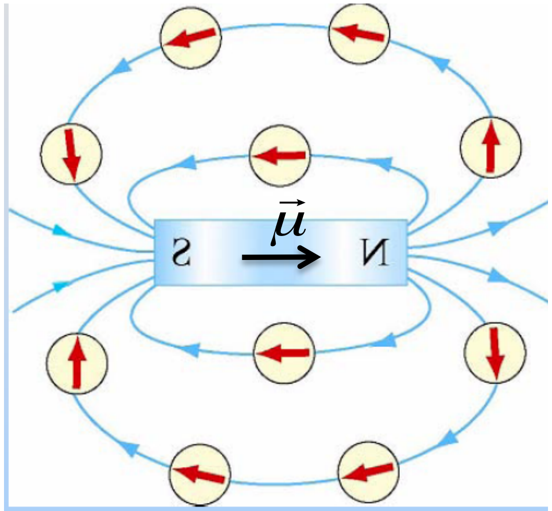
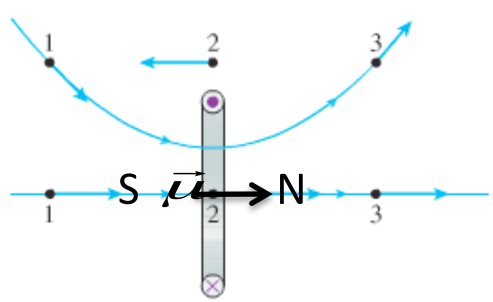
Note 1
Complement to Chapter 33

PHYS 270
SPRING 2011
Dennis Papadopoulos



(a) A single loop



1. Look at the magnet. We have defined North the point that the field lines start on the outside. Equivalently – if we had field lines inside the magnet they would align with the S-N direction. This would be the direction of its magnetic moment $\vec{\mu}$

2. Now look at the loop. Current comes out of the page on top and into the page at the bottom. Inside the loop lines go from left to right (RH rule) but outside they return the opposite way. Namely the field configuration is similar to the magnet. Therefore South pole to the left and North pole to the right.

8.6.2 Mass Spectrometer

Various methods can be used to measure the mass of an atom. One possibility is through the use of a mass spectrometer. The basic feature of a *Bainbridge* mass spectrometer is illustrated in Figure 8.6.2. A particle carrying a charge $+q$ is first sent through a velocity selector.

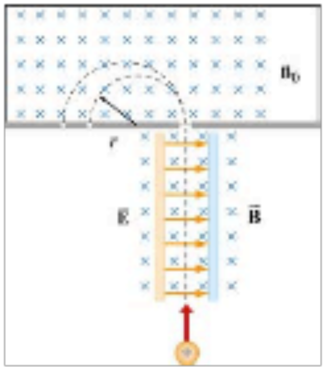


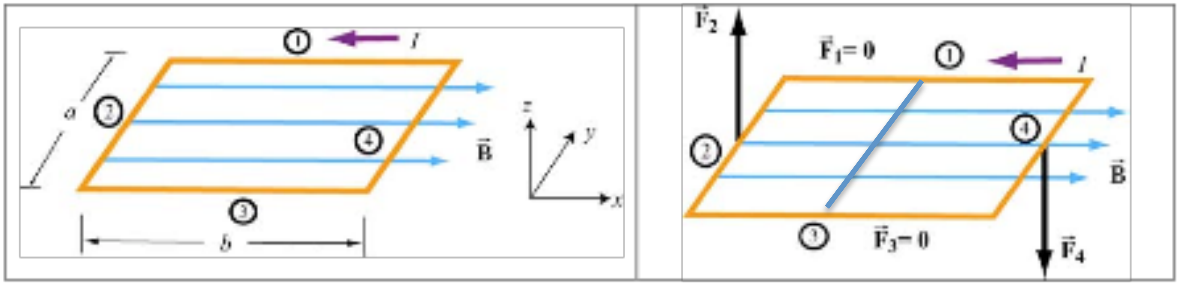
Figure 8.6.2 A Bainbridge mass spectrometer

