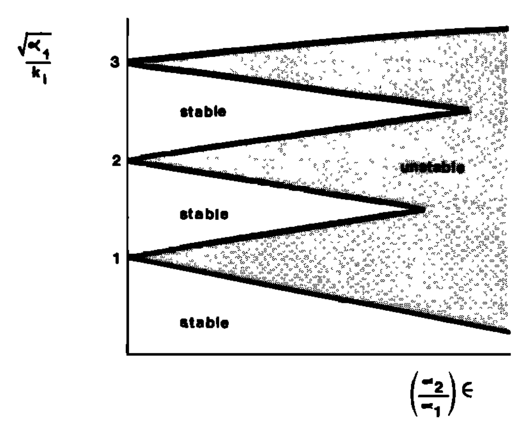


Fig. 2. Stability diagram for the wave equation of the X mode. Instability occurs whenever $(\alpha_1)^{1/2} \cong pk_i$, where p is an integer. As the relative density fluctuation amplitude ϵ increases, the less strictly this criterion must be satisfied.

gram (denoted as unstable) are actually the modes that are excited and are causal depends on the physics, which determines the direction of energy transfer. When unstable solutions exist, the $p = 1$ mode is the fastest growing.

The conditions for unstable roots of the X mode may be rewritten

$$p^2\left(\frac{ck_i}{\omega}\right)^2 + n_z^2 = K_\perp + \frac{K_H^2}{(K_\perp - n_z^2)} \tag{18}$$

Generally, $ck_i/\omega \gtrsim 1$, since the ion wavelength tends to be shorter than the free space electromagnetic wavelength (but not of the electromagnetic wavelength in the plasma medium). Thus for there to be a solution of the X mode stability criterion, we must have

$$\frac{\omega_b^2}{\omega_{ce}^2 - (\omega - k_z v_b)^2} \gtrsim \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \tag{19}$$

There are two frequency regimes where this is satisfied:

1. $\omega \gtrsim \omega_{ce}$. This frequency regime has waves that are not accessible to free space because of the cutoff layer between the upper hybrid and right-hand cutoff. Also, an analysis of the stability of the wave near $(\alpha_1)^{1/2} \sim pk_i$ in this frequency regime shows that only evanescent waves exist, so no growth can occur (see Appendix B).

2. $\omega \gtrsim \omega_{ce} + k_z v_b$. This is the Doppler-shifted beam cyclotron resonance. An analysis shows that the wave is indeed amplifying when $(\alpha_1)^{1/2} \sim pk_i$ in this frequency regime (see Appendix B). We will discuss the physical mechanism for this in the next section. There are waves in this regime that are accessible to free space provided the beam cyclotron frequency is greater than the right-hand cutoff frequency ω_R:

$$\omega_{ce} + k_z v_b > \omega_R = \frac{\omega_{ce}}{2} + \left(\omega_{pe}^2 + \frac{\omega_{ce}^2}{4}\right)^{1/2} \tag{20}$$

Expanding the right-hand side for $\omega_{pe}^2 \ll \omega_{ce}^2$, this becomes

$$\omega_{pe}^2 < k_z v_b \omega_{ce} \tag{21}$$

Now *Matthews et al.* [1976] reported beam energies of $E \sim$ 6–13 keV, corresponding to velocities $v_b \sim 0.1$–$0.2c$. If we take the small k_z component to be $k_z \sim 0.2$–$0.3(\omega/c)$, we find

$$\omega_{pe} < 1.5\text{–}2.5\omega_{ce} \tag{22}$$

This is to be compared with $\omega_{pe} < 0.2\omega_{ce}$, which was observed in the source of AKR. Thus the density depletion observed by Isis 1 at the source is of just the amount predicted by theory to be necessary for radiation. When the condition in (22) is satisfied, we get radiation in the narrow band

$$\omega_{ce} + \omega_{pe}^2/\omega_{ce} < \omega < \omega_{ce} + k_z v_b \tag{23}$$

just above the right-hand cutoff. This frequency regime is then the most probable regime of the kilometric radiation. It should be noted that there is a range of values of the plasma and cyclotron frequencies at the source of AKR in the aurora, and the frequencies that lie in the narrow band given by (23) are different for each source point. Thus the total radiation received has the broad spectrum between 50 and 500 kHz observed. Also, (21) shows that radiation will not be observed in regions where beams of sufficiently high energy are not present (such as outside the region of discrete arcs and inverted V's), where the plasma density is too high (such as outside the density-depleted region), or where the magnetic field is too low (such as at large distances R from the earth, since the earth's dipole field goes as $B_0 \sim R^{-3}$). Thus the restriction on the plasma and beam parameters, coupled with the temporal variations of these parameters in the auroral region, could explain the sporadic nature and the restricted locality of the AKR.

