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Model of red sprite optical spectra

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Abstract. A synthetic spectrum of red sprites due to electron energization by the electric field from lightning is computed by using the electron energy spectrum obtained from a Fokker-Planck code, which includes various inelastic losses. The results are compared with observed sprite spectra. Implications to models of red sprites are presented.

1 Introduction

Observations of optical emissions at altitudes between 60–90 km associated with giant thunderstorms have been the focus of many recent ground and aircraft campaigns [*Lyons*, 1994; *Sentman et al.*, 1995; *Winckler et al.*, 1996]. While the gross phenomenology of the emissions, termed red sprites, has been known for over one year, their spectroscopic structure is only currently emerging [*Mende et al.*, 1995; *Hampton et al.*, 1996]. The objective of this letter is to present a theoretical model of the spectroscopically resolved emissions and compare them with the observations. Constraints on the electron energy spectrum and the power density locally absorbed in the emission region are derived.

2 Model Description

The model is based on energization of ionospheric electrons by fields generated by conventional lightning and applies equally well to energization by electromagnetic or quasi-static fields [*Milikh et al.*, 1995; *Rowland et al.*, 1995; *Pasko et al.*, 1995]. It capitalizes on the fact that there are two distinct timescales. A fast timescale on which a steady state electron distribution function $f(v)$ is established by balancing the electron energization rate with inelastic losses [*Tsang et al.*, 1991], and a slow radiation timescale dominated by interlevel transfer and collisional quenching. The model starts with an electric field E_ω having relevant frequencies ω below the electron cyclotron frequency Ω and the field dependent electron collision frequency $\nu(v)$ at altitudes between 60–90 km. Here E_0 is defined as $E_0 = \sqrt{\sum_\omega E_\omega^2}$. For a given value of the ambient electron-neutral collisional frequency ν_o and E_0 the asymptotic stationary state of the electron distribution function (edf) $f(v)$ is found by solving numerically the local kinetic Fokker-Planck equation [*Tsang et al.*, 1991] for altitudes such that $\cos^2\theta < \nu^2/\Omega^2$

$$\frac{\partial f(v)}{\partial t} = \frac{e^2 E_0^2}{3m}\frac{1}{v^2}\frac{\partial}{\partial v}\left(\frac{v^2\nu(v)}{\nu^2(v)+\Omega^2}\frac{\partial f}{\partial v}\right) - L(v). \quad (1)$$

Here $L(v)$ is the operator which describes the effect of the inelastic collisions, while θ is the angle between the electric and geomagnetic fields. Note that the edf's obtained for different conditions could be found in [*Tsang et al.*, 1991]. In this letter we assume for the sake of definiteness that the sprite is located at z=80 km, and solved Eq. (1) for different values of the electric field. The detailed height dependent analysis will be presented elsewhere.

Radiative deexcitation of the excited molecules produces optical flashes that superficially resemble those observed during auroras. However, unlike auroras which last for hours, and in which even forbidden transitions need to be considered, red sprites have millisecond duration, so that only N_2 transitions faster than a millisecond excited by direct electron impact or through cascades need to be retained. The emission intensity for a particular radiative transition is calculated from value of $f(v)$ found above. In the current model we have retained only the excited electronic states of N_2 shown in Fig. 1. The computation of the intensity of a radiative transition connecting the v-th and v′-th vibrational levels of the electronic states α and β is accomplished as following: We first compute the excitation rate ν_{ex}^α of the α electronic state of N_2 by electron impact $\nu_{ex}^\alpha = 4\pi N_{N_2}\int f(v)v^3\sigma_{ex}^\alpha(v)\,dv$, using the excitation cross section of the B, B', W, C, D and E states of N_2 and $N_2^+(B)$ by the electron impact from *Cartwright et al.* [1977], and *Van Zyl and Pendleton* [1995]. Figure 2

