IS THE MAGNETOSPHERE A LENS FOR MHD WAVES?

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Abstract. A novel viewpoint of the magnetosphere as a lens for MHD waves is presented. Using a simple model of the variation of the Alfven speed as proportional to the local magnetic field value given by the Earth's dipole field and that due to the magnetopause currents represented by a current loop, it is found that the near-Earth magnetotail, in the range 8 - 16 R_E, is the focus of the magnetospheric lens. This location is found to be quite insensitive to a wide variation of parameters. By using simple diffraction theory analysis it is found that the focal region extends about 1 R_E about the neutral sheet in the north-south plane and 0.2 - 0.5 R_E along the Sun-Earth line. Compressive MHD waves carried by the solar wind or created by the interaction of the wind with the magnetopause can be amplified by a factor of about 100 in the focal region and this has potentially important implications to substorm activity.

Introduction

The magnetic field and plasma configuration of the magnetosphere can be viewed as an optical system through which MHD waves propagate. These waves can be focused or defocused depending on the variation of the refractive index which is inversely proportional to the Alfven speed $v_A = B/\sqrt{4\pi\rho}$, where B is the magnetic field and ρ the mass density. It will be shown below that when the interplanetary magnetic field (IMF) convected to the magnetosphere by the solar wind has a southward component, the configuration can act as an optical system that focuses MHD waves transported by the solar wind, as well as the ones generated at upstream discontinuities. This lensing action can then focus the MHD waves by guiding their Poynting flux towards the near-Earth magnetotail leading to local enhancement of the wave energy density. As discussed by Lui et al. [1992] such wave energy enhancements have been observed at about 10 R_E in the magnetotail and can have significant implications for the dynamics of substorms.

The objective of this letter is to present the basic physics underlying the idea of the magnetosphere as a lens by using a simplified analysis that relies on conventional techniques of optics. More detailed analysis and modeling using MHD codes is currently under way and will appear at a later time. The next section presents the basic optical properties of wave propagation in the combined magnetic fields of the Earth's dipole and the magnetopause currents. By utilizing a simple model of the magnetic field as a superposition of the dipole field and that of a coil representing the magnetopause currents, the focal distance, the beam width and the Raleigh length are computed. The observational evidence, model pitfalls and limitations, and implications for magnetospheric

processes such as the substorms are presented in the final section.

Focusing of MHD Waves by the Magnetosphere

The total magnetic field of the magnetosphere can be modeled as a superposition of the dipole magnetic field and the field due to the currents in the magnetopause and magnetotail. For the study of MHD wave propagation from the dayside magnetosphere to the near-Earth magnetotail, the effect of the magnetotail currents is neglected. This would correspond to the situation when the neutral sheet does not extend into the near-Earth region. The field due to the magnetopause currents can then be represented by the field due to an equivalent solenoid if the field near its axis and edge effects are neglected. The main advantage of the solenoid model is the convenient parametric representation of the effect of the magnetopause currents, allowing us to explore the robustness of the model. We characterize the solenoid by three parameters, its position x_0 relative to the Earth, its radius a and its aspect ratio $v = L/a$, where L is the axial length of the solenoid. Throughout the paper all distances are given in units of the Earth radius R_E. Figure 1 shows the magnetic field lines in the upper half of the noon-midnight meridian plane from the combined dipole/solenoid system for the case a = 10, x_0 = 3 and v = 2 corresponding to an axial length L = 20. The value of a = 10 corresponds to the average location of the magnetopause [Fairfield, 1991]. This field resembles closely the field obtained from magnetospheric field models [e.g., Mead, 1964; Tsyganenko, 1990]. In our model the field in the lower half is taken as an image of that in the upper half. The dipole field by itself, due to its symmetry, does not produce focusing in the axial direction, while the transverse gradients away from the Earth are too weak to produce any significant defocusing effect. On the other hand there are
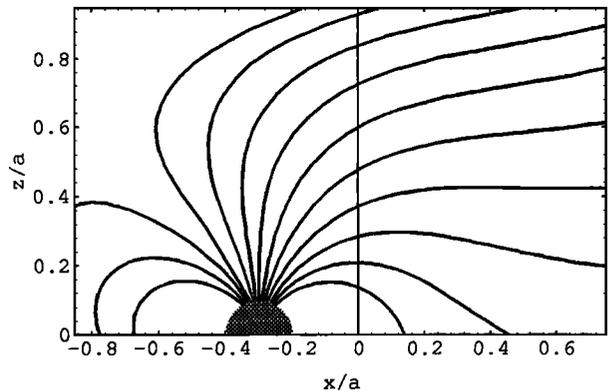
Fig. 1. The magnetospheric configuration in the noon-midnight meridian plane with the magnetopause currents represented by a solenoid of radius a = 10 R_E, length L = 20 R_E located at x_0 = 3 R_E tail side of the Earth.

