Lecture 13
Wave Optics-1 Chapter 22

PHYSICS 270

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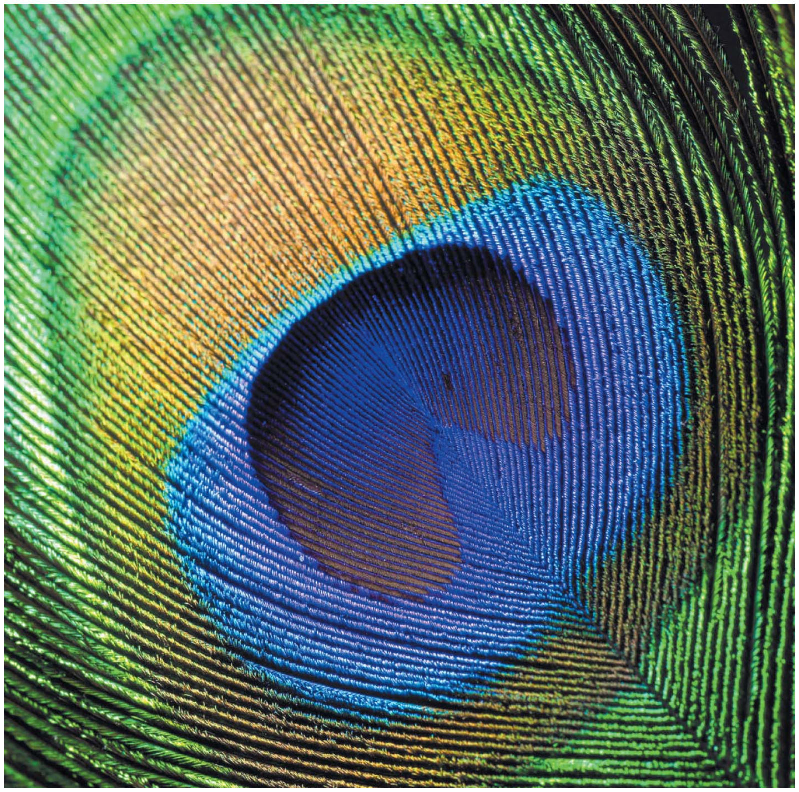
March 15, 2011

Chapter 22. Wave Optics

Light is an electromagnetic wave(!!!).The interference of light waves produces the colors reflected from a CD, the iridescence of bird feathers, and the technology underlying supermarket checkout scanners and optical computers.



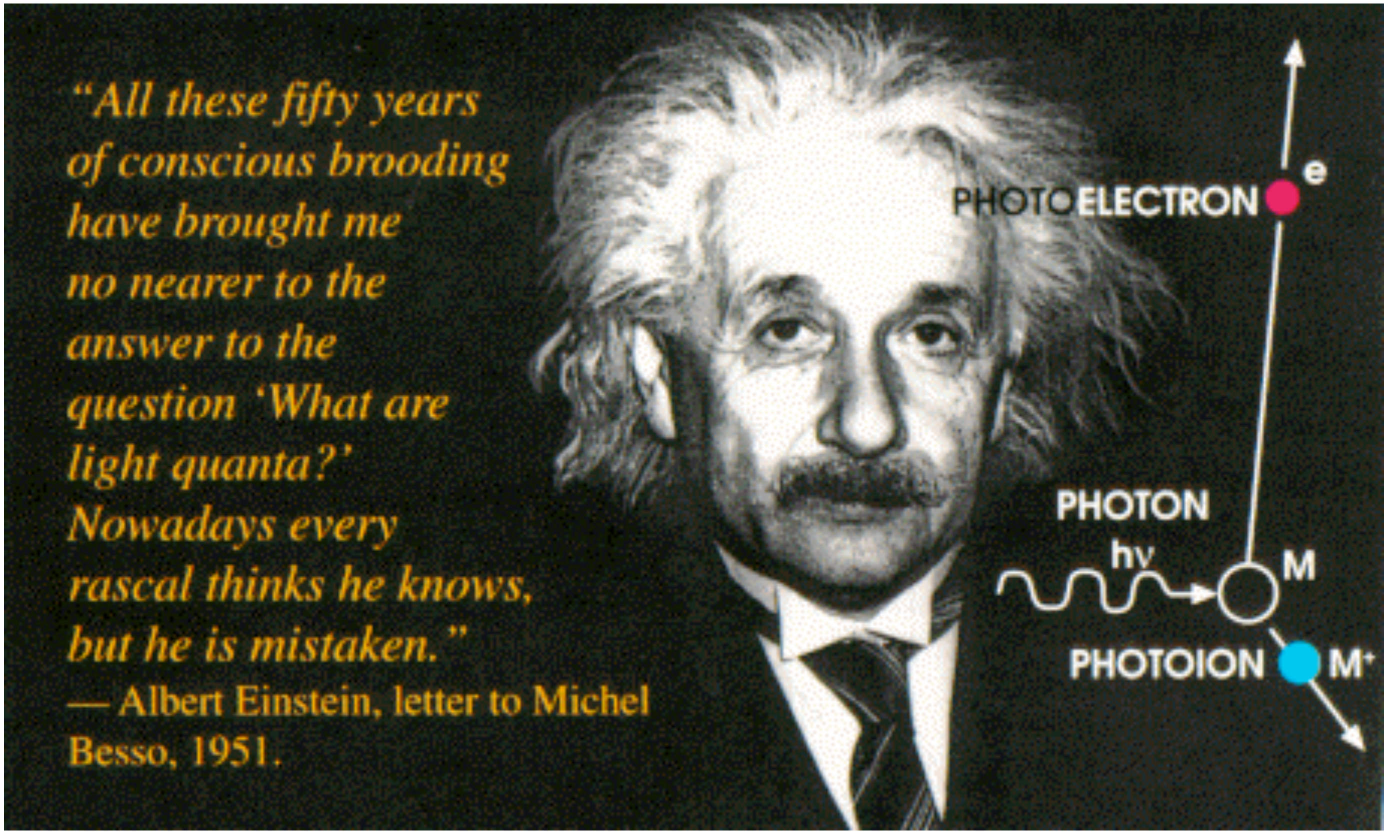
Chapter Goal: To understand and apply the wave model of light.

Wave Optics

Topics:

- Light and Optics
- The Interference of Light
- The Diffraction Grating
- Single-Slit Diffraction
- Circular-Aperture Diffraction
- Interferometers

What is the nature of Light ?



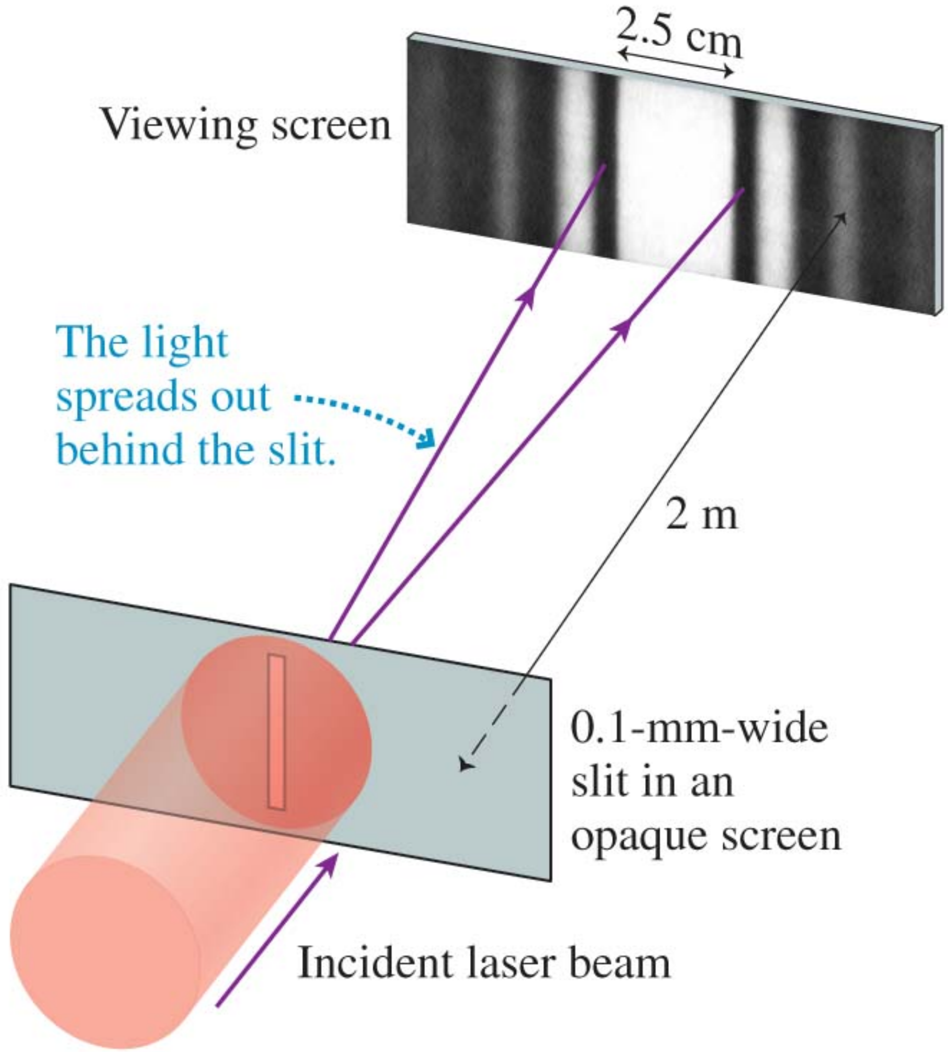
Models of Light

- **The wave model:** under many circumstances, light exhibits the same behavior as sound or water waves. The study of light as a wave is called *wave optics***.**
- **The ray model:** The properties of prisms, mirrors, and lenses are best understood in terms of *light rays*. The ray model is the basis of *ray optics*.
- **The photon model:** In the quantum world, light behaves like neither a wave nor a particle. Instead, light consists of *photons* that have both wave-like and particle-like properties. This is the *quantum theory* of light.

Light diffracts when traversing apertures like the water waves depicted below

At a beach in Tel Aviv, Israel, plane water waves pass through two openings in a breakwall. Notice the diffraction effect—the waves exit the openings with circular wave fronts, as in Figure 37.1b. Notice also how the beach has been shaped by the circular wave fronts.

FIGURE 22.2 Light, just like a water wave, does spread out behind a hole *if* the hole is sufficiently small.

