PHYS 270 – SUPPL. #9

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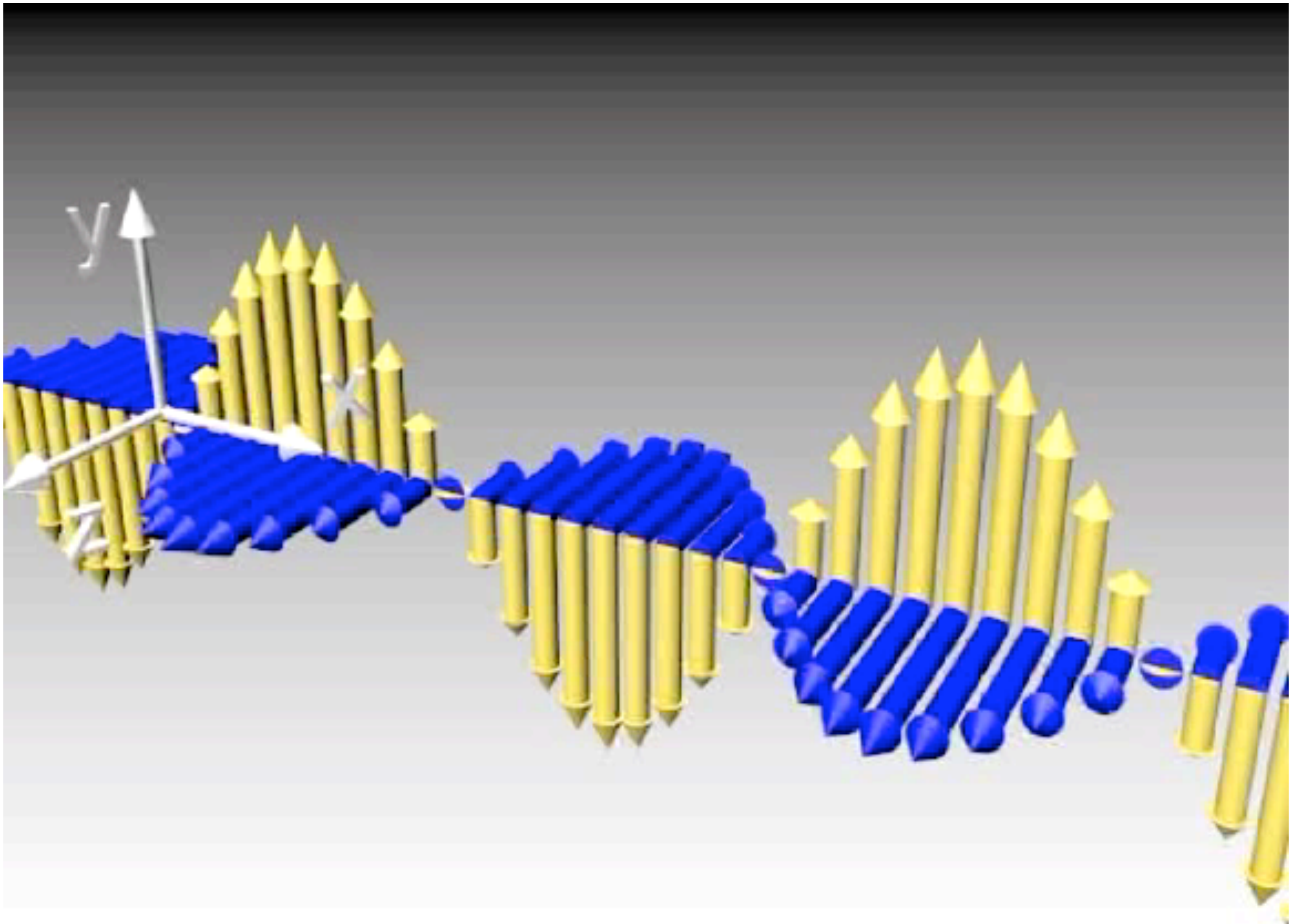
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Problem-Solving Strategy: Traveling Electromagnetic Waves

This chapter explores various properties of the electromagnetic waves. The electric and the magnetic fields of the wave obey the wave equation. Once the functional form of either one of the fields is given, the other can be determined from Maxwell's equations. As an example, let's consider a sinusoidal electromagnetic wave with

$$\vec{\mathbf{E}}(z,t) = E_0 \sin(kz - \omega t)\hat{\mathbf{i}}$$

The equation above contains the complete information about the electromagnetic wave:

1. Direction of wave propagation: The argument of the sine form in the electric field can be rewritten as $(kz - \omega t) = k(z - vt)$, which indicates that the wave is propagating in the $+z$-direction.

2. Wavelength: The wavelength λ is related to the wave number k by $\lambda = 2\pi / k$.

3. Frequency: The frequency of the wave, f, is related to the angular frequency ω by $f = \omega / 2\pi$.

4. Speed of propagation: The speed of the wave is given by

$$v = \lambda f = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\omega}{k}$$

   In vacuum, the speed of the electromagnetic wave is equal to the speed of light, c.

5. Magnetic field $\vec{\mathbf{B}}$: The magnetic field $\vec{\mathbf{B}}$ is perpendicular to both $\vec{\mathbf{E}}$ which points in the $+x$-direction, and $+\hat{\mathbf{k}}$, the unit vector along the $+z$-axis, which is the direction of propagation, as we have found. In addition, since the wave propagates in the same direction as the cross product $\vec{\mathbf{E}} \times \vec{\mathbf{B}}$, we conclude that $\vec{\mathbf{B}}$ must point in the $+y$-direction (since $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$).

Since $\vec{\mathbf{B}}$ is always in phase with $\vec{\mathbf{E}}$, the two fields have the same functional form. Thus, we may write the magnetic field as

$$\vec{\mathbf{B}}(z,t) = B_0 \sin(kz - \omega t)\hat{\mathbf{j}}$$

where B_0 is the amplitude. Using Maxwell's equations one may show that $B_0 = E_0(k/\omega) = E_0/c$ in vacuum.