significant gradients related to the field of the magnetopause currents, represented by a solenoid in this model, which can lead to focusing. The important parameter controlling the focusing properties is the variation of the Alfven speed. In view of the dependence of the Alfven speed on the square root of the plasma density, compared to the direct dependence on the magnetic field, we neglect the density variation and assume that the gradients of the Alfven speed arise from magnetic field gradients alone. This is motivated in part by the lack of observed density profiles or models of the inner magnetosphere at 10 - 15 R_E. The index of refraction can then be written as

$$\eta = \sqrt{1 + \frac{4\pi n M c^2}{B^2}} \sim \frac{1}{B}.$$

The magnetic field B is a function of the distance x with respect to the Earth along the Sun-Earth line, the distance z from the x axis along the south-north direction, and the aspect ratio $v = L/a$. From the magnetic field of a solenoid, the spatial variation of the refractive index in the Sun-Earth meridian plane can be computed by using the expressions of B for a solenoid. A numerical fit of the contours yields, for small x,

$$\eta(z, v) \simeq \eta_0 \left(1 - \frac{f(v)}{a^2} z^2\right), \tag{1}$$

where f(v) is a function of the aspect ratio alone. A simple expression for f(v) is obtained by considering the fact that $\eta \to \eta_0$ for large v and this yields $f(v) = exp(0.07 - 1.05v) \approx exp(-v)$. The magnetosphere is then represented by an equivalent optical system with a refractive index $\eta(z, v)$ and the propagation of the incident MHD waves can be analyzed using standard optical techniques. A convenient method is the matrix propagator technique [Yariv, 1989, Chap. 6] in which, in the paraxial approximation, the propagation of a ray at position z with slope z' ($\equiv dz/dx$) to (z_1, z_1') is given by

$$\begin{bmatrix} z_1 \\ z_1' \end{bmatrix} = M \begin{bmatrix} z \\ z' \end{bmatrix}. \tag{2}$$

The matrix M propagates the ray from one point to the next and is a function of the properties of the medium. For the solenoid model in which the dependence of the refractive index on z is quadratic as given by (1), M is given by [Yariv, 1989, p. 113]

$$M_s(v, a, \Delta x) = \begin{bmatrix} cos(\theta \Delta x) & \frac{1}{\theta} sin(\theta \Delta x) \\ -\theta sin(\theta \Delta x) & cos(\theta \Delta x) \end{bmatrix}, \tag{3}$$

where

$$\theta = \frac{\sqrt{2f(v)}}{a}.$$

A ray propagates through the solenoid for a distance $\Delta x = L$, emerges at $x = L/2$ and then propagates through a medium of constant refractive index. The action of the dipole field is represented by two adjacent refracting surfaces with opposite curvatures but the same radius of curvature, and does not yield a focusing effect. The rays emerging from the solenoid propagate a distance $d = x - L/2$ without any change in their slope and is represented by the propagator

$$M_1(d) = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}. \tag{4}$$

The final effect on the MHD waves is given by

$$M(x) = M_1(x - L/2)M_s, \tag{5}$$

which is the product of the lensing effect of the solenoid represented by M_s and the subsequent rectilinear propagation in a medium of constant refractive index η_0 for the distance $(x - L/2)$ represented by $M_1(x - L/2)$.

