Chapter 22. Wave Optics

Light is an electromagnetic wave(!!!). The interference of light waves produces the colors reflected from a CD, the iridescence of bird feathers, and the technology underlying supermarket checkout scanners and optical computers.

Chapter Goal: To understand and apply the wave model of light.
Wave Optics

Topics:

- Light and Optics
- The Interference of Light
- The Diffraction Grating
- Single-Slit Diffraction
- Circular-Aperture Diffraction
- Interferometers
What is the nature of Light?

“All these fifty years of conscious brooding have brought me no nearer to the answer to the question ‘What are light quanta?’ Nowadays every rascal thinks he knows, but he is mistaken.”

— Albert Einstein, letter to Michel Besso, 1951.
Models of Light

• **The wave model:** under many circumstances, light exhibits the same behavior as sound or water waves. The study of light as a wave is called *wave optics*.

• **The ray model:** The properties of prisms, mirrors, and lenses are best understood in terms of *light rays*. The ray model is the basis of *ray optics*.

• **The photon model:** In the quantum world, light behaves like neither a wave nor a particle. Instead, light consists of *photons* that have both wave-like and particle-like properties. This is the *quantum theory* of light.
Light diffracts when traversing apertures like the water waves depicted below.

At a beach in Tel Aviv, Israel, plane water waves pass through two openings in a breakwall. Notice the diffraction effect—the waves exit the openings with circular wave fronts, as in Figure 37.1b. Notice also how the beach has been shaped by the circular wave fronts.
FIGURE 22.2 Light, just like a water wave, does spread out behind a hole if the hole is sufficiently small.

The light spreads out behind the slit.

Incident laser beam

0.1-mm-wide slit in an opaque screen

2.5 cm

2 m
The invention of radio?

Hertz proves that light is really an electromagnetic wave.

Waves could be generated in one circuit, and electric pulses with the same frequency could be induced in an antenna some distance away.

These electromagnetic waves could be reflected, and refracted, focused, polarized, and made to interfere—just like light!
“Okay...what you have to realize is...to first order...everything is a simple harmonic oscillator. Once you've got that, it's all downhill from there.”
Waves 101 Part 1: Oscillations

An oscillation is a time-varying disturbance.

oscillation

Phasor Representation

restoring force: \( F = -kx \)

\[
E = KE + PE = \frac{1}{2}kA^2
\]
A wave is a time-varying disturbance that also propagates in space.

The wave advances one wavelength $\lambda$ while the float executes one period $T$. The relationship

$$\lambda = vT$$

along with

$$T = \frac{1}{f}$$

gives the usual form of the wave relationship

$$v = f\lambda$$

$y = A \sin \omega t$ becomes

$$y = A \sin (kx - \omega t)$$
Waves vs. Particles

Interference

waves can interfere (add or cancel).

Consequences are that:

1+1 not always 2, refraction (bend corners), diffraction (spread out of hole), reflection..

waves are not localized and can be polarized

particles 1+1 makes 2, localized
Interference – Phase Shift

What can introduce a phase shift?

1. From different, out of phase sources
2. Sources in phase, but travel different distances
   1. Thin films
   2. Microwave Demonstration
   3. Double-slit or Diffraction grating
The Mathematics of Interference

\[ R = A \cos(\omega t + \phi_1) + A \cos(\omega t + \phi_2) \]
\[ \cos(a + b) = 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) \]
\[ R = 2A \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \cos(\omega t + \frac{\phi_1 + \phi_2}{2}) \]
\[ A_R = 2A \cos\left(\frac{\phi_1 - \phi_2}{2}\right) = 2A \cos\left(\frac{\Delta \phi}{2}\right) \]

\[ R = A_R \cos(\omega t + \frac{\phi_1 + \phi_2}{2}) \]
\[ I \propto A_R^2 \left[ \frac{1}{T} \int_0^T \cos^2(\omega T + \frac{\phi_1 + \phi_2}{2}) \right] = A_R^2 / 2 \]
\[ I \propto A_R^2 = 4A^2 \cos^2(\Delta \phi / 2) \]