Having established the frequency regime for the radiation, it must also be pointed out that (19) also gives a minimum required beam density n_b, namely,

$$\left(\frac{n_b}{n_0}\right) = \left(\frac{\omega_b}{\omega_{pe}}\right)^2 > \frac{\nu^2 + \omega_{ce}^2 - (\omega - k_z v_b)^2}{\omega^2 - \omega_{ce}^2} \tag{24}$$

where we have included the finite beam width and damping term ν. On resonance this reduces to

$$\left(\frac{n_b}{n_0}\right)_{\min} > \left(\frac{\nu^2}{\omega^2 - \omega_{ce}^2}\right) = \left(\frac{k_z(\Delta v)^2}{2\omega_{ce} v_b}\right) \tag{25}$$

which for $\Delta v \sim 0.3v_b$, $k_z \sim 0.2$–$0.3k$, $v_b \sim 0.1$–$0.2c$ gives

$$n_b \gtrsim 10^{-3} n_0 \tag{26}$$

Recall n_0 is the reduced plasma density due to density depletion in the area of the discrete auroral arcs.

For the O mode we may make the same analysis as we did for the X mode. For instability for this mode we require

$$p^2\left(\frac{ck_i}{\omega}\right)^2 + n_z^2 = \left(\frac{K_{\perp 0} - n_z^2}{K_{\perp 0}}\right)\left(1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_b^2}{(\omega - k_z v_b)^2}\right) \tag{27}$$

If we analyze this equation, we see that in general the right side is less than positive unity for the high frequencies of interest. Using the fact that the EIC wavelength $2\pi/k_i$ is of the order of the ion cyclotron radius $2\pi v_i/\omega_{ci}$, we find that (27) cannot be satisfied unless we have ion temperatures $T_i > 150$ eV. Since typical ion temperatures in the auroral zone are less than 1% of this value, high-intensity radiation in the O mode would not be produced by our theory. It should be recalled that (27) breaks down in a very narrow band near the upper hybrid. Thus the presence of $k_z \neq 0$ makes it conceivable that there might be an instability and some radiation in the O mode near $\omega = \omega_{uh}$. However, even if such an instability can exist, the band is very narrow, and the wave equation is not nearly as unstable as the Mathieu equation. Thus, in any case, the X mode radiation would definitely dominate, and we will concentrate on that mode from now on.

We may now calculate the growth rate for our X mode radiation. For example, the dispersion relation for the wave equation including the first harmonic interactions, (14), will yield an instability provided the equation has a complex (nonreal) solution. The spatial growth rate for the first harmonic instability that occurs when $(\alpha_1)^{1/2} \approx k_i$ ($p = 1$ mode) is for $\epsilon \ll 1$ (see Appendix A):

$$\kappa_1 \cong (\alpha_1)^{1/2}\left[\left(\frac{\alpha_2}{\alpha_1}\right)^2 \frac{\epsilon^2}{4} - \left(\frac{(\alpha_1)^{1/2}}{k_i} - 1\right)^2\right]^{1/2} \tag{28}$$

Note that as ϵ increases in size, both the maximum growth rate and the size of the unstable region increase. Similarly, for the $p = 2$ mode we have a growth rate

$$\kappa_2 \cong (\alpha_1)^{1/2}\left[\left(\frac{\alpha_2}{\alpha_1}\right)^2 \frac{\epsilon^2}{4} - \left(\frac{(\alpha_1)^{1/2}}{k_i} - 2\right)^2\right] \tag{29}$$

Plots of typical curves for the growth length $L = \kappa^{-1}$ for these two modes are shown in Figures 3–6. As p increases, the maxi-

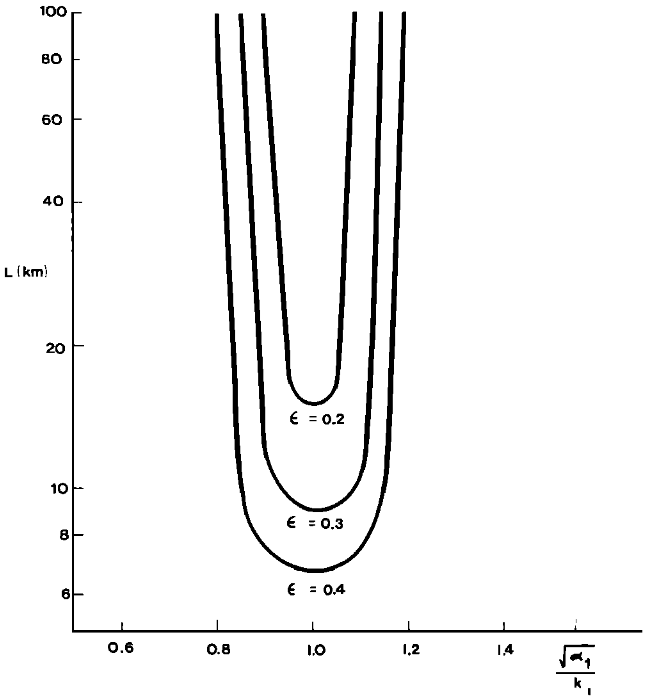


Fig. 3. Growth length L for the $p = 1$ mode for various density fluctuation ratios ϵ and frequency $\omega \cong \omega_{ce} + k_z v_b$. The plasma parameters we have chosen are $f \cong 178$ kHz, $\omega_{pe} \cong 0.3\omega_{ce}$, $k_z \cong 0.3k$, $v_b \cong 0.2c$, $\Delta v \cong 0.3\ v_b$, and $n_b = 6 \times 10^{-3} n_0$.

mum growth rate and the frequency width of the instability both decrease. Thus we are most interested in the two lowest modes. The graph shows that under the right conditions the wave has a growth length of $L \sim 5$–15 km for the $p = 1$ mode and $L \sim 10$–30 km for the $p = 2$ mode.

