Excitation of quasi-electrostatic whistler mode waves by electromagnetic whistler mode waves in the topside ionosphere.

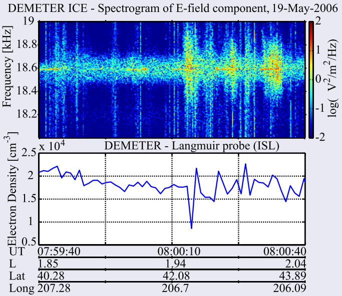
F. Foust, U.S. Inan, and T.F. Bell

December 11, 2010

Density irregularities and spectral broadening

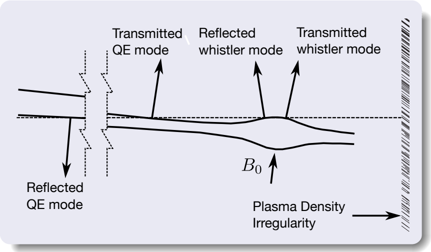
- Demeter passes over VLF transmitters often show spectral broadening of narrowband VLF pulses.
- Believed to be caused by QE whistler mode waves excited by transmitter signals as they propagate through small-scale plasma density irregularities.
- The very fine-scale structure of the irregularities is *unknown* (insufficient sampling rate).
- The input wave energy loss due to the excitation of QE waves may be related to the apparent 20 dB wave power deficit reported by *Starks et al.* [2008].

Demeter pass



Mode conversion in an anisotropic medium

- Any inhomogeneities can excite additional *modes*.
- Snell's law - tangential component of the refractive is conserved.
- The presence of the resonance cone makes the process a lot more interesting.
  - Short-wavelength (QE) modes.
  - Very different group and phase velocity directions.
  - Strong angle dependence.



Questions

Questions we wish to answer:

- What conditions are necessary for strong excitation of QE modes?
- How much power can be converted into QE modes through linear scattering alone?
- How much power is lost to collisions and damping?
- What is the actual fine-scale structure of the irregularities (need better data!).
- How do irregularities above powerful transmitters develop?

Questions

Questions we wish to answer:

- What conditions are necessary for strong excitation of QE modes?
- How much power can be converted into QE modes through linear scattering alone?
- How much power is lost to collisions and damping?
- What is the actual fine-scale structure of the irregularities (need better data!).
- How do irregularities above powerful transmitters develop?

Questions

Questions we wish to answer:

- What conditions are necessary for strong excitation of QE modes?
- How much power can be converted into QE modes through linear scattering alone?
- How much power is lost to collisions and damping?
- What is the actual fine-scale structure of the irregularities (need better data!).
- How do irregularities above powerful transmitters develop?

Questions

Questions we wish to answer:

- What conditions are necessary for strong excitation of QE modes?
- How much power can be converted into QE modes through linear scattering alone?
- How much power is lost to collisions and damping?
- What is the actual fine-scale structure of the irregularities (need better data!).
- How do irregularities above powerful transmitters develop?

Questions

Questions we wish to answer:

- What conditions are necessary for strong excitation of QE modes?
- How much power can be converted into QE modes through linear scattering alone?
- How much power is lost to collisions and damping?
- What is the actual fine-scale structure of the irregularities (need better data!).
- How do irregularities above powerful transmitters develop?

Questions

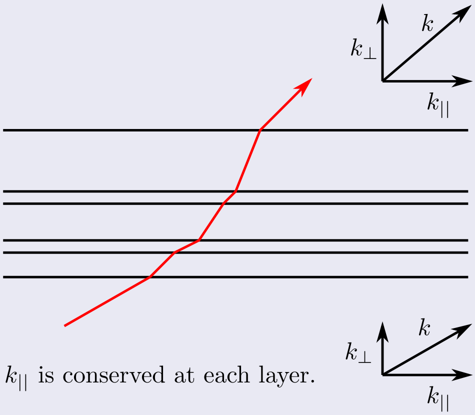
Questions we wish to answer:

- What conditions are necessary for strong excitation of QE modes?
- How much power can be converted into QE modes through linear scattering alone?
- How much power is lost to collisions and damping?
- What is the actual fine-scale structure of the irregularities (need better data!).
- How do irregularities above powerful transmitters develop?

Full-wave method

- Snell's law: $k_{||}$ is conserved.
- Reflection and transmission coefficients: yield relationship between mode amplitudes at each layer.
- System is solved to yield all mode amplitudes.
- Modes are summed.
- Repeat for all possible k in the system.
- Documented in *Lehtinen and Inan* [2008].