The focal length of the optical system representing the magnetosphere can be computed from (2) — (5) for a ray incident parallel to the x axis. The focal length f is the distance along the x-axis from the center of the lens (x=0) to the point where the ray intersects the axis, i.e. $z_1 = 0$ at $x = f$. Using (1) — (5) we have computed the focal length as a function of the aspect ratio v, for values of the solenoid radius (magnetopause location) a between 7 and 12 R_E and the results are shown in Figure 2. The most prominent aspect of Figure 2, is the relative invariance of the focal length f for a variation of the magnetopause position in the range from 7 to 12 R_E and values of the aspect ratio v > 0.3. The magnetic field configuration for v < 0.3 does not correspond to that of the magnetosphere as the tail fields flare out close to the Earth (at about 3 R_E for a = 10 R_E) and thus is not relevant. The
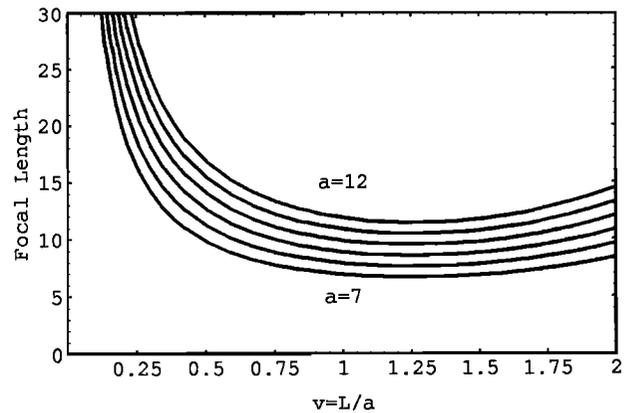
Fig. 2. The focal length (in R_E) as a function of the aspect ratio v for the magnetopause radius a = 7 (bottom curve), 8, 9, 10, 11 and 12 (top curve), and $x_0 = 0$. Since x_0 is only a linear displacement, the focal length f becomes f + x_0 for nonzero x_0. The focal length is insensitive to changes in v and a over a wide range of values.

value of the focal length f from Figure 2 is between 8 and 14 R_E. The actual location of the focus is f + x_o, x_o being the relative distance between the solenoid and the Earth, and since x_o < 3, the expected focal position will still be 8 - 17 R_E. The focal length can also be estimated by considering the magnetic field lines to coincide with the refracting surfaces of an equivalent lens. Using the magnetic field model of Mead [1964] the radii of curvature of these surfaces can be obtained and this yields focal lengths of about 10 R_E, in agreement with the more detailed computations presented here.

The analogy between the magnetosphere and optical systems can be carried further to estimate the spatial properties of the focused wave beam such as width, divergence and shape for diffraction limited propagation. These quantities allow us to estimate the amplification of the wave energy density and the dimensions of the focusing area and can be obtained from the Gaussian beam analysis given below.

The solution of the wave equation for a Gaussian beam under paraxial approximation is [Yariv, 1989, p. 118]

$$\psi \sim exp\left\{-i\left[p(x) + \frac{k}{2q(x)}z^2\right]\right\}exp\{i(wt - kx)\}$$

where ω is the angular frequency, k = $\eta_0\omega/c$ and p(x) is a phase term. The complex function q(x) is related to the width of the beam w(x) at the location x in the perpendicular plane by

$$\frac{1}{q(x)} = \frac{1}{R(x)} - \frac{2i}{kw^2(x)},$$

where R is the radius of curvature of the beam. The profile of the beam along x is characterized by the Raleigh length r_0, defined as $w^2(x) = w^2_{min}\left[1 + (x - x_0 - f)^2/r_0^2\right]$. In terms of the matrix propagators q(x) is given by

$$q(x) = \frac{q_oM_{11} + M_{12}}{q_oM_{21} + M_{22}},$$

where $q_o = ikw_0^2/2$, w_0 = a being the initial beam width
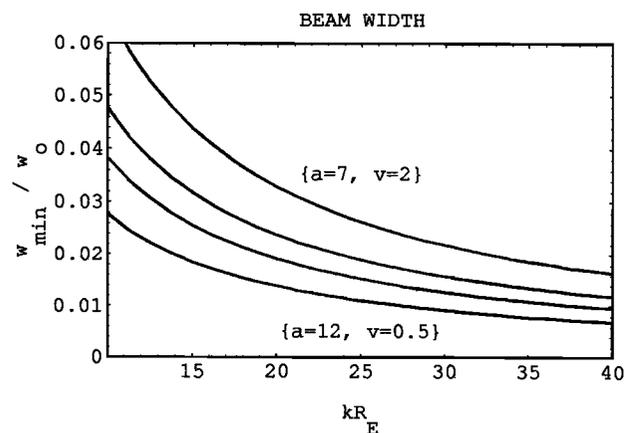
BEAM WIDTH