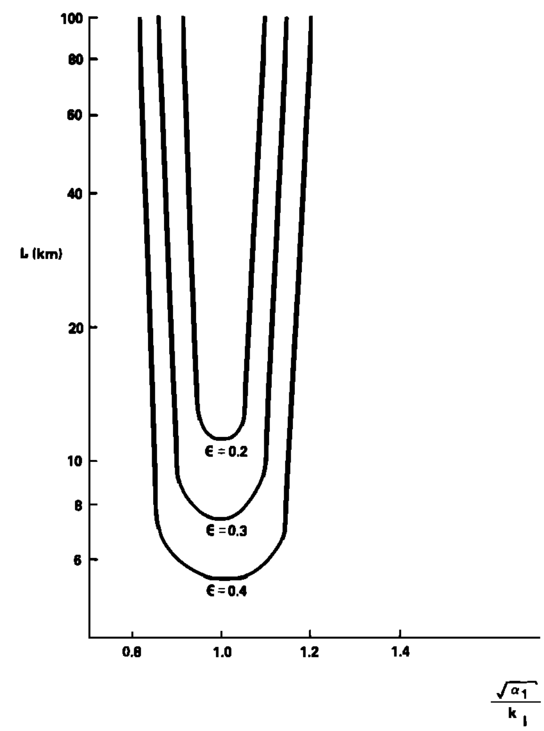


Fig. 4. Growth length for $p = 1$ mode for the following set of plasma parameters: $\omega \cong \omega_{ce} + k_z v_b$, $f \cong 178$ kHz, $\omega_{pe} \cong 0.3\omega_{ce}$, $k_z \cong 0.3k$, $v_b \cong 0.3c$, $\Delta v \cong 0.1 v_b$, and $n_b \cong 10^{-2} n_0$.

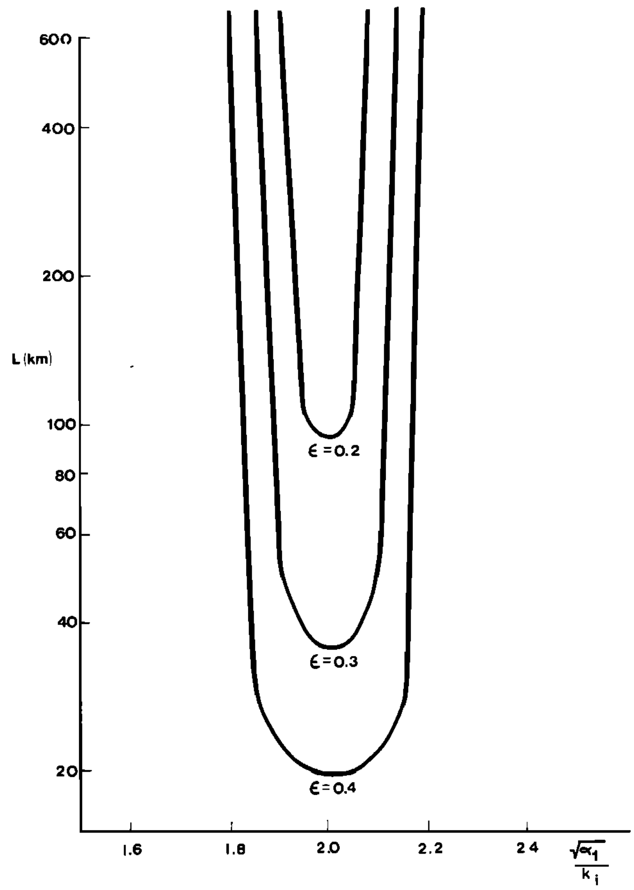


Fig. 5. Growth length for $p = 2$ mode with plasma parameters the same as in Figure 3.

Recent observations made on Isis 1 [*Benson and Calvert*, 1979] show an apparent harmonic band structure accompanying the AKR fundamental. The harmonic bands could be the various p mode harmonics found above. However, an

