PHYS 270-SPRING 2011

**Dennis Papadopoulos**

# LECTURE # 21

## RELATIVITY II

LENGTH CONTRACTION

SPACE TIME INTERVALS

LORENTZ TRANSFORMATIONS

MASS-ENERGY EQUIVALENCE

APRIL 26, 2011

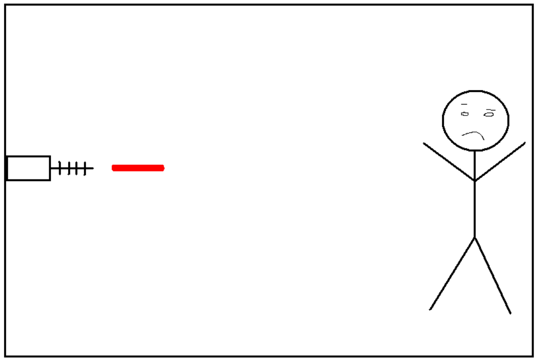
# Special Relativity

- Einstein's postulates X
- Simultaneity X
- Time dilation X
- Length contraction
- Space-Time intervals
- Lorentz transformation
- Examples

Mr. Tompkins by George Gamow

# The laser gun experiment

- Suppose there is a laser gun at one end of spacecraft, targeted at a victim at the other end.



- Laser gun fires (event A) and then victim gets hit (event B).
- Can we change the order of these events by changing the frame of reference? i.e., can the victim get hit **before** the gun fires?

# Causality

- This  is a question of **causality**.
- The events described are **causally-connected** (i.e. one event can, and does, affect the other event).
- In fact, it is **not possible** to change the order of these events by changing frames, according to Special Relativity theory.
- This is true provided that
  - The laser bolt does not travel faster than the speed of light
  - We do not change to a frame of reference that is going faster than the speed of light
- To preserve the Principle of Causality (cause precedes effect, never vice versa), the speed of light must set the upper limit to the speed of anything in the Universe.

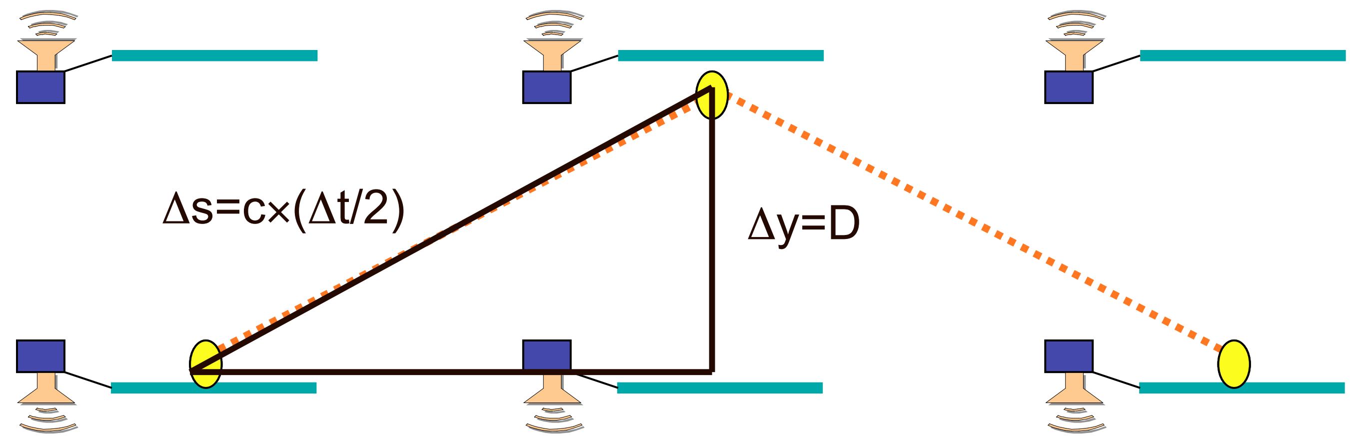
# EINSTEIN'S POSTULATES OF RELATIVITY

- **Postulate 1** – The laws of physics are the same in all inertial frames of reference
- **Postulate 2** – The speed of light in a vacuum is the same in all inertial frames of reference.

Simultaneity relative to the observer; causality restricts speeds to below c;

Time dilation – proper time shortest time interval can be measured by a single clock.

# Time Dilation




$\Delta t = 2\Delta s/c$

An astronaut will measure $\Delta t_o$**=**$2D/c$

You can easily show that

$\Delta t/\Delta t_o = \Delta s/D = \gamma$

# Time Dilation

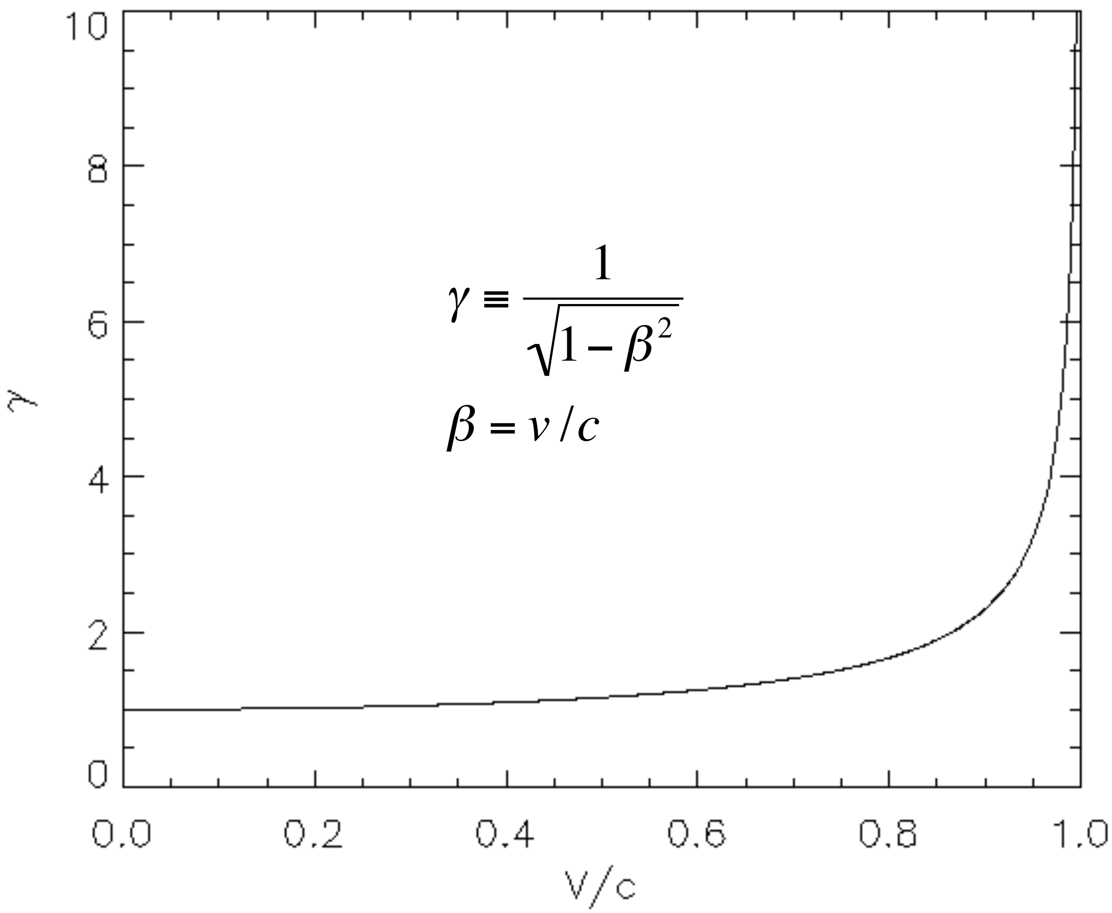
The time interval between two events that occur at the *same position* is called the **proper time** $\Delta\tau$. In an inertial reference frame moving with velocity $v = \beta c$ relative to the proper time frame, the time interval between the two events is

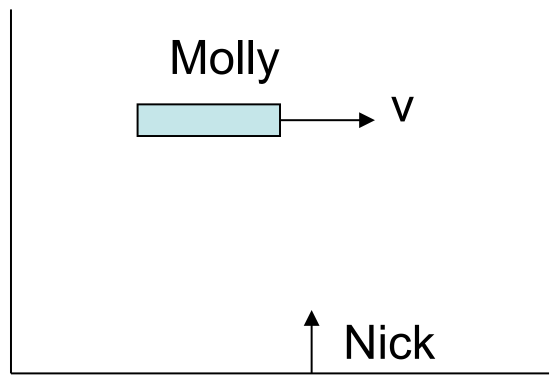
$$\Delta t = \frac{\Delta\tau}{\sqrt{1-\beta^2}} \geq \Delta\tau \qquad \text{(time dilation)}$$