The applied electric and magnetic fields satisfy the relation $E = vB$ so that the trajectory of the particle is a straight line. Upon entering a region where a second magnetic field $\vec{\mathbf{B}}_0$ pointing into the page has been applied, the particle will move in a circular path with radius r and eventually strike the photographic plate. Using Eq. 8.5.2, we have

$$r = \frac{mv}{qB_0} \tag{8.6.5}$$

Since $v = E / B$, the mass of the particle can be written as

$$m = \frac{qB_0 r}{v} = \frac{qB_0 B r}{E} \tag{8.6.6}$$



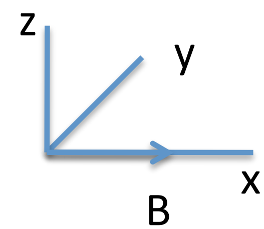


Figure 8.4.1 (a) A rectangular current loop placed in a uniform magnetic field. (b) The magnetic forces acting on sides 2 and 4.

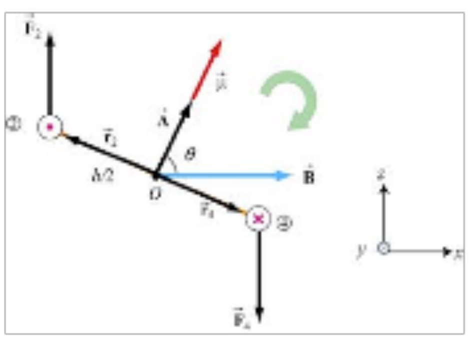
$$\begin{cases} \vec{\mathbf{F}}_2 = I(-a\,\hat{\mathbf{j}})\times(B\,\hat{\mathbf{i}}) = IaB\,\hat{\mathbf{k}} \\ \vec{\mathbf{F}}_4 = I(a\,\hat{\mathbf{j}})\times(B\,\hat{\mathbf{i}}) = -IaB\,\hat{\mathbf{k}} \end{cases}$$

$$\vec{\mathbf{F}}_{net} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = \vec{\mathbf{0}}$$

$$\vec{\boldsymbol{\tau}} = \left(-\frac{b}{2}\hat{\mathbf{i}}\right)\times\vec{\mathbf{F}}_2 + \left(\frac{b}{2}\hat{\mathbf{i}}\right)\times\vec{\mathbf{F}}_4 = \left(-\frac{b}{2}\hat{\mathbf{i}}\right)\times\left(IaB\,\hat{\mathbf{k}}\right) + \left(\frac{b}{2}\hat{\mathbf{i}}\right)\times\left(-IaB\,\hat{\mathbf{k}}\right)$$

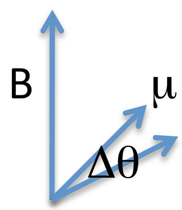
$$= \left(\frac{IabB}{2} + \frac{IabB}{2}\right)\hat{\mathbf{j}} = IabB\,\hat{\mathbf{j}} = IAB\,\hat{\mathbf{j}}$$

$$\vec{\boldsymbol{\tau}} = I\vec{\mathbf{A}}\times\vec{\mathbf{B}}$$



For a loop consisting of N turns, the magnitude of the toque is

$$\tau = NIAB\sin\theta$$



$$\boxed{\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}} \tag{8.4.9}$$

The above equation is analogous to $\vec{\boldsymbol{\tau}} = \vec{\mathbf{p}} \times \vec{\mathbf{E}}$ in Eq. (2.8.3), the torque exerted on an electric dipole moment $\vec{\mathbf{p}}$ in the presence of an electric field $\vec{\mathbf{E}}$. Recalling that the potential energy for an electric dipole is $U = -\vec{\mathbf{p}} \cdot \vec{\mathbf{E}}$ [see Eq. (2.8.7)], a similar form is expected for the magnetic case. The work done by an external agent to rotate the magnetic dipole from an angle θ_0 to θ is given by

$$\begin{aligned} W_{ext} &= \int_{\theta_0}^{\theta} \tau \, d\theta' = \int_{\theta_0}^{\theta} (\mu B \sin\theta') d\theta' = \mu B \left(\cos\theta_0 - \cos\theta \right) \\ &= \Delta U = U - U_0 \end{aligned} \tag{8.4.10}$$

Once again, $W_{ext} = -W$, where W is the work done by the magnetic field. Choosing $U_0 = 0$ at $\theta_0 = \pi/2$, the dipole in the presence of an external field then has a potential energy of

$$U = -\mu B \cos\theta = -\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}} \tag{8.4.11}$$

The configuration is at a stable equilibrium when $\vec{\boldsymbol{\mu}}$ is aligned parallel to $\vec{\mathbf{B}}$, making U a minimum with $U_{min} = -\mu B$. On the other hand, when $\vec{\boldsymbol{\mu}}$ and $\vec{\mathbf{B}}$ are anti-parallel, $U_{max} = +\mu B$ is a maximum and the system is unstable.