Fig. 3. Beam width at the focusing region as a function of k = $\eta_0\omega/c$, for different values of a and v: {v = 0.5, a = 12} (bottom curve), {v = 2, a = 12}, {v = 0.5, a = 7} and {v = 2, a = 7} (top curve). Again the insensitivity on the parameters is striking.
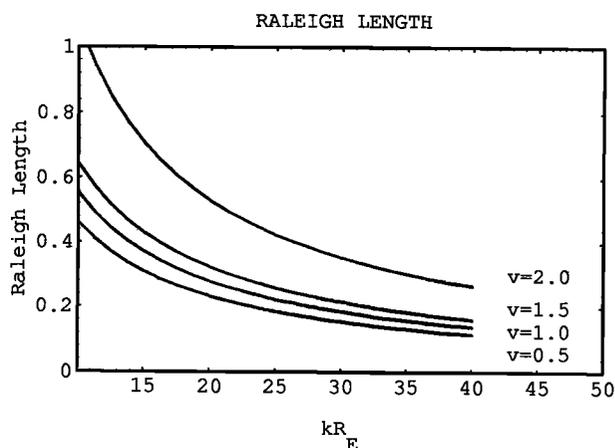
RALEIGH LENGTH

Fig. 4. Raleigh length (in R_E) as a function of k = $\eta_0\omega/c$, for different values of a in the range 7 - 12 R_E and v = L/a in the range 0.5 - 2. Each curve corresponds to the specified value of v and all the values of a, and thus a has almost no effect on the Raleigh length.

and M_{ij} are the elements of the matrix M(x) given by (5). For our case we assume that the incident beam corresponds to a plane wave with infinite radius of curvature. We obtain w(x) from the imaginary part of 1/q(x) and compute the beam width at the focal length, where it is a minimum (which is also where 1/R(x)=0). The normalized transverse beam width at the focus w_{min}/a is shown in Figure 3 as a function of the wavelength kR_E of the incident Alfven waves. Under typical conditions $kR_E \simeq 10 - 20$ giving a width w_{min} in the range 0.02 - 0.04 w_o, which for $w_o\approx 10$ correspond to 0.5 - 1.0 R_E. The Raleigh length r_0 represent the axial width of the beam in the focal region and is shown in Figure 4 for the same range of parameters, and has typical values in the range 0.2 - 0.5 R_E. These values are consistent with simple diffraction theory estimates. The enhancement of the MHD energy flux can be estimated by assuming comparable inflow and outflow speeds. It will then be given by

$$\left(\frac{a}{w_{min}}\right)^2\left(\frac{\Delta}{a}\right)$$

where Δ is the width about the North-South plane over which focusing occurs. Taking $\Delta/a \approx 0.2$, we find enhancement of the wave energy due to focusing of the order of 10^2.

Summary and Conclusions

We have presented an admittedly simplified analysis of the Earth's magnetosphere as a lens for MHD waves. The analysis shows that the near-Earth magnetotail location, in the range of 8 - 15 R_E, is indeed a special location representing the natural focal point of the magnetosphere. The width of the focus is about 1 R_E on each side of the equatorial plane and the activity extends about 0.2 - 0.5 R_E about the focus along the Sun-Earth line, leading to enhancements of about 100 in the fluctuation energy density of the MHD waves. The most important aspect of our analysis is the "robustness" of the focal location and its characteristics over a broad range of parameter variations. It is this insensitivity of the main result to details that makes the lensing concept

attractive. In addition to the parametric studies presented here, we have varied a number of other parameters, involving the dipole field and the solenoid with little influence on the final results. While we expect that detailed models including density variations and three dimensional aspects will reveal important aspects, we do not expect that the focal range will change significantly.

On the observational side we should note that enhanced MHD activity by almost two orders of magnitude in the near tail was observed by AMPTE/CCE [Lopez et al., 1990; Lui et al., 1992]. Furthermore, focusing of the Poynting flux in the region of 8 - 12 R_E was found in three dimensional MHD simulations of substorms (J. Lyon, private communication, 1993).

In concluding we should emphasize that a key objective of this letter is to provide an alternative viewpoint of the magnetosphere and to induce the experimental and modeling community to examine their data and simulation models from the viewpoint of the Earth's and other magnetospheres as lenses of incident MHD waves.

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