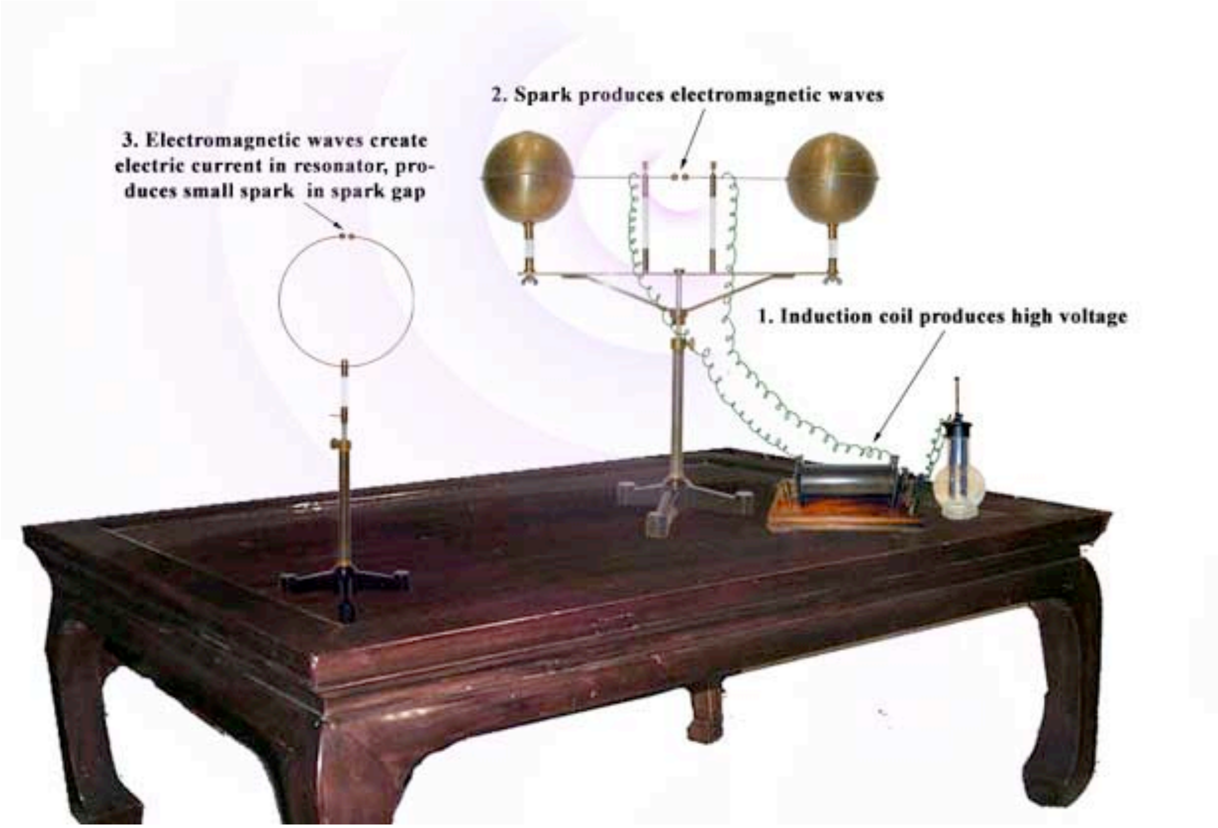


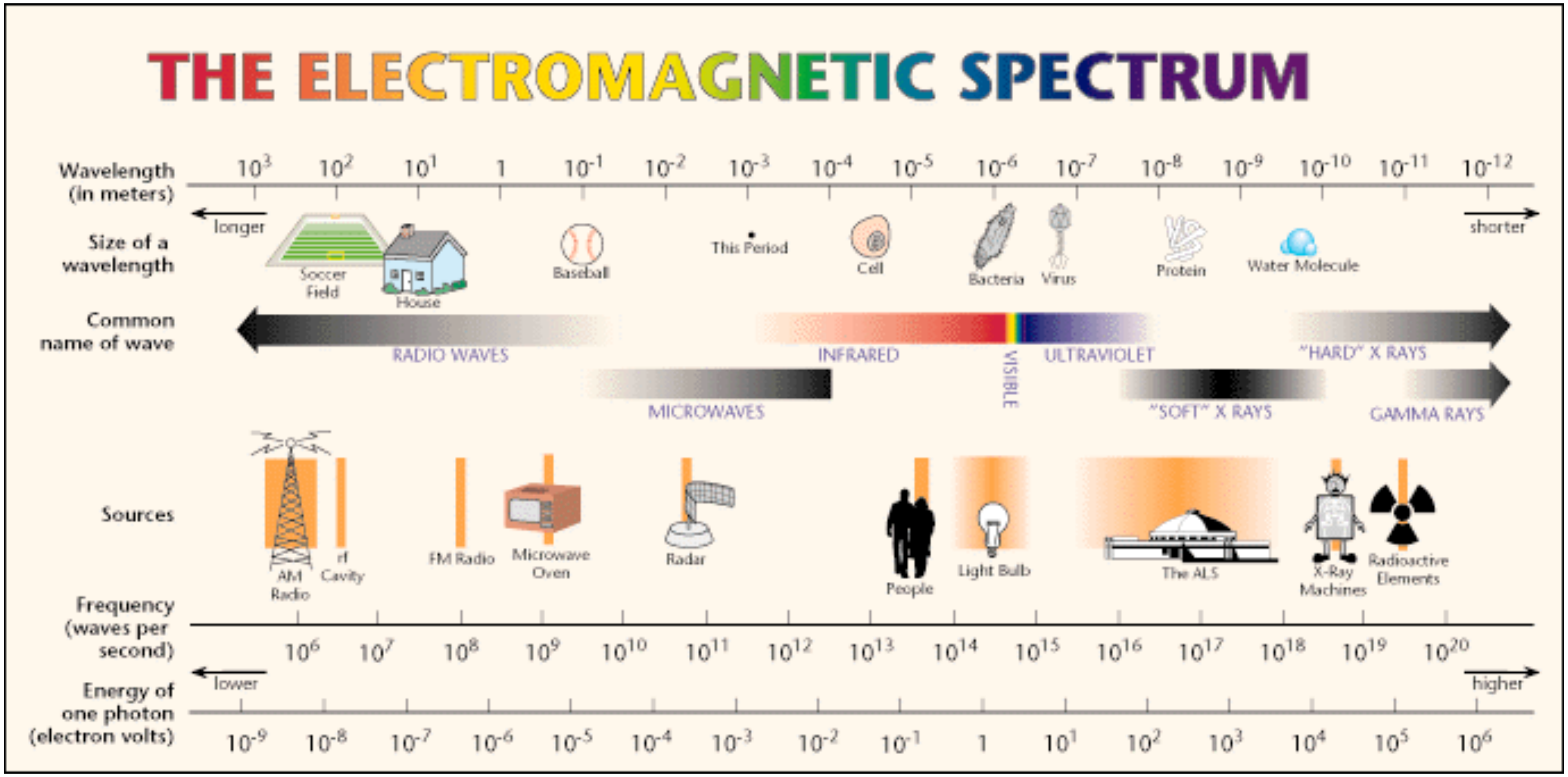
The invention of radio?

Hertz proves that light is really an electromagnetic wave.

Waves could be generated in one circuit, and electric pulses with the same frequency could be induced in an antenna some distance away.

These electromagnetic waves could be reflected, and refracted, focused, polarized, and made to interfere—just like light!




THE ELECTROMAGNETIC SPECTRUM
Wavelength (in meters)
10^3 10^2 10^1 1 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9} 10^{-10} 10^{-11} 10^{-12}
longer
shorter
Size of a wavelength
Soccer Field
House
Baseball
This Period
Cell
Bacteria
Virus
Protein
Water Molecule
Common name of wave
RADIO WAVES
INFRARED
VISIBLE
ULTRAVIOLET
"HARD" X RAYS
MICROWAVES
"SOFT" X RAYS
GAMMA RAYS
Sources
AM Radio
rf Cavity
FM Radio
Microwave Oven
Radar
People
Light Bulb
The ALS
X-Ray Machines
Radioactive Elements
Frequency (waves per second)
10^6 10^7 10^8 10^9 10^{10} 10^{11} 10^{12} 10^{13} 10^{14} 10^{15} 10^{16} 10^{17} 10^{18} 10^{19} 10^{20}
lower
higher
Energy of one photon (electron volts)
10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 1 10^1 10^2 10^3 10^4 10^5 10^6

Everything is a wave

"Okay…what you have to realize is…to first order…everything is a simple harmonic oscillator. Once you've got that, it's all downhill from there."

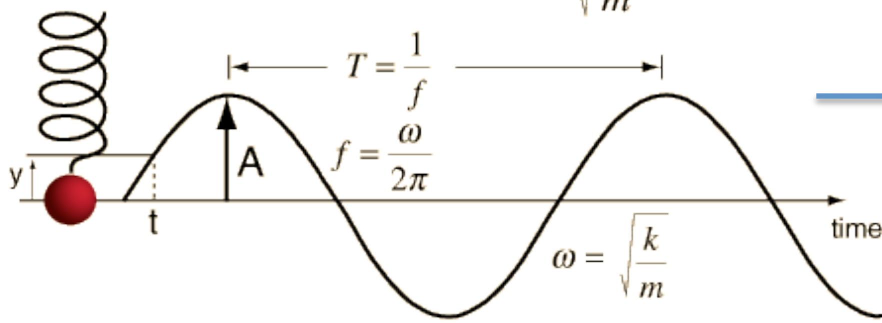
Waves 101 Part 1: Oscillations

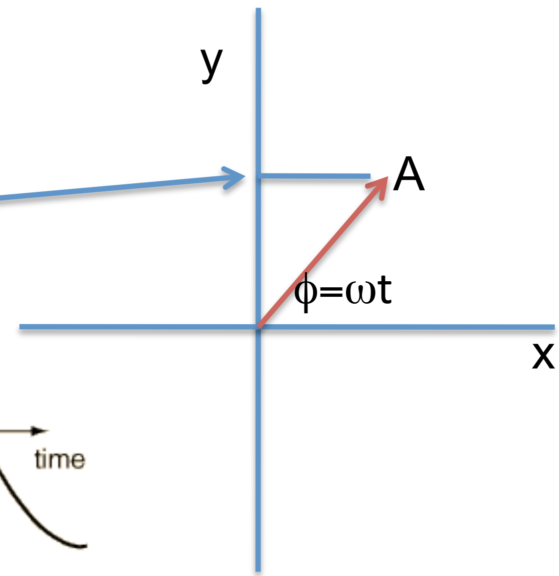
An oscillation is a time-varying disturbance.