FOR UNEQUAL AMPLITUDES

\[ A_R^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\phi_2 - \phi_1) \]
\[ R = A \cos(kx - \omega t) + A \cos(ky - \omega t) \]
\[ R = 2A \cos\left(\frac{k(x-y)}{2}\right) \cos\left[\omega t + \frac{k(x+y)}{2}\right] = A_r \cos[\omega t + \frac{k(x+y)}{2}] \]
\[ A_r = 2A \cos\left(\frac{k(x-y)}{2}\right) = 2A \cos\left(\frac{2\pi(x-y)}{2\lambda}\right) \]
\[ I \propto A_r^2 = 4A^2 \cos^2\left[\frac{2\pi(x-y)}{\lambda}\right] = 4A^2 \cos^2\left[\frac{\Delta r}{\lambda}\right] \]
Standing Waves

(a) Constructive interference

(b) Destructive interference

Interference

Waves can interfere (add or cancel)

\[ A = a_1 \cos(kx_1 - \omega t) + a_2 \cos(kx_2 - \omega t) \]

Vector addition of phasors
\[ \Delta L = m\lambda \quad (m=0, \pm1, \pm2 \ldots) \]

\[ I \propto 4A^2 \cos^2\left[\pi \frac{\Delta L}{\lambda}\right] = 4A^2 \cos^2\left[\pi \frac{m\lambda}{\lambda}\right] = 4A^2 \cos^2[m\pi] \]
\[ \Delta L = (m + \frac{1}{2}) \lambda \]

\[ I \propto 4A^2 \cos^2[\pi \frac{\Delta L}{\lambda}] = 4A^2 \cos^2[\pi \frac{(m + 1/2)\lambda}{\lambda}] = 4A^2 \cos^2[m\pi + \pi/2] = 0 \]
Interference in Waves

FIGURE 22.3 A double-slit interference experiment.

The drawing is not to scale: the distance to the screen is actually much greater than the distance between the slits.
FIGURE 22.3 A double-slit interference experiment.

(b)

1. A plane wave is incident on the double slit.
2. Waves spread out behind each slit.
3. The waves interfere in the region where they overlap.
4. Bright fringes occur where the antinodal lines intersect the viewing screen.
\[
\Delta \phi = k(r_2 - r_1) = k\Delta r = kd \sin \theta = 2\pi \frac{d}{\lambda} \sin \theta
\]

Maximum \(\rightarrow\) \(\Delta \phi = 2\pi m, m = 0,1,2,3\)

\[\rightarrow 2\pi \frac{d}{\lambda} \sin \theta = 2\pi m \rightarrow \sin \theta_m = m \frac{\lambda}{d}\]

Minimum \(\rightarrow\) \(\Delta \phi = \pi m, m = 1,2,3\)

\[\rightarrow 2\pi \frac{d}{\lambda} \sin \theta = \pi m \rightarrow \sin \theta_m = m \frac{2\lambda}{d}\]

\[
A_r = 2A \cos\left(\frac{\phi_1 - \phi_2}{2}\right) = 2A \cos\left(\frac{\Delta \phi}{2}\right)
\]

\[\theta \ll 1 \rightarrow \sin \theta \approx \theta\]

Maximum \(\rightarrow\) \(\theta_m \approx m \frac{\lambda}{d}, m = 0,1,2,\ldots\)

Minimum \(\rightarrow\) \(\theta_m \approx m \frac{2\lambda}{d}, m = 1,2,\ldots\)
\[ \theta \ll 1 \rightarrow \sin \theta \approx \theta \]

**Maximum** \( \theta_m \approx \frac{m \lambda}{d} \), \( m = 0, 1, 2, \ldots \)

**Minimum** \( \theta_m \approx \frac{2m \lambda}{d} \), \( m = 1, 2, \ldots \)

\[ y = L \tan \theta \approx L \theta \]

\[ y_m = L \theta_m = m \frac{\lambda L}{d}, m = 0, 1, 2, \ldots \]

\[ \Delta y = y_{m+1} - y_m = [(m + 1) - m] \frac{\lambda L}{d} = \frac{\lambda L}{d} \]

*Independent of \( m \) - Same spacing*

**Positions of dark fringes**

\[ y_m^d = (m + \frac{1}{2}) \frac{\lambda L}{d}, m = 0, 1, 2 \]

Dark fringes exactly halfway between bright fringes
**Double Slit experiment for light**

