Precipitation of Energetic Protons from the Radiation Belts using Lower Hybrid Waves

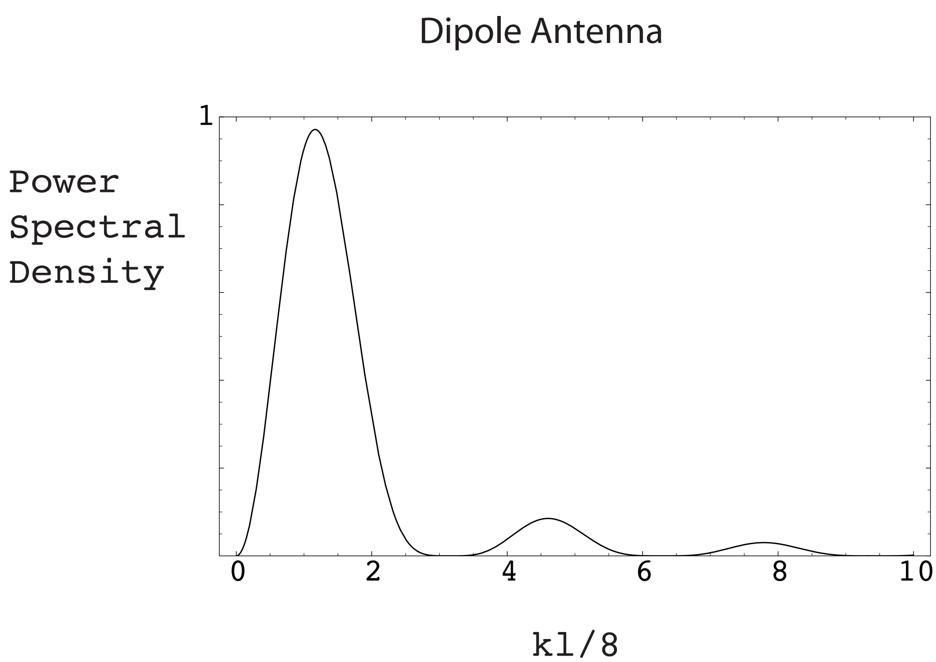
- Lower hybrid waves are quasi-electrostatic whistler mode waves whose wave normal direction is very close to the whistler mode resonance cone.

- The refractive index of ELF/VLF whistler mode waves in the radiation belts propagating along the Earth's magnetic field $\mathbf{B_o}$ generally lies in the range 10 -100.

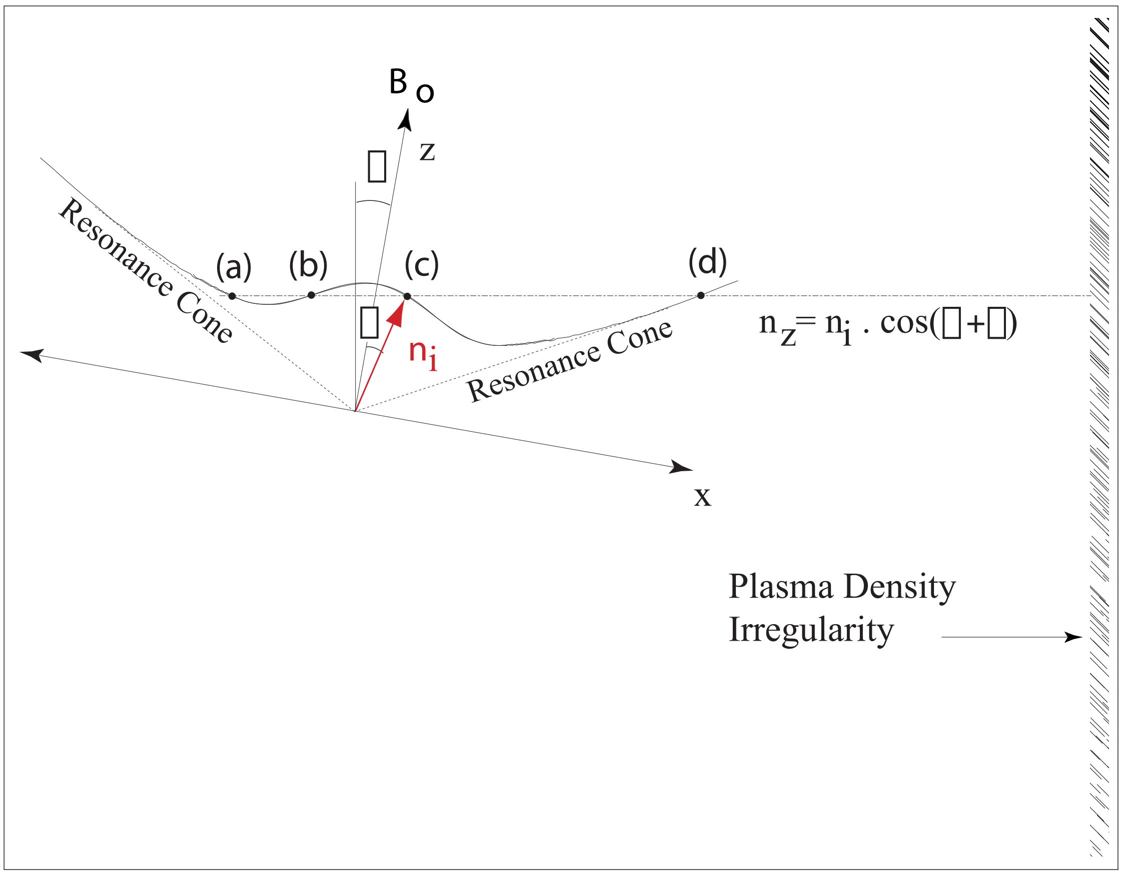
- The refractive index of ELF/VLF lower hybrid waves in the radiation belts generally lies in the range $10^3 - 10^4$.