$$\Delta t = \gamma\Delta\tau$$

The "stretching out" of the time interval is called **time dilation.**

# Lorentz factor





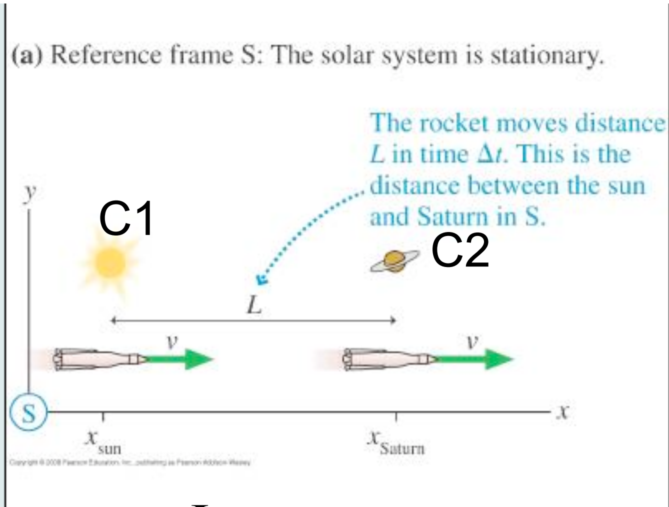

1. Nick measures the time it takes for Molly's rocket from nose to tail to fly past him as $dt_N$

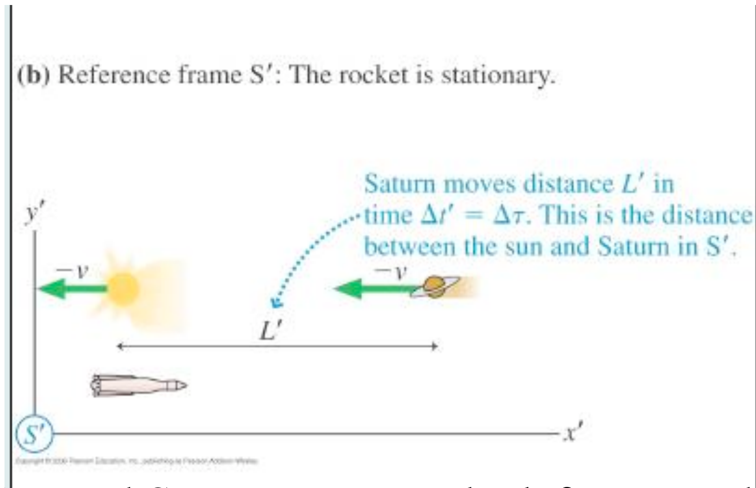
2.Molly measures the time it takes for the rocket to fly past Nick as $dt_M$

3. Do they measure the same time and if not which is shorter?

Nick's time is the proper time since his clock does not change position. Molly on the other hand had to use two clocks one in the nose and one in the tail. $dt_M=\gamma dt_N$. $dt_N$ is proper time and the shortest interval. It is the one and only reference frame that the clock is at rest.

# Length Contraction




$$L = x_{Sat} - x_{Sun}$$

$$v = L / \Delta t$$

Two clocks $L, \Delta t$ in frame S

Sun and Saturn move to the left at speed

$$v = L' / \Delta t'$$

In rocket frame single clock

$v$ is relative speed of systems S and S'

Must be the same when measured from either frame

$$v = L / \Delta t = L' / \Delta t'$$

$$\Delta t' = \Delta \tau = \Delta t / \gamma$$

$$L' = L / \gamma = L\sqrt{1 - \beta^2} < L$$

Proper length l is distance between two points measured in a reference frame at rest to the objects

# Length Contraction

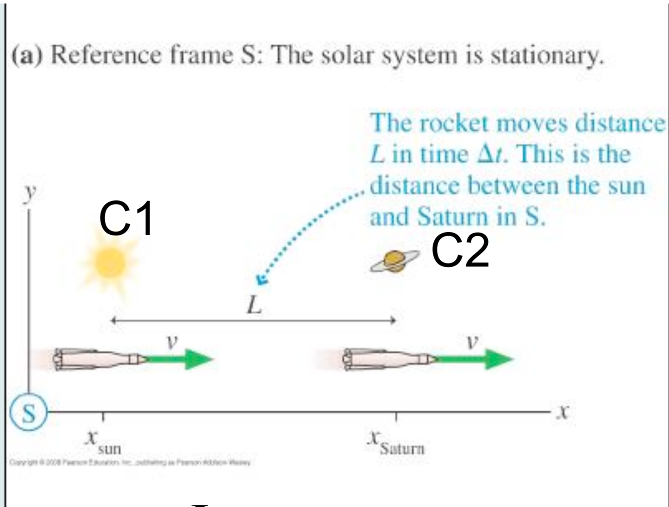
The distance $L$ between two objects, or two points on one object, measured in the reference frame S in which the objects are at rest is called the **proper length** $\ell$. The distance $L'$ in a reference frame S′ is

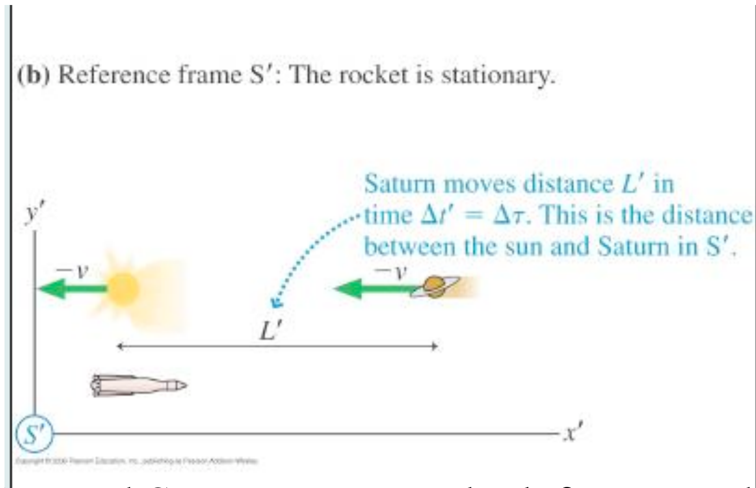
$$L' = \sqrt{1-\beta^2}\,\ell \le \ell$$

$$L' = l/\gamma$$

- So, moving observers see that objects contract in the direction of motion.
- **Length contraction**… also called
  - Lorentz contraction
  - FitzGerald contraction

# Length Contraction




$$L = x_{Sat} - x_{Sun}$$

$$v = L/\Delta t$$

Two clocks $L, \Delta t$ in frame S

*Sun* and Saturn move to the left at speed

$$v = L'/\Delta t'$$

*In* rocket frame single clock

S exps: L=1.43x$10^{12}$ m, it takes 5300 sec to travel the distance L->v=.9c=2.7x$10^{8}$ m/sec. S' exps: It takes only 2310 s to reach Saturn after they passed the Sun. No conflict because the distance is only .62x$10^{12}$ m. Saturn's speed towards them is again .9 c.