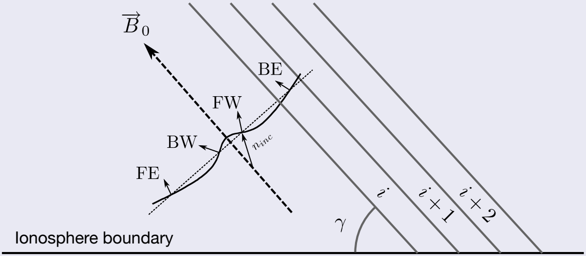
Layered media approximation



$k_{||}$ is conserved at each layer.

Problem geometry

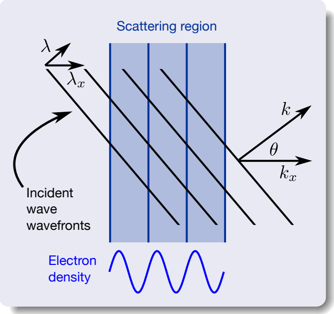
Scattering and mode conversion

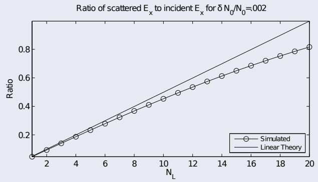


- By Snell's law, the component of $\overrightarrow{k}$ parallel to the layers is conserved.
- Four possible scattered modes: forward and backward whistler (FW and BW) and forward and backward quasielectrostatic (FE and BE).
- Periodic layers can preferentially excite one mode.

Preferential mode excitation

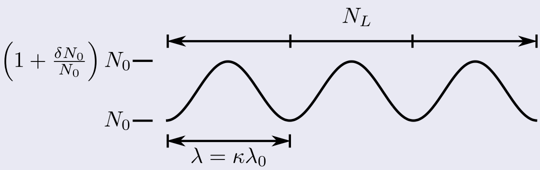
- Right: illustration of Bragg-like scattering.
- Consider the case of a single Fourier mode of the plasma density distribution.
- Perturbation theory predicts a linear increase in QE wave power when the irregularity period matches the target QE wave.
- In reality the process quickly saturates.
- Perturbation theory fails at even moderate perturbation amplitudes (right bottom).





Perturbation

Perturbation function

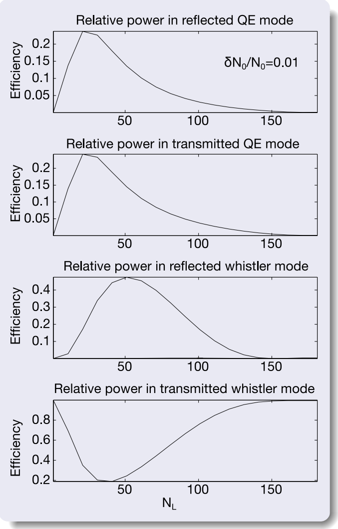


- N_L - Length of scattering region (periods)
- δN_0 - density perturbation magnitude
- λ_0 - nominal spatial wavelength of the excited QE mode
- κ - variation about the nominal irregularity period λ_0
- λ - spatial wavelength of the irregularities

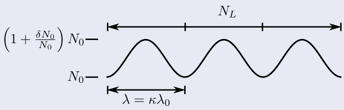
Saturation

Saturation is caused by two competing processes:

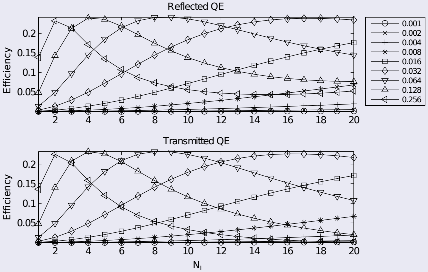
- **conversion to the QE mode** by linear scattering, and
- **conversion back into the whistler mode**, by exactly the same process. This is evident from the plot on the right.
- The initial power loss of the input wave is 7 dB after the wave has propagated 1km across the irregularities.
- If the QE waves were heavily damped, the input wave would continue to lose power.