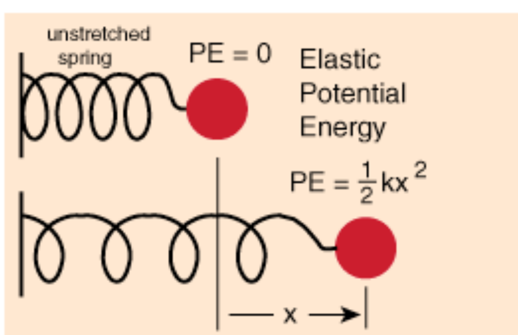
oscillation

Phasor Representation

$$y = A\sin\omega t = A\sin\sqrt{\frac{k}{m}}\,t$$







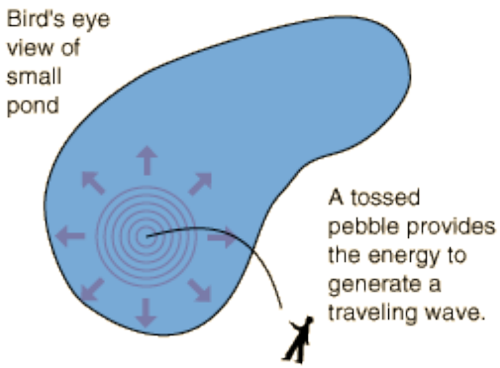
restoring force: $F = -kx$

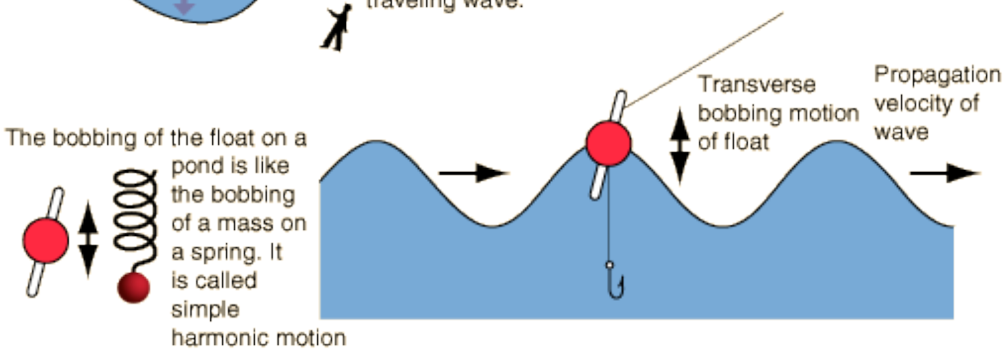
$$E = KE + PE = \frac{1}{2}kA^2$$

Travelling Waves

wave

A wave is a time-varying disturbance that also propagates in space.





The wave advances one wavelength λ while the float executes one period T. The relationship

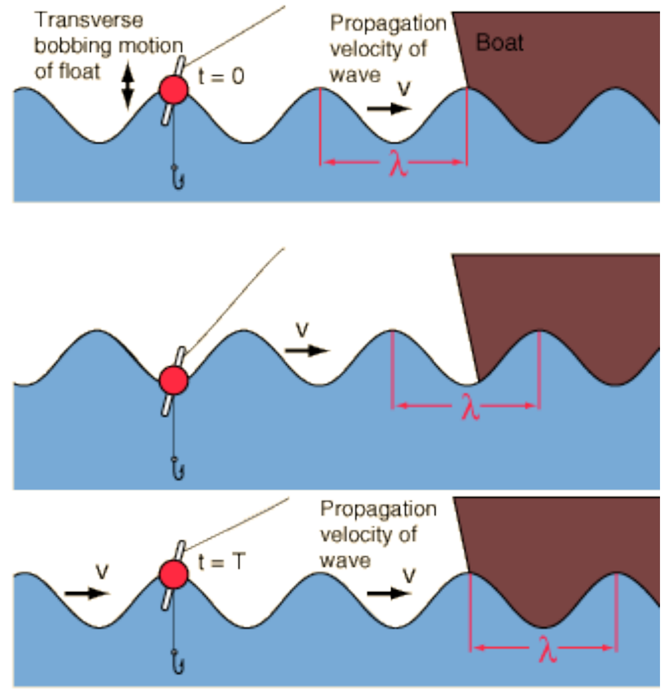
$$\lambda = vT$$

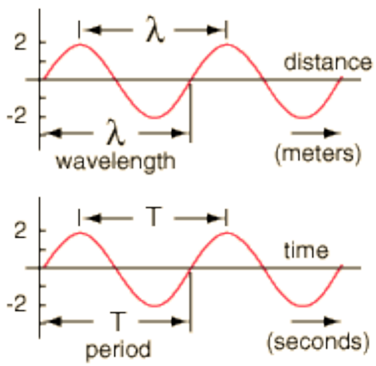
along with

$$T = 1/f$$

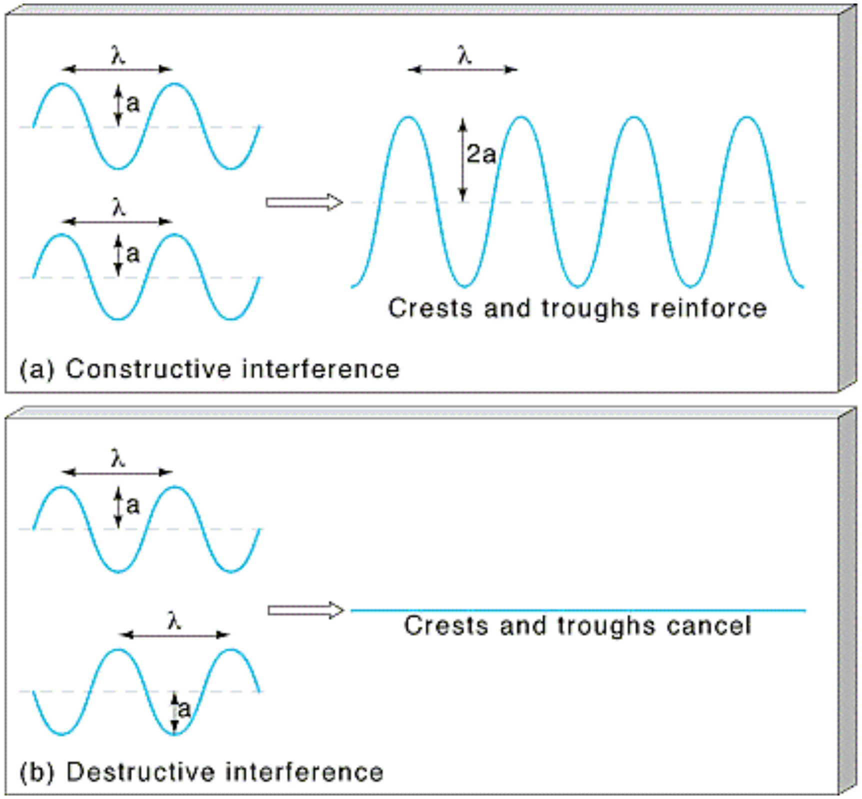
gives the usual form of the wave relationship

$$v = f\lambda$$



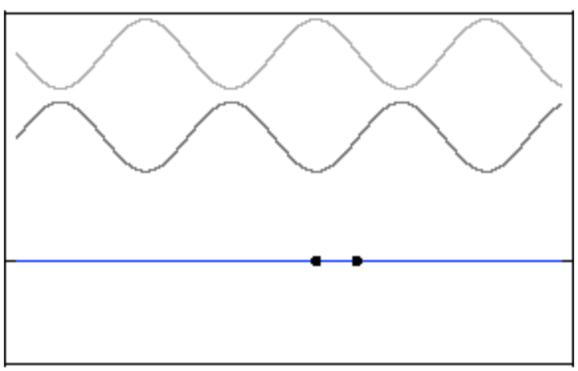


$$y = A\sin\omega t \text{ becomes}$$
$$y = A\sin(kx - \omega t)$$



Waves vs.Particles

Interference



waves can interfere (add or cancel).

Consequences are that:

1+1 not always 2 , refraction (bend corners), diffraction (spread out of hole), reflection..

waves are not localized and can be polarized

particles 1+1 makes 2, localized

Interference – Phase Shift

What can introduce a phase shift?

1. From different, out of phase sources
2. Sources in phase, but travel different distances
    1. Thin films
    2. Microwave Demonstration
    3. Double-slit or Diffraction grating

The Mathematics of Interference

$$R = A\cos(\omega t + \phi_1) + A\cos(\omega t + \phi_2)$$

$$\cos(a+b) = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$$

$$R = 2A\cos(\frac{\phi_1 - \phi_2}{2})\cos(\omega t + \frac{\phi_1 + \phi_2}{2})$$

$$A_R = 2A\cos(\frac{\phi_1 - \phi_2}{2}) \equiv 2A\cos(\frac{\Delta\phi}{2})$$

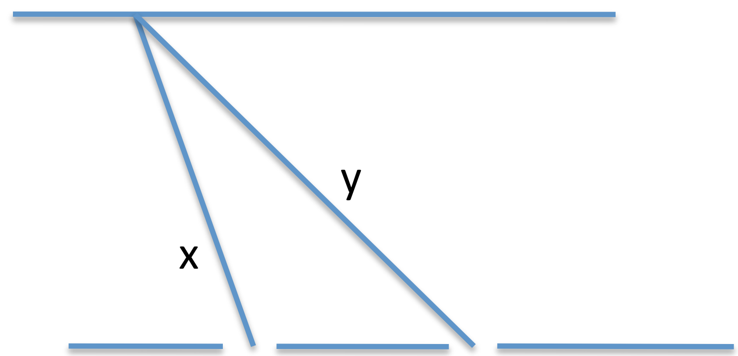
$$R = A_R\cos(\omega t + \frac{\phi_1 + \phi_2}{2})$$

$$I \propto A_R^2[\frac{1}{T}\int_0^T \cos^2(\omega T + \frac{\phi_1 + \phi_2}{2})] = A_R^2/2$$