$$L = L_p / \gamma$$

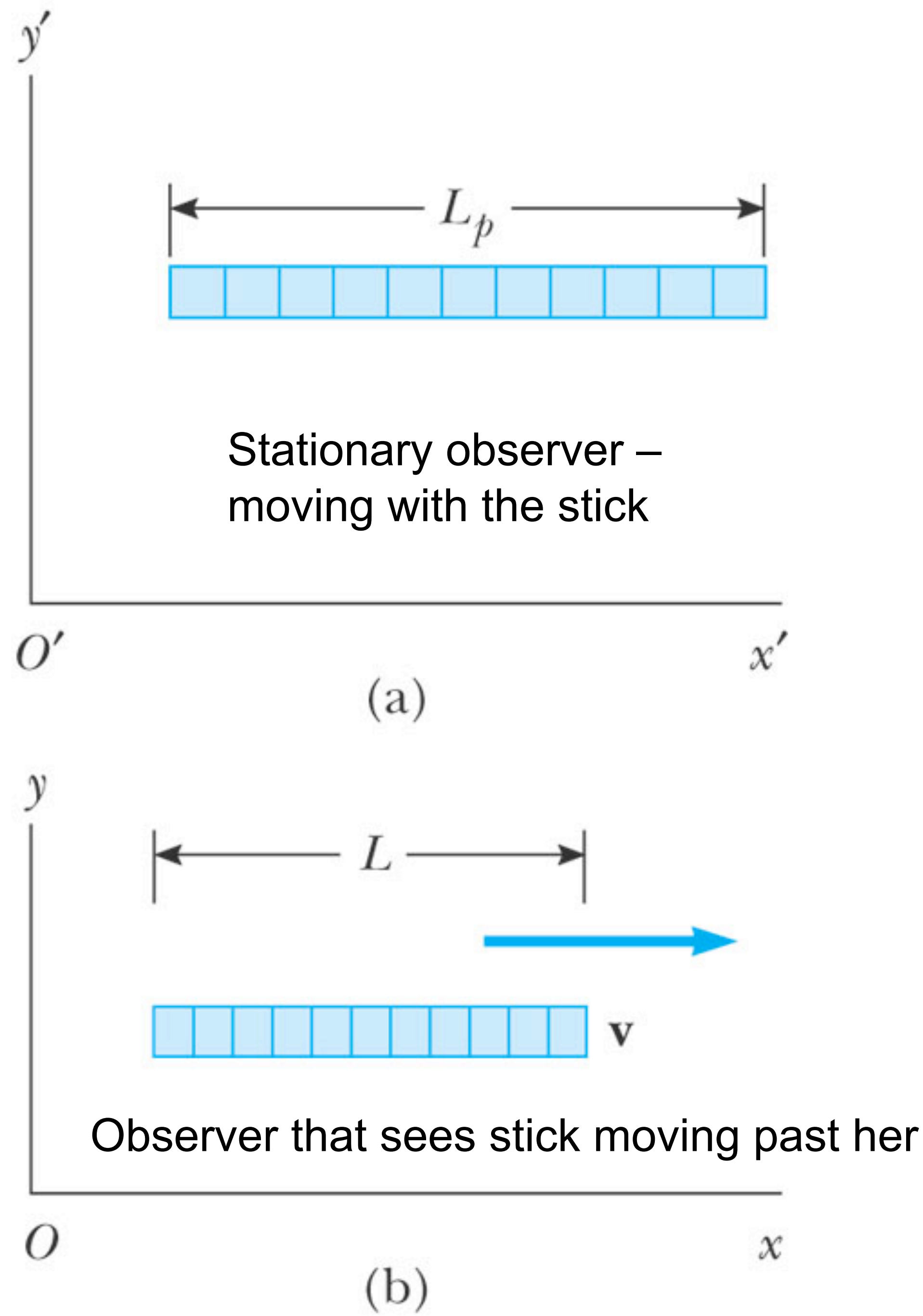





Fig. 1-13, p. 19

Contraction ONLY in the direction of motion
Transverse direction do not change

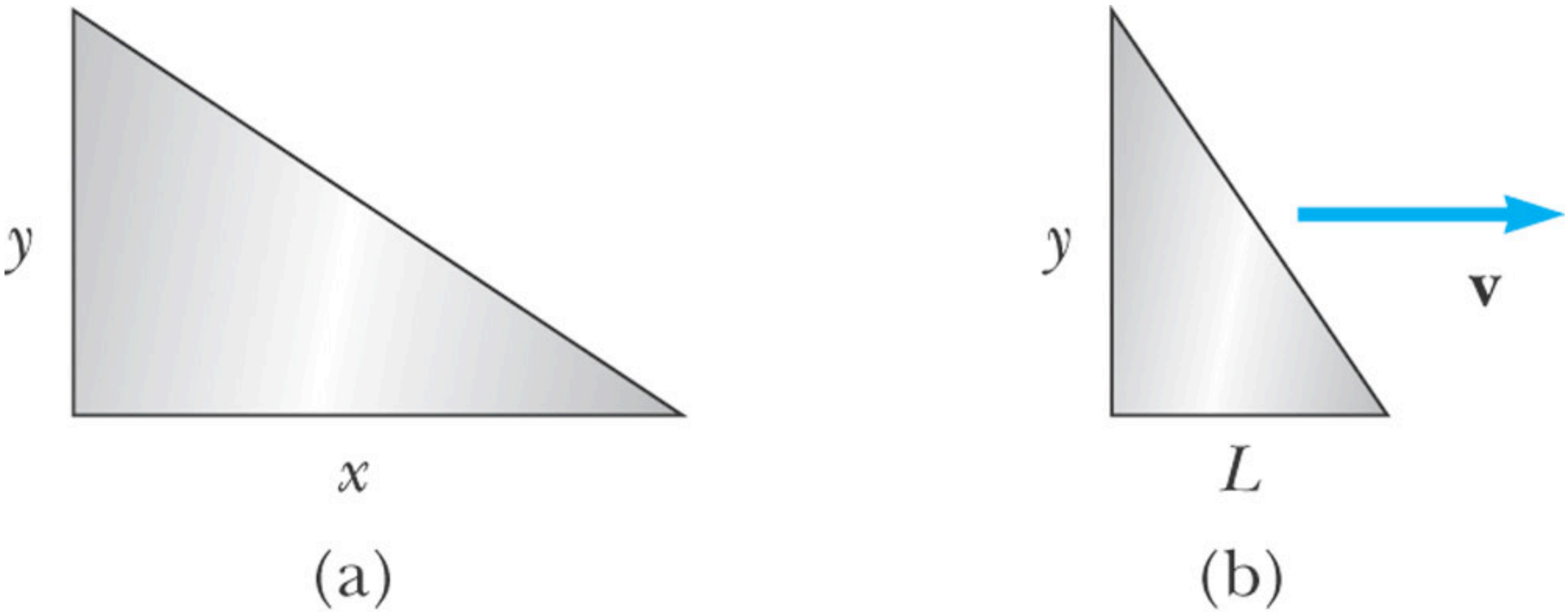





Fig. 1-15, p. 21

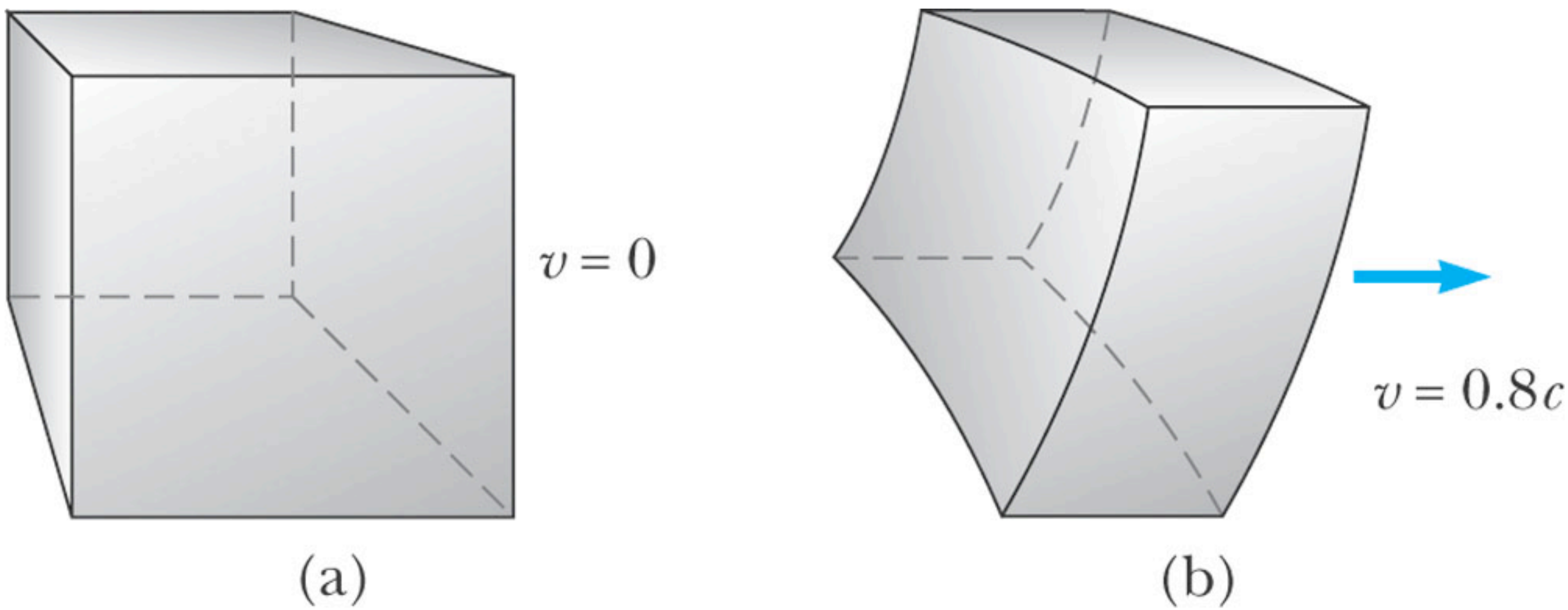





Fig. 1-14, p. 20

Tom Sceptic has been transferred to a world in which the velocity of light, c, amounts to only 20 mls/hour. Suddenly he passes in the road a car which appears to be completely compressed. At the steering wheel sits a driver who appears to be squashed flat.

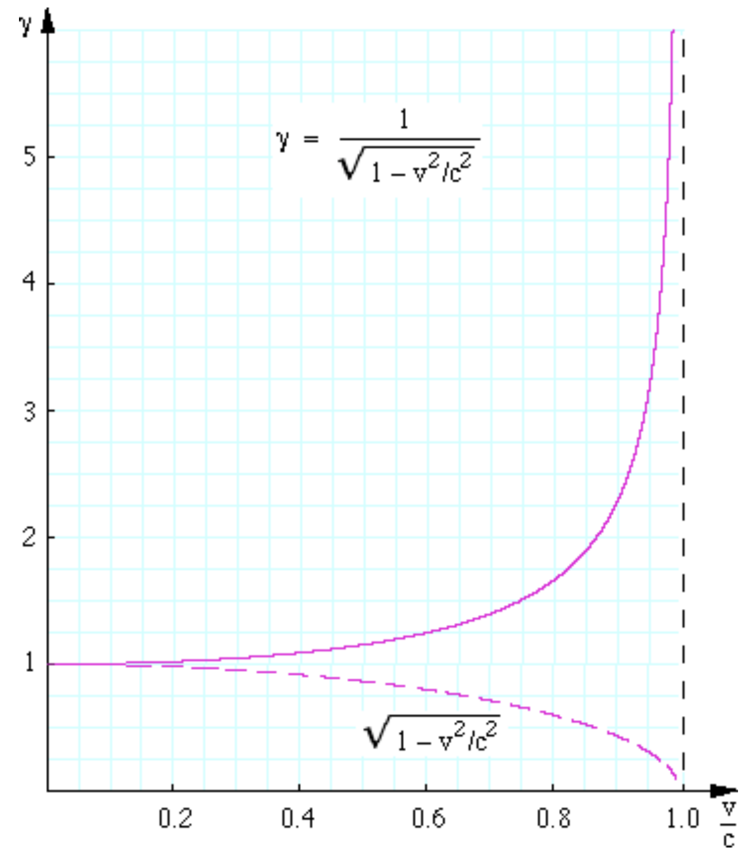
## 6. An Encounter with Several Dimensions

This was the scene that presented itself to Sceptic as he drove at a speed of approximately 17 mls/hour through Einstein-town.