Ideal matching: $\kappa = 1.0$

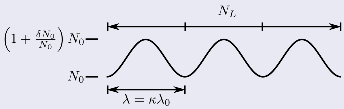


$\kappa = 1.0$ (ideal scatterer)

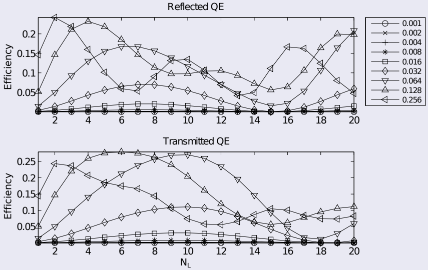


- Figure of merit is the ratio of power converted into the QE mode.
- Nominal scattering period λ_0 is about 30 meters (perpendicular to B_0.
- The QE mode propagates approximately 30 km parallel to B_0 for every 1 km perpendicular to B_0.
- General trend - increasing conversion to a maximum, then leveling out and decaying.
- Attained maximum is relatively constant.

Nonideal matching

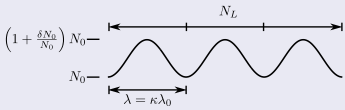


$\kappa = 0.94$ (shorter than ideal)

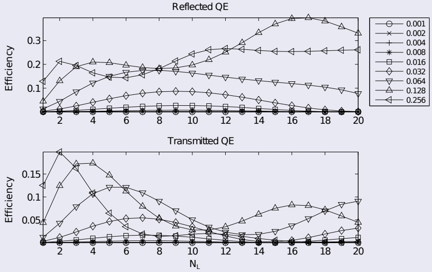


- Attained maximum in the forward mode is increased slightly (less than 5%).
- No smooth decay - energy sloshes back and forth between the QE and whistler modes.
- Loss of matching at high β slightly ameliorated by shortening the scattering period.

Nonideal matching



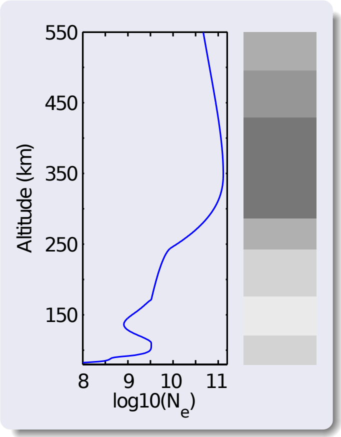
κ = 1.06 (longer than ideal)



- Attained maximum is decreased.
- No smooth decay - energy sloshes back and forth between the QE and whistler modes.
- Forward matching completely destroyed (no modes available at this scattering length).
- Relatively large backscattered mode.

Simple scattering

- Simple sinusoidal scatterers obviously aren’t realistic.
- Actual distribution is *unknown* down to the very fine scale.
- Two new ideas are needed:
  - **Irregularities with a bandwidth** - Real irregularities should have some range of length scales.
  - **Natural “coherence length”** - A coherent interaction is only valid so long as the properties of the background medium *don’t change much*.



Randomized trials - Fourier mode sums

The irregularities are constructed as a sum of M Fourier modes:

$$N(x) = N_0 + \sum_{i=1}^{M} A_i \cos(k_i x - \phi_i),$$

- $N(x)$ - the plasma density.
- N_0 - the background plasma density.
- A_i - The mode amplitude for mode i.
- k_i - The wavenumber for mode i.
- ϕ_i - Phase shift for mode i.

Procedure: Select from uniform distributions over some reasonable range. Repeat for 400000 random trials.

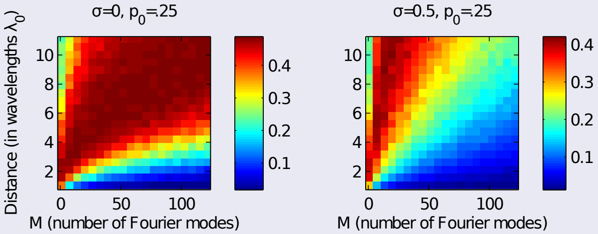
Randomized trials - Fourier mode sums

Our figure of merit is the ratio of power converted into the QE mode, α.
We plot:

$$\textbf{prob}(\alpha_{\mathrm{QE,f}} + \alpha_{\mathrm{QE,b}} \geq \alpha) \geq p_0$$

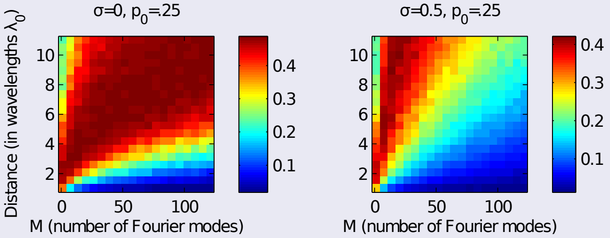
for some fixed value of p_0, e.g. $p_0 = .25$.
This is a **proxy** for relative likelihood of a given conversion efficiency over some set of parameters.

Fourier mode sums



Randomized trials - Fourier mode sums

Fourier mode sums



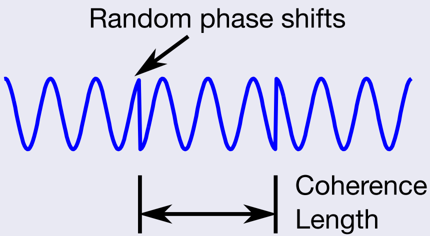
For the Fourier mode sum experiments:

- A **large number of modes** do not favor strong QE mode scattering.
- **Saturation** is still present.
- Likely QE power conversion ratios are still in the range of 50%.

Randomized trials - Random Phase Shifts

Idea: disrupt conversion *back* into the whistler mode by introducing a random phase shift between coherent segments:

Randomized coherence length

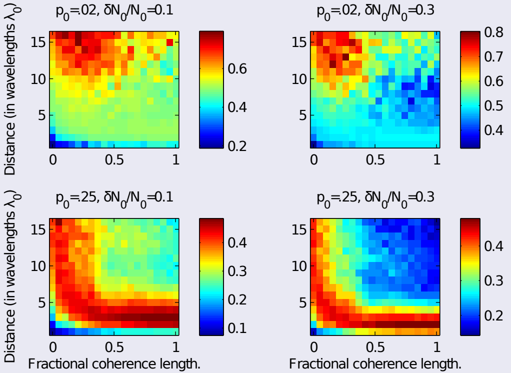


Justification: The real world is not 1D and the plasma properties change with altitude.

We conduct a similar set of trials, this time randomizing the **phase shift** and the **average length** of a coherent segment.

Randomized trials - Random Phase Shifts

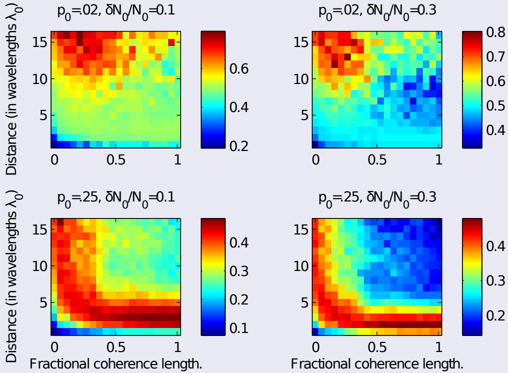
Random phase shifts



Results show the possibility of **strong scattering** into the QE mode, in excess of 90%, but at a **relatively low probability** of less than 2%.

Randomized trials - Random Phase Shifts

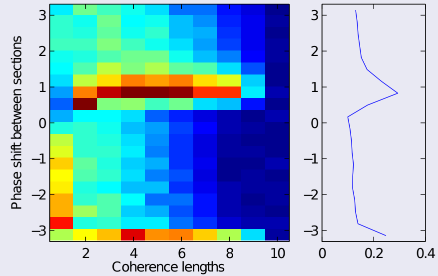
Random phase shifts



Results also hint that **partly incoherent scatter** over **long scattering regions** may be important.

Randomized trials - Random Phase Shifts

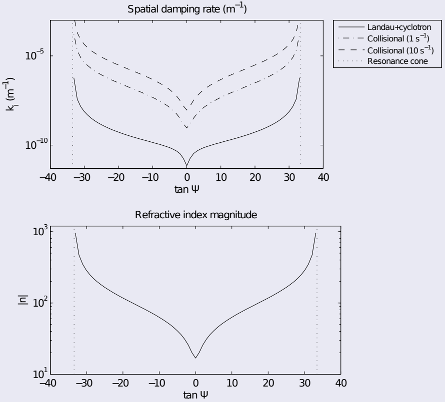
- Interesting trend when we plot the phase shifts for strong scattering configurations (right).
- Histogram shows relative phase shifts of π and $\pi/4$ favor strong scattering.
- Interesting result, but probably not practically important.