$$I \propto A_R^2 = 4A^2\cos^2(\Delta\phi/2)$$

FOR UNEQUAL AMPLITUDES

$$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_2 - \phi_1)$$



$$R = A\cos(kx - \omega t) + A\cos(ky - \omega t)$$

$$R = 2A\cos[\frac{k(x-y)}{2}]\cos[\omega t + \frac{k(x+y)}{2}] =$$

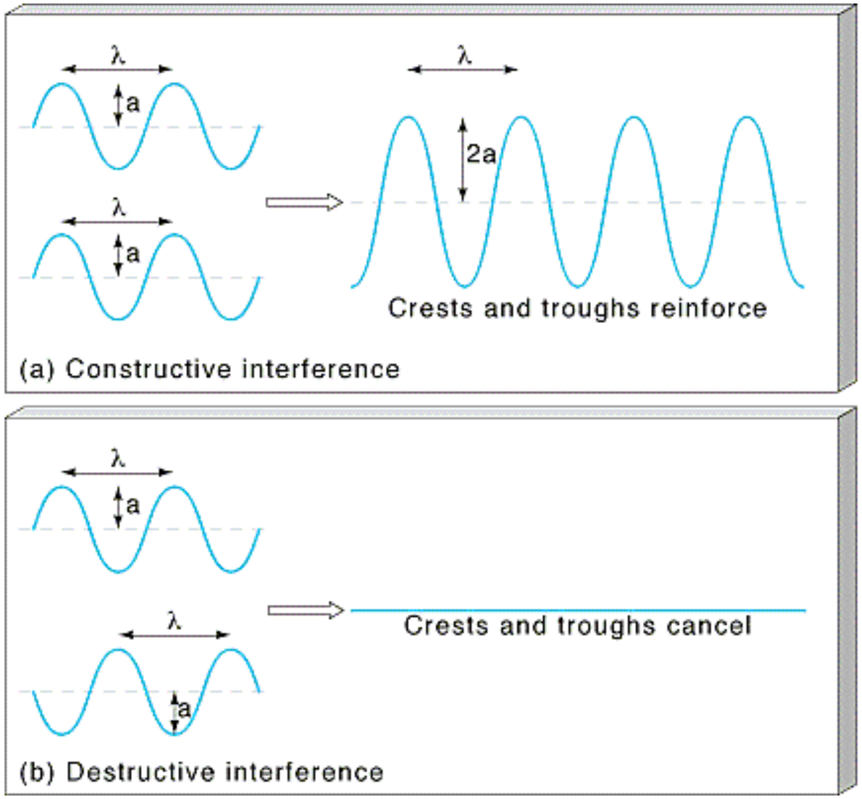
$$= A_R\cos[\omega t + \frac{k(x+y)}{2}]$$

$$A_R = 2A\cos[\frac{k(x-y)}{2}] = 2A\cos[\frac{2\pi(x-y)}{2\lambda}]$$

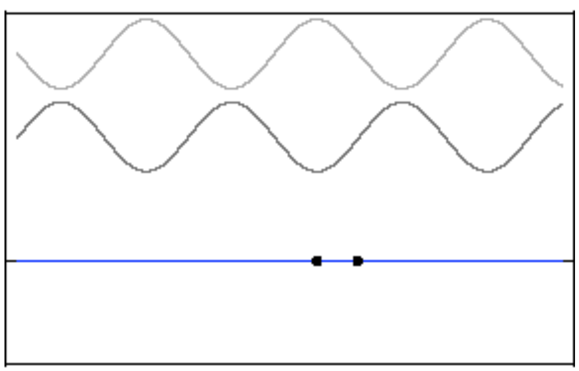
$$I \propto A_R^2 = 4A^2\cos^2[\pi\frac{(x-y)}{\lambda}] = 4A^2\cos^2[\pi\frac{\Delta r}{\lambda}]$$

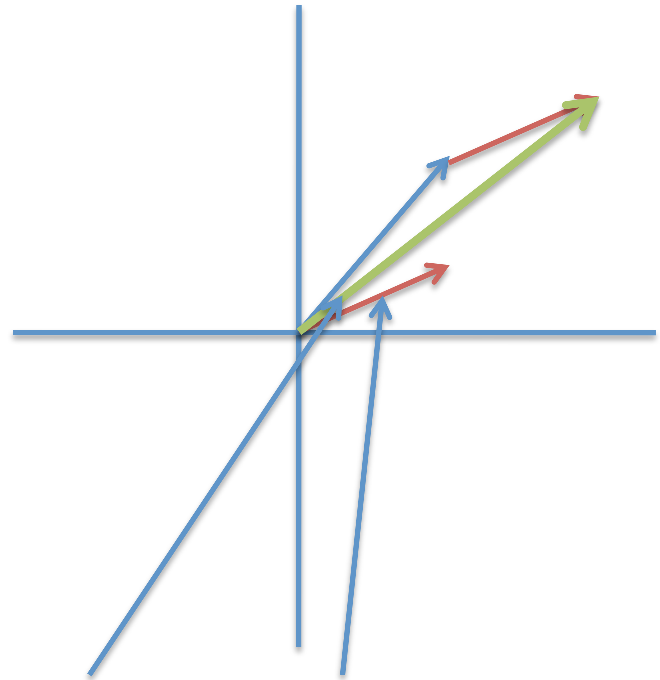
Interference

waves can interfere (add or cancel)



Standing Waves

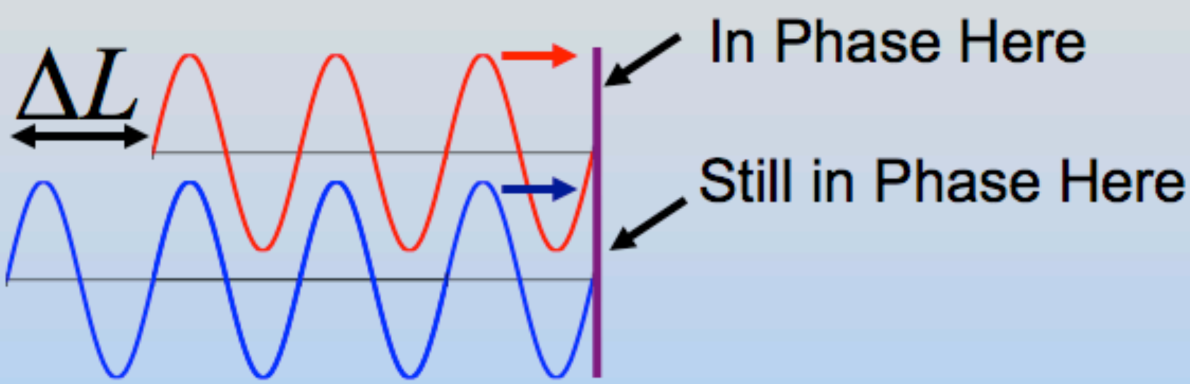




$$A = a_1 \cos(kx_1 - \omega t) + a_2 \cos(kx_2 - \omega t)$$

Vector addition of phasors

Extra Path Length

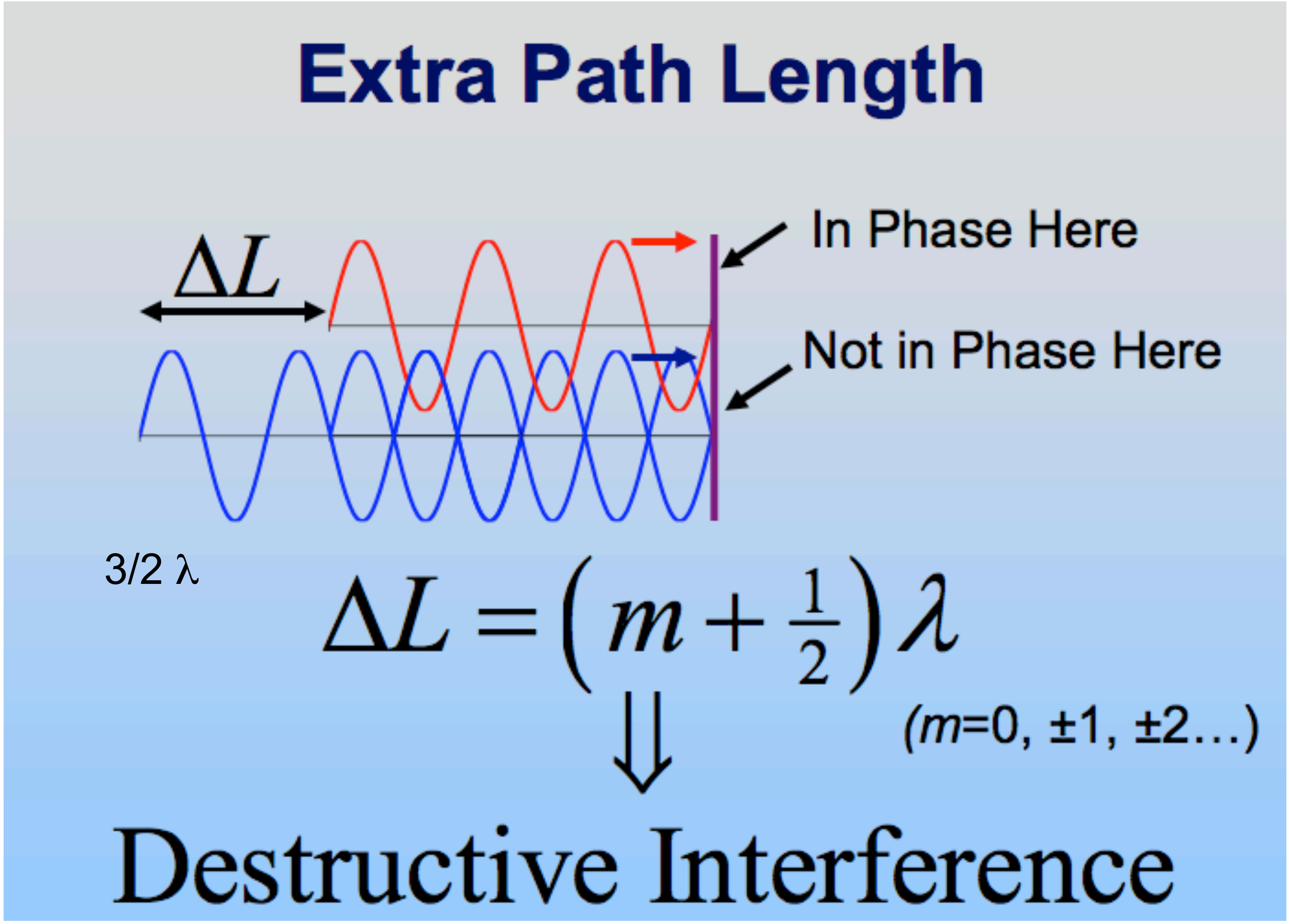


$$\Delta L = m\lambda \quad (m=0, \pm1, \pm2\ldots)$$

$$\Downarrow$$

Constructive Interference

$$I \propto 4A^2\cos^2[\pi\frac{\Delta L}{\lambda}] = 4A^2\cos^2[\pi\frac{m\lambda}{\lambda}] = 4A^2\cos^2[m\pi]$$