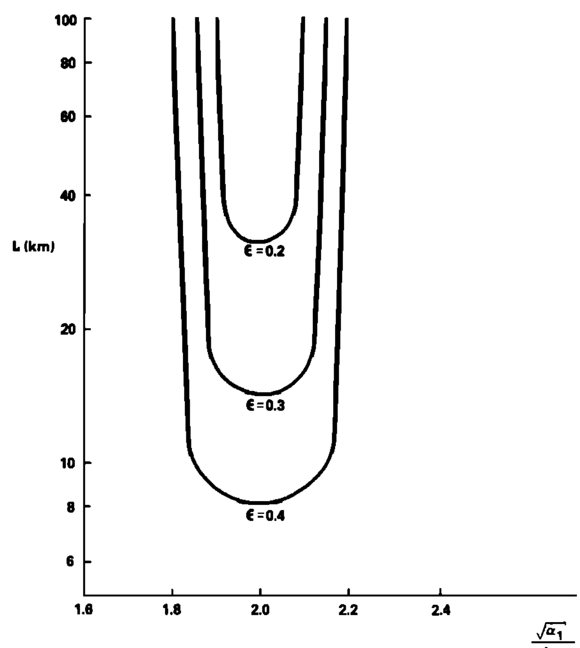


Fig. 6. Growth length for $p = 2$ mode with plasma parameters the same as in Figure 4.

even more likely explanation of the inferred band structure could result from a full Vlasov treatment of the AKR problem. Such a treatment should reveal possible radiation at the Doppler-shifted beam cyclotron harmonic frequencies $n\omega_{ce} + k_z v_b$, where $n = 1, 2, 3, \cdots$.

A schematic diagram of a possible ray path in the auroral density-depleted cavity is shown in Figure 7. If we assume the density depletion is caused by the inverted V events, the cavity width may be ~200 km, which is the typical width of the inverted V high-energy beam region [*Frank and Ackerson*, 1971]. Radiation may then undergo multiple reflection off the density cavity boundaries, where the density is higher, if the right-hand cutoff layer is encountered there, and amplify on each pass. The cavity length may typically be a few thousand kilometers, since the inverted V region length is much greater than its width. This might then be enough to allow the wave to propagate to a region where the background density (outside the density-depleted region) is such that at the cavity boundary the wave is accessible to free space. Note that waves propagating in directions toward higher magnetic fields and higher densities will eventually reach the right-hand cutoff layer and reflect back toward lower field and density regions. Thus this radiation may also reach a region of escape to free space.

We may make an estimate of the path lengths required for the amplifying wave to grow from the noise levels present in the inverted V region to the power levels of AKR observed coming from that region. We know that total estimated power flux to be $\int \mathbf{S} \cdot d\mathbf{A} = 10^9$ W [*Gurnett*, 1974]. If we now model the source region as a cylinder with radius 100 km and length 2000 km (consistent with the density cavity and inverted V dimensions just discussed) and take S to be roughly constant on the cylinder surface, the radiated power per unit area is $S \sim 10^{-3}$ W/m^2. On the other hand, we will take the wave noise level to be at the minimum that from the contribution from the galactic spectrum and the continuum radiation that happens to lie in the AKR frequency band. From Figure 3 of *Gurnett* [1974] this is $S_n \gtrsim 10^{-15}$ W/m^2. Thus we find the ratio $S/S_n = \exp(2\kappa\ell) \lesssim 10^{12}$, where κ is the spatial growth rate and ℓ is the path length required. Thus for $L = \kappa^{-1} \sim$ 5–10 km we find $\ell \gtrsim$ 70–140 km as the path length required to bring the observed radiation levels out of the noise. In any case, the path lengths available for the density cavity dimensions just discussed appear to be quite adequate to produce the total wave growth required from the growth rates we calculated.

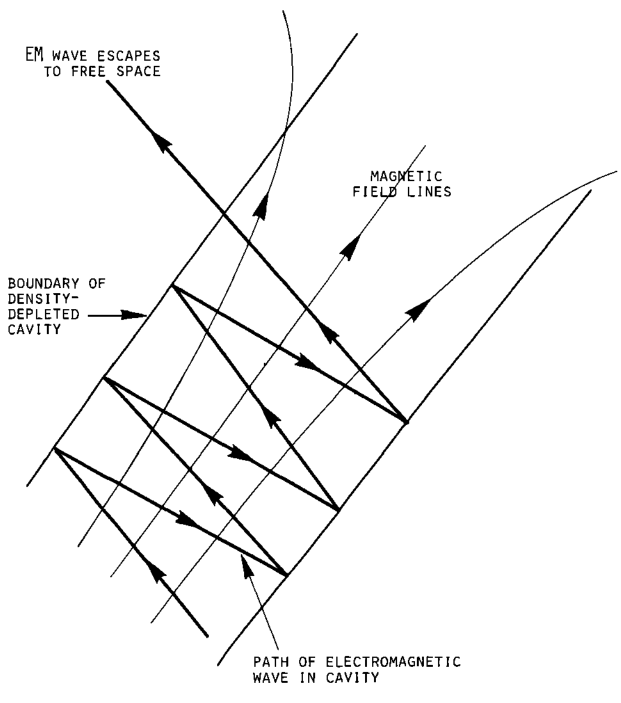


Fig. 7. Path of electromagnetic ray in density-depleted region. Multiple reflections off of cavity boundaries may provide long paths for growth. The way may eventually reach a weak field and/or low plasma density region where it is accessible to free space.

4. PHYSICAL MECHANISM

Now that we have found the unstable solutions to the wave equation and shown that they are adequate to explain the kilometric radiation observed, it is informative to look at the physics of the amplification in more detail.