*Everything is slowed/contracted by a factor of:*

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

*in a frame moving with respect to the observer.*

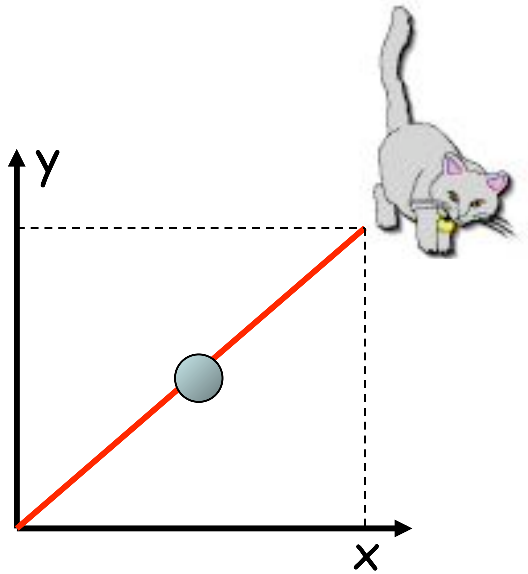



**Time always runs slower when measured by an observer moving with respect to the clock.**

**The length of an object is always shorter when viewed by an observer who is moving with respect to the object.**

# Coordinate systems

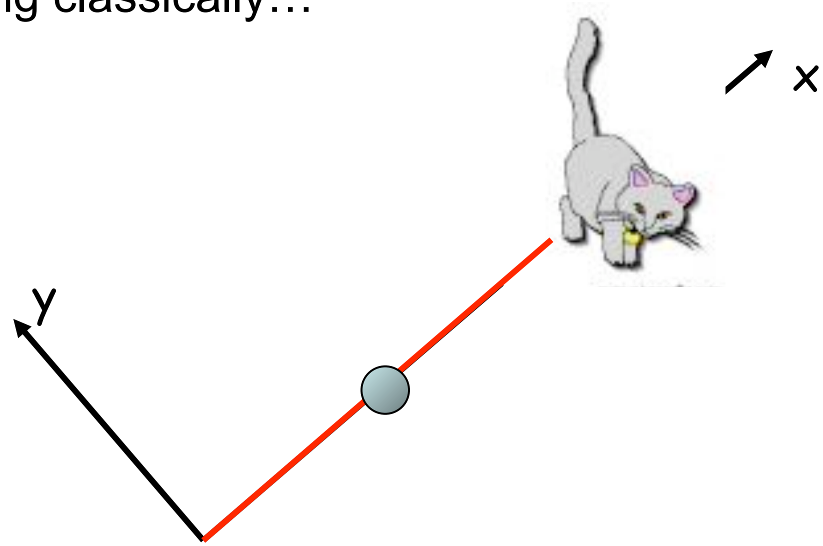
Let's look at coordinate systems:  thinking classically…




You rolls a cat toy across the floor towards her cat.

By Pythagorean's theorem, the toy rolls:

$$d = \sqrt{x^2 + y^2}$$




The same cat, the same cat toy, different (arbitrary) choice of coordinate systems.

Easier visualization, easier calculation —You are happy (although her cat probably doesn't care as long as she gets his toy.)

This is an example of rotating your coordinate axes in space.

# Space-time Intervals

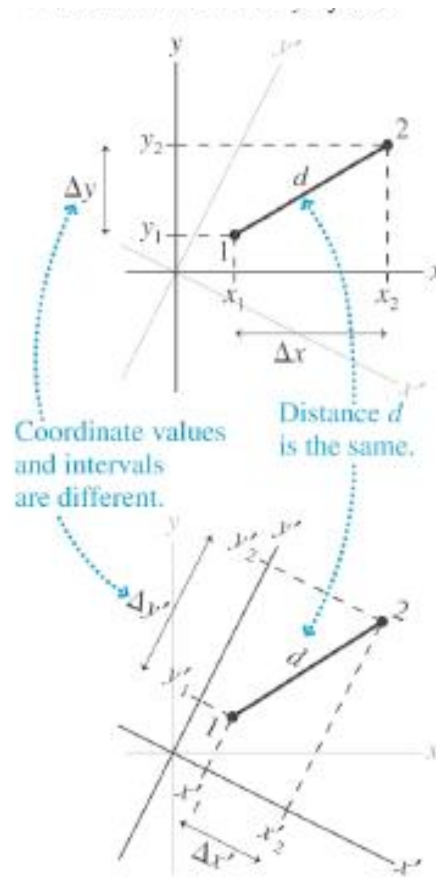
$$x_1 \neq x_1'$$

$$\Delta x_1 \neq \Delta x_1'$$

$$etc$$

$$d^2 = (\Delta x)^2 + (\Delta y)^2 = (\Delta x')^2 + (\Delta y')^2$$

D² invariant. Same value in any Cartesian coordinate system. Rotation and translation preserve it.




Measurements in the $x'y'$-system

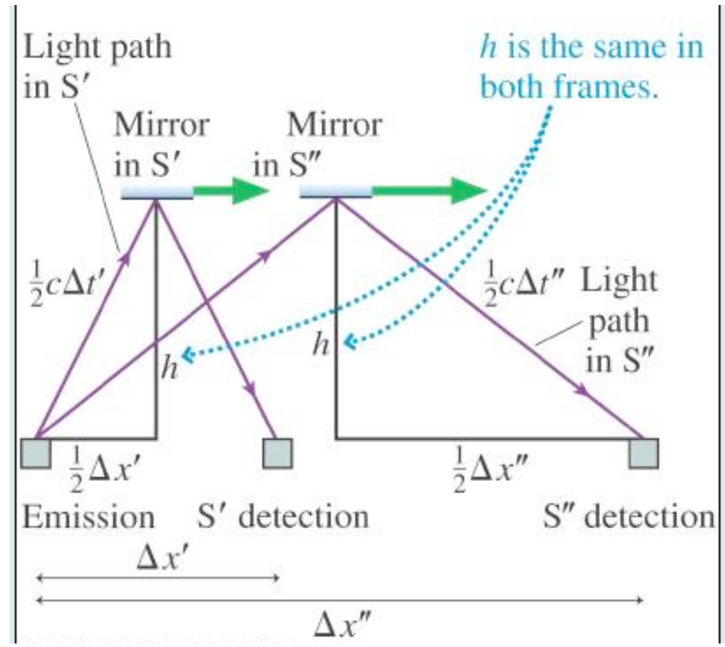
# Invariant space-time interval

Take the space and time distance of two events. The value of h is the same in both frames

$$h^2 = (\frac{c\Delta t'}{2})^2 - (\frac{\Delta x'}{2})^2 = (\frac{c\Delta t''}{2})^2 - (\frac{\Delta x''}{2})^2$$

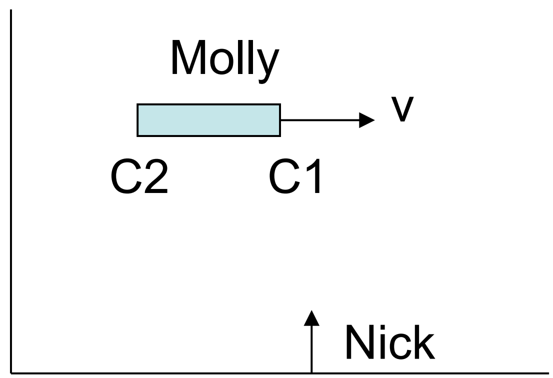
$$c^2(\Delta t')^2 - (\Delta x')^2 = c^2(\Delta t'')^2 - (\Delta x'')^2$$

$$s^2 \equiv c^2(\Delta t)^2 - (\Delta x)^2$$




**Spacetime interval- INVARIANT –**

**Same for all inertial frames**




$$For \text{ Nick, } (\Delta t_N, \Delta x_N = 0)$$

$$For \text{ Molly } (\Delta t_M, \Delta x_M = L)$$

$$c^2 \Delta t_N^2 = c^2 \Delta t_M^2 - L^2$$

$$\Delta t_N^2 < \Delta t_M^2$$

1. Nick measures the time it takes for Molly's rocket from nose to tail to fly past him as $dt_N$ while he is in the same position.
2. Molly measures the time it takes for the rocket to fly past Nick as $dt_M$ while her position changed by L.

3. Do they measure the same time and if not which is shorter?

Nick's time is the proper time since his clock does not change position. Molly on the other hand had to use two clocks one in the nose and one in the tail. $dt_2 = \gamma dt_1$. $dt_1$ is proper time and the shortest interval.

# The Lorentz Transformations

Consider two reference frames S and S′. An event occurs at coordinates *x*, *y*, *z, t* as measured in S, and the same event occurs at *x*′, *y*′, *z*′, *t*′ as measured in S′ . Reference frame S′ moves with velocity *v* relative to S, along the *x*-axis.