6. The Poytning vector: the Poynting vector can be obtained as

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\left[E_0\sin(kz-\omega t)\hat{\mathbf{i}}\right]\times\left[B_0\sin(kz-\omega t)\hat{\mathbf{j}}\right] = \frac{E_0B_0\sin^2(kz-\omega t)}{\mu_0}\hat{\mathbf{k}}$$

7. Intensity: The intensity of the wave is equal to the average of S:

$$I = \langle S\rangle = \frac{E_0B_0}{\mu_0}\langle\sin^2(kz-\omega t)\rangle = \frac{E_0B_0}{2\mu_0} = \frac{E_0^2}{2c\mu_0} = \frac{cB_0^2}{2\mu_0}$$

8. Radiation pressure: If the electromagnetic wave is normally incident on a surface and the radiation is completely *reflected*, the radiation pressure is

$$P_R = \frac{2I}{c} = \frac{E_0B_0}{c\mu_0} = \frac{E_0^2}{c^2\mu_0} = \frac{B_0^2}{\mu_0}$$

If it is completely absorbed $P_R = I/c$

1-5 solve 5.1

EXAMPLE 35.5 Solar sailing

A low-cost way of sending spacecraft to other planets would be to use the radiation pressure on a solar sail. The intensity of the sun's electromagnetic radiation at distances near the earth's orbit is about 1300 W/m^2. What size sail would be needed to accelerate a 10,000 kg spacecraft toward Mars at 0.010 m/s^2?

MODEL Assume that the solar sail is perfectly absorbing.

$$F = ma$$

$$P_R = I/c$$

$$F_R = P_R A = IA/c = ma$$

$$(I/c)A = ma$$

$$A = mac/I = \frac{(3x10^8 m/s)x(10^8 kg)x.01m/s^2}{1300W/m^2} = 2.3x10^7 m^2$$

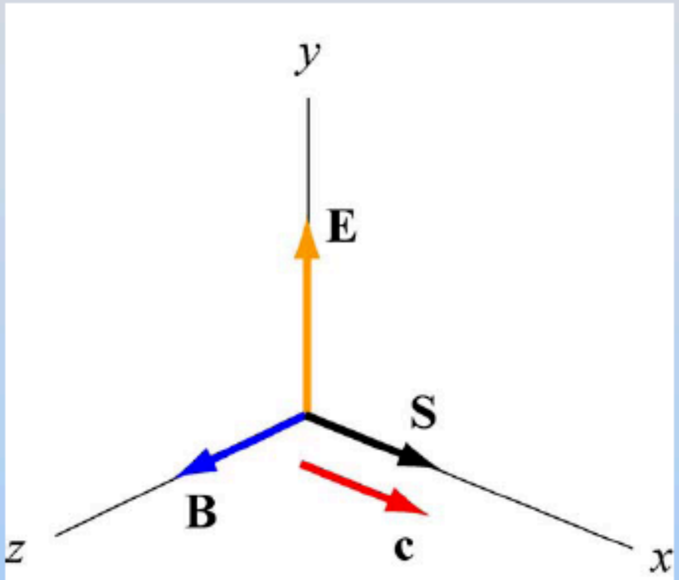
35.51 similar – need area of earth as viewed from Sun R_E=6.4 Mm

35.53 similar balance gravity force against F_R=(IA/c) Nt

35.55 Momentum balance F_R=IA/c=P/c P is power. Find acceleration etc

Poynting Vector and Intensity

Direction of energy flow = direction of wave propagation



$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$: Poynting vector

units: Joules per square meter per sec

Intensity *I*:

$$I \equiv \langle S \rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2\mu_0 c} = \frac{c B_0^2}{2\mu_0}$$

Energy Flow: Resistor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of resistor is INWARD