- The **E** field of the lower hybrid waves is much larger than, and the **B** field of the lower hybrid waves is much smaller than, the cooresponding fields of whistler mode waves propagating along $\mathbf{B_o}$.

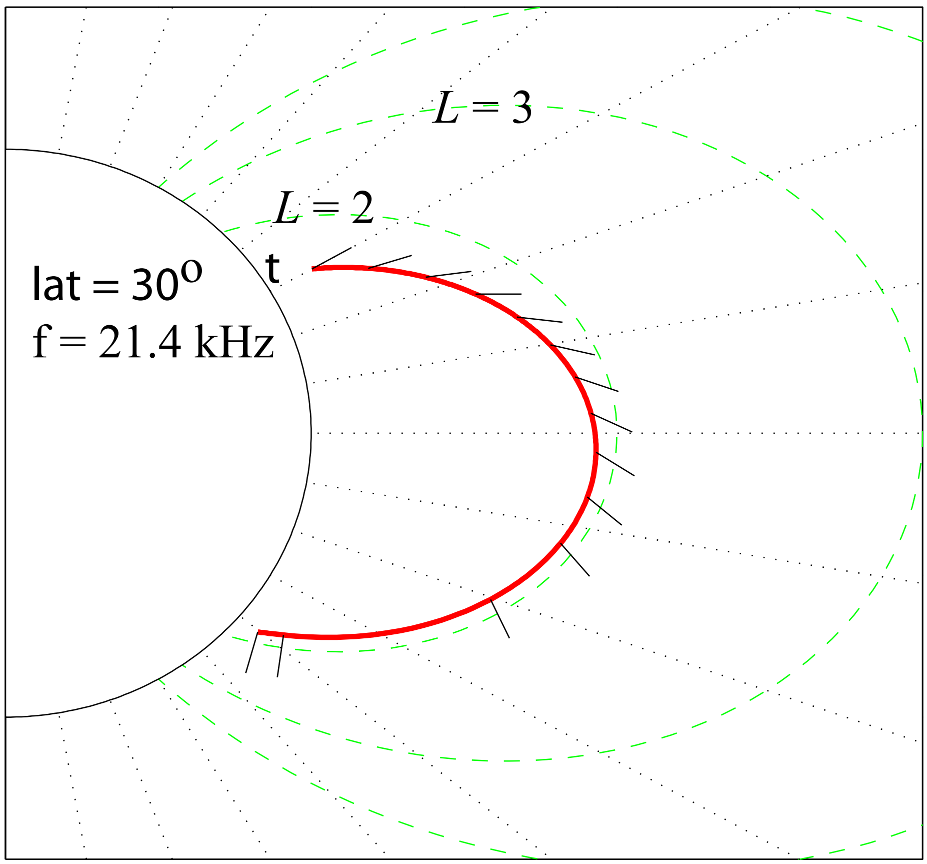
- Lower hybrid waves can be generated by many different means:
    1. ELF/VLF transmitters
    2. Linear mode coupling
    3. Propagation transformation
    4. Plasma instabilities



Power spectral density as a function of wave number k. The dipole antenna length is equal to l. The antenna preferentially radiates whistler mode waves whose wave length is approximately equal to the antenna length.



An electromagnetic whistler mode wave reflecting from a plasma density irregularity and exciting lower hybrid waves.