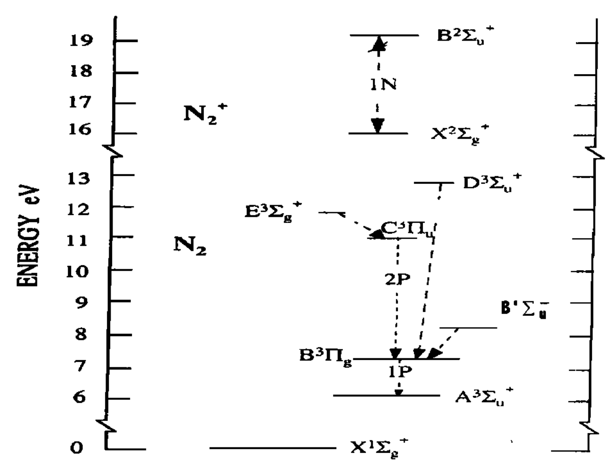

Figure 1. Energy levels diagram for the nitrogen electronic levels considered in the discussed model. The relevant radiative transitions are shown by arrows.

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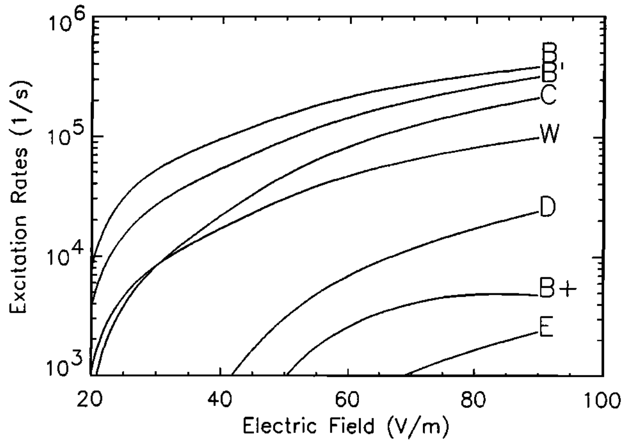

Figure 2. Excitation rate coefficients of the electronic levels of nitrogen relevant to red sprites versus the electric field amplitude at the source region located at 80 km.

shows the excitation rates for the relevant excited states of N_2 as a function of the electric field amplitude E_0. We then obtain the population of the excited states by solving the following set of stationary equations [*Cartwright*, 1978]

$$q_{ov}^{x\alpha}\nu_{ex}^{\alpha}n_e+\sum_{\beta,j}A_{vj}^{\alpha\beta}\left(n_j^{\beta}-n_v^{\alpha}\right)-k_{q,v}^{\alpha}N_m n_v^{\alpha}=0, \quad (2)$$

here n_e is the electron density, n_v^α is the number density of the v-th vibrational level of the electronic state α, $q_{ov}^{x\alpha}$ is the Franck-Condon factor which shows the transition probability to the v-th vibrational level of the α electronic state from the 0 vibrational level of the ground state X (in a cold ambient gas only the lowest vibrational level is populated), $A_{vv'}^{\alpha\beta}$ is the Einstein spontaneous transition probability, and $k_{q,v}^\alpha$ is the rate constant of collisional quenching of the v-th vibrational level of the α electronic state, and N_m is the air density.

Therefore, the first term in the right side of Eq. (2) shows the direct pumping of the v vibrational level of the α electronic state by the electron impact. While the second term shows cascade excitation and radiation losses. The last term reveals losses due to the collisional quenching. Note that the usage of the stationary equations for the population of vibrational level is justified by the fact that the radiative lifetimes of the relevant excited electronic states have to be shorter than the duration of an electromagnetic pulse from lightning in order to be effectively pumped. Therefore, a stationary distribution of n_v^α is established during the pulse. From Eq. (2) we obtain now the population of the $B^3\Pi g$ state related to the N_2(1P) band

$$n_v^B=\nu_{ex}^B n_e F_{1,v}^B+\sum_{\alpha}\nu_{ex}^{\alpha}n_e F_{2,v}^{\alpha}\,;\alpha=B,B',C,D;$$