Losses

- Collision frequency = 1 and 10 s^{-1}.
- Electron distribution taken from averaged satellite observations within the plasmasphere.
- Losses not dominant unless the refractive index n is large.
- Prediction: will manifest as a sharp “cutoff” on propagation to the conjugate region.

Loss rates - damping and collsions

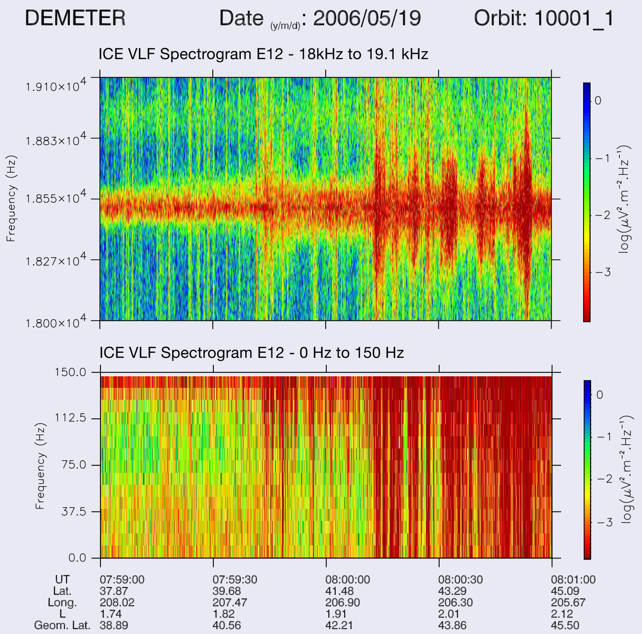


Density Measurements

Recent spacecraft plasma density measurements

Spacecraft	**Sampling rate**	**Altitude**
DEMETER	1/sec	˜700 km
ISIS 1	60/sec	570 - 3500 km
FREJA	100/sec	600 - 1760 km
AUREOL 3	1000/sec	400 - 3000 km

- AUREOL 3 observations at high latitude showed that when small scale plasma density irregularities were present, so also was a band of ELF turbulence in the 25 - 100 Hz frequency range.
- It was found that the ELF turbulence was due to quasi-electrostatic E fields associated with the small scale irregularities.
- The power spectral density of the ELF turbulence and the plasma density fluctuations were very similar, varying as $k^{-1.8}$.
- These high latitude results may not hold at low- to mid- latitudes.
- Simulations are needed to determine the relationship between small scale plasma density irregularities and ELF turbulence at low- to mid-latitudes.


DEMETER
Date (y/m/d): 2006/05/19
Orbit: 10001_1
ICE VLF Spectrogram E12 - 18kHz to 19.1 kHz
1.910×10^4
1.883×10^4
1.855×10^4
1.827×10^4
1.800×10^4
Frequency (Hz)
0
−1
−2
−3
log(μV².m⁻².Hz⁻¹)
ICE VLF Spectrogram E12 - 0 Hz to 150 Hz
150.0
112.5
75.0
37.5
0.0
Frequency (Hz)
0
−1
−2
−3
log(μV².m⁻².Hz⁻¹)
UT 07:59:00 07:59:30 08:00:00 08:00:30 08:01:00
Lat. 37.87 39.68 41.48 43.29 45.09
Long. 208.02 207.47 206.90 206.30 205.67
L 1.74 1.82 1.91 2.01 2.12
Geom. Lat. 38.89 40.56 42.21 43.86 45.50

Conclusions

- Linear mode conversion to QE modes is **consistent** with observations of spectral broadening.
- Power lost to QE modes is most probably in the range of **3-10 dB**.
- Probability of the $>$10 dB cases is rather low over the parameters investigated, **less than 2 %.**
- Short coherence lengths favor strong QE mode conversion.
- Results suggest that some power loss by QE wave excitation is **pervasive and common**, but may only account for a portion of the 20 dB deficit reported by *Starks et al.* [2008].
- The actual distribution of small scale irregularities may favor more power loss than our general model indicates.

Limitations

- Layered media approximations can never capture the full range of scattering effects in 3D (spreading loss, diffraction).
- Full 3D simulation of QE scattering remains a *very hard problem*. FDTD and other similar techniques have difficulty resolving very high wavenumbers.
- No heating or other sources of loss (aside from Landau damping and collisions) are considered.
- The parameters chosen are *reasonable* but may not adequately reflect the actual spatial gradients in the electron density. **Higher resolution** measurements are needed.