**Minima**

\[ d \sin \theta_m = (m + 1/2)\lambda, \ m = 0,1,2., \ \text{dark} \]

\[ \theta_m \approx (m + 1/2)\lambda \]

**Maxima**

\[ d \sin \theta_m = m\lambda, \ m = 0,1,2,3,..,\text{bright} \]

\[ \theta_m = m\lambda / d \]

Dark fringes exactly midway of bright ones

How to measure the wavelength of light?
How we measure 1/10,000 of a cm

**Question:** How do you measure the wavelength of light?

**Answer:** Do the same experiment we just did (with light)

First \( y_{destructive} = \frac{\lambda L}{2d} \)

\( \lambda \) is smaller by 10,000 times.

But \( d \) can be smaller (0.1 mm instead of 0.24 m)

So \( y \) will only be 10 times smaller – **still measurable**
Analyzing Double-Slit Interference

The $m$th bright fringe emerging from the double slit is at an angle

$$\theta_m = m \frac{\lambda}{d} \quad m = 0, 1, 2, 3, \ldots \quad \text{(angles of bright fringes)}$$

where $\theta_m$ is in radians, and we have used the small-angle approximation. The $y$-position on the screen of the $m$th fringe is

$$y_m = \frac{m\lambda L}{d} \quad m = 0, 1, 2, 3, \ldots \quad \text{(positions of bright fringes)}$$

while dark fringes are located at positions

$$y'_m = \left( m + \frac{1}{2} \right) \frac{\lambda L}{d} \quad m = 0, 1, 2, \ldots$$

(positions of dark fringes)
Double slit intensity pattern

\[ I \propto A_R^2 = 4 A^2 \cos^2(\Delta\phi/2) \]

\[ \Delta\phi = 2\pi \frac{\Delta r}{\lambda} = 2\pi \frac{d \sin\theta}{\lambda} \approx 2\pi \frac{d \tan\theta}{\lambda} \]

\[ I_{\text{double}} = 4I_1 \cos^2\left(\frac{\pi d}{\lambda L} y\right) \]
Double slit intensity pattern

If there was no interference intensity for two slits → 2I₁

\[ \Delta \phi = k(r_2 - r_1) = k\Delta r = kd \sin \theta = 2\pi \frac{d}{\lambda} \sin \theta \approx 2\pi \frac{d}{\lambda} \tan \theta = 2\pi \frac{d}{\lambda L} y \]

\[ A = \left| 2a \cos \left( \frac{\Delta \phi}{2} \right) \right| = \left| 2a \cos \left( \frac{\pi d}{\lambda L} y \right) \right| \]

\[ I = cA^2 = 4ca^2 \cos^2 \left( \frac{\pi d}{\lambda L} y \right) \]

\[ I_2 = 4I_1 \cos^2 \left( \frac{\pi d}{\lambda L} y \right) \]

I₁ light intensity of single slit
The **Diffraction Grating**

Suppose we were to replace the double slit with an opaque screen that has \( N \) closely spaced slits. When illuminated from one side, each of these slits becomes the source of a light wave that diffracts, or spreads out, behind the slit. Such a multi-slit device is called a **diffraction grating**. Bright fringes will occur at angles \( \theta_m \), such that

\[
d \sin \theta_m = m\lambda \quad m = 0, 1, 2, 3, \ldots
\]

The \( y \)-positions of these fringes will occur at

\[
y_m = L \tan \theta_m \quad \text{(positions of bright fringes)}
\]
**FIGURE 22.6** Top view of a diffraction grating with \( N = 10 \) slits.

\[
d\sin\theta_m = m\lambda ~ \quad m = 0, 1, 2, 3, \ldots
\]

\[
y_m = L\tan\theta_m
\]

\( m \) is the order of diffraction
The Physics of Two Slit Interference

\[ I = A^2 \]

\[ \lambda \]

\[ \lambda / 2 \]

\[ \lambda / 4 \]
FIGURE 22.6 Top view of a diffraction grating with $N = 10$ slits.

$$d \sin \theta_m = m \lambda \quad m = 0, 1, 2, 3, \ldots$$

Key dependence $N^2$

$A_{\text{total}} = NA = 10A$

$I_{\text{total}} = (NA)^2 = N^2 I_1 = 100I_1$