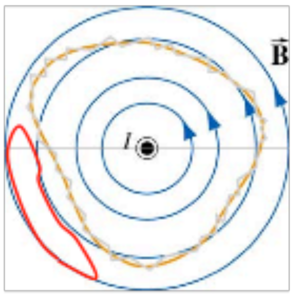
Summary 2
Ampere's law – The equivalent of Gauss's law

Ampere's Law
With electric fields we saw that rather than always using Coulomb's law, which gives a completely generic method of obtaining the electric field from charge distributions, when the distributions were highly symmetric it became more convenient to use Gauss's Law to calculate electric fields. The same is true of magnetic fields – Biot-Savart does not always provide the easiest method of calculating the field. In cases where the current source is very symmetric it turns out that Ampere's Law, another of Maxwell's four equations, can be used, greatly simplifying the task.

Ampere's law rests on the idea that if you have a curl in a magnetic field (that is, if it wraps around in a circle) the field must be generated by some current source inside that circle (at the center of the curl). So, if we walk around a loop and add up the magnetic field heading in our direction, then if, when we finish walking around, we have seen a net field wrapping in the direction we walked, there must be some current penetrating the loop we just walked around. Mathematically this idea is expressed as: $\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{\text{penetrate}}$, where on the left we are integrating the magnetic field as we walk around a closed loop, and on the right we add up the total amount of current penetrating the loop.



In the example pictured here, a single long wire carries current out of the page. As we discussed in class, this generates a magnetic field looping counter-clockwise around it (blue lines). On the figure we draw two "Amperian Loops." The first loop (yellow) has current I penetrating it. The second loop (red) has no current penetrating it. Note that as you walk around the yellow loop the magnetic field always points in roughly the same direction as the path: $\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} \neq 0$, whereas around the red loop sometimes the field points with you, sometimes against you: $\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = 0$.

We use Ampere's law in a very similar way to how we used Gauss's law. For highly symmetric current distributions, we know that the produced magnetic field is constant along certain paths. For example, in the picture above the magnetic field is constant around any blue circle. The integral then becomes simple multiplication along those paths $\left(\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = B \cdot \text{Path Length}\right)$, allowing you to solve for B. For details and examples see the course notes.

Solving Problems using Ampere's law

Ampere's law provides a powerful tool for calculating the magnetic field of current distributions that have radial or rectangular symmetry. The following steps are useful when applying Ampere's law:

(1) Identify the symmetry associated with the current distribution, and the associated shape of "Amperian loops" to be used.
(2) Divide space into different regions associated with the current distribution, and determine the exact Amperian loop to be used for each region. The magnetic field must be constant, perpendicular to or known (i.e. zero) along each part of the loop.
(3) For each region, calculate $I_{\text{penetrate}}$, the current penetrating the Amperian loop.
(4) For each region, calculate the integral $\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}}$ around the Amperian loop.
(5) Equate $\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}}$ with $\mu_0 I_{\text{penetrate}}$, and solve for the magnetic field in each region.

Important Equations

Ampere's Law: $\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{\text{penetrate}}$