Consider the situation where a small dipole $\vec{\boldsymbol{\mu}}$ is placed along the symmetric axis of a bar magnet, as shown in Figure 8.4.4.

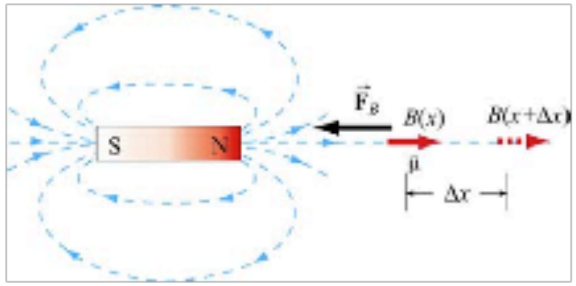


Figure 8.4.4 A magnetic dipole near a bar magnet.

The dipole experiences an attractive force by the bar magnet whose magnetic field is non-uniform in space. Thus, an external force must be applied to move the dipole to the right. The amount of force F_{ext} exerted by an external agent to move the dipole by a distance Δx is given by

$$F_{ext}\Delta x = W_{ext} = \Delta U = -\mu B(x+\Delta x) + \mu B(x) = -\mu[B(x+\Delta x) - B(x)] \qquad (8.4.12)$$

where we have used Eq. (8.4.11). For small Δx, the external force may be obtained as

$$F_{ext} = -\mu \frac{[B(x+\Delta x) - B(x)]}{\Delta x} = -\mu \frac{dB}{dx} \qquad (8.4.13)$$

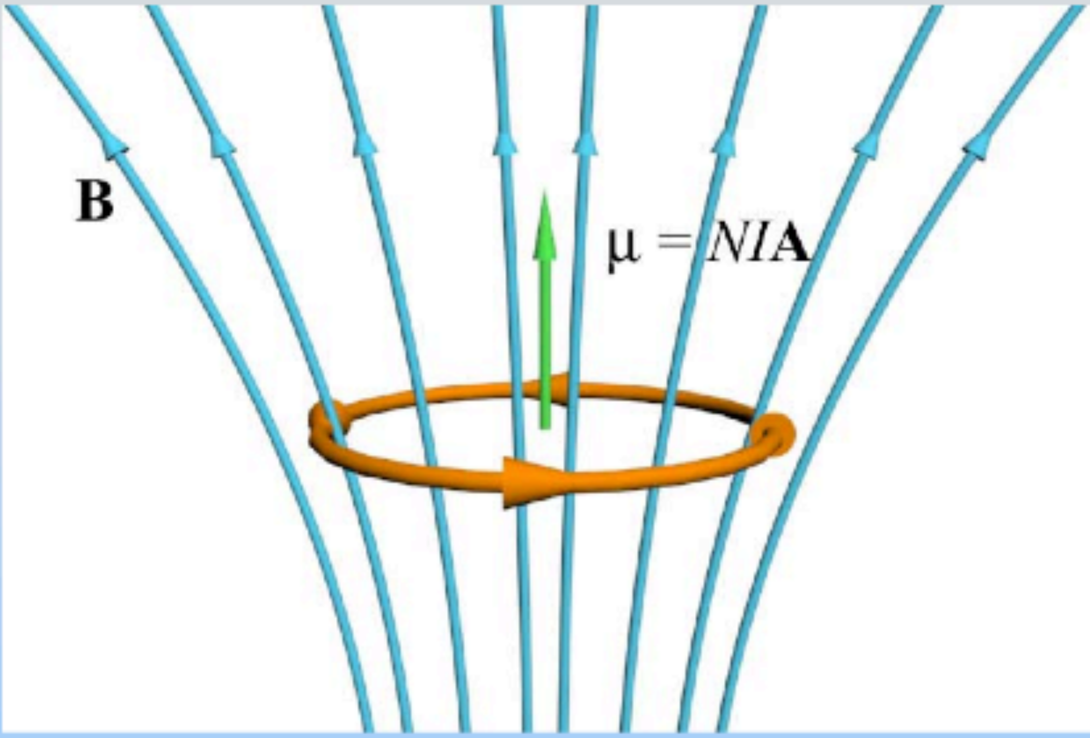
which is a positive quantity since $dB/dx < 0$, i.e., the magnetic field decreases with increasing x. This is precisely the force needed to overcome the attractive force due to the bar magnet. Thus, we have