$$F_{1,v}^{\alpha}=\frac{q_{ov}^{x\alpha}\tau_v^{\alpha}}{1+\tau_v^{\alpha}k_{qv}^{\alpha}N_m}, \quad (3)$$

$$F_{2,v}^{\alpha}=\frac{\tau_v^B}{1+\tau_v^B k_{qv}^B N_m}\sum_i F_{1,i}^{\alpha}A_{iv}^{\alpha B},$$

here $\tau_v^\alpha = 1/\sum_{j,\beta} A_{vj}^{\alpha\beta}$ is the lifetime of the v-th vibrational level of the electronic state α. Moreover the quenching factor $\left(1+\tau_v^\alpha k_{qv}^\alpha N_m\right)^{-1}$ which appears in the coefficients $F_{1,v}^B$, $F_{2,v}^{B'}$, $F_{2,v}^C$ and $F_{2,v}^D$ can be taken as unity for heights above 70 km as calculated with the data from *Morrill and Benesch* [1996]. However, the collisional transfer of excitation which couples resonant vibrational levels of overlapping *B, A, W* and *B'* electronic states of N_2 could affect sprite spectra at the height where this process occurs on a time scale shorter than the sprite duration [*Morrill and Benesch*, 1996]. This effect has to be considered in a complete model which will be presented elsewhere. The coefficients $F_{1,v}^B$, $F_{2,v}^{B'}$, $F_{2,v}^C$ and $F_{2,v}^D$ are calculated from (3) by using data from *Gilmore et al.*, [1992]. The intensity of the radiative transition in Rayleighs connecting the v-th and v'-th vibrational levels of electronic states α and β is given by $I_{vv'}^{\alpha\beta} = (4\pi)^{-1}10^{-6}\int dz\, n_v^\alpha A_{vv'}^{\alpha\beta}$. A similar scheme is applied to obtain the intensities of the N_2(2P) and the N_2^+(1N) bands. Note that since we consider the shape of the sprite spectrum, only relative band intensities are of interest. Furthermore, the vibrational-electronic population depends linearly on the electron density (see Eq. (3)); as a result the spectrum is not affected by the possible increase in the electron density due to the ionization of the neutral gas by "hot" electrons.

The observed spectrum depends on the location of the detector. If observed from space, we obtain the source spectrum, while if observed from the ground it will be distorted by atmospheric attenuation. Atmospheric attenuation depends on the zenith angle χ of the optical source, the altitude of the detector, and on the properties of the atmosphere, such as relative humidity and aerosol density. We consider the following contributions to the attenuation: absorption by ozone, oxygen and water vapor, the Rayleigh scattering by air molecules, and Mie scattering by aerosols. The total attenuation of the optical emission is the result of the above contributions and is given by

$$I=I_{spr}\,exp\left\{-\sec\chi\sum_s\sigma_{a,s}(\lambda)\int_{z_0}^{z_{spr}}N(z)dz\right\}, \quad (4)$$

where z_0 and z_{spr} are the altitude of the optical detector and sprite respectively, $I_{spr}=I(z_{spr})$, $\sigma_{a,s}$ is the corresponding effective cross section, and $N(z)$ is the density of particles that absorb or scatter the photons. The absorption due to molecular oxygen has four narrow peaks centered at λ = 687, 689, 760 and 763 nm [*Greenblatt et al.*, 1990]. The absorption by ozone is computed using the absorption cross section from *Lenoble* [1993], and the mid-latitude ozone model [*Brasseur and Solomon*, 1984]. The absorption caused by the water vapor is calculated using the cross section from *Lenoble* [1993, pg. 303] with 80% relative humidity. The Rayleigh scattering was calculated using the wavelength dependence of the cross section given by *Nicolet et al.* [1982]. Finally the Mie scattering was calculated by assuming the vertical distribution of the aerosol attenuation (at 0.55 μm) from the background spring-summer tropospheric aerosol model [*Handbook of Geophysics*, 1985, Fig. 18–21] with a number density of 5,000 particles/cm^3.