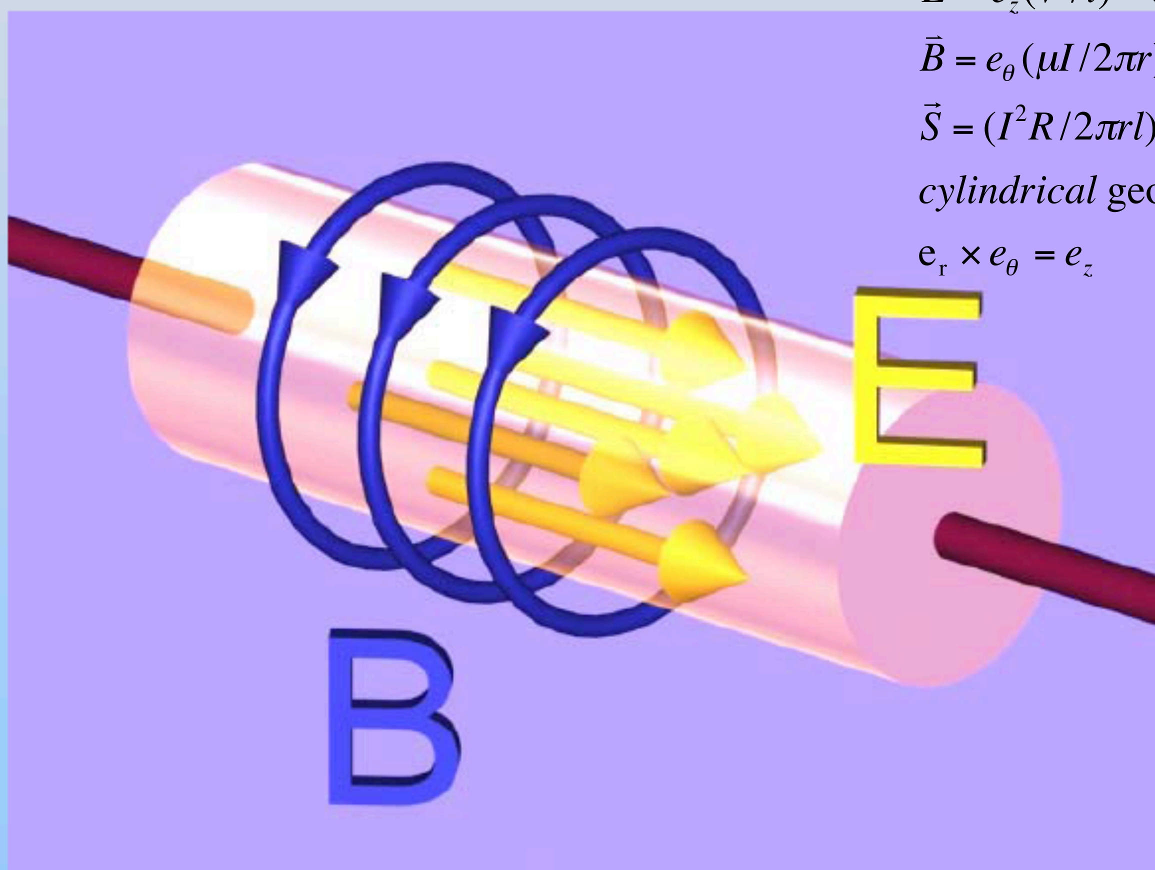
$\vec{E} = e_z(V/l) = e_z(IR/l)$

$\vec{B} = e_\theta(\mu I/2\pi r)$

$\vec{S} = (I^2R/2\pi rl)(e_z \times e_\theta) = -(I^2R/2\pi rl)e_r$

cylindrical geometry

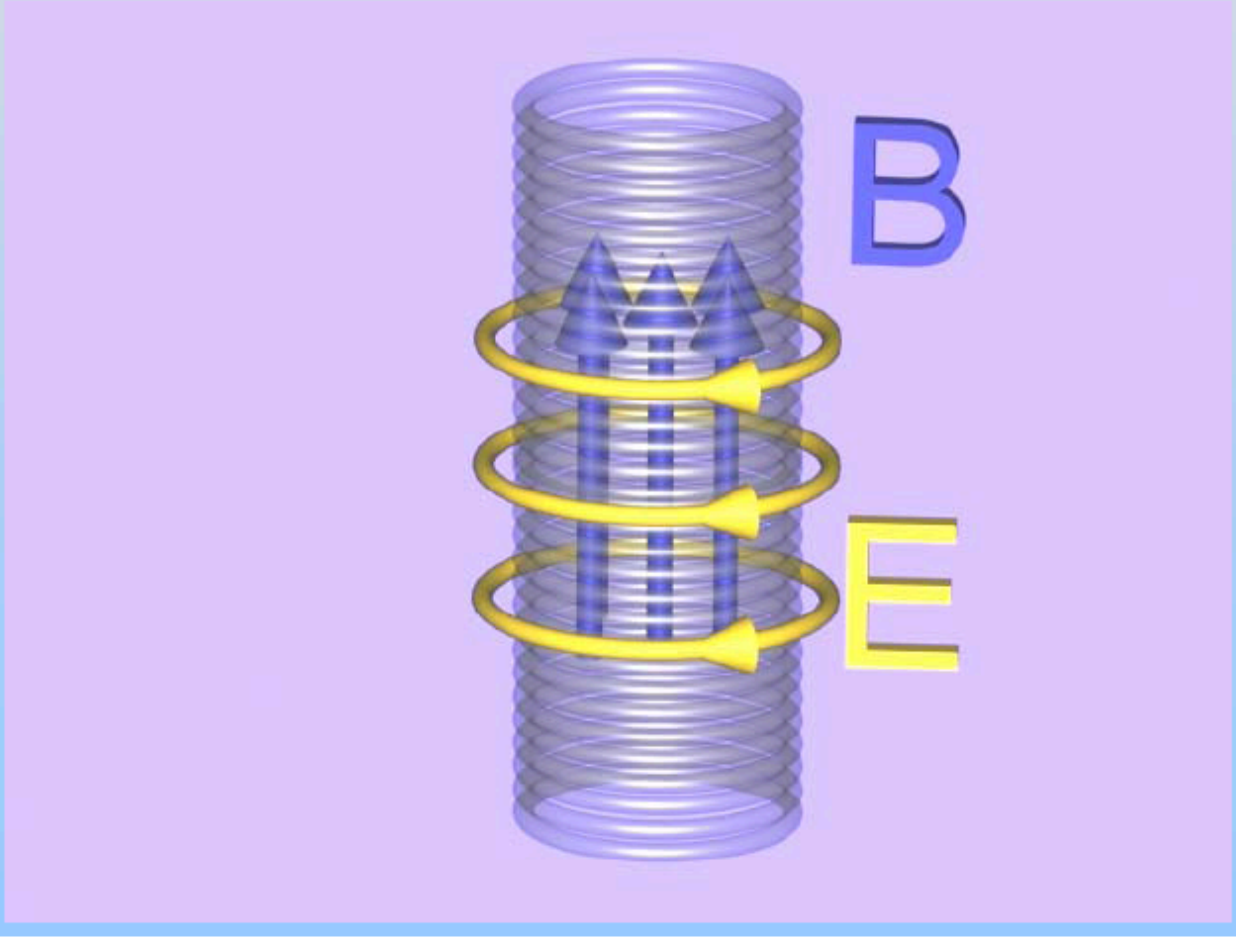
$\mathrm{e_r} \times e_\theta = e_z$



Energy Flow: Inductor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of inductor with increasing current is INWARD