L = 3
L = 2
t
lat = 30o
f = 21.4 kHz

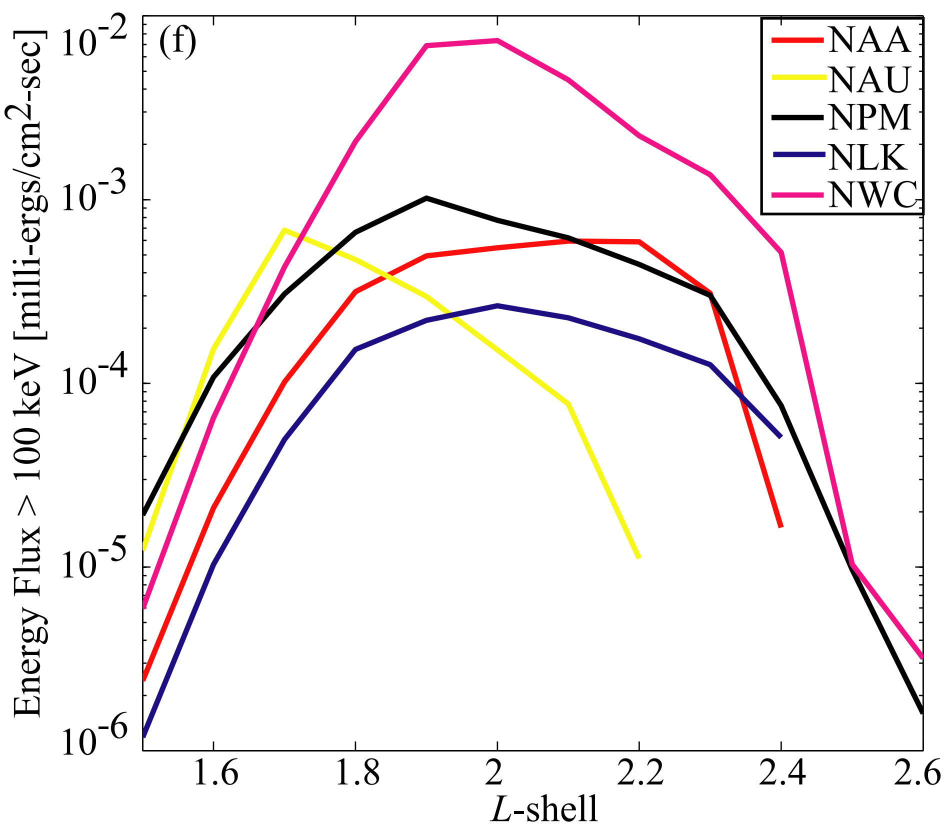
Proton Resonance Interactions with Lower Hybrid Waves

- We are currently modifying our WIPP (Wave Induced Particle Precipitation) code to include energetic radiation belt protons.

- The WIPP code is a deterministic code which presently uses the Lorentz force law to determine the changes in energetic electon pitch angle and energy which are caused by known whistler mode waves in the radiation belts:

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m}\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right]$$

- An example of the WIPP code output for energetic electrons is shown on the next page.



(f)
NAA
NAU
NPM
NLK
NWC
Energy Flux > 100 keV [milli-ergs/cm2-sec]
10-2
10-3
10-4
10-5
10-6
1.6
1.8
2
2.2
2.4
2.6
L-shell

- From inspection of the Lorentz force law, $\frac{d\mathbf{v}}{dt} = \frac{e}{m}[\mathbf{E} + \mathbf{v} \text{ x } \mathbf{B}]$, it may appear that one only needs to change m_e to m_p in order to use the WIPP code to find the proton precipitation. However, this is not the case.

- The largest pitch angle changes occur during gyroresonance and Landau resonance interactions:

$$v_z = (n\,\omega_c - \omega)/k_z$$

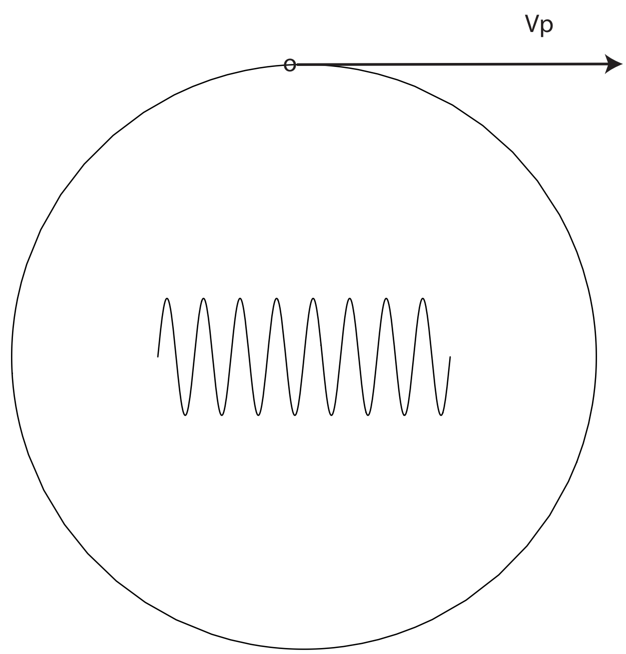
where ω_c is the particle gyrofrequency, k_z is the component of the wave vector $\mathbf{k}$ parallel to $\mathbf{B_o}$, where n is an integer, and where setting $n = 0$ yields the parallel Landau resonance.

- The fundamental (n = 1) gyro-resonance interaction between a 100 keV electron and a 5 kHz whistler mode wave near the magnetic equatorial plane takes place over a distance of $\simeq$ 600 km.

- Over this same distance a 100 keV proton in gyro-resonace with the same wave would experience approximately 20 resonances.