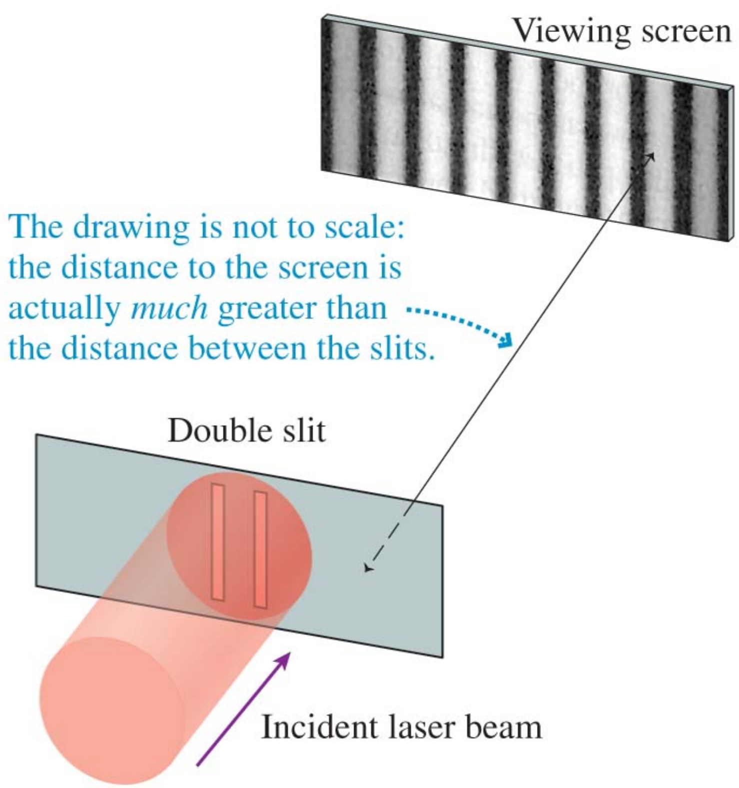


$$I \propto 4A^2\cos^2[\pi\frac{\Delta L}{\lambda}] = 4A^2\cos^2[\pi\frac{(m+1/2)\lambda}{\lambda}] = 4A^2\cos^2[m\pi + \pi/2] = 0$$

Interference in Waves

FIGURE 22.3 A double-slit interference experiment.

(a)



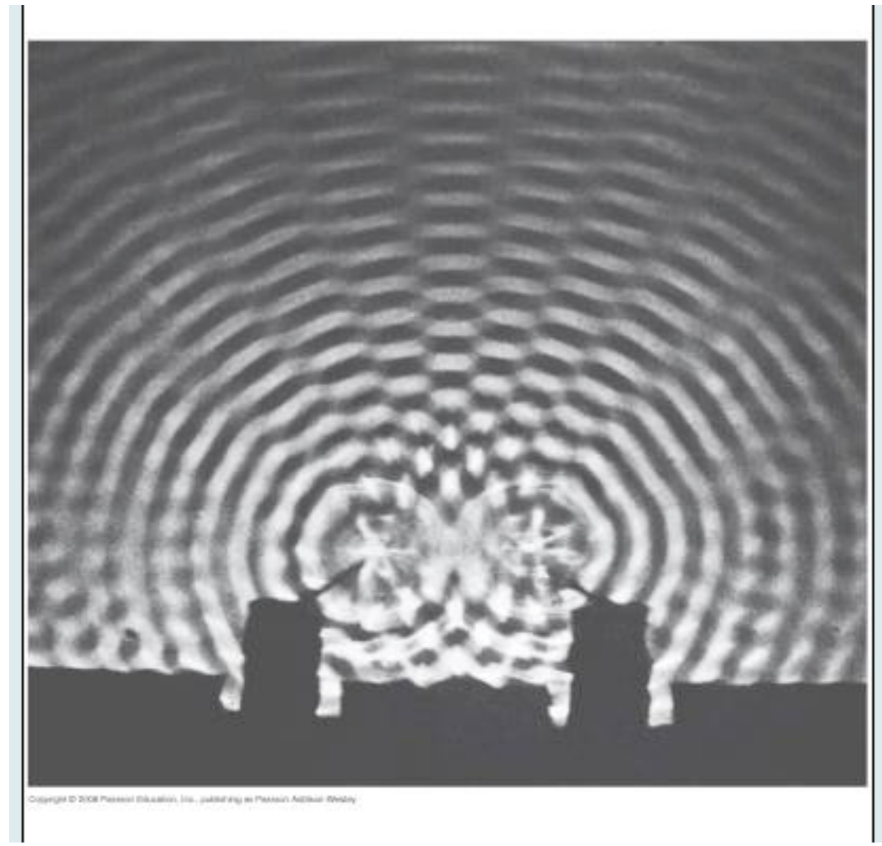


FIGURE 22.3 A double-slit interference experiment.

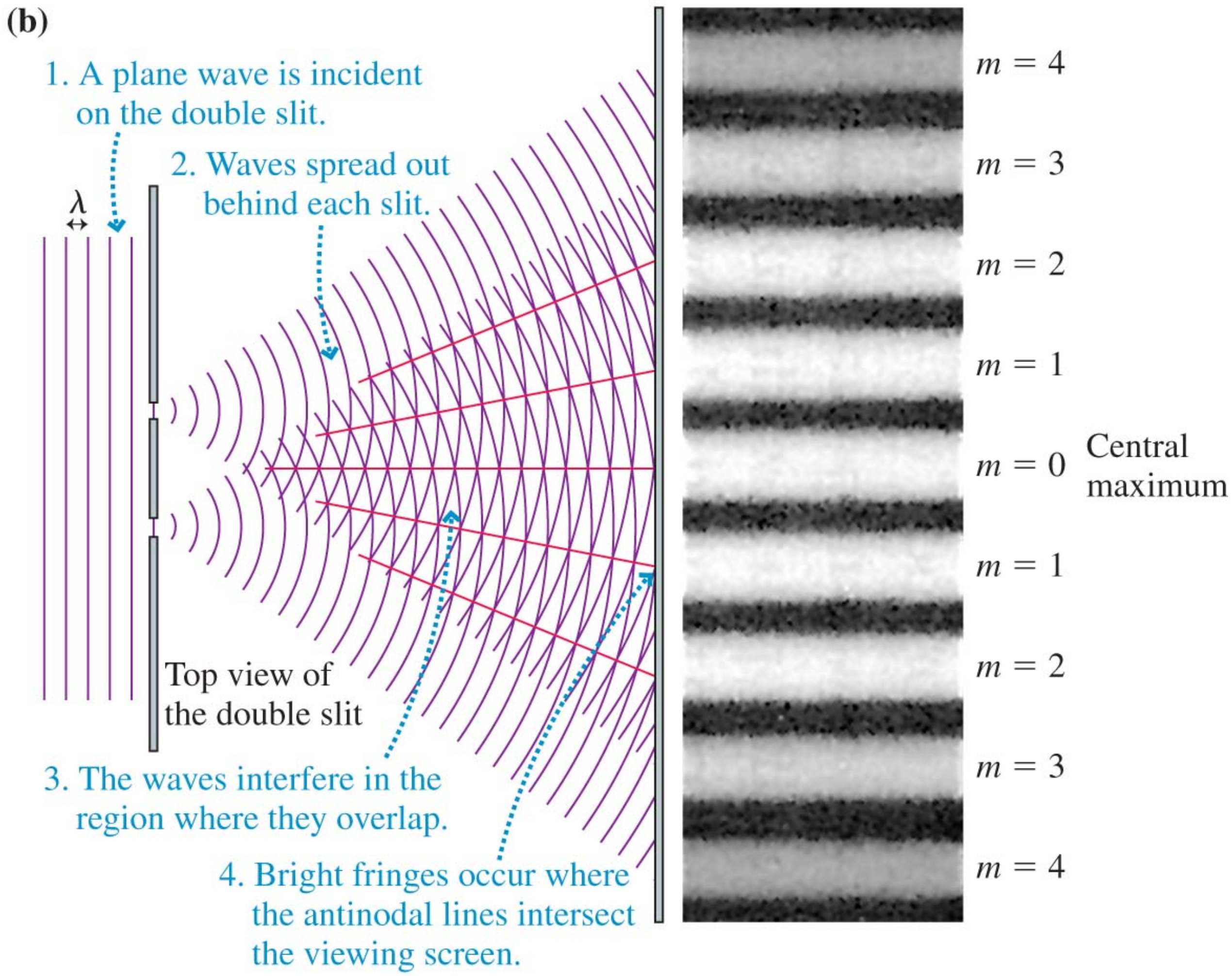
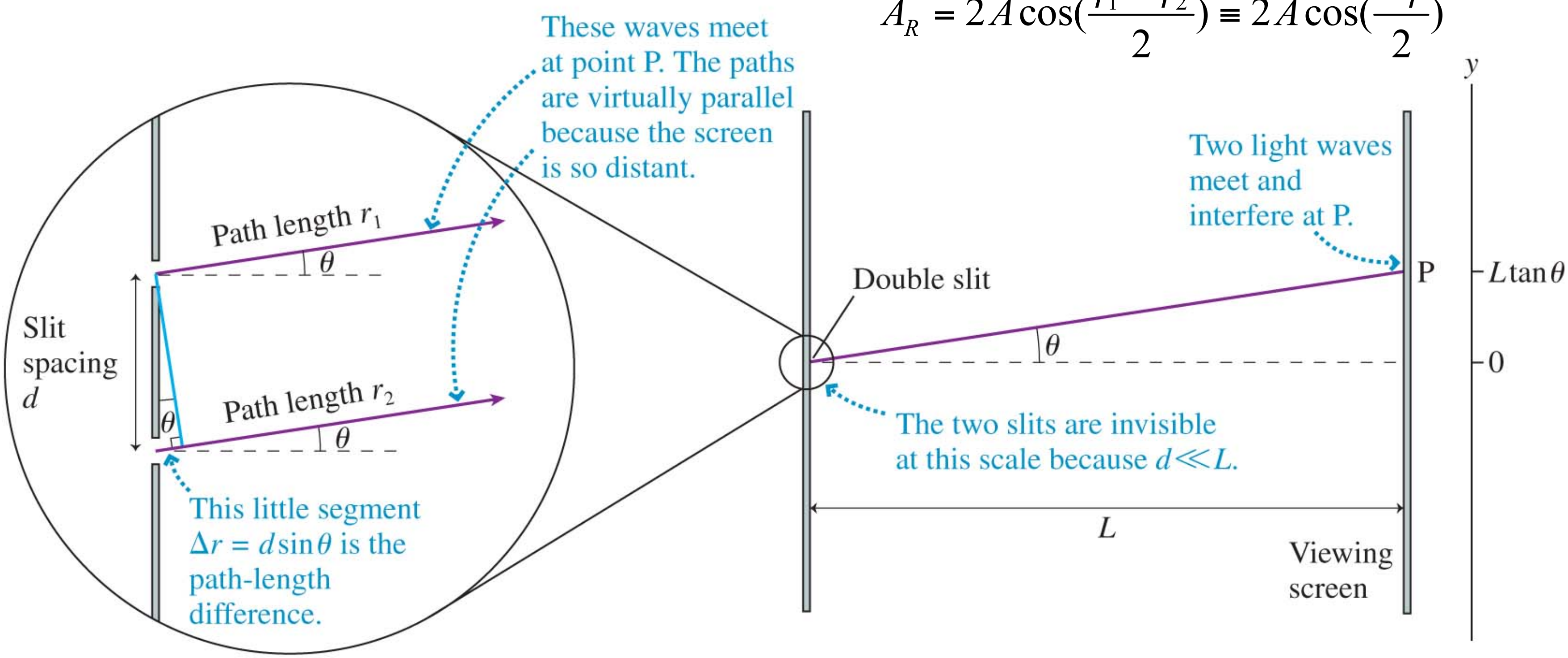