Energy Flow: Inductor

$$\vec{\mathbf{S}} = \frac{\vec{\mathbf{E}} \times \vec{\mathbf{B}}}{\mu_0}$$

On surface of inductor with decreasing current is OUTWARD

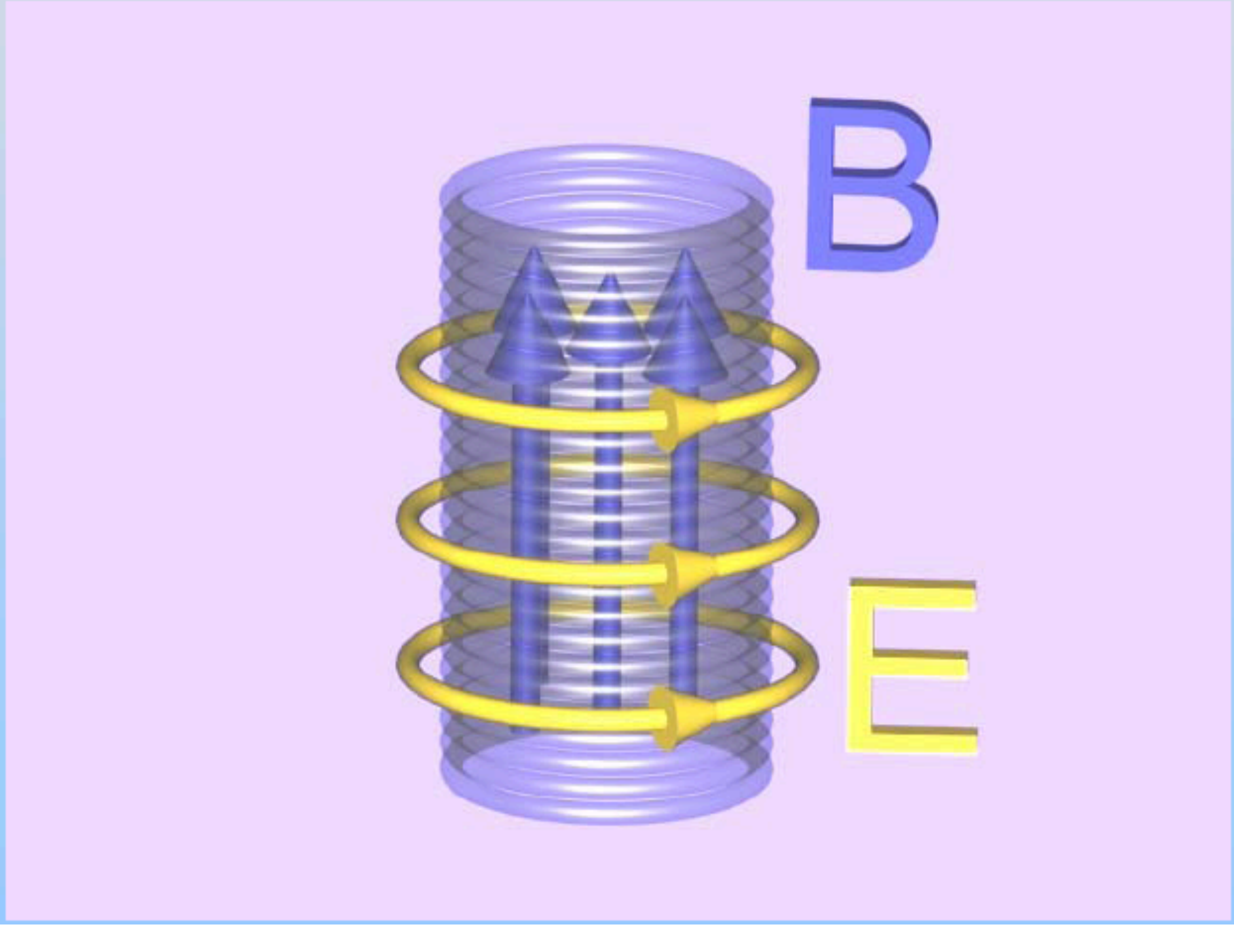


FIGURE 35.25 An electric dipole creates an electric field that reverses direction if the dipole charges are switched.

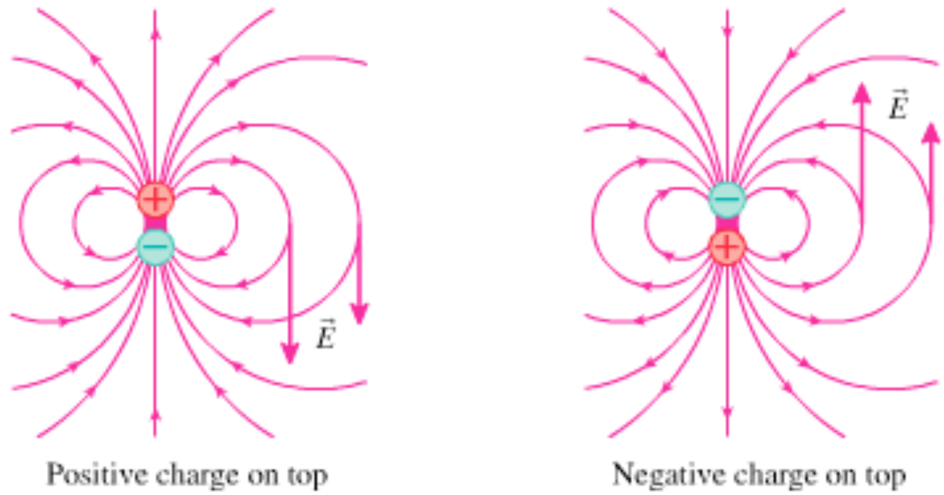
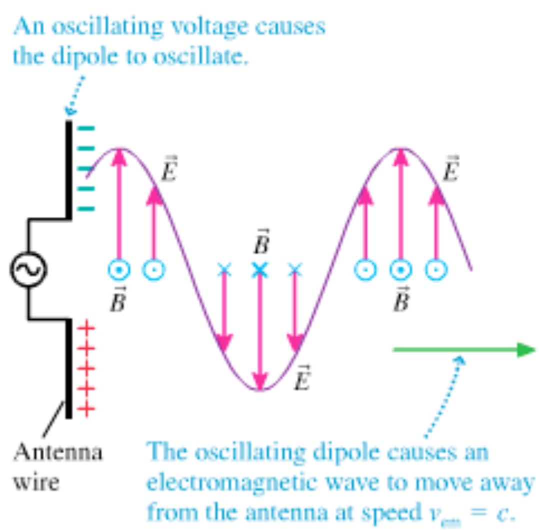


FIGURE 35.26 An antenna generates a self-sustaining electromagnetic wave.



STOP TO THINK 35.5 The amplitude of the oscillating electric field at your cell phone is 4.0 μV/m when you are 10 km east of the broadcast antenna. What is the electric field amplitude when you are 20 km east of the antenna?

a. 1.0 μV/m
b. 2.0 μV/m
c. 4.0 μV/m
d. There's not enough information to tell.

FIELD OF MY POINTER

HE-NEON POWER 1 mWatt, diameter 1 mm^2. How big is the electric field near the aperture?

$$I = \frac{P}{A} = S_{avg} = \frac{1}{2c\mu_0}E_0^2 = \frac{c\epsilon_0}{2}E_0^2$$

$A=\pi r^2=\pi(5x10^{-4})^2\ m^2$

$$E_0 = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(1270\ \mathrm{W/m^2})}{(3.00 \times 10^8\ \mathrm{m/s})(8.85 \times 10^{-12}\ \mathrm{C^2/N\,m^2})}}$$

$$= 980\ \mathrm{V/m}$$

POLARIZATION

FIGURE 35.27 The plane of polarization is the plane in which the electric field vector oscillates.

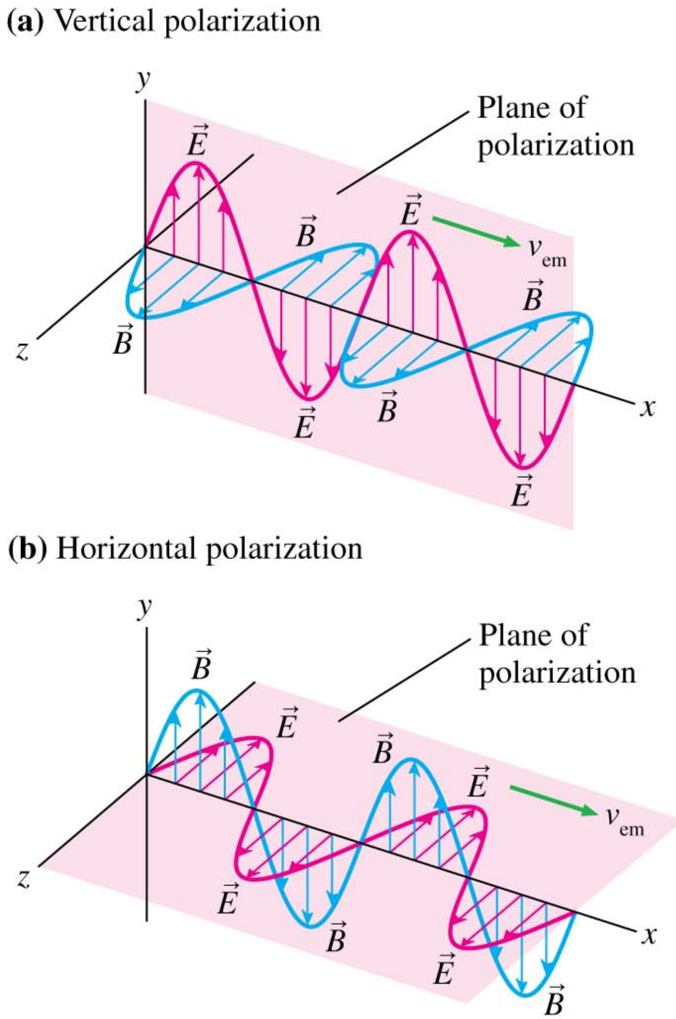


FIGURE 35.28 A polarizing filter.