- This means that the step size in the WIPP code needs to be reduced by a factor of $\simeq$ 20.

- Since the gyro-radius of the protons is so large, we must now include radial gradients in $\mathbf{B_o}$ and N_o.



The gyro - motion of the protons can be neglected when:

$$\square \ll r_g$$

- If we can assume that the protons are unmagnetized, the Lorentz force law has the simple form:

$$\frac{d\mathbf{v}}{dt} = \frac{e}{m_p}\mathbf{E_o}\ e^{i(\omega t - \mathbf{k}\cdot\mathbf{r})}$$

where $\mathbf{E_o}$ is the amplitude of the lower hybrid wave.

- A resonance will occur when the phase of $\mathbf{E}$ is constant:

$$\omega = \mathbf{k} \cdot \mathbf{v} = \mathbf{k}_\perp \cdot \mathbf{v}_\perp + k_z v_z \simeq \mathbf{k}_\perp \cdot \mathbf{v}_\perp$$

- Thus for the case of unmagnetized protons there is only a single resonance, the transverse Landau resonance.

Distribution of k values

- If we assume that the k vectors of the lower-hybrid waves are uniformly distributed around $\mathbf{B_o}$, the pitch angle diffusion coefficient can be written: $D_\alpha = D_\perp/v_\perp^2$, where [*Retterer et al.*, 1980]:

$$D_\perp = (e/m)^2 \int \frac{I(\omega)\,\omega^2 d\omega}{8\pi^2 v_\perp^2} \int_{k_o}^{\infty} \frac{S(k_\perp) dk_\perp}{k_\perp \sqrt{k_\perp^2 v_\perp^2 - \omega^2}}$$

where $I(\omega)$ is the spectral intensity of the LH waves in the frequency domain, $S(k_\perp)$ is the normalized spectral density of the LH waves in the k domain, $v_\perp$ is the proton perpendicular velocity, and $k_o = \omega/v_\perp$.

- We assume that the LH wave energy is uniformly distributed over wavelengths from 20 m to 200 m, and that the illumination region extends to 30° latitude on both sides of the magnetic equator.

- We also assume the average **E** field amplitude of the LH waves to be 1 mV/m.

- In this case we find for a 50 keV proton with an initial pitch angle α of 45°: $\Delta\alpha_{\text{total}} \simeq \sqrt{D_\alpha T} \simeq 45°$, where T =6 days.

Use of existing VLF Transmitters

- Ray tracing results suggest that $\simeq 20$ kHz signals from ground based VLF transmitters can transform into lower hybrid waves as the signals propagate into the conjugate hemisphere.

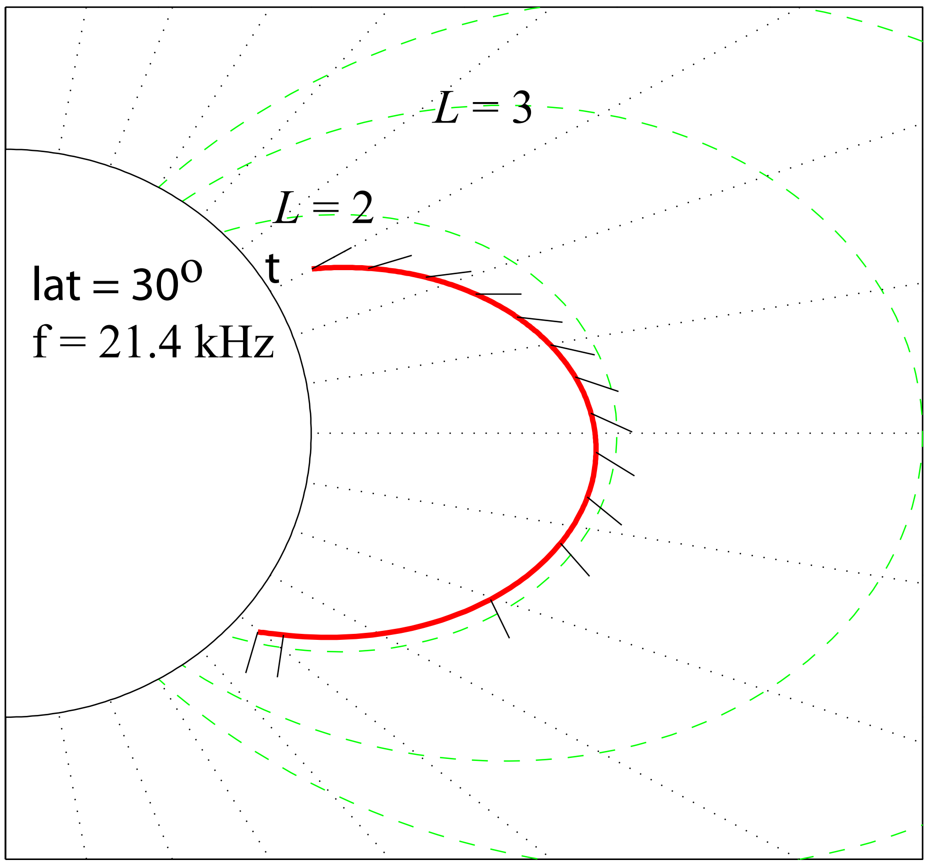
- However, the transformation is completed at low altitudes where energetic proton pitch angle scattering is very inefficient.