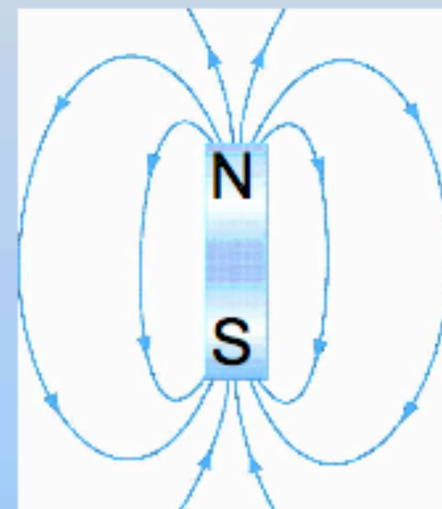
$$F_B = \mu \frac{dB}{dx} = \frac{d}{dx}(\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}) \qquad (8.4.14)$$

More generally, the magnetic force experienced by a dipole $\vec{\boldsymbol{\mu}}$ placed in a non-uniform magnetic field $\vec{\mathbf{B}}$ can be written as

$$\vec{\mathbf{F}}_B = \nabla(\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}) \qquad (8.4.15)$$

Force on Magnetic Dipole





Bar magnet below dipole, with N pole on top
It is aligned with the dipole pictured, they attract!

$$\vec{\mathbf{M}} = \frac{1}{V}\sum_i \vec{\boldsymbol{\mu}}_i$$

Ferromagnetism

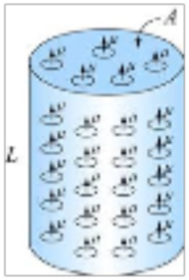
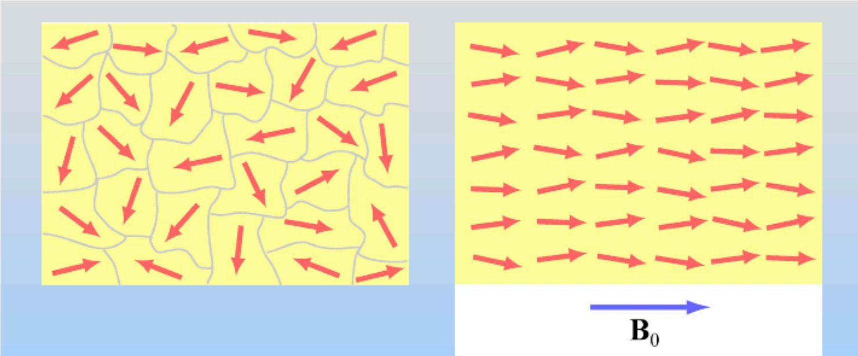


Figure 9.6.1 A cylinder with N magnetic dipole moments



Applied external field B_0 tends to align the atomic magnetic moments (unpaired electrons)

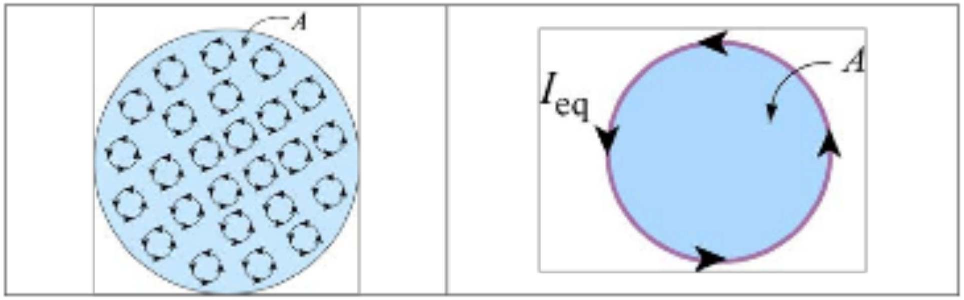


Figure 9.6.2 (a) Top view of the cylinder containing magnetic dipole moments. (b) The equivalent current.