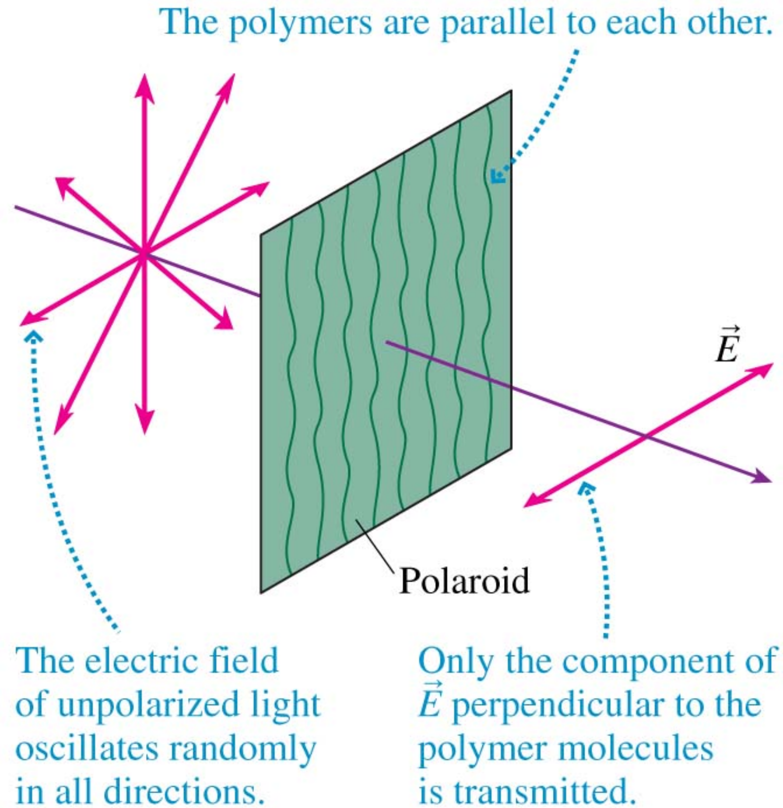


FIGURE 35.28 A polarizing filter.

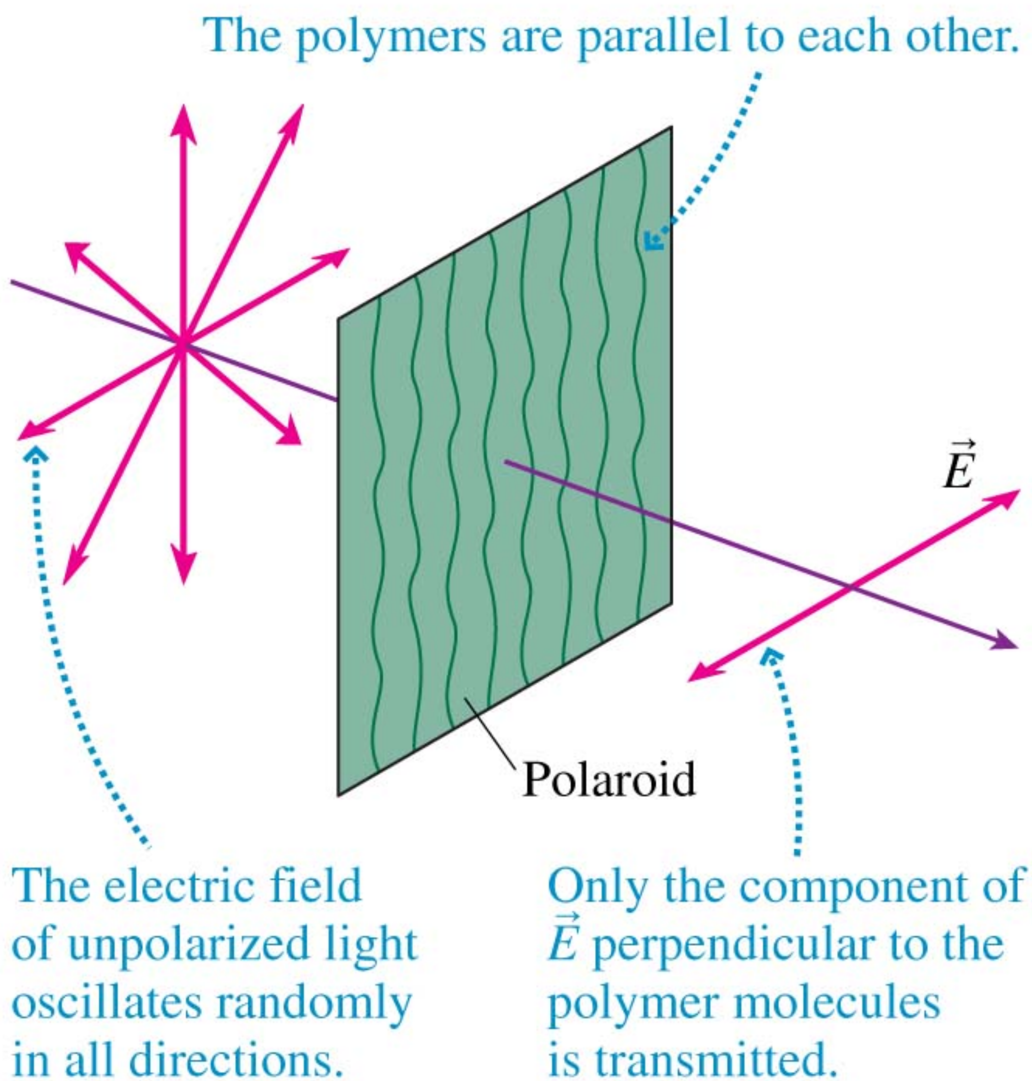
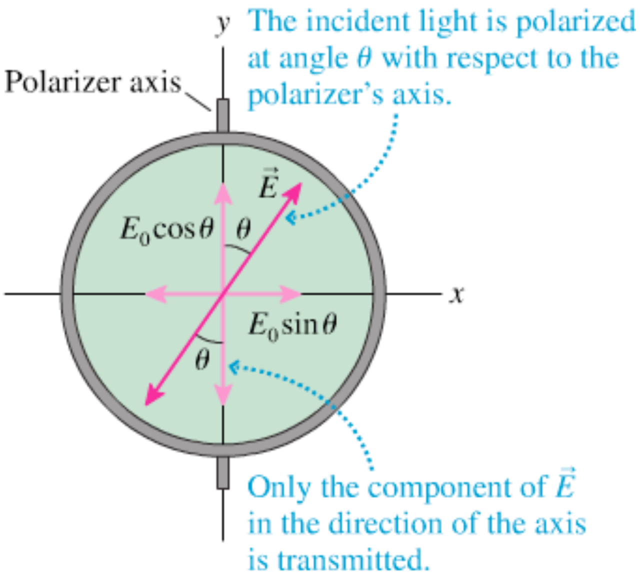