- This situation could be significantly improved if a chemical release took place in the ionosphere above the transmitter, creating plasma density irregularities.

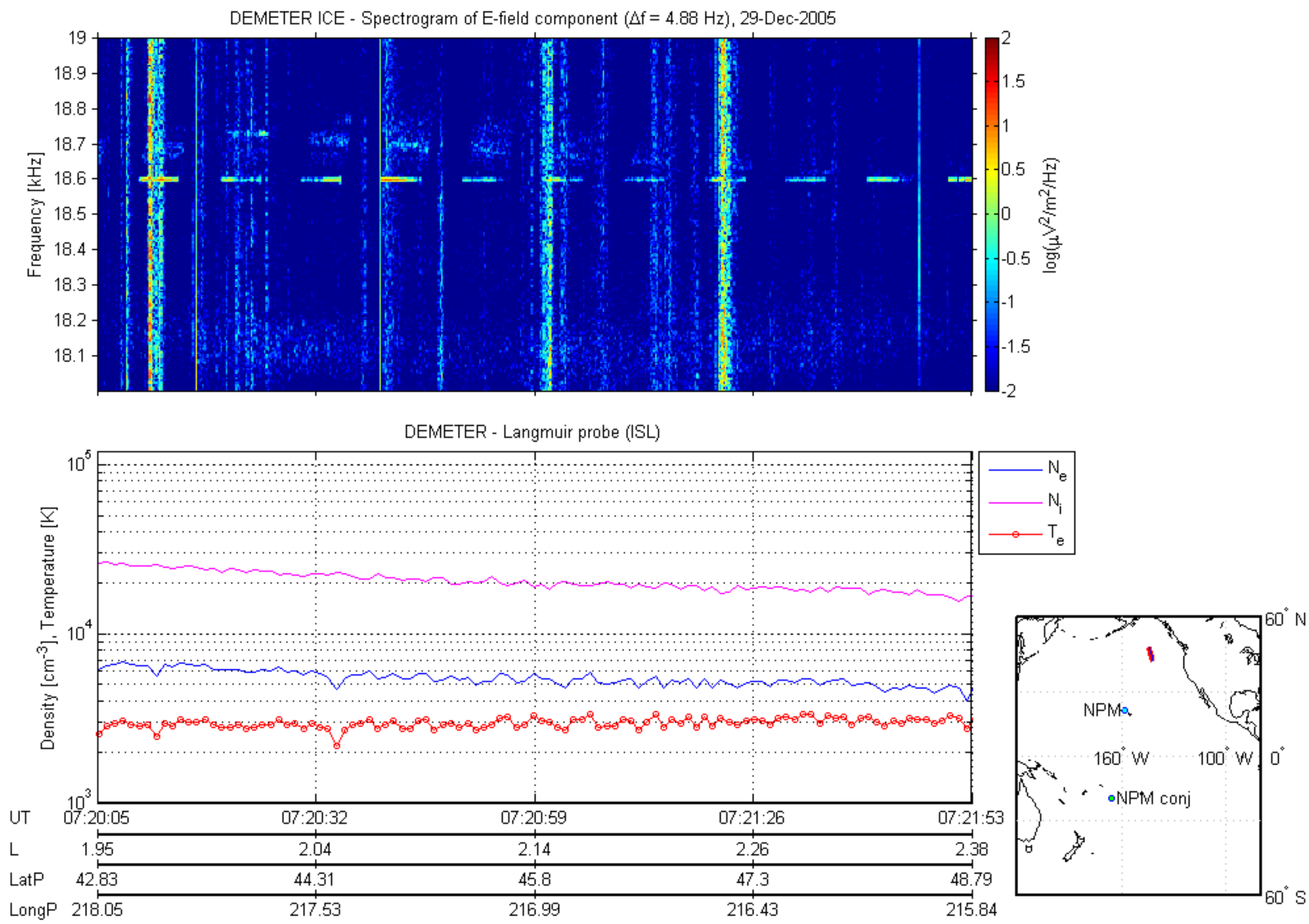
- The transmitter signals propagating upward within these irregularities would transform into lower hybrid waves through linear mode coupling.

- The lower hybrid waves would then propagate upward along $\mathbf{B_o}$, filling the plasmasphere with waves which can change the pitch angles of energetic protons at every point along their trajectory.