The **Lorentz transformations** for the coordinates of one event are:

Rules in deriving LT:

1. Agree with GT for v<<c
2. Transform not only space but time
3. Ensure that speed of light is constant

$$x' = \gamma(x - vt) \qquad x = \gamma(x' + vt')$$

$$y' = y \qquad y = y'$$

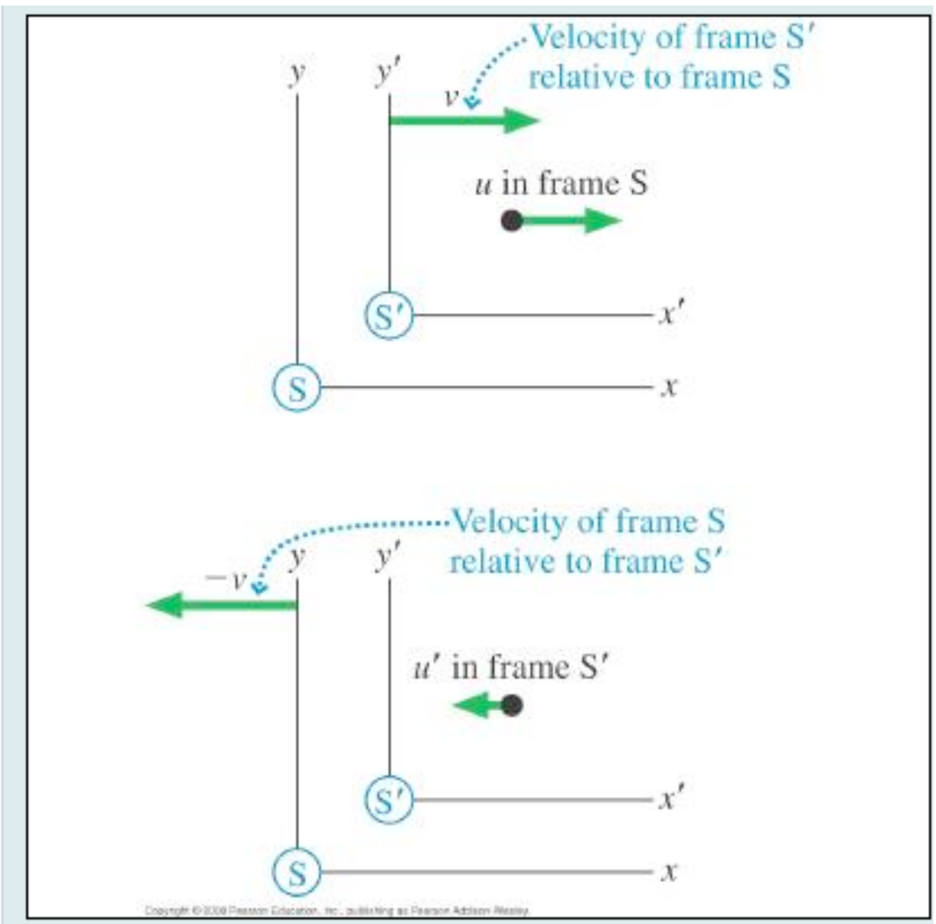
$$z' = z \qquad z = z'$$

$$t' = \gamma(t - vx/c^2) \qquad t = \gamma(t' + vx'/c^2)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

# Velocity Transformation

In GT u'=u-v




$$u' = \frac{dx'}{dt'} = \frac{d[\gamma(x - vt)]}{d[\gamma(t - xv/c^2)]}$$

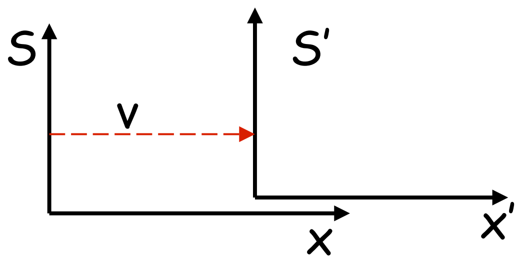
$$u' = \frac{dx - vdt}{dt - vdx/c^2} = \frac{dx/dt - v}{1 - v(dx/dt)/c^2}$$

$$u' = \frac{u - v}{1 - uv/c^2}$$

$$S' \rightarrow S, v \rightarrow -v$$

$$u' = \frac{u - v}{1 - uv/c^2} \quad \text{and} \quad u = \frac{u' + v}{1 + u'v/c^2}$$

# The Lorentz transformations!




The transformation:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

To transform from S' back to S:

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

# Definition of length in moving frames

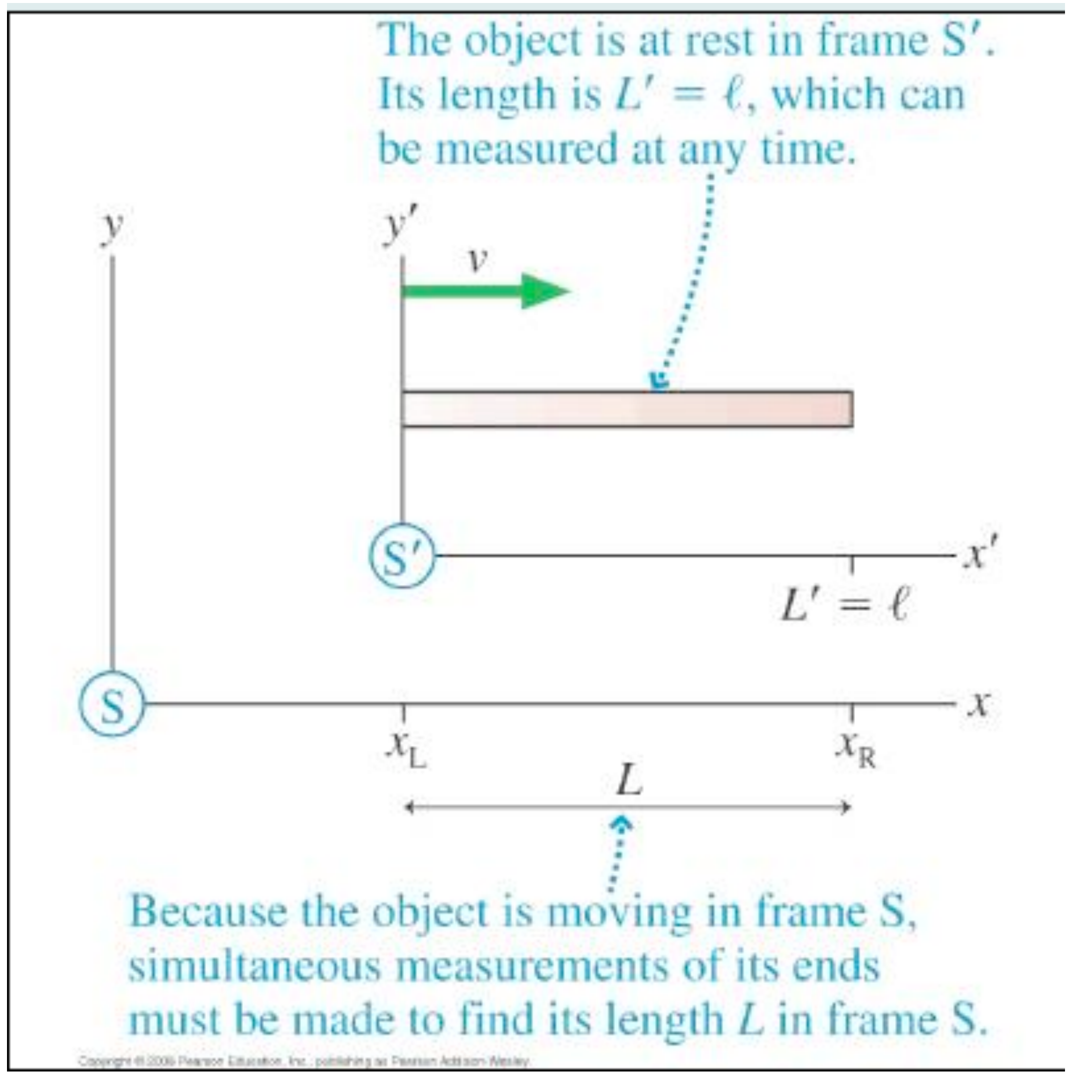
$$\Delta x' = x'_R - x'_L = l$$

$$\Delta x = x_R - x_L = L$$

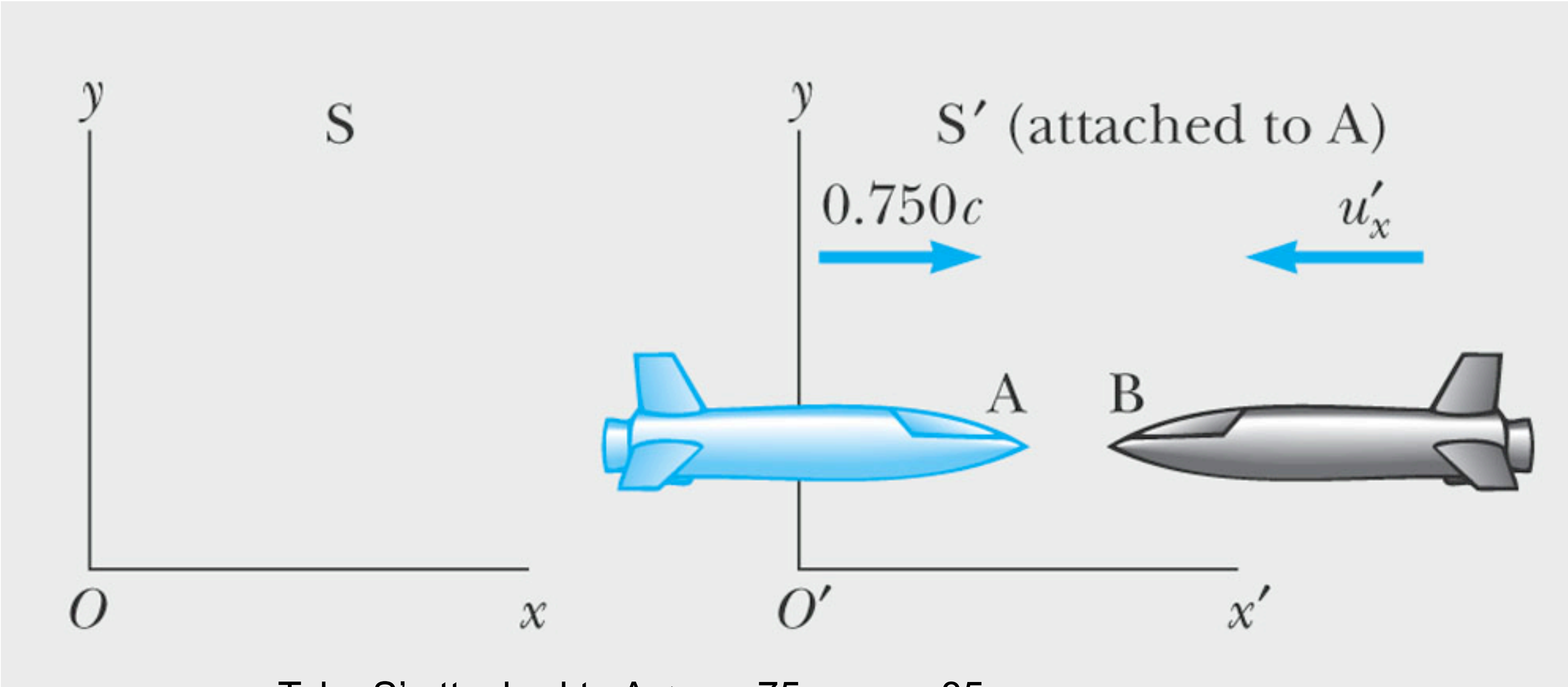
$$x'_R = \gamma(x_R - vt)$$

$$x'_L = \gamma(x_L - vt)$$

$$x'_R - x'_L = l = \gamma(x_R - x_L) = \gamma L$$



Find speed of B wr to A




 Take S' attached to A -> v=.75 c. $u_B$=-.85 c

Speed of B wr A=u'= $$\frac{-.85c-.75c}{1-\frac{(-.85c)(-.75c)}{c^2}}=.9771c$$

Fig. 1-19, p. 30

# Relativistic Energy

Let a particle of mass $m$ move through distance $\Delta x$ during a time interval $\Delta t$, as measured in reference frame S. The spacetime interval is

$$s^2 = c^2(\Delta t)^2 - (\Delta x)^2 = \text{invariant}$$

We can turn this into an expression involving momentum if we multiply by $(m/\Delta\tau)^2$, where $\Delta\tau$ is the proper time (i.e., the time measured by the particle). Doing so gives

$$(mc)^2\left(\frac{\Delta t}{\Delta\tau}\right)^2 - \left(\frac{m\Delta x}{\Delta\tau}\right)^2 = (mc)^2\left(\frac{\Delta t}{\Delta\tau}\right)^2 - p^2 = \text{invariant} \qquad (37.37)$$

where we used $p = m(\Delta x/\Delta\tau)$ from Equation 37.32.

Now $\Delta t$, the time interval in frame S, is related to the proper time by the time-dilation result $\Delta t = \gamma_p \Delta\tau$. With this change, Equation 37.37 becomes

$$(\gamma_p mc)^2 - p^2 = \text{invariant}$$

Finally, for reasons that will be clear in a minute, we multiply by $c^2$, to get

$$(\gamma_p mc^2)^2 - (pc)^2 = \text{invariant} \qquad (37.38)$$

# Relativistic Energy

$$(\gamma_p mc^2)^2 - (pc)^2 = \text{invariant}$$

$$\underbrace{(\gamma_p mc^2)^2 - (pc)^2}_{\text{frame } S} = \underbrace{(\gamma_p' mc^2)^2 - (p'c)^2}_{\text{frame } S'}$$

$$(\gamma_p mc^2)^2 - (pc)^2 = \underbrace{(mc^2)^2}_{\text{"Particle at Rest" frame}}$$

$(p' = 0 \Rightarrow \gamma_p' = 1)$

# Relativistic Energy

$$(\gamma_p mc^2)^2 - (pc)^2 = \text{invariant}$$

$$\gamma_p mc^2 = \frac{mc^2}{\sqrt{1 - u^2/c^2}} \approx \left(1 + \frac{1}{2}\frac{u^2}{c^2}\right)mc^2 = mc^2 + \frac{1}{2}mu^2$$

$u \ll c$

New!

KE

An inherent energy associated with the particles rest mass!

Call it "rest" Energy of particle

$$(\gamma_p mc^2)^2 - (pc)^2 = (mc^2)^2$$

# Relativistic Energy

The **total energy** *E* of a particle is

$$E = \gamma_p mc^2 = E_0 + K = \text{rest energy} + \text{kinetic energy}$$

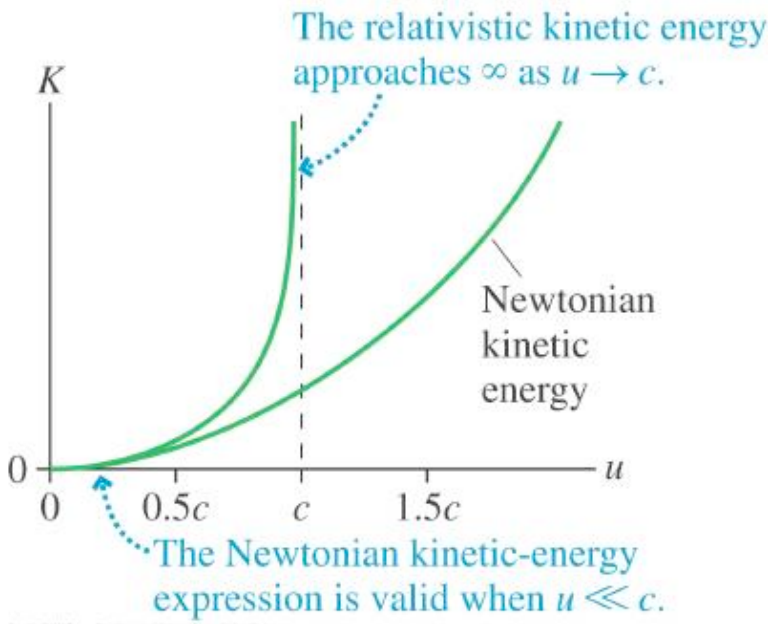
This total energy consists of a **rest energy**

$$E_0 = mc^2$$

and a relativistic expression for the *kinetic energy*

$$K = (\gamma_p - 1)mc^2 = (\gamma_p - 1)E_0$$

This expression for the kinetic energy is very nearly $½mu^2$ when $u << c$.




$$K = \int_0^u dW = \int_0^u F dx = \int_0^u \frac{dp}{dt} dx$$

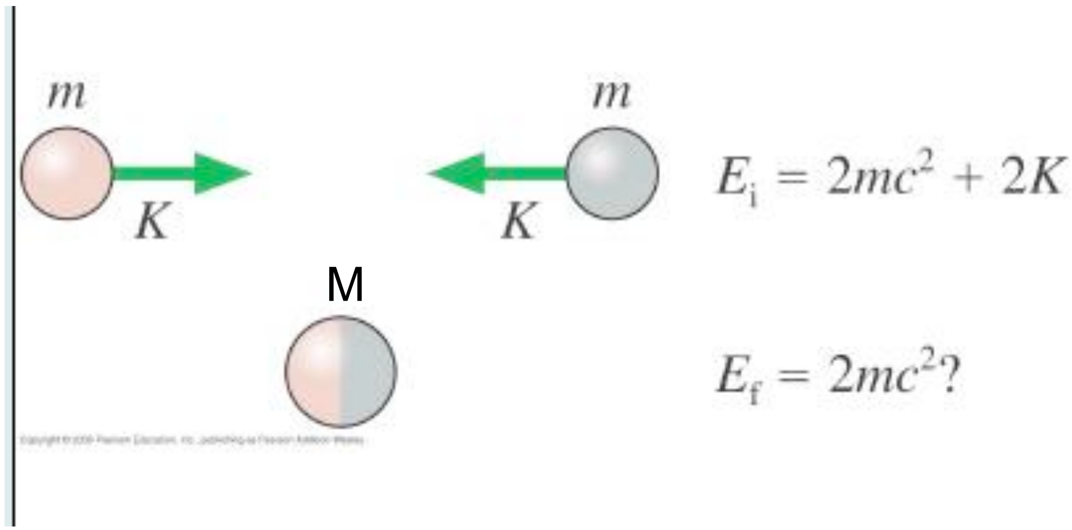
$$If \rightarrow dp/dt = m dv/dt$$

$$K = \frac{1}{2} m v^2$$

*Otherwise*

$$K = mc^2(\gamma - 1)$$

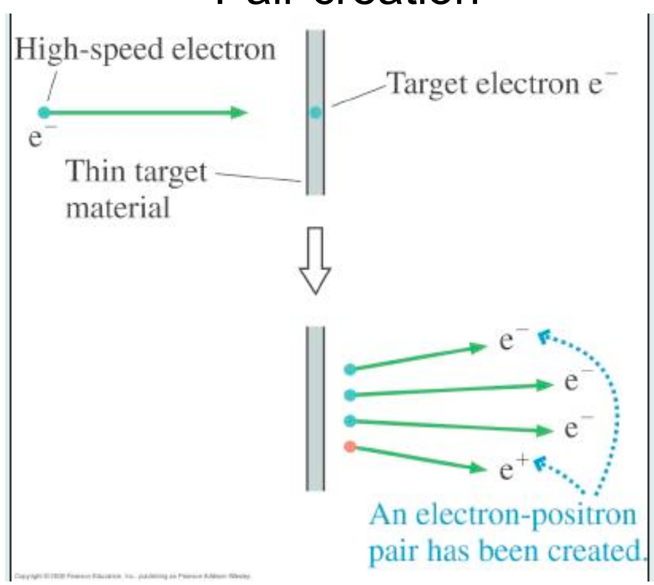
$E_o = mc^2$ Invariant




Energy conservation requires that
$M=2m+2K/c^2$
Mass is not conserved

$M=2m+2K/c^2$

Pair creation




$\Delta K=2m_ec^2$

$e^-(fast)+e^-(at\ rest) \rightarrow e^-+e^-+e^-+e^+$

Law of conservation of total energy

$$E = \sum_i E_i = \sum_i (\gamma_p)_i m_i c^2$$

# Conservation of Energy

The creation and annihilation of particles with mass, processes strictly forbidden in Newtonian mechanics, are vivid proof that neither mass nor the Newtonian definition of energy is conserved. Even so, the *total* energy—the kinetic energy *and* the energy equivalent of mass—remains a conserved quantity.