FIGURE 35.29 An incident electric field can be decomposed into components parallel and perpendicular to a polarizer's axis.



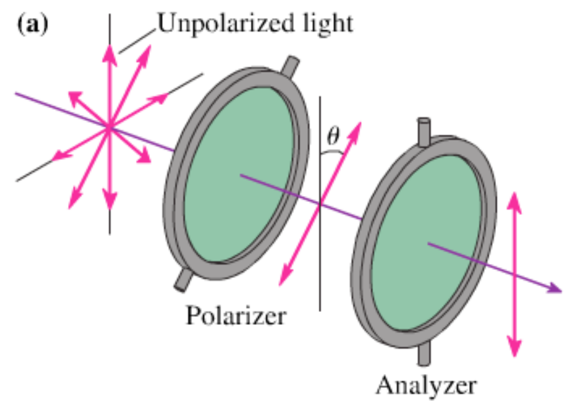
$$\vec{E}_{inc} = E_x\hat{i} + E_y\hat{j} = \hat{i}E_o\sin\theta + \hat{j}E_o\cos\theta$$

Transmitted

$$\vec{E}_{transm} = E_y\hat{j} = E_o\cos\theta\hat{j}$$

$$I_{transm} = I_o\cos^2\theta$$

polarizing



Reflection has horizontal polarization

Average of random theta 1/2

Malus's Law

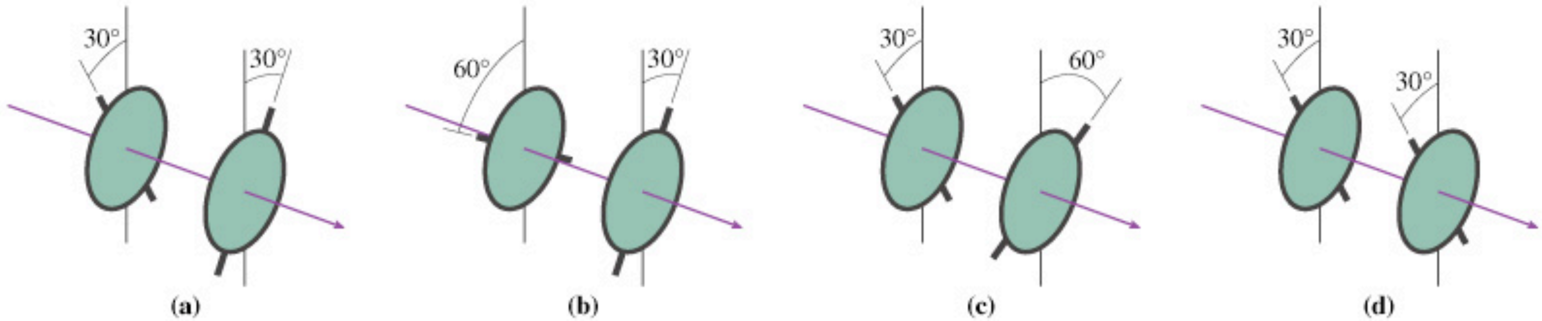
Suppose a *polarized* light wave of intensity I_0 approaches a polarizing filter. θ is the angle between the incident plane of polarization and the polarizer axis. The transmitted intensity is given by Malus's Law:

$$I_{\text{transmitted}} = I_0 \cos^2\theta \qquad \text{(incident light polarized)}$$

If the light incident on a polarizing filter is *unpolarized*, the transmitted intensity is

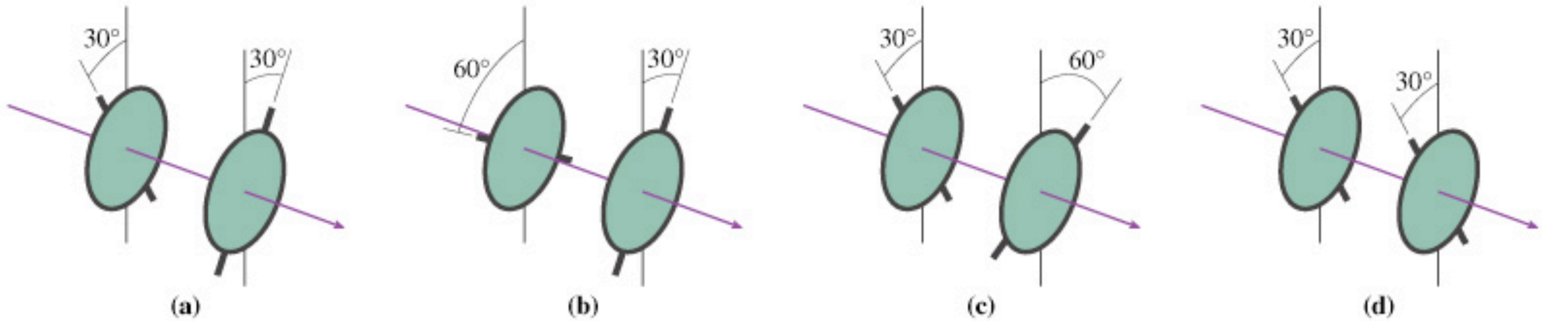
$$I_{\text{transmitted}} = \frac{1}{2} I_0 \qquad \text{(incident light unpolarized)}$$

In other words, a polarizing filter passes 50% of unpolarized light and blocks 50%.



Unpolarized light of equal intensity is incident on four pairs of polarizing filters. Rank in order, from largest to smallest, the intensities I_a to I_d transmitted through the second polarizer of each pair.