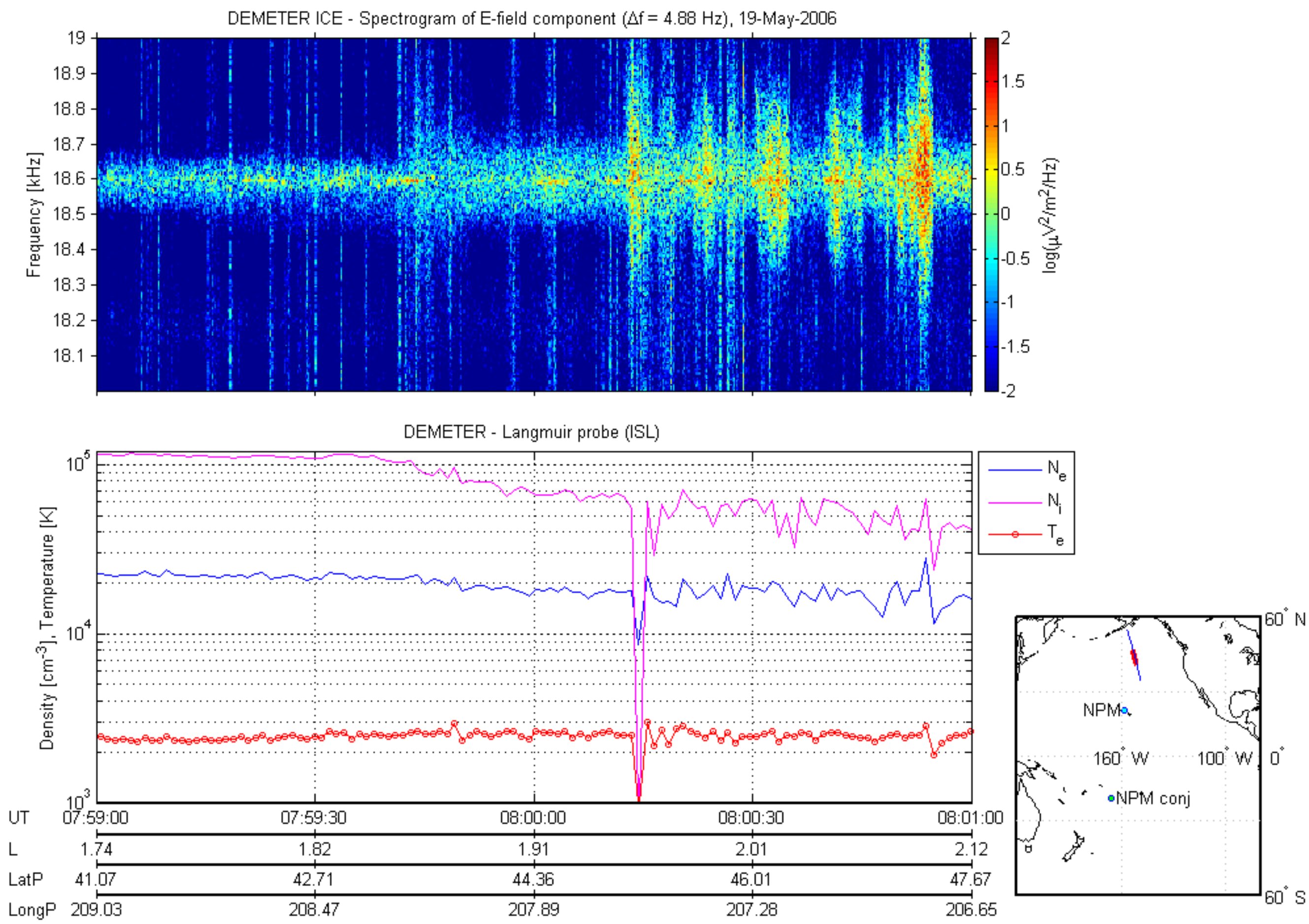
- Recent DEMETER data suggests that this technique might be very successful.

