Summary 5

Faraday's Law & Lenz's Law

Recall: Faraday's Law says that a changing magnetic flux generates an EMF $\mathcal{E} = -d\Phi_B/dt$

Lenz's Law says that the direction of that EMF is so as to oppose the ***change*** in magnetic flux.

Important Equations

Faraday's Law: $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$

Magnetic Flux: $\Phi_B = \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$

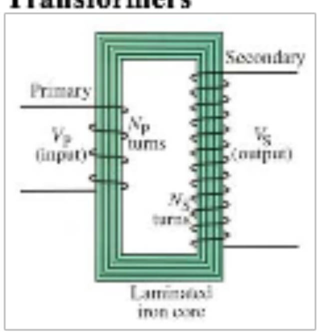
EMF: $\mathcal{E} = \oint \vec{\mathbf{E}}' \cdot d\vec{\mathbf{s}}$

Mutual Inductance: $\mathcal{E}_2 = -M\dfrac{dI_1}{dt}$

Self Inductance, L: $L = \dfrac{\Phi_B}{I}$

EMF Induced by Inductor: $\mathcal{E} = -L\dfrac{dI}{dt}$

Transformers



A major application of mutual inductance is the transformer, which allows the easy modification of the voltage of AC (alternating current) signals. At left is the schematic of a step up transformer. An input voltage V_P on the primary coil creates an oscillating magnetic field, which is "steered" through the iron core (recall that ferromagnets like iron act like wires for magnetic fields) and through the secondary coils, which induces an EMF in them. In the ideal case, the amount of flux generated and received is proportional to the number of turns in each coil. Hence the ratio of the output to input voltage is the same as the ratio of the number of turns in the secondary to the number of turns in the primary. As pictured we have more turns in the secondary, hence this is a "step up transformer," with a larger output voltage than input.

The ease of creating transformers is a strong argument for using AC rather than DC power. Why? Before sending power across transmission lines, voltage is stepped way up (to 240,000 V), leading to smaller currents and losses in the lines. The voltage is then stepped down to 240 V before going into your home.

Self Inductance

Self inductance is very similar to mutual inductance, obeying a similar equation: $\mathcal{E} = -L\frac{dI}{dt}$, and the same concept: when a circuit has a current in it, it creates a magnetic field, and hence a flux, through itself. If that current changes, then the flux will change and hence an EMF will be induced in the circuit. The action of that EMF will be to *oppose the change* in current (if the current is decreasing it will try to make it bigger, if increasing it will try to make it smaller). For this reason, we often refer to the induced EMF as the "back EMF."

To calculate the self inductance (or inductance, for short) of an object, imagine that a current I flows through it, and determine how much magnetic field and hence flux Φ_B that makes through the object. The self inductance is then $L = \Phi_B / I$.

An inductor is a circuit element whose main characteristic is its inductance, L. It is drawn as a coil  in circuit diagrams. The strong resemblance to a solenoid is intentional – solenoids make very good inductors both because of their ability to make a strong field inside themselves, and also because the field they produce is fairly well contained, and hence doesn't produce much flux (and induce EMFs) in other, nearby circuits.

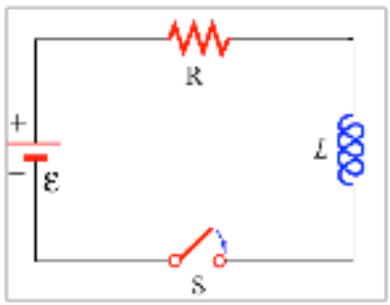
The role of an inductor is to oppose changing currents. At steady state, in a DC circuit, an inductor is off – it induces no EMF as long as the current through it is constant. As soon as you try to change the current through an inductor though, it will fight back. In this sense an inductor is the opposite of a capacitor. If a capacitor is placed in a steady state current it will eventually fill up and "open" the circuit, whereas an inductor looks like a short in this case. On the other hand, when starting from its uncharged state, a capacitor looks like a short when you first try to move current through it, while an inductor looks like an open circuit, as it prevents the change (from no current to some current).