A. $I_a = I_d > I_b = I_c$
B. $I_b = I_c > I_a = I_d$
C. $I_d > I_a > I_b = I_c$
D. $I_b = I_c > I_a > I_d$
E. $I_d > I_a > I_b > I_c$



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B. $I_b = I_c > I_a = I_d$
✔ **C. $I_d > I_a > I_b = I_c$**
D. $I_b = I_c > I_a > I_d$
E. $I_d > I_a > I_b > I_c$

$$I_{transmitted} = I_0 \cos^2\theta \qquad \text{(incident light polarized)}$$

Experimenter A creates a magnetic field in the laboratory. Experimenter B moves relative to A. Experimenter B sees

A. just the same magnetic field.
B. a magnetic field of different strength.
C. a magnetic field pointing the opposite direction.
D. just an electric field.
E. both a magnetic and an electric field.

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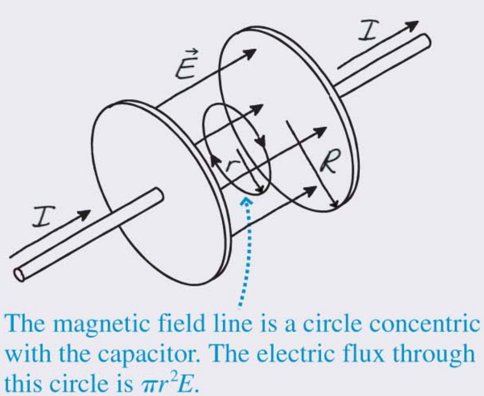
B. a magnetic field of different strength.
C. a magnetic field pointing the opposite direction.
D. just an electric field.
✔ **E. both a magnetic and an electric field.**

B fields inside a charging capacitor

DISPLACEMENT CURRENT

FIGURE 35.18 The magnetic field strength is found by integrating around a closed curve of radius r.



Find B at position r<R if the capacitor is charging with a current I.

Apply Ampere's law. Inside the capacitor the only current is the displacement current

$$\oint \vec{B}(r)\bullet d\vec{s} = \mu_o I_{displ}(\text{current inside r})$$

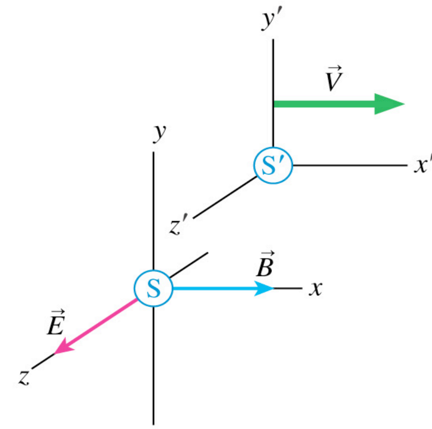
$$I_{displ} = \varepsilon_o \frac{d\Phi_E}{dt} = \varepsilon_o \pi r^2 \frac{dE}{dt}$$

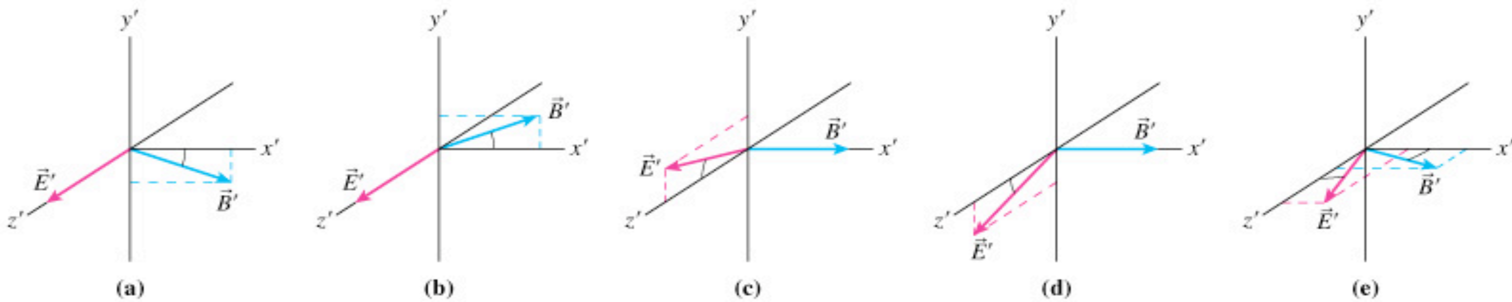
$$E = \frac{Q}{\varepsilon_o A} = \frac{Q}{\varepsilon_o \pi R^2}$$

$$2\pi r B_\theta(r) = \mu_o \frac{r^2}{R^2}\frac{dQ}{dt} = \mu_o \frac{r^2}{R^2} I$$

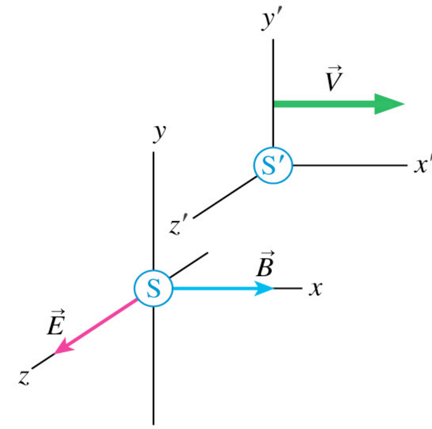
$$B_\theta(r) = (\mu_o / 2\pi)\frac{r}{R^2} I$$

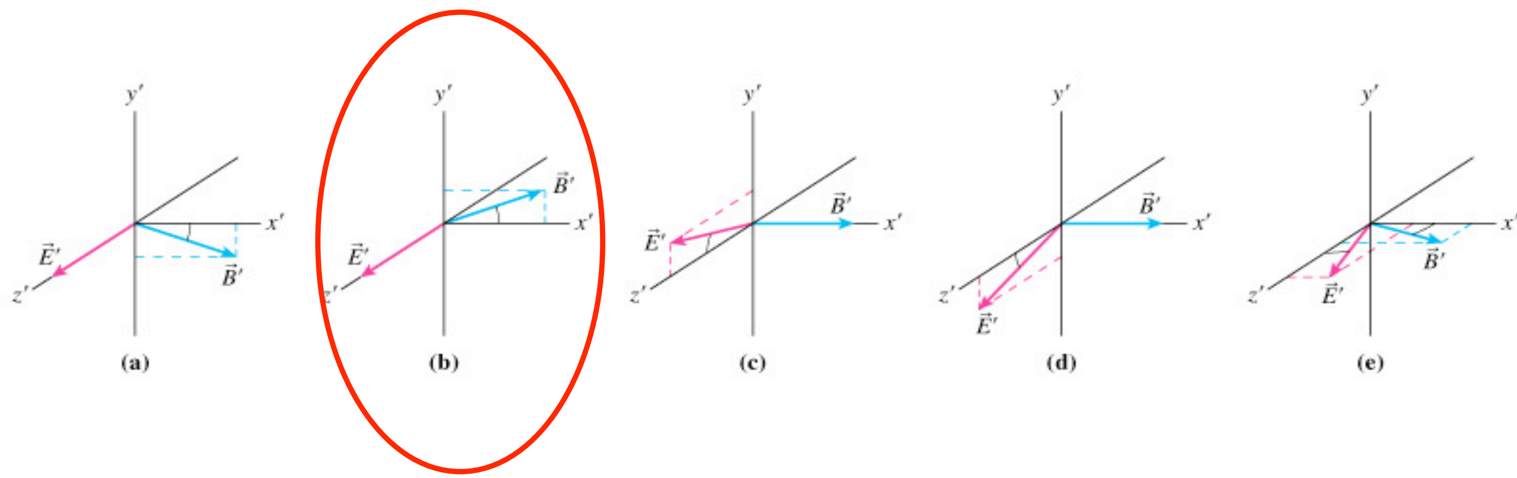
Which diagram shows the fields in frame S´?



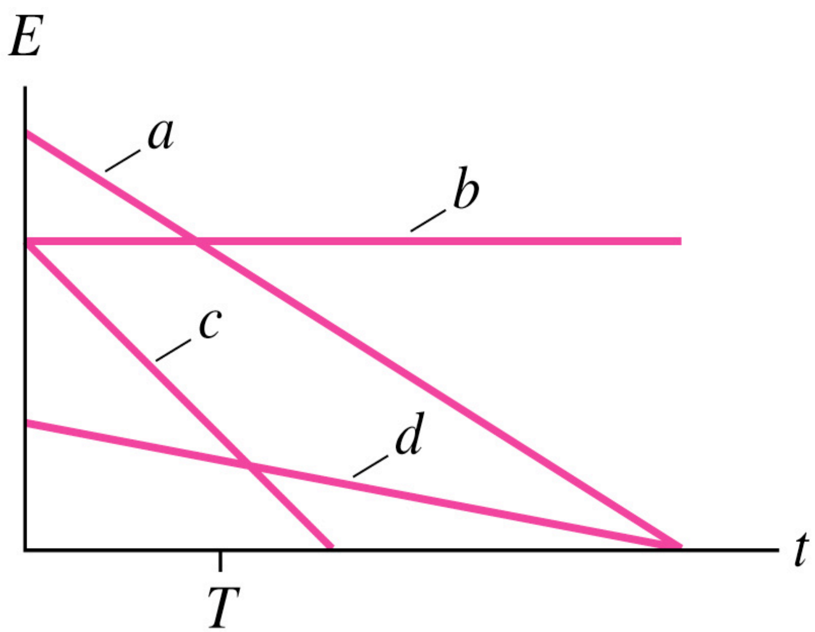


Which diagram shows the fields in frame S´?



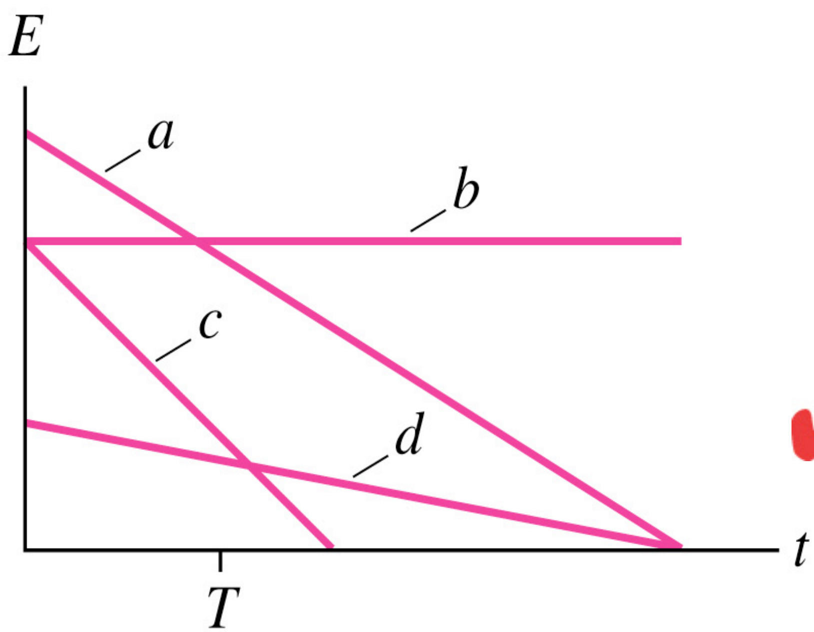


The electric field in four identical capacitors is shown as a function of time. Rank in order, from largest to smallest, the magnetic field strength at the outer edge of the capacitor at time *T*.



A. $B_a = B_b > B_c = B_d$
B. $B_d > B_c > B_a = B_b$
C. $B_a > B_b > B_c > B_d$
D. $B_a = B_a > B_c > B_d$
E. $B_c > B_a > B_d > B_b$

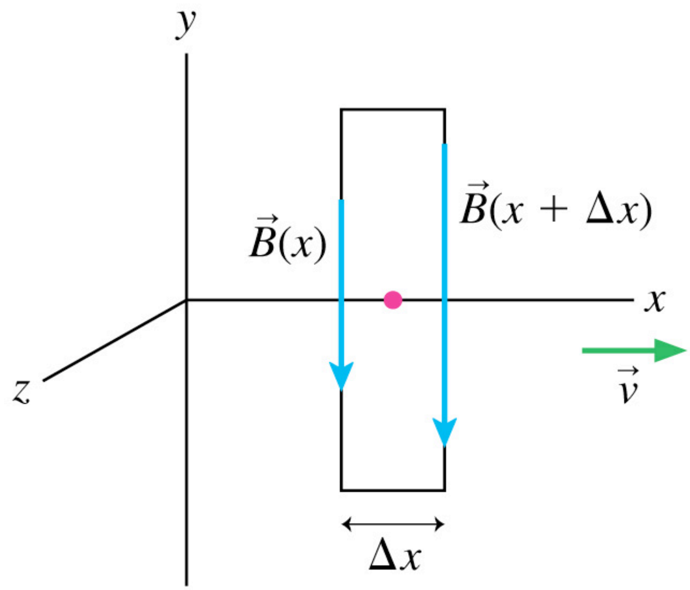
The electric field in four identical capacitors is shown as a function of time. Rank in order, from largest to smallest, the magnetic field strength at the outer edge of the capacitor at time *T*.



A. $B_a = B_b > B_c = B_d$
B. $B_d > B_c > B_a = B_b$
C. $B_a > B_b > B_c > B_d$
D. $B_a = B_a > B_c > B_d$