As was shown in section 3, there were two frequency regimes in which unstable solutions could exist: $\omega \lesssim \omega_{ce}$ and $\omega \lesssim \omega_{ce} + k_z v_b$. The former was discounted because of inaccessibility and a causality of the growing solution. Let us consider the energy density of the wave in the two frequency ranges as seen by the electrons in their rotating frame:

$$U = \frac{\epsilon_0}{2} \mathbf{E}^* \cdot \frac{\partial}{\partial \omega'}(\omega' \mathbf{K}_0') \cdot \mathbf{E} = 0 \tag{30}$$

where

$$\mathbf{K}_0' = \left[1 - \frac{\omega_{pe}^2}{\omega'^2} - \frac{\omega_b^2}{(\omega' - k_z v_b)^2} \right] \mathbf{I} \tag{31}$$

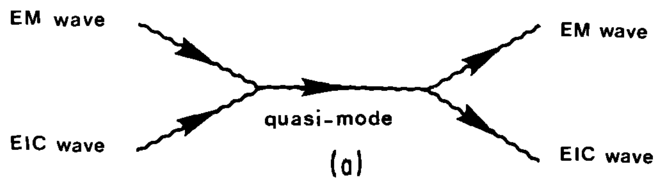
with $\mathbf{I}$ being the unit tensor and $\omega' = \omega - \omega_{ce}$ being the Doppler-shifted frequency seen by rotating electrons;

$$U = \frac{\epsilon_0}{2}\left[\left(1 + \frac{\omega_{pe}^2}{\omega'^2} + \frac{\omega_b^2(\omega' + k_z v_b)}{(\omega' - k_z v_b)^3} \right) (|E_x|^2 + |E_y|^2) \right] \tag{32}$$

Now U is negative when $\omega' \lesssim k_z v_b$; i.e., near the Doppler-shifted cyclotron resonance $\omega \lesssim \omega_{ce} + k_z v_b$ the X mode is a negative energy mode. However, for $\omega \lesssim \omega_{ce}$ it is not possible to get a negative energy mode for auroral conditions. Thus the former mode can absorb energy from the beam, whereas the latter cannot, because of the relation of the phase velocity in the electron frame ω'/k_z to the beam velocity v_b.

Since the frequency range $\omega \lesssim \omega_{ce}$ is not a negative energy mode, any energy it absorbs must come from the EIC density fluctuations via resonant interaction. The Feynman diagram for such an interaction is shown in Figure 8*a*. Ordinarily, energy flow from the electromagnetic wave to the lower-energy ion waves would be thermodynamically favored, and the X mode is more likely to be absorbed in this frequency range. However, in the frequency range $\omega \lesssim \omega_{ce} + k_z v_b$, energy flow from the beam into the negative energy X mode via coherent interaction with the ion mode is thermodynamically favored. (See Figure 8*b* for the Feynman diagram in this case.)

In Figure 8*b* it is seen that the beam interacts with and effectively amplifies the nonlinear beat wave between the EIC mode and the X mode. This mode is a driven or virtual mode, also called a quasi-mode, since it is not necessarily a natural mode of the system. We may learn more about this nonlinearly driven mode by looking at its phase, which is the



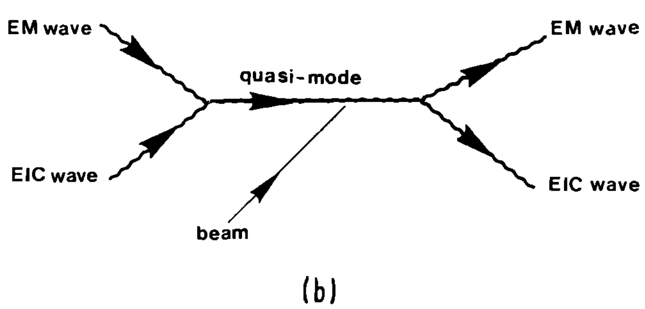


Fig. 8. (*a*) Simple resonant scattering of the electromagnetic (EM) and EIC waves. Energy transfer takes place in this scattering, normally from the EM wave to the EIC wave. (*b*) Resonant scattering of EM and EIC waves in which the beat 'quasi-mode' interacts with the beam at resonance. When the quasi-mode is negative energy in the beam frame, the beam transfers energy to it. When the electromagnetic wave is negative energy, energy flows from EIC to EM waves preferentially.

phase difference between the modes producing it. For interaction with the pth spatial harmonic,

$$\phi(x, t) = (k_x - pk_i)x - (\omega - \omega_i)t + k_z z \tag{33}$$

Now the EIC wave frequency $\omega_i \ll \omega$, and for instability we require $k_x = (\alpha_1)^{1/2} \cong pk_i$. Thus

$$\phi(x, t) \cong k_z z - \omega t \tag{34}$$

Thus the quasi-mode is essentially almost a right-hand circularly polarized (R) mode travelling nearly along the magnetic field. Being a driven mode, it does not have to satisfy the dispersion relation for an R mode exactly, but being almost a normal mode of the plasma enhances the coupling efficiency of energy into this mode. The R mode has an unstable resonance at the Doppler-shifted beam cyclotron frequency, and thus our quasi-mode is similarly amplified by the beam. The amplification of the quasi-mode then amplifies the parent waves that are driving it, since the process of driving the quasi-mode is reversible.