3 Discussion and Conclusions

Using the above computational scheme we find first the synthetic source spectrum of N_2 localized at 80 km which includes the first and second positive and first negative bands. It is presented in Fig. 3 for two values of the electric field amplitude E_0 = 35 and 70 V/m. Notice that only the N_2(1P) and N_2(2P) bands give a distinctive contribution to the source spectrum, while the N_2^+(1N) band plays a minor role since it can be excited only by the "tail" electrons having energy in excess of 19 eV. In comparison, the N_2^+(1N) band is among the brightest in auroras since it is caused by high energy electrons.

From the optical spectrum one can retrieve the intensity of the pumping electric field. This is accomplished by comparing bands either belonging to different transitions or to the same transition. In the first case the ratio of excitation rates of the corresponding electronic levels depends on the direct pumping of the levels (mainly) and from cascade excitation. In the second case this ratio is controlled by the cascade excitation only. Since only a few bands belonging to the 1P spectrum have been observed so far [*Mende et al.*, 1995; *Hampton et al.*, 1996], in what follows we consider bands v - v$'$ and $v_1 - v_1'$ belonging to the N_2(1P) spectrum. For given values of the relative intensities of two chosen bands $I_{vv'}$ and $I_{v_1v_1'}$ we obtain that

$$\frac{I_{vv'}}{I_{v_1v_1'}} = \frac{A_{vv'}\left(\nu_{ex}^B F_{1,v'} + \sum_\alpha \nu_{ex}^\alpha F_{2,v'}^\alpha\right) e^{-\tau_{vv'}}}{A_{v_1v_1'}\left(\nu_{ex}^B F_{1,v_1'} + \sum_\alpha \nu_{ex}^\alpha F_{2,v_1'}^\alpha\right) e^{-\tau_{v_1v_1'}}}, \quad (5)$$

where the summation is over the B', C and D electronic states representing the effect of excitation and cascade; $\tau_{vv'}$ and $\tau_{v_1v_1'}$ account for the atmospheric absorption for the corresponding bands. Generally speaking, one can obtain ratios $\nu_{ex}^{B'}/\nu_{ex}^B$, ν_{ex}^C/ν_{ex}^B and ν_{ex}^D/ν_{ex}^B by using intensities of three different bands. This allows the evaluation of the electric field amplitude from Fig. 2. This procedure requires knowledge of the atmospheric attenuation, which includes the zenith angle, as well as the relative humidity, and the aerosol number density. However, if the detector is boarded on a high altitude airplane, the absorption caused by the water vapor and aerosols becomes negligible. Thus the retrieval procedure is simplified.

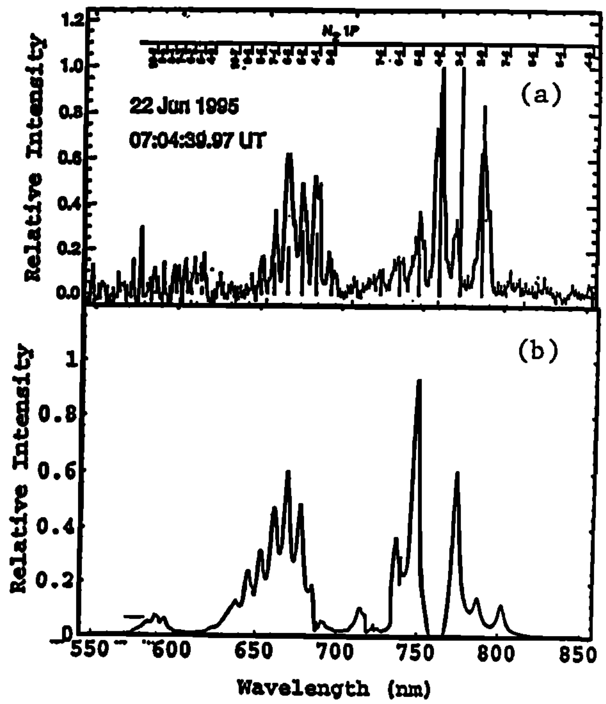

Figure 4. Sprite spectra observed by *Hampton et al.* [1996] at z_0 = 4.3 km above the sea level, at zenith angle χ = 80°(a) along with the spectrum calculated for similar conditions (b).

In order to illustrate the opportunities given by the proposed method we made some estimates using data from *Hampton et al.* [1996] presented in Fig. 4a. In this spectrum we choose the 6–3 and the 7–4 bands. Their intensities relate as 0.62/0.4. We take into account that for the chosen bands the largest role is played by the direct pumping of the B level and by the cascade from the B' level. The difference in the atmospheric attenuations, as we check with our model, was less than a few percent for the zenith angle about 80°. Substituting into Eq. (5) the ratio of the intensities we obtain that $\nu_{ex}^{B'}/\nu_{ex}^B \simeq 0.3$, and according to Fig. 2 this corresponds to the electric field amplitude E_0 = 35 V/m which at z = 80 km is near the breakdown threshold. Note that this estimate was made using noisy data which are not spatially resolved, and can be considered only as an illustrative example. In order to retrieve the electric field due to the lightning from observed sprite spectra some methods such as spectral fitting tech-

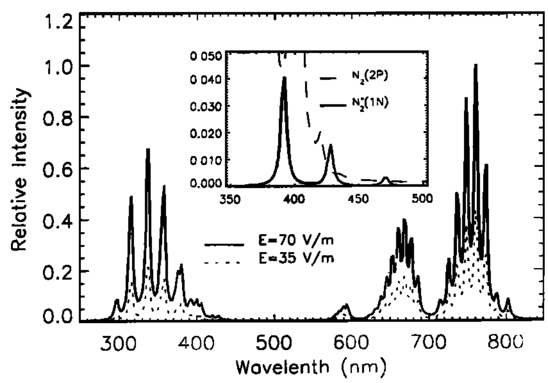

Figure 3. Synthetic source spectrum, which includes the 1P and 2P bands of N_2 along with the N_2^+(1N) band. Those are obtained for z = 80 km, and for the electric field amplitudes E_0 = 35 and 70 V/m, shown by the broken and solid lines correspondingly. The small box distinguishes the N_2^+(1N) spectrum obtained for E_0 = 70 V/m.

nique [*Green et al.*, 1996] have to applied. However, the synthetic spectrum calculated for the electric field amplitude E_0 = 35 V/m, the zenith angle of 80°, and for the detector location of 4.3 km above the sea level, which is shown in Fig. 4b, resembles that observed by *Hampton et al.* [1996] at similar conditions, as revealed by Fig. 4a.

The spectrum observed from the ground differs significantly from the synthetic source spectrum. First, the of N_2(2P) and N_2^+(1N) bands are attenuated more significantly than the of N_2(1P) band. Second, the 3–1 band centered at 760 nm is significantly absorbed in the atmosphere. These effects are stronger for longer optical paths. By comparing the two peaks of, say, the 5–2 and 4–2 bands which undergo different absorption, one could estimate the zenith angle of the observed sprite.

In conclusion, a model of the red sprite spectrum due to molecular excitation by ionospheric electrons accelerated by the electric field from lightning was developed. The model allows us to evaluate the electric field amplitude by comparing the intensities of different spectral bands. The model reveals some differences between the aurora and sprite spectra: in the aurora both permitted and forbidden transitions play a noticeable role, while in sprites only permitted transitions are important. Sprites are normally observed at high zenith angle, so the spectrum is highly influenced by the atmospheric attenuation. Finally, it seems that sprites are produced by electrons of much lesser energy than that of auroral electrons.

For a given zenith angle and atmospheric constitution (i.e. humidity and aerosol density), the collisional quenching and the atmospheric attenuation can be computed accurately. As a result, if the measurements have good spatial resolution, the model output could in principle yield the spatial profile of the amplitude of the electric field causing the sprite.

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