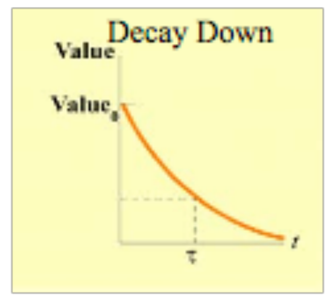
Energy in B Fields

Where do inductors get the energy to source current when they need to? In capacitors we found that energy was stored in the electric field between their plates. In inductors, energy is similarly stored, only now its in the magnetic field. Just as with capacitors, where the electric field was created by a charge on the capacitor, we now have a magnetic field created when there is a current through the inductor. Thus, just as with the capacitor, we can discuss both the energy in the inductor, $U = \frac{1}{2}LI^2$, and the more generic energy density $u_B = \frac{B^2}{2\mu_0}$, stored in the magnetic field. Again, although we introduce the magnetic field energy density when talking about energy in inductors, it is a generic concept – whenever a magnetic field is created it takes energy to do so, and that energy is stored in the field itself.

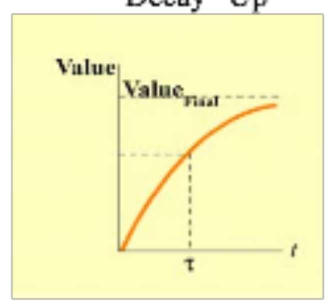
RL Circuits

A simple RL circuit is shown below. When the switch is closed, no current will initially flow in the circuit,





"Decay" Up



but as time goes on this current will increase. We can write down the differential equation for current flow by using Faraday's Law. We begin by calculating the line integral of the electric field around the loop. There are two contributions to this integral: the electric field in the wire and the electric field in the battery. If we orient the line integral in the direction of the electric field in the wire (the electric field in the battery points in the opposite direction) then

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = IR - \mathcal{E}_{battery}.$$

oriented

We now use Faraday's law:

$$\oint_{oriented} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = \boldsymbol{\mathcal{E}}_{electromotive} = -\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}}.$$

For a loop with only self induction,

$$-\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{a}} = -L\frac{dI}{dt}.$$

So combining the two results in Faraday's Law yields

$$IR - \boldsymbol{\mathcal{E}}_{battery} = -L\frac{dI}{dt}$$

which is first order linear differential equation with constant coefficients. The solution to this differential equation

$$I(t) = \frac{\boldsymbol{\mathcal{E}}_{battery}}{R}(1 - e^{-t/\tau})$$

shows that the current increases exponentially from its initial value of zero to a final constant value of $\boldsymbol{\mathcal{E}}_{battery} / R$. The rate at which this change happens is dictated by the "time constant" τ, which for this circuit is given by $\tau = L / R$.

Now the current starts at zero and "decays" upward to a constant value, now with a time constant $\tau = L/R$ (a big inductance slows down the circuit as it is more effective at opposing changes, but now a big resistance reduces the size of the current and hence changes in the current that the inductor will see, and thus decreases the time constant – speeds things up).

Interestingly, in RL circuits any value that you could ask about (current, potential drop across the resistor, …) "decays" exponentially (either down or up). You should be able to determine which of the two plots (above and to the right a value) will follow just by thinking about it.

Important Equations

Self Inductance, L: $L = \frac{\Phi_B}{I}$

EMF Induced by Inductor: $\mathcal{E} = -L\frac{dI}{dt}$

Energy stored in Inductor: $U = \frac{1}{2}LI^2$

Energy Density in B Field: $u_B = \frac{B^2}{2\mu_0}$

Exponential Decay: $Value = Value_{initial}e^{-t/\tau}$

Exponential "Decay Upwards": $Value = Value_{final}\left(1 - e^{-t/\tau}\right)$

Simple RL Time Constant: $\tau = L/R$