**Law of conservation of total energy** The energy $E = \sum E_i$ of an isolated system is conserved, where $E_i = (\gamma_p)_i m_i c^2$ is the total energy of particle *i*.

Mass and energy are not the same thing, but they are *equivalent* in the sense that mass can be transformed into energy and energy can be transformed into mass as long as the total energy is conserved.

- **Explosion :** In ordinary, e.g. TNT, explosion chemical energy is transformed in kinetic energy of the fragments, acoustic energy of the snow-plowed air (shock or blast wave) and some radiation from the heated air. In an explosion we deliver the energy *fast*.

**Power:** P= Energy/time. Units are Watt = J/sec. Hair dryer 1 kW, Light bulb 100 W.

Power worldwide 1.3 Twatts,
In a year multiply by $2x10^7$ secs to get $3x10^{19}$ J or $10^4$ MT (MT=$4x10^{15}$ J)

# CHEMICAL BINDING ENERGY

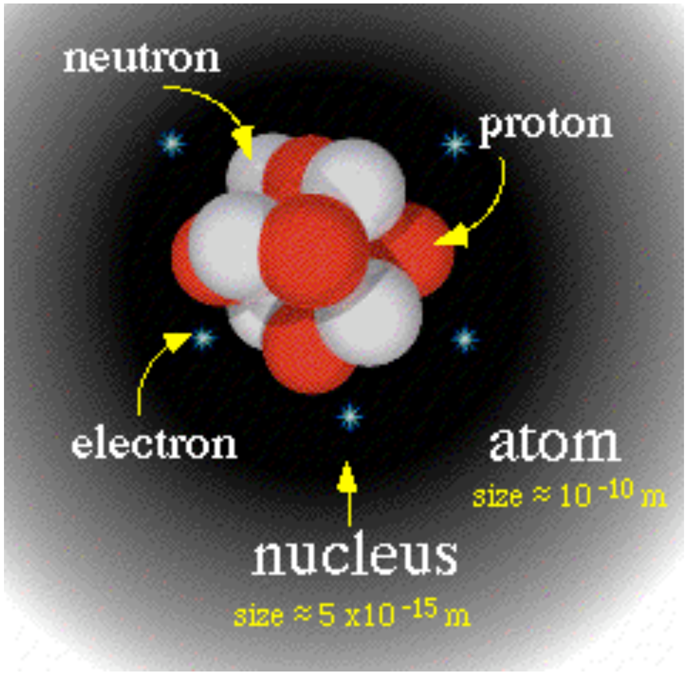
TWO OXYGEN ATOMS ATTRACT EACH OTHER TO FORM $O_2$ WHILE RELEASING 5 eV OF ENERGY.
THEREFORE 2 OXYGEN ATOMS ARE HEAVIER THAN AN OXYGEN MOLECULE BY

$$\Delta m = 5 \text{ eV}/c^2 = 9\text{x}10^{-36}\text{kg}$$

MASS OF OXYGEN MOLECULE IS $5\text{x}10^{-26}$kg.
$\Delta m/m = 2\text{x}10^{-10}$
FORM 1 GRAM OF $O_2$ AND GET $2\text{x}10^{4}$ JOULES

**All matter is an Assembly of Atoms**

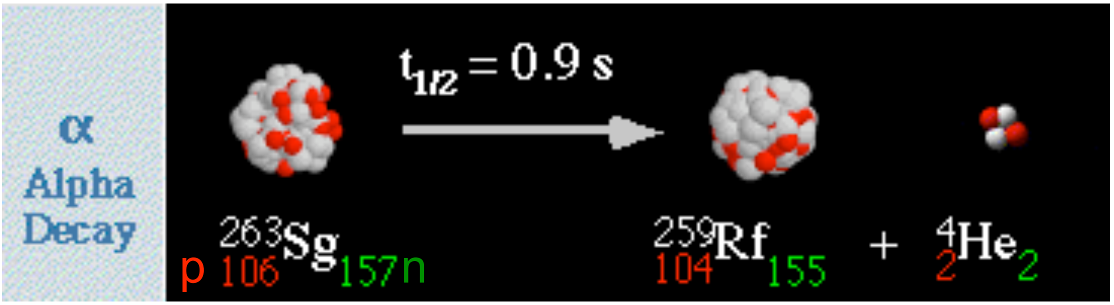
Atomic number vs.
Mass number



Radioactivity alpha decay

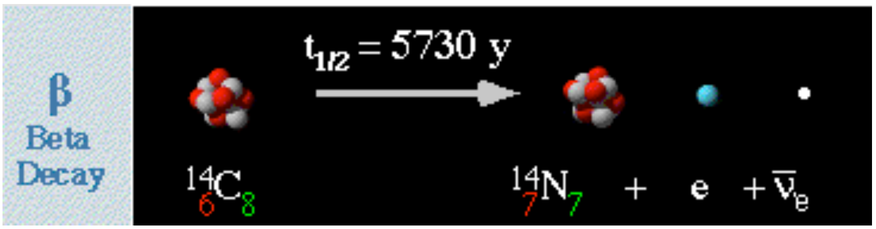
Ra(226,88)->Rn(222,86)+He(4,2)

U(238,92)->Th(234,90)+He(4,2)




Beta decay

C(14,6)->N(14,7)+$e^-$+ν



## Uranium-235 fission scenario

1) Initial state: the neutron and the uranium-235 nucleus are almost at rest, at room temperature.

2) Intermediate state: the neutron has been incorporated into the nucleus, which vibrates like a drop of water before breaking up. The reader can fill a balloon with water, tap on it, and see how it vibrates.

3) The nuclear "droplet" has split, giving way to two lighter radioactive nuclei whose kinetic energy is 150 MeV, and to two or three neutrons whose energy is 2 MeV each.

The electron volt (eV) corresponds to the energy acquired by an electron accelerated in vacuum by a potential of one volt, about the voltage of the familiar dry cell. It requires about ten electron volts (10 eV) to extract an electron from an atom, and about ten million electron volts (10 MeV) to extract a neutron or a proton from a nucleus.

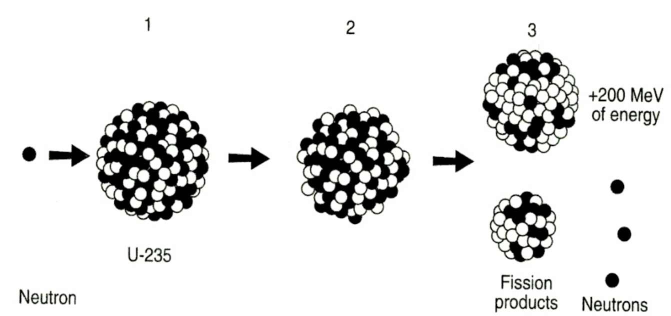



*Fig. 1.1. Chain reaction scenario.*

The energy of the fission products corresponds to the energy of the atoms in a medium raised to a temperature of many billions of degrees. The sum of the kinetic energies of the two fission-product nuclei is almost equal to the energy $E = Mc^2$, where $M$ is the difference in mass between the initial nucleus and the sum of the final nuclei.

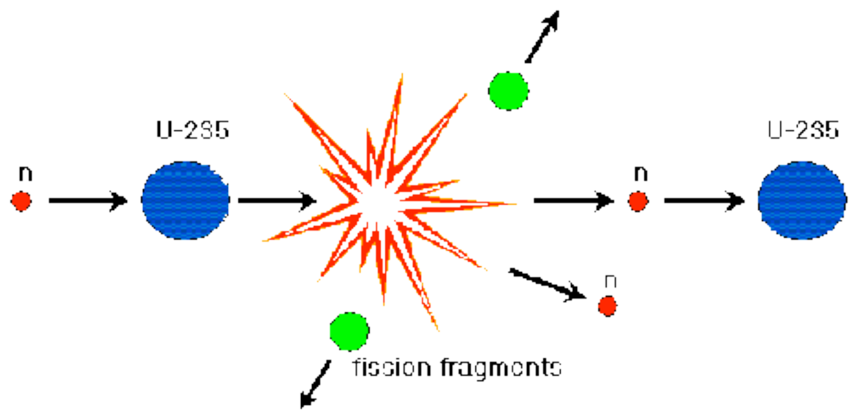
$E=mc^2$

1 kg has the potential to generate $9\times10^{16}$ Joules
Could provide electric power to city of 800000 for 3 years

$\eta=1$ 300 kg/year

Efficiency=$\eta\ mc^2$

| | |
|---|---|
| Chemical reactions (0il, coal, etc) $\eta = 10^{-9}$ | $10^{10}$ tons/year |
| Nuclear power – Fission $\eta = 10^{-3}$ | 300 tons/year |
| Nuclear power – Fusion $\eta=10^{-2}$ | 30 tons/year |