5. Summary and Conclusions

Nonlinear amplification of electromagnetic waves through coherent interaction with EIC waves and an electron beam is a promising candidate for explaining auroral kilometric radiation and predicts a number of the observed properties of the radiation. The theory predicts that the radiation must originate in the X mode and must propagate nearly perpendicular to the magnetic field (for maximum interaction with EIC waves), in agreement with observed data [*Benson and Calvert*, 1979; *Kaiser et al.*, 1978]. The principal requirements found for the X mode amplification are (1) frequency, $\omega \lessapprox \omega_{ce} + k_z v_b$; (2) minimum beam density (Δv is beam width), $(n_b/n_0) > [k_z(\Delta v)^2]/(2\omega c e v_b)$, when $\omega \lessapprox \omega_{ce} + k_z v_b$; and (3) accessibility to free space, $\omega_{pe}^2 < k_z v_b \omega_{ce}$. When these three conditions are satisfied, the theory predicts radiation may be produced in a narrow frequency band just above the right-hand cutoff, in agreement with data reported by *Gurnett and Green* [1978] and *Benson and Calvert* [1979]. That band is given by (23).

The accessibility conditions require the existence of local plasma density depletion, in agreement with the conclusion of Benson and Calvert from the Isis 1 data. For typical beam velocities we found the limitation on the plasma frequency to be $\omega_{pe} \lessapprox 0.2\omega_{ce}$, in excellent agreement with the values reported by Benson and Calvert. It was found that for reasonable noise levels (X mode waves at frequencies in the AKR band present in this density-depleted 'cavity') a growth length of ~100 km or so was adequate to produce the AKR power levels observed. For cavity dimensions inferred from the size of the inverted V region we concluded that growth lengths of a few times this value should easily be available.

We noted that two other properties of AKR may be explained by this model. One is the apparent harmonic band structure reported by Benson and Calvert. The other is the bursty nature of AKR, which could easily result from temporal variations in plasma density, magnetic field, and beam energy in regions of marginal instability of the AKR.

One limitation of the present work is that a steady state model was used in which the density fluctuation amplitude associated with the EIC waves was taken to be constant in time. However, our Feynman diagram interpretation of the growth mechanism indicates that a more dynamical process is occurring, since EIC waves are both being absorbed by the X mode and being produced by the beam. A steady state is reached when the absorption and production rates balance. The full dynamical model will be analyzed in a future paper.

It should be noted that there are possible future applications of the proposed mechanism for generation of radiation to laboratory sources of high-intensity radiation and possibly even plasma-based masers or lasers. This is related to the *Lin et al.* [1973] proposal for a plasma laser. A future paper on that area is planned.

Appendix A: Derivation of Dispersion Relation Roots

Equation (14) is the dispersion relation of the X mode in the fluctuating plasma density that includes the coupling of the fundamental X mode ($n = 0$ in (12)) with the first harmonics of the density fluctuations ($n = \pm 1$ in (12)). We want to solve the equation for density fluctuations small in comparison with background ($\epsilon^2 \ll 1$). To do this properly, we first note that in the case $\epsilon = 0$ we can factor the equation as follows:

$$(q^2 - \alpha_1)[q^2 - ((\alpha_1)^{1/2} - k_i)^2][q^2 - ((\alpha_1)^{1/2} + k_i)^2] = 0 \tag{A1}$$

which can be verified by multiplying out these factors. Thus we have the following solutions for $\epsilon = 0$: $\pm q$, $\pm q_2$, $\pm q_3$, where

$$q_1^2 = \alpha_1 \qquad q_2^2 = ((\alpha_1)^{1/2} - k_i)^2 \qquad q_3^2 = ((\alpha_1)^{1/2} + k_i)^2 \tag{A2}$$

Now we want to expand the actual solution about these zero-order solutions for $\epsilon^2 \ll 1$, the case considered in the paper. Thus we let $q_\alpha^2 \rightarrow q_\alpha^2 + r_\alpha^2 \epsilon^2 + O(\epsilon^4)$ for $\alpha = 1, 2, 3$. Thus our $\epsilon \neq 0$ equation is

$$(q^2 - q_1^2 - \epsilon^2 r_1^2)(q^2 - q_2^2 - \epsilon^2 r_2^2) \times (q^2 - q_3^2 - \epsilon^2 r_3^2) = 0 + O(\epsilon^4) \tag{A3}$$

which expands to

$$\begin{aligned} &q^6 - q^4[(q_1^2 + q_2^2 + q_3^2) + \epsilon^2(r_1^2 + r_2^2 + r_3^2)] \\ &+ q^2[(q_1^2 q_2^2 + q_1^2 q_3^2 + q_2^2 q_3^2) \\ &+ \epsilon^2(q_2^2 r_3^2 + q_3^2 r_2^2 + q_1^2 r_3^2 + q_3^2 r_1^2 \\ &+ q_1^2 r_2^2 + q_2^2 r_1^2)] - [(q_1^2 q_2^2 q_3^2) \\ &+ \epsilon^2(r_1^2 q_2^2 q_3^2 + r_2^2 q_1^2 q_3^2 + r_3^2 q_1^2 q_2^2)] + O(\epsilon^4) = 0 \end{aligned} \tag{A4}$$

