Summary 1
B field creation

Topic Introduction

Today we begin a major new topic in the course – magnetism. In some ways magnetic fields are very similar to electric fields: they are generated by and exert forces on electric charges. There are a number of differences though. First of all, magnetic fields only interact with (are created by and exert forces on) charges that are moving. Secondly, the simplest magnetic objects are not monopoles (like a point charge) but are instead dipoles.

This week we will focus on the *creation* of magnetic fields. The presentation is analogous to our discussion of charges creating electric fields. We first describe the magnetic field generated by a single charge and then proceed to collections of moving charges (currents), the fields from which we will calculate using superposition – just like for continuous charge distributions.

Dipole Fields

We will begin the class by noting that the magnetic fields you are most familiar with, those generated by bar magnets and by the Earth, act like magnetic dipoles. Magnetic dipoles create magnetic fields identical in shape to the electric fields generated by electric dipoles. We even describe them in the same way, saying that they consist of a North pole (+) and a South pole (-) some distance apart, and that magnetic field lines flow from the North pole to the South pole. Despite these similarities, magnetic dipoles are different from electric dipoles, in that if you cut an electric dipole in half you will find a positive charge and a negative charge, while if you cut a magnetic dipole in half you will be left with two new magnetic dipoles. There is no such thing as an isolated "North magnetic charge" (a magnetic monopole).

Field from a Single Moving Charge

Next we turn to the creation of magnetic fields. Just as a single electric charge creates an electric field which is proportional to charge q and falls off as r^{-2}, a single moving electric charge additionally creates a magnetic field given by

$$\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$

Note the similarity to Coulomb's law for the electric field – the field is proportional to the charge q, obeys an inverse square law in r, and depends on a constant, the permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ T m/A. The difference is that the field no longer points along $\hat{\mathbf{r}}$ but is instead perpendicular to it (because of the cross product).

If you haven't worked with cross products in a while, you should read the vector analysis review module. Rapid calculation of at least the direction of cross-products will dominate the rest of the course so you need to understand what they mean and how to compute them.

Field from a Current: Biot-Savart Law

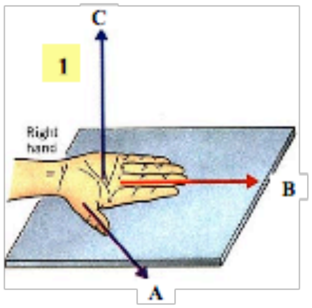
We can immediately switch over from discrete charges to currents by replacing $q\vec{\mathbf{v}}$ with $Id\vec{\mathbf{s}}$:

$$d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

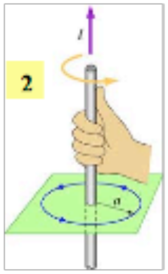
This is the Biot-Savart formula, and, like the differential form of Coulomb's Law, provides a generic method for calculating fields – here magnetic fields generated by currents. The *ds* in this formula is a small length of the wire carrying the current *I*, so that *I ds* plays the same role that *dq* did when we calculated electric fields from continuous charge distributions. To find the total magnetic field at some point in space you integrate over the current distribution (e.g. along the length of the wire), adding up the field generated by each little part of it *ds*.

Right Hand Rules

Because of the cross product in the Biot-Savart Law, the direction of the resulting magnetic field is not as simple as when we were working with electric fields. In order to quickly see what direction the field will be in, or what direction the force on a moving particle will be in, we can use a "Right Hand Rule." At times it seems that everyone has their own, unique, right hand rule. Certainly there are a number of them out there, and you should feel free to use whichever allow you to get the correct answer. Here I describe the three that I use (including one we won't come to until next week).



The important thing to remember is that cross-products yield a result which is perpendicular to both of the input vectors. The only open question is in which of the two perpendicular directions will the result point (e.g. if the vectors are in the floor does their cross product point up or down?). Using your RIGHT hand:



1) For a generic cross-product (C = AxB): open your hand perfectly flat. Put your thumb along A and your fingers along B. Your palm points along C.

2) For determining the direction of the magnetic field generated by a current: fields wrap around currents the same direction that your fingers wrap around your thumb. At any point the field points tangent to the circle your fingers will make as you twist your hand keeping your thumb along the current.

3) For determining the direction of the dipole moment of a coil of wire: wrap your fingers in the direction of current. Your thumb points in the direction of the North pole of the dipole (in the direction of the dipole moment μ of the coil).

Important Equations

Biot-Savart – Field created by moving charge; current: $\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{q\,\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$; $d\vec{\mathbf{B}} = \frac{\mu_o}{4\pi}\frac{I\,d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$