FIGURE 22.4 Geometry of the double-slit experiment.

$$A_R = 2A\cos\left(\frac{\phi_1 - \phi_2}{2}\right) \equiv 2A\cos\left(\frac{\Delta\phi}{2}\right)$$



$$\Delta\phi = k(r_2 - r_1) = k\Delta r = kd\sin\theta = 2\pi\frac{d}{\lambda}\sin\theta$$

$$\theta << 1 \rightarrow \sin\theta \approx \theta$$

$$Maximum \rightarrow \Delta\phi = 2\pi m, m = 0,1,2,3$$

$$\rightarrow 2\pi\frac{d}{\lambda}\sin\theta = 2\pi m \rightarrow \sin\theta_m = m\frac{\lambda}{d}$$

$$Maximum \rightarrow \theta_m \approx m\frac{\lambda}{d}, m = 0,1,2..$$

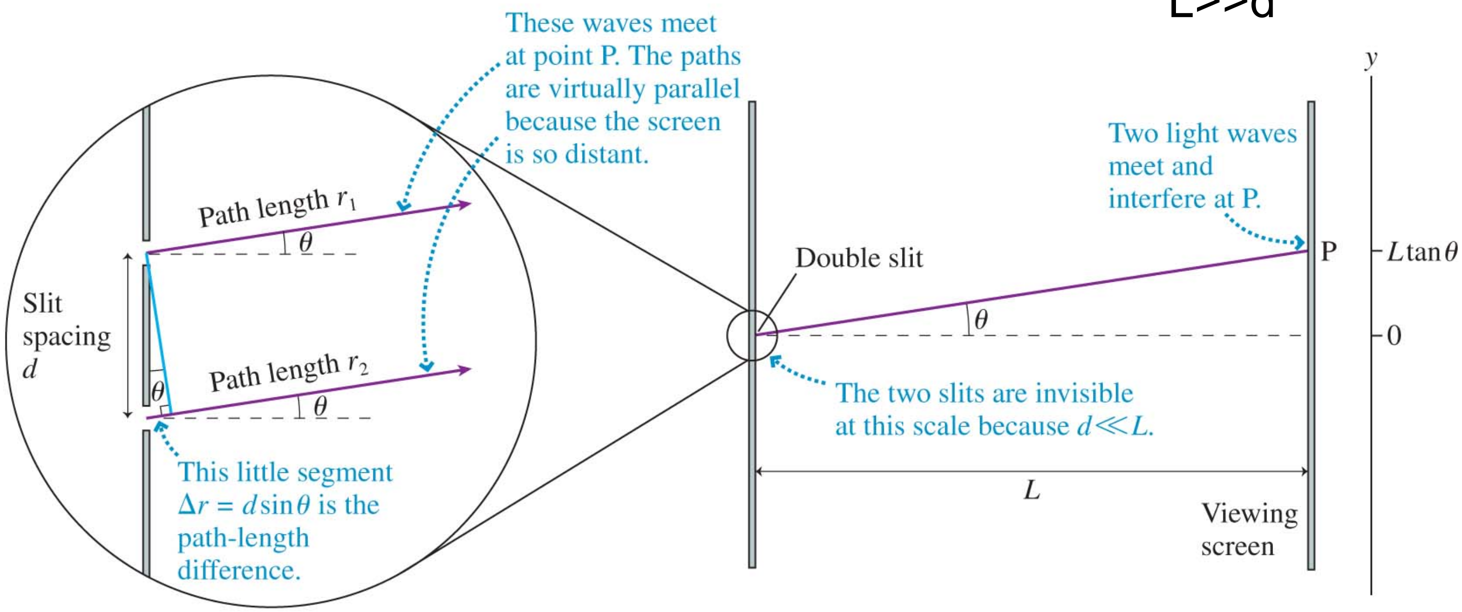
$$Minimum \rightarrow \Delta\phi = \pi m, m = 1,2,3$$

$$\rightarrow 2\pi\frac{d}{\lambda}\sin\theta = \pi m \rightarrow \sin\theta_m = m\frac{2\lambda}{d}$$

$$Minimum \rightarrow \theta_m \approx m\frac{2\lambda}{d}, m = 1,2,..$$

FIGURE 22.4 Geometry of the double-slit experiment.

L>>d



$$\theta << 1 \rightarrow \sin\theta \approx \theta$$

$$Maximum \rightarrow \theta_m \approx m\frac{\lambda}{d}, m = 0,1,2..$$

$$Minimum \rightarrow \theta_m \approx m\frac{2\lambda}{d}, m = 1,2,..$$

Positions of bright fringes

$$y = L\tan\theta \approx L\theta$$

$$y_m \approx L\theta_m = m\frac{\lambda L}{d}, m = 0,1,2,$$

$$\Delta y = y_{m+1} - y_m = [(m+1) - m]\frac{\lambda L}{d} = \frac{\lambda L}{d}$$

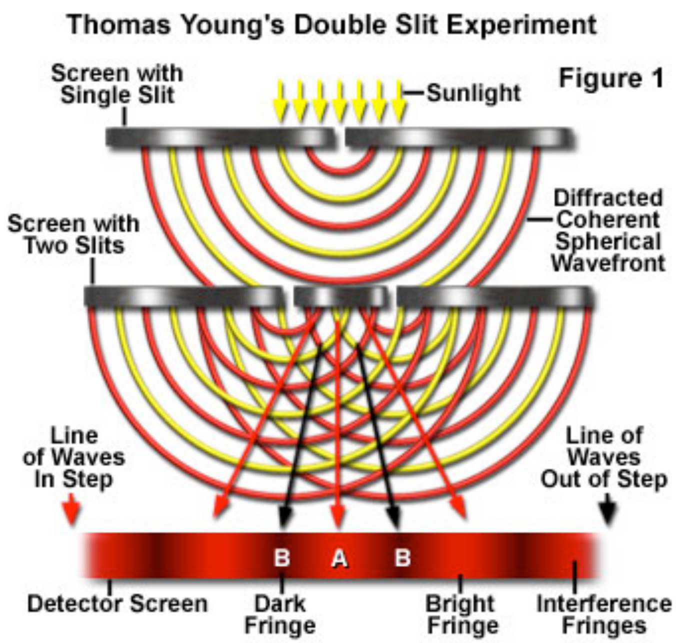
Independent of m - Same spacing

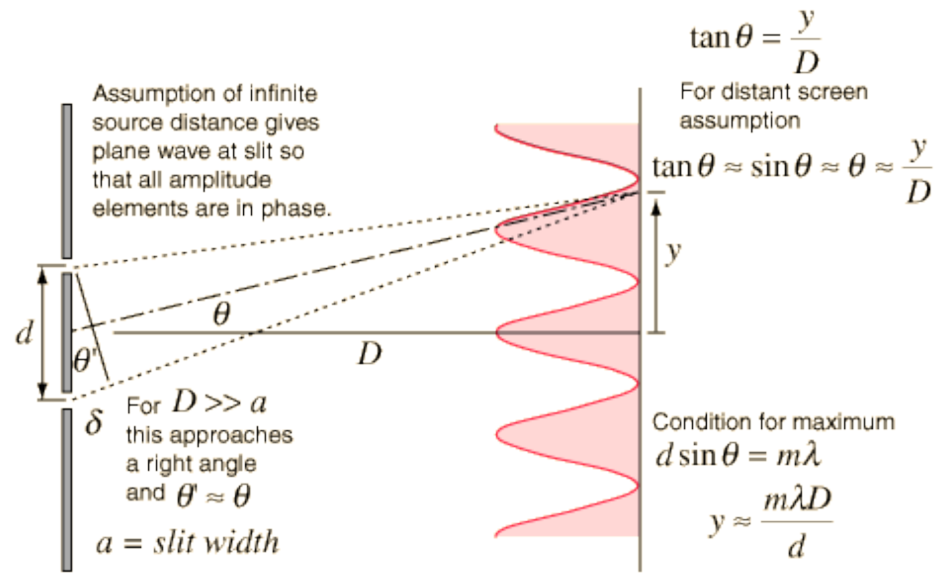
Positions of dark fringes

$$y_m^d \approx (m + \tfrac{1}{2})\frac{\lambda L}{d}, m = 0,1,2$$

Dark fringes exactly halfway between bright fringes

Double Slit experiment for light





Minima

$$d\sin\theta_m = (m+1/2)\lambda,\ \mathrm{m} = 0,1,2.,\ \text{dark}$$

$$\theta_m \approx (m+1/2)\lambda$$

Maxima

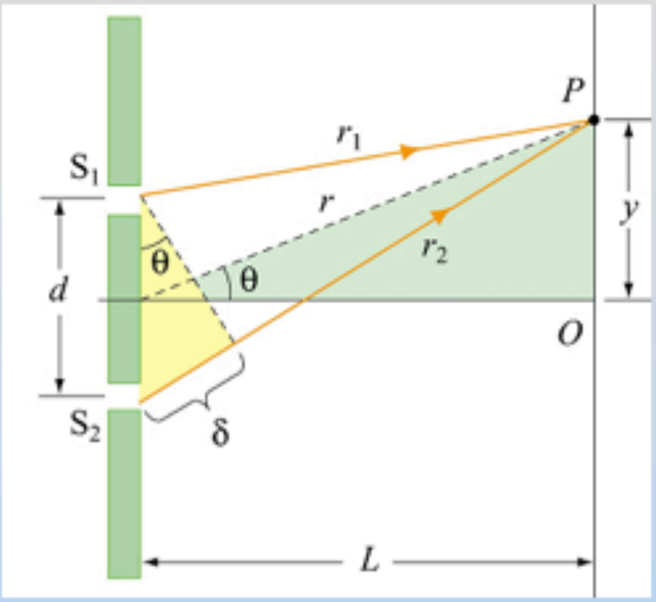
$$d\sin\theta_m = m\lambda,\ \ m = 0,1,2,3,..,bright$$

$$\theta_m = m\lambda/d$$

Dark fringes exactly midway of bright ones

How to measure the wavelength of light ?