Now we can equate the $O(\epsilon^2)$ terms in (A4) with those in the original equation, (14), to get three equations in the three unknowns r_1^2, r_2^2, and r_3^2:

$$r_1^2 + r_2^2 + r_3^2 = 0 \tag{A5}$$

$$(q_2^2 + q_3^2) r_1^2 + (q_1^2 + q_3^2) r_2^2 + (q_1^2 + q_2^2) r_3^2 = -\alpha_2^2/2 \equiv s_2 \tag{A6}$$

$$q_2^2 q_3^2 r_1^2 + q_1^2 q_3^2 r_2^2 + q_1^2 q_2^2 r_3^2 = (-\alpha_2^2/2)(\alpha_1 + k_i^2) \equiv s_3 \tag{A7}$$

Solving the three linear algebraic equations for the r's is simple. We will only give the solution for r_2^2, since that is the case of interest in the paper:

$$r_2^2 = \frac{(s_2 q_2^2 - s_3^2)}{(q_1^2 - q_2^2)(q_2^2 - q_3^2)} = \frac{-\alpha_2^2}{4k_i[2(\alpha_1)^{1/2} - k_i]} \tag{A8}$$

Thus we have found the root

$$q^2 = [(\alpha_1)^{1/2} - k_i]^2 - \alpha_2^2 \epsilon^2 / 4[2(\alpha_1)^{1/2} - k_i] \tag{A9}$$

This root can give an imaginary q_2 so that growing and decaying solution exists, provided $(\alpha_1)^{1/2} \approx k_i$ so that $q^2 < 0$. In that case we may write $q = i\kappa$ and find

$$\kappa = \pm(\alpha_1)^{1/2}\left[\left(\frac{\alpha_2}{\alpha_1}\right)^2 \frac{\epsilon^2}{4} - \left(\frac{(\alpha_1)^{1/2}}{k_i} - 1\right)^2\right]^{1/2} \tag{A10}$$

We choose the positive rather than the negative root in this case because of the arguments in Appendix B and section 4.

APPENDIX B: PROOF OF INSTABILITY OF THE X MODE

To verify that the X mode is indeed unstable for $\omega \gtrsim \omega_{ce} + k_z v_b$, we use the criterion of *Sturrock* [1958], which was further amplified by *Briggs* [1964]. Consider the plane electromagnetic wave $\sim\exp[i(\mathbf{k} \bullet \mathbf{r} - \omega t)]$ produced by a steady state source: $\omega = \omega_0 =$ real. Then the wave is evanescent if Im $k(\omega_0) > 0$ and amplifying if Im $k(\omega_0) < 0$. Now let the source grow in time: $\omega = \omega_0 + i\gamma$ with $\gamma > 0$. If we let the source grow infinitely fast ($\gamma \rightarrow \infty$), then all waves produced by the source must be evanescent, since none can grow spatially as fast as the source grows temporally. Thus Im $k(\omega_0 + i\gamma) > 0$ in all cases as $\gamma \rightarrow \infty$. Therefore for a wave to be amplifying, Im $k(\omega)$ must change sign as we substitute $\omega = \omega_0$ by $\omega = \omega_0 + i\gamma$ with $\gamma \rightarrow \infty$.

For the wave solutions in this paper it is sufficient to consider the case $p = 1$, the fastest (or decaying) mode, and the case $(\alpha_1)^{1/2} \equiv k_i$, the condition for maximum growth (or decay) of this mode. In that case we have from Appendix A (also (28)):

$$\text{Im } k(\omega) = \pm\kappa_1 = \pm\frac{\alpha_2 \epsilon}{2(\alpha_1)^{1/2}} \tag{B1}$$

If we take the dielectric tensor components from (6), then take the limit for $\omega = \omega_0 + i\gamma$ as $\gamma \rightarrow \infty$, and retain only the lowest-order term in $(1/\gamma)$ for both the real and imaginary parts, we find

$$K_{\perp 0} \rightarrow 1 + 2i\left(\frac{\Omega}{\gamma}\right)^3 \tag{B2}$$

$$K_{\parallel 0} \rightarrow 1 + 2i\left(\frac{\Omega}{\gamma}\right)^3 \tag{B3}$$

$$K_{H0} \rightarrow -2\left(\frac{\omega_{ce}}{\gamma}\right)\left(\frac{\Omega}{\gamma}\right)^3\left(1 + i\frac{\omega_0}{\gamma}\right) \tag{B4}$$

where $\Omega^3 \equiv [\omega_{pe}^2 \omega_0 + \omega_b^2(\omega_0 - k_z v_b)]$. Substituting these into the alphas and again taking the limit gives to lowest order

$$\alpha_1 \rightarrow \left(\frac{\omega}{c}\right)^2\left[(1 - n_z^2) + 2i\left(\frac{\Omega}{\gamma}\right)^2\right] \tag{B5}$$

$$\alpha_2 \rightarrow -\left(\frac{\omega}{c}\right)^2\left(\frac{\omega_{pe}^2}{\gamma^2}\right)\left(1 + \frac{2i\omega_0}{\gamma}\right) \tag{B6}$$

