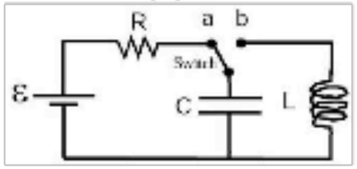
Summary #10

Mass on a Spring: Simple Harmonic Motion

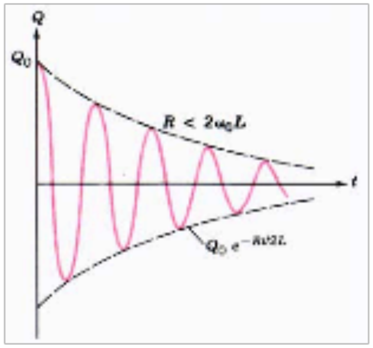
In a simple system consisting of a mass hanging on a spring, when the mass is pulled down and released it oscillates up and down. We think about this in a couple of ways. One way is to look at the forces on the mass and write a differential equation for its motion, $F = m\ddot{x} = -kx$, where $\ddot{x}$ means two time derivatives of the displacement (acceleration). The solution to this is simple harmonic motion: $x = x_0 \cos(\omega t)$ where $\omega = \sqrt{k/m}$.

We can also think about the energy in the system. As the mass moves, energy oscillates between kinetic energy of the mass and potential energy stored in the spring. If there is no damping (friction) in the system to dissipate energy, the oscillation will continue forever.

Undriven L(R)C Circuits



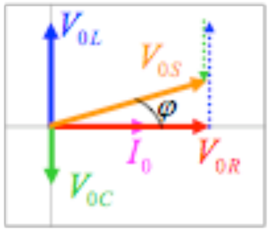
Consider the LC circuit at left, where the switch is at "a" until the capacitor is fully charged and then thrown to "b." This is analogous to pulling down a mass and releasing it. Here the capacitor will want to discharge and will drive a current through the inductor. Eventually all the charges will run off of the capacitor (spring), so it won't "push" anymore, but now the inductor will want to keep the current flowing through it that it already has (inductors, like masses, have inertia). It will keep the current flowing, but that will eventually fill up the capacitor which will stop the current and send it back the other direction. Our differential equation is thus analogous, $V = -L\ddot{q} = q/C$, and has the same solution: $q = q_0 \cos(\omega t)$ where $\omega = \sqrt{1/LC}$.



We can also think about energy here, where it oscillates between being stored in the electric field in the capacitor and the magnetic field in the inductor. As long as there is no dissipation (resistance) is the circuit the oscillations will continue forever.

If we add a resistor in series with the capacitor and inductor we provide a method of energy loss, through joule heating in the resistor as current flows. The oscillations will thus damp out to zero. The exact path the charge will take as it oscillates to zero depends on the relative sizes of L, R and C, but will typically look something like the curve above, where the oscillations are bounded by an "envelope" which is exponentially decaying to zero as a function of time.

Seeing it Mathematically – Phasors



To do this mathematically we will use phasor diagrams. A phasor is a vector whose magnitude is the amplitude of either voltage or current and whose angle corresponds to its phase. Phasors rotate CCW about the origin with time as their phase evolves, and their current amplitude is their component along the y-axis, which oscillates as it should. Phasors allow us to add voltages that are not in phase with each other. For example, the phasor diagram above illustrates the relationship of voltages in a series LRC circuit. The current *I* is assigned to be at "0 phase" (along the x-axis). The phase of the voltage across the resistor is the same. The voltage across the inductor L leads (is ahead of *I*) and the voltage across the capacitor C lags (is behind *I*). If you add up (using vector arithmetic) the voltages across R, L & C (the red and dashed blue & green lines respectively) you arrive at the voltage across the power supply. This then gives you a rapid way of understanding the phase between the drive (the power supply) and the response (the current) – here labeled ϕ.