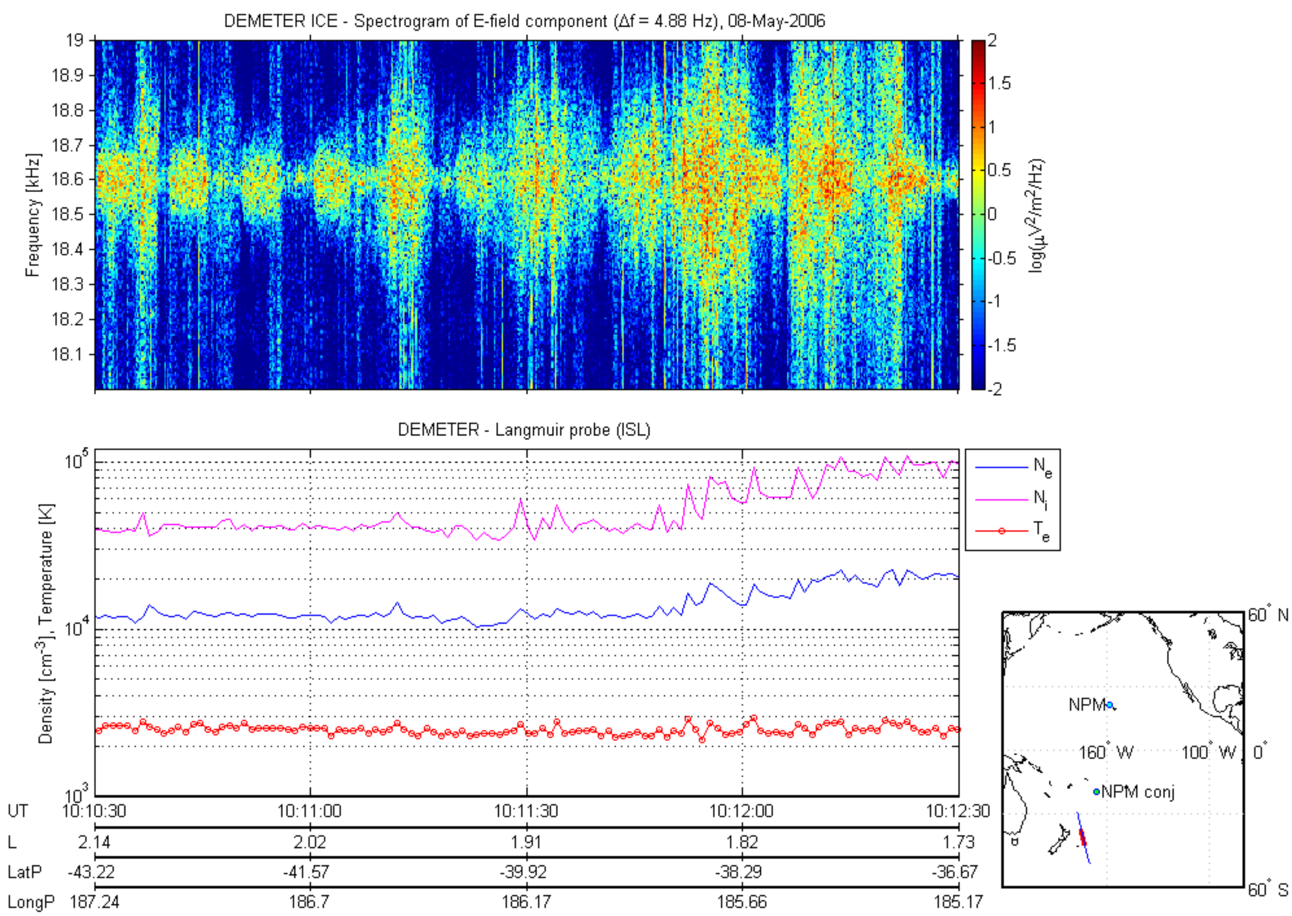


L = 3
L = 2
t
lat = 30o
f = 21.4 kHz



DEMETER ICE - Spectrogram of E-field component (Δf = 4.88 Hz), 29-Dec-2005
Frequency [kHz]
log(μV²/m²/Hz)
DEMETER - Langmuir probe (ISL)
Density [cm⁻³], Temperature [K]
Ne
Ni
Te
UT 07:20:05 07:20:32 07:20:59 07:21:26 07:21:53
L 1.95 2.04 2.14 2.26 2.38
LatP 42.83 44.31 45.8 47.3 48.79
LongP 218.05 217.53 216.99 216.43 215.84
NPM
NPM conj
160° W
100° W
0°
60° N
60° S


DEMETER ICE - Spectrogram of E-field component (Δf = 4.88 Hz), 19-May-2006
Frequency [kHz]
log(μV²/m²/Hz)
DEMETER - Langmuir probe (ISL)
Density [cm⁻³], Temperature [K]
N_e
N_i
T_e
UT 07:59:00 07:59:30 08:00:00 08:00:30 08:01:00
L 1.74 1.82 1.91 2.01 2.12
LatP 41.07 42.71 44.36 46.01 47.67
LongP 209.03 208.47 207.89 207.28 206.65
NPM
NPM conj
160° W
100° W
0°
60° N
60° S