Substituting this in (B1), replacing ω, and again expanding to lowest order, we find

$$\text{Im } k(\omega_0 + i\gamma) = \mp\frac{\omega_{pe}^2}{c\gamma} \tag{B7}$$

We may compare (B1) and (B7) for the two cases found in section 3:

1. $\omega \gtrsim \omega$. Then $\alpha_2 < 0$ and $\alpha_1 > 0$, so

$$\begin{aligned} &\text{Im } k(\omega_0) = \mp|\kappa_1| \\ &\text{Im } k(\omega_0 + i\gamma) = \mp\frac{\omega_{pe}^2}{c\gamma} \qquad \gamma \rightarrow \infty \end{aligned} \tag{B8}$$

Thus Im k does not change signs when a large imaginary part is added to ω, and the only causal solution in this case is evanescent.

2. $\omega_{ce} \lesssim \omega_{ce} + k_z v_b$. Then $\alpha_1, \alpha_2 > 0$ and

$$\begin{aligned} &\text{Im } k(\omega_0) = \pm|\kappa_1| \\ &\text{Im } k(\omega_0 + i\gamma) = \mp\frac{\omega_{pe}^2}{c\gamma} \qquad \gamma \rightarrow \infty \end{aligned} \tag{B9}$$

This Im k changes sign when a large imaginary part is added to ω, and the causal solution is amplifying. This case, then, gives us the electromagnetic instability needed to produce auroral kilometric radiation.

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REFERENCES

Benson, R. F., Source mechanism for terrestrial kilometric radiation, *Geophys. Res. Lett., 2,* 52, 1975.

Benson, R. F., and W. Calvert, Isis I observations at the source of auroral kilometric radiation, *Geophys. Res. Lett., 6,* 479, 1979.

Briggs, R., *Electron Stream Interactions With Plasmas,* MIT Press, Cambridge, Mass., 1964.

Dunckel, N., B. Ficklin, L. Rorden, and R. A. Helliwell, Low-frequency noise observed in the distant magnetosphere with Ogo 1, *J. Geophys. Res., 75,* 1854, 1970.

Frank, L., and K. Ackerson, Observations of charged particle precipitation in the auroral zone, *J. Geophys. Res., 76,* 3612, 1971.

Gurnett, D. A., The earth as a radio source: Terrestrial kilometric radiation, *J. Geophys. Res., 79,* 4227, 1974.

Gurnett, D. A., and J. L. Green, On the polarization and origin of the auroral kilometric radiation, *J. Geophys. Res., 83,* 697, 1978.

Kaiser, M., J. Alexander, A. Riddle, J. Pearce, and J. Warwick, Direct measurements by Voyager 1 and 2 of the polarization of terrestrial kilometric radiation, *Geophys. Res. Lett., 5,* 857, 1978.

Lin, A. T., P. K. Kaw, and J. M. Dawson, A possible plasma laser, *Phys. Fluids, 8,* 2618, 1973.

Matthews, D., M. Pongratz, and K. Papadopoulos, Nonlinear production of suprathermal tails in auroral electrons, *J. Geophys. Res., 81,* 123, 1976.

Melrose, D. B., An interpretation of Jupiter's decametric radiation and the terrestrial kilometric radiation as direct amplified gyroemission, *Astrophys. J., 207,* 651, 1976.

Oya, H., Origin of Jovian decametric wave emissions—Conversion from the electron cyclotron plasma wave to the *O* mode electromagnetic wave, *Planet. Space Sci., 22,* 687, 1974.

Palmadesso, P., T. P. Coffey, S. Ossakow, and K. Papadopoulos, Generation of terrestrial radiation by a beam-driven electromagnetic instability, *J. Geophys. Res., 81,* 1762, 1976.

Roux, A., and R. Pellat, Coherent generation of the auroral kilometric radiation by nonlinear beatings between electrostatic waves, *J. Geophys. Res., 84,* 5189, 1979.

Sturrock, P., Kinematics of growing waves, *Phys. Rev., 112,* 1488, 1958.

Temerin, M., P. M. Kintner, and F. S. Mozer, The coherence of electrostatic ion cyclotron waves (abstract), *Eos Trans. AGU, 59,* 1155, 1978.

Whittaker, E., and G. Watson, *A Course of Modern Analysis,* 4th ed., pp. 413–414, Cambridge University Press, New York, 1927.

Wu, C. S., and L. C. Lee, A theory of terrestrial kilometric radiation, *Astrophys. J., 230,* 621, 1979.

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