How we measure 1/10,000 of a cm



Question: How do you measure the wavelength of light?

Answer: Do the same experiment we just did (with light)

$$\text{First } y_{destructive} = \lambda L / 2d$$

λ is smaller by 10,000 times.

But d can be smaller (0.1 mm instead of 0.24 m)

So y will only be 10 times smaller – **<u>still measurable</u>**

Analyzing Double-Slit Interference

The *m*th bright fringe emerging from the double slit is at an angle

$$\theta_m = m\frac{\lambda}{d} \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(angles of bright fringes)}$$

where θ_m is in radians, and we have used the small-angle approximation. The *y*-position on the screen of the *m*th fringe is

$$y_m = \frac{m\lambda L}{d} \qquad m = 0, 1, 2, 3, \ldots \qquad \text{(positions of bright fringes)}$$

while dark fringes are located at positions

$$y'_m = \left(m + \frac{1}{2}\right)\frac{\lambda L}{d} \qquad m = 0, 1, 2, \ldots$$

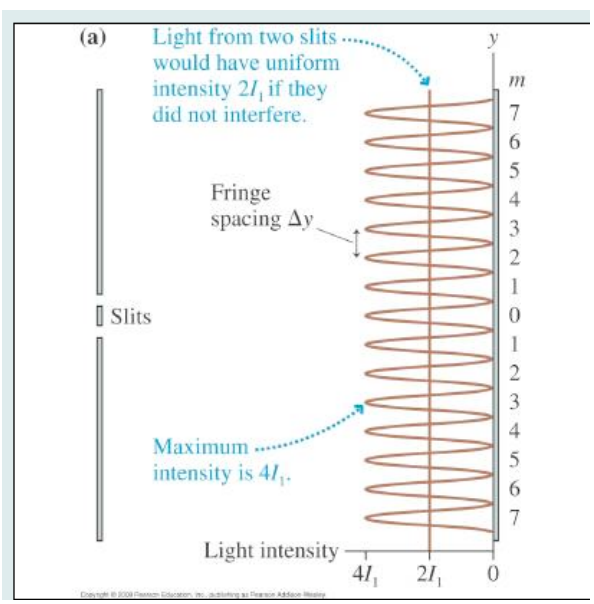
(positions of dark fringes)

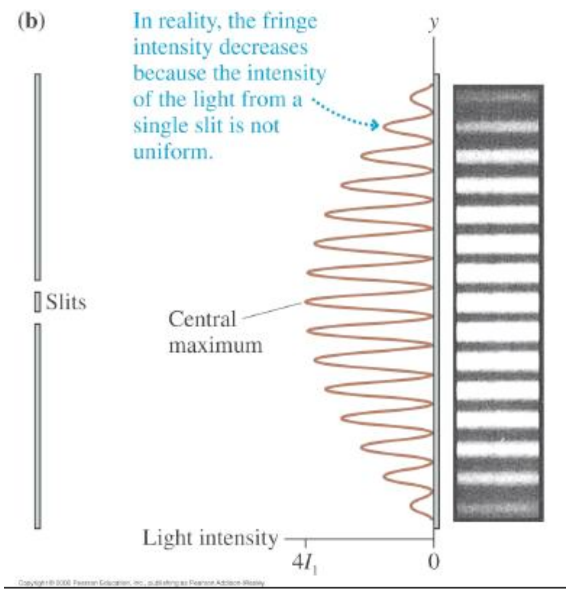
Double slit intensity pattern

$$I \propto A_R^2 = 4A^2 \cos^2(\Delta\phi/2)$$

$$\Delta\phi = 2\pi \frac{\Delta r}{\lambda} = 2\pi \frac{d\sin\theta}{\lambda} \approx 2\pi \frac{d\tan\theta}{\lambda}$$

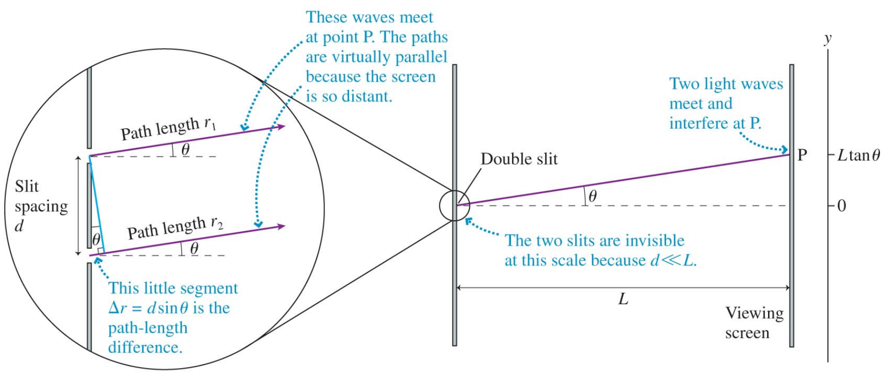
$$I_{double} = 4I_1 \cos^2(\frac{\pi d}{\lambda L} y)$$





Double slit intensity pattern

FIGURE 22.4 Geometry of the double-slit experiment.



If there was no interference intensity for two slits → $2I_1$

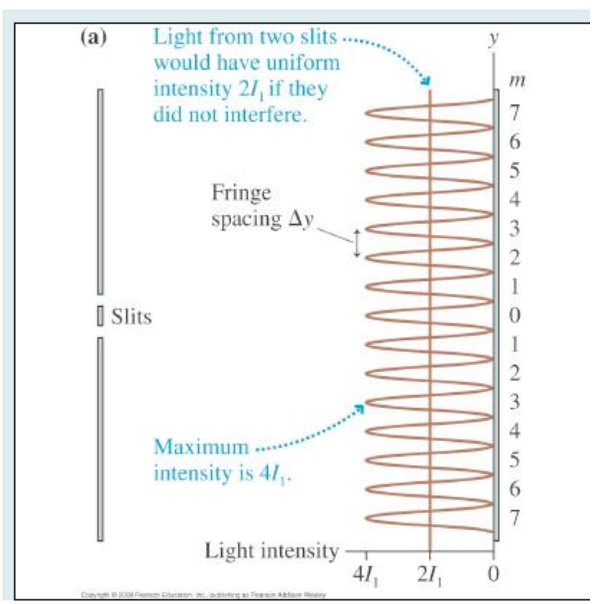
$$\Delta\phi = k(r_2 - r_1) = k\Delta r = kd\sin\theta = 2\pi\frac{d}{\lambda}\sin\theta \approx 2\pi\frac{d}{\lambda}\tan\theta = 2\pi\frac{d}{\lambda L}y$$

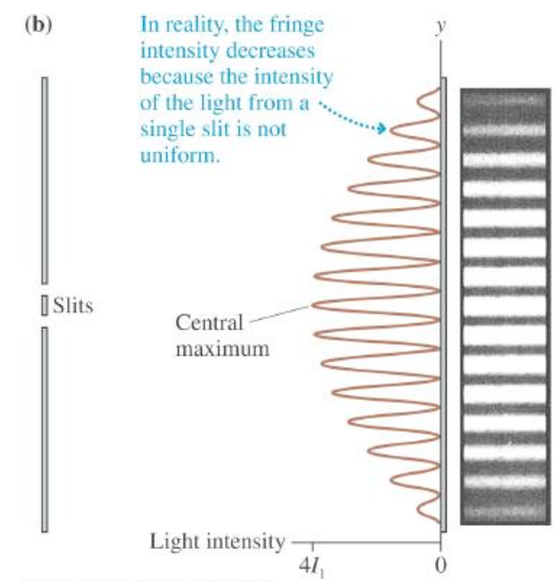
$$A = \left|2a\cos(\frac{\Delta\phi}{2})\right| = \left|2a\cos(\frac{\pi d}{\lambda L}y)\right|$$

$$I = cA^2 = 4ca^2\cos^2(\frac{\pi d}{\lambda L}y)$$

$$I_2 = 4I_1\cos^2(\frac{\pi d}{\lambda L}y)$$

I_1 light intensity of single slit





Interference of N overlapped waves

The Diffraction Grating

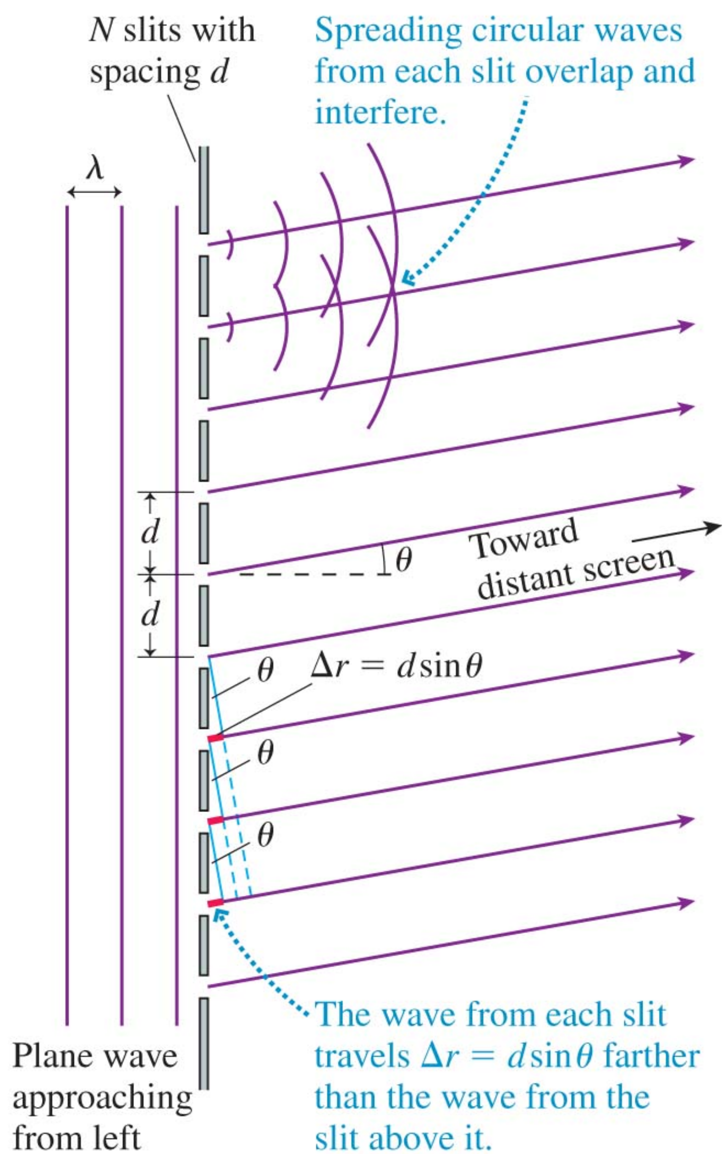
Suppose we were to replace the double slit with an opaque screen that has *N* closely spaced slits. When illuminated from one side, each of these slits becomes the source of a light wave that diffracts, or spreads out, behind the slit. Such a multi-slit device is called a **diffraction grating.** Bright fringes will occur at angles ϑ_m, such that