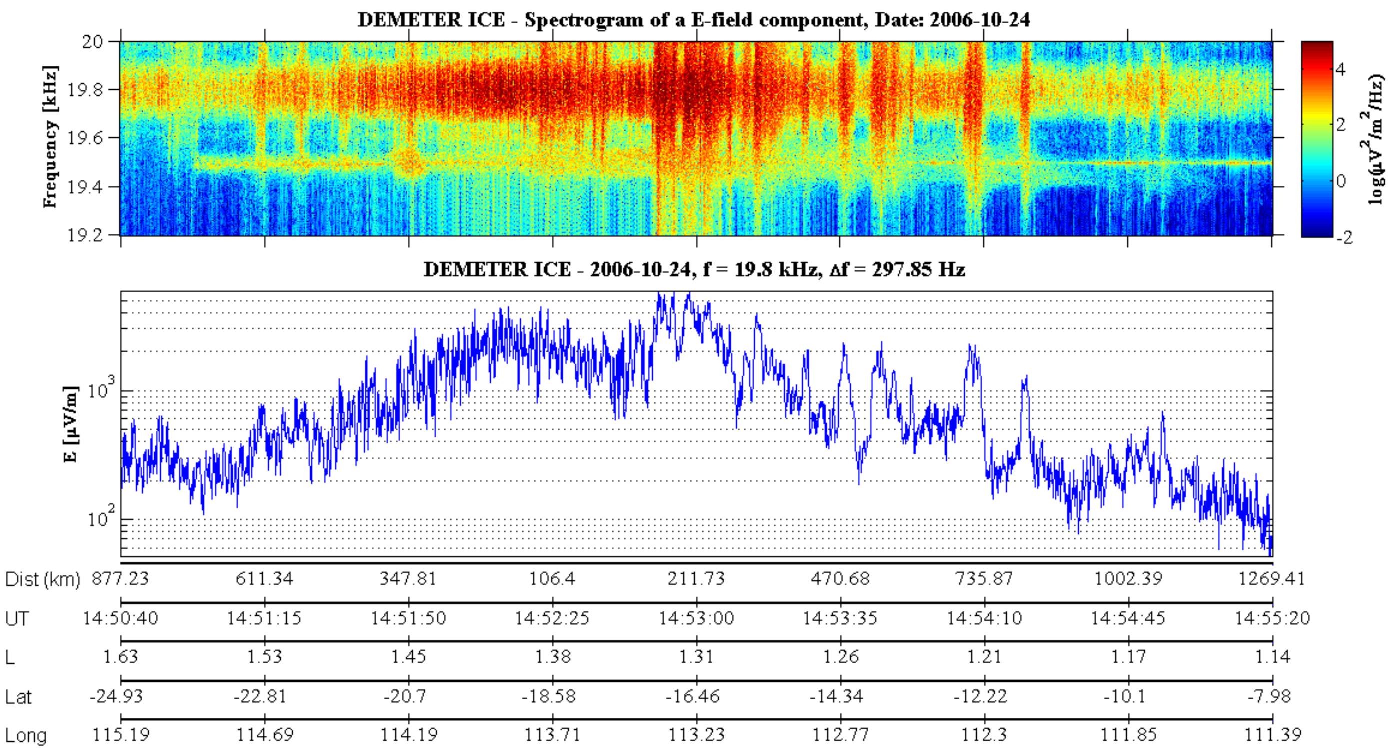
DEMETER ICE - Spectrogram of E-field component (Δf = 4.88 Hz), 08-May-2006
Frequency [kHz]
19
18.9
18.8
18.7
18.6
18.5
18.4
18.3
18.2
18.1
log(μV²/m²/Hz)
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
DEMETER - Langmuir probe (ISL)
Density [cm⁻³], Temperature [K]
10⁵
10⁴
10³
Nₑ
Nᵢ
Tₑ
UT 10:10:30 10:11:00 10:11:30 10:12:00 10:12:30
L 2.14 2.02 1.91 1.82 1.73
LatP -43.22 -41.57 -39.92 -38.29 -36.67
LongP 187.24 186.7 186.17 185.66 185.17
60° N
NPM
160° W
100° W
0°
NPM conj
60° S

Energetic Proton Precipitation produced by Lower Hybrid Waves

Stanford University

- We are currently modifying our WIPP (Wave Induced Particle Precipitation) code to include energetic radiation belt protons.
- The WIPP code is a deterministic code which uses the Lorentz force law to determine the changes in energetic electon pitch angle and energy which are caused by known whistler mode waves in the radiation belts.
- We are also developing a new code which determines proton pitch angle and energy changes in a lower hybrid wave field with a wide spectrum of wave vectors **k**.
- We will treat a number of cases:

1. Lower hybrid wave fields produced in the radiation belts by a space based ELF/VLF transmitter .
2. Lower hybrid wave fields produced in the radiation belts by existing ground based VLF transmitters.
3. Lower hybrid wave fields produced in the radiation belts by a ground based VLF transmitter operating in the 5 - 10 kHz frequency range.



DEMETER ICE - Spectrogram of a E-field component, Date: 2006-10-24
Frequency [kHz]
20
19.8
19.6
19.4
19.2
log(μV²/m²/Hz)
4
2
0
-2
DEMETER ICE - 2006-10-24, f = 19.8 kHz, Δf = 297.85 Hz
E [μV/m]
10³
10²
Dist (km) 877.23 611.34 347.81 106.4 211.73 470.68 735.87 1002.39 1269.41
UT 14:50:40 14:51:15 14:51:50 14:52:25 14:53:00 14:53:35 14:54:10 14:54:45 14:55:20
L 1.63 1.53 1.45 1.38 1.31 1.26 1.21 1.17 1.14
Lat -24.93 -22.81 -20.7 -18.58 -16.46 -14.34 -12.22 -10.1 -7.98
Long 115.19 114.69 114.19 113.71 113.23 112.77 112.3 111.85 111.39