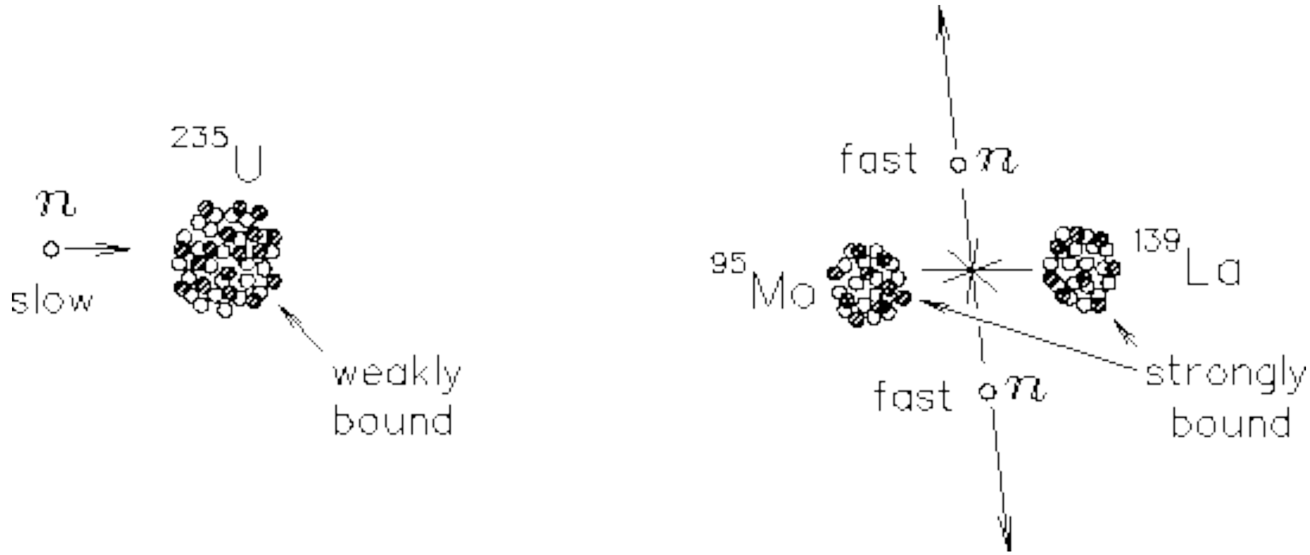
# EXAMPLES OF CONVERTING MASS TO ENERGY

- Nuclear fission (e.g., of Uranium)
  - Nuclear Fission – the splitting up of atomic nuclei
  - E.g., Uranium-235 nuclei split into fragments when smashed by a moving neutron. One possible nuclear reaction is

$$^{235}U + 1n \rightarrow 3n + ^{89}Kr + ^{144}Ba$$

  - Mass of fragments slightly **less** than mass of initial nucleus + neutron
  - That mass has been converted into energy (gamma-rays and kinetic energy of fragments)

# FISSION




One case of the fission of $^{236}U$. The net mass of the initial neutron plus the $^{235}U$ nucleus is 219,883 MeV/c$^2$. The net mass of the fission products (two neutrons, a $^{95}Mo$ nucleus and a $^{139}La$ nucleus) is 219,675 MeV/c$^2$ - smaller because of the stronger *binding* of the Mo and La nuclei. The "missing mass" of 208 MeV/c$^2$ goes into the *kinetic energy* of the *fragments* (mainly the *neutrons*), which of course adds up to 208 MeV.

# Fusion

- Nuclear fusion (e.g. hydrogen)
  - Fusion – the sticking together of atomic nuclei
  - Much more important for  Astrophysics than fission
    - e.g. power source for stars such as the Sun.
    - Explosive mechanism for particular kind of supernova
  - Important example – hydrogen fusion.
    - Ram together 4 hydrogen nuclei to form helium nucleus
    - Spits out couple of “positrons” and “neutrinos” in process

$$4\ {}^{1}H \rightarrow {}^{4}He + 2e^{+} + 2\upsilon$$

# Fusion

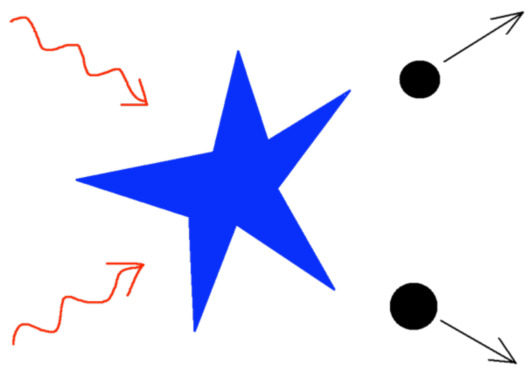
  - Mass of final helium nucleus plus positrons and neutrinos is less than original 4 hydrogen nuclei
  - Mass has been converted into energy (gamma-rays and kinetic energy of final particles)
- This (and other very similar) nuclear reaction is the energy source for…
  - Hydrogen Bombs (about 1kg of mass converted into energy gives 20 Megaton bomb)
  - The Sun (about $4\times10^9$ kg converted into energy per second)

# Annihilation

- Anti-matter
  - For every kind of particle, there is an antiparticle…
    - Electron ↔ anti-electron (also called positron)
    - Proton ↔ anti-proton
    - Neutron  anti-neutron
  - Anti-particles have opposite properties than the corresponding particles (e.g., opposite charge)… but exactly same mass.
  - When a particle and its antiparticle meet, they can completely annihilate each other… **all of their mass is turned into energy (gamma-rays)!**

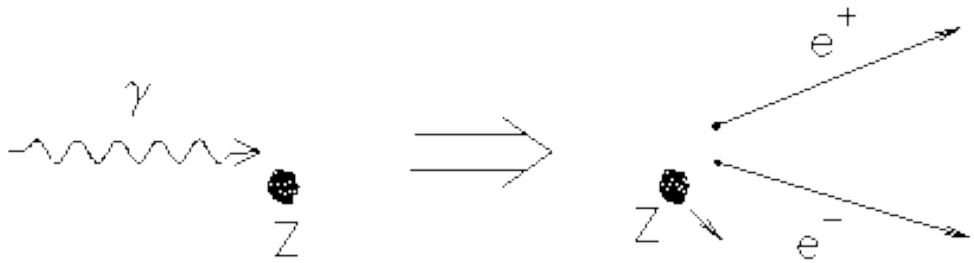
# EXAMPLES OF CONVERTING ENERGY TO MASS

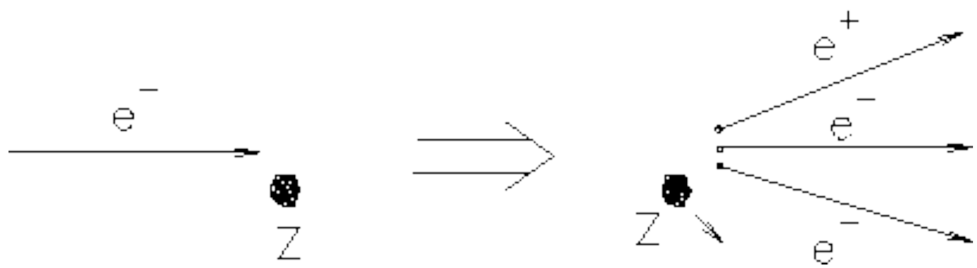
- Particle/anti-particle production
  - Opposite process to that just discussed!
  - Energy (e.g., gamma-rays) can produce particle/ anti-particle pairs



  - **Very fundamental process in Nature**… shall see later that this process, operating in early universe, is responsible for all of the mass that we see today!

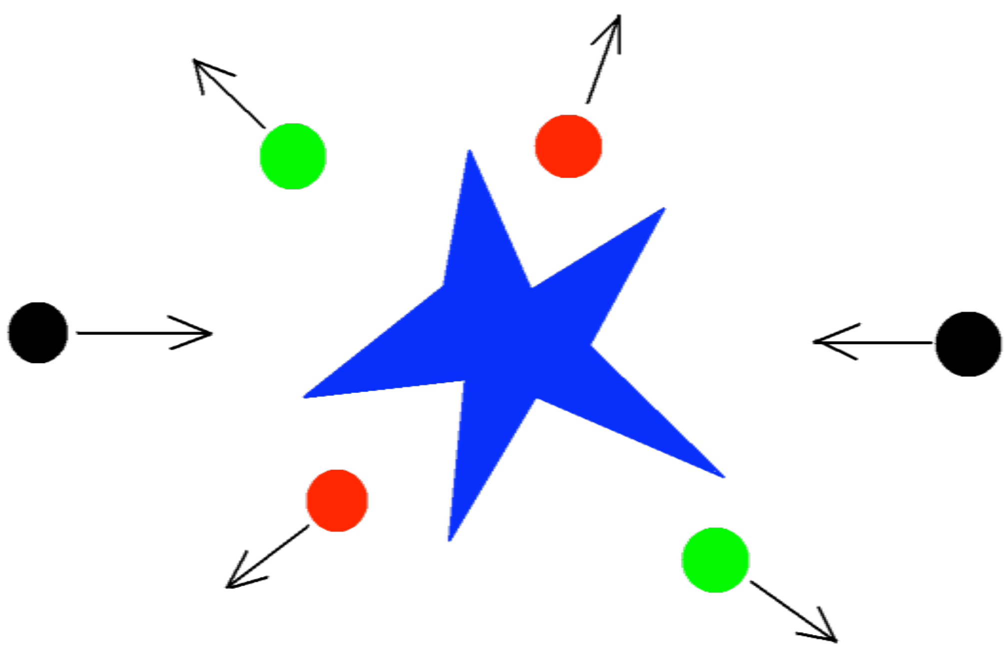
# PAIR PRODUCTION




Electron-positron PAIR PRODUCTION by gamma rays (above) and by electrons (below). The positron ($e^+$) is the ANTIPARTICLE of the electron ($e^-$). The gamma ray ($\gamma$ ) must have an energy of at least 1.022 MeV [twice the rest mass energy of an electron] and the pair production must take place near a heavy nucleus ($Z$) which absorbs the momentum of the $\gamma$ .

- Particle production in a particle accelerator
  - Can reproduce conditions similar to early universe in modern particle accelerators…



# A real particle creation event