$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

The *y*-positions of these fringes will occur at

$$y_m = L \tan\theta_m \qquad \text{(positions of bright fringes)}$$

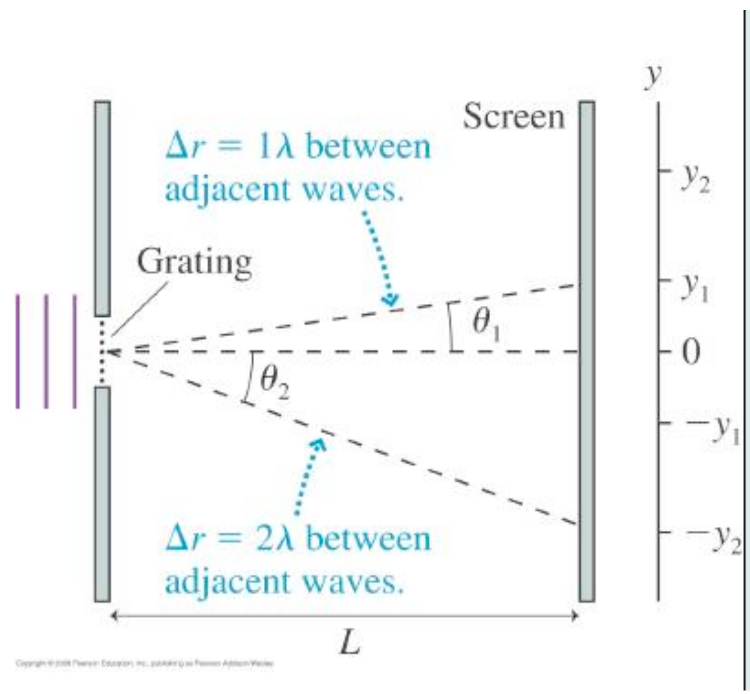
FIGURE 22.6 Top view of a diffraction grating with $N = 10$ slits.



$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$

$$y_m = L\tan\theta_m$$

m is the order of diffraction



$$I \propto A_R^2 = 4A^2\cos^2(\Delta\phi/2)$$

$$\Delta\phi = 2\pi\frac{\Delta r}{\lambda} = 2\pi\frac{d\sin\theta}{\lambda} \approx 2\pi\frac{d\tan\theta}{\lambda}$$

$$I_N = N^2 I_1 \cos^2\left(\frac{\pi d}{\lambda L}y\right)$$

The Physics of Two Slit Interference

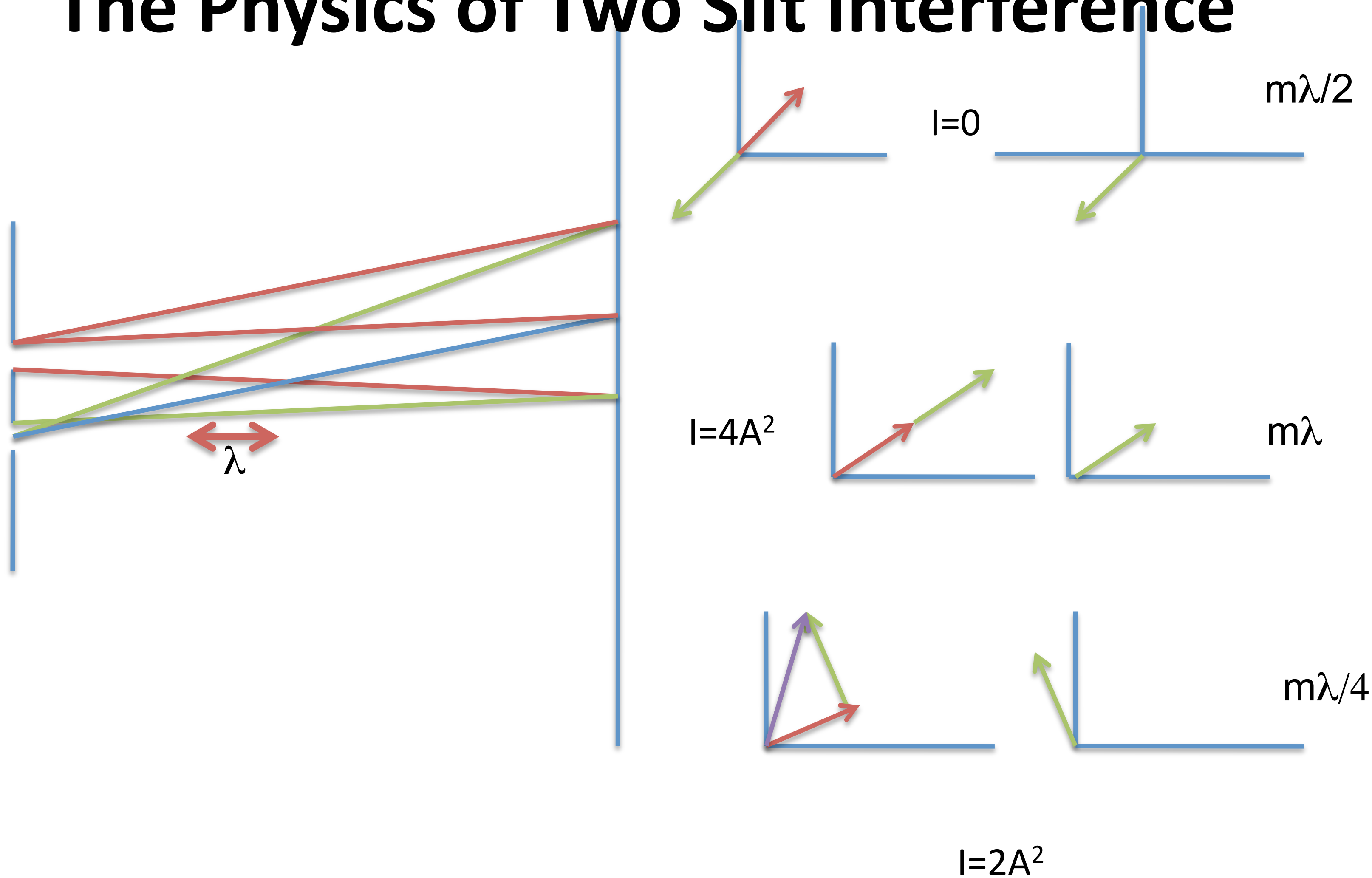
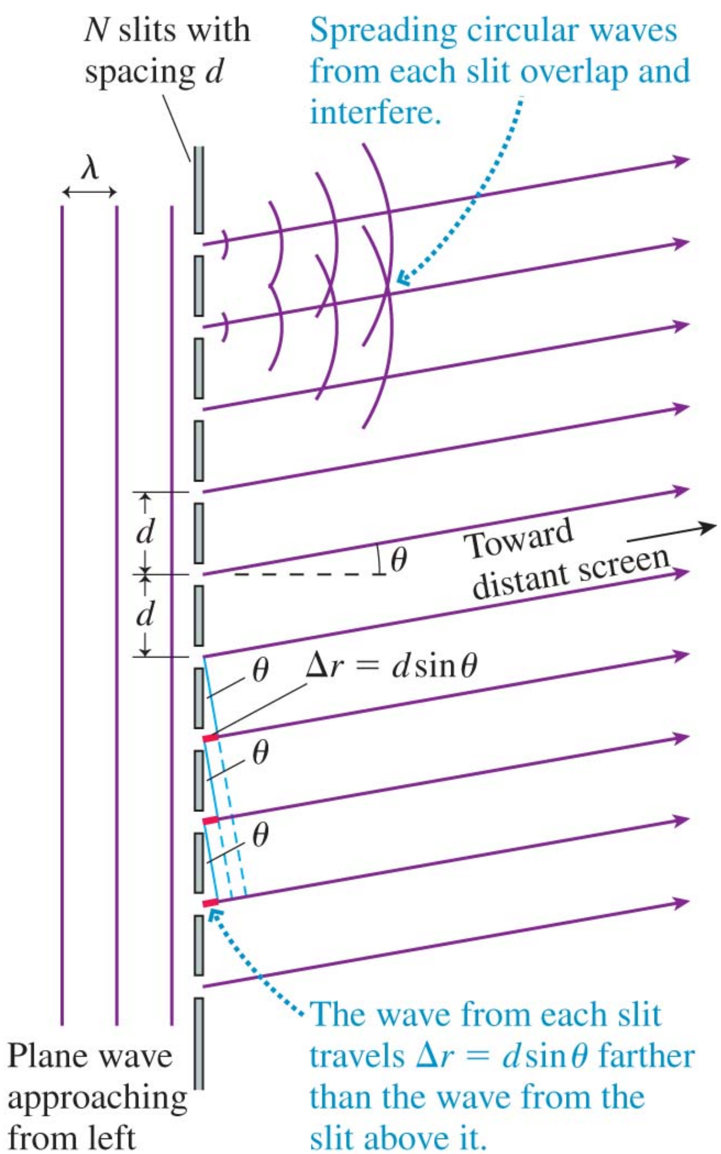
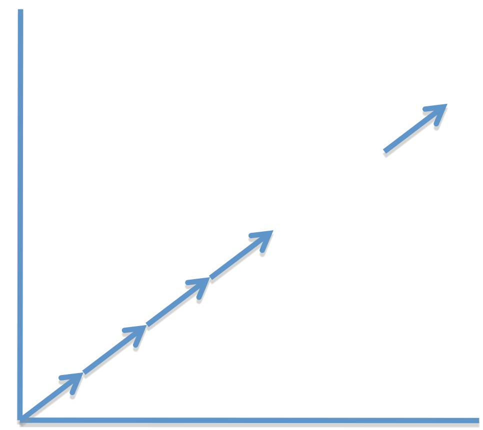


FIGURE 22.6 Top view of a diffraction grating with $N = 10$ slits.



$$d \sin\theta_m = m\lambda \qquad m = 0, 1, 2, 3, \ldots$$



$A_{total} = NA = 10A$

$I_{tot} = (NA)^2 = N^2 I_1 = 100 I_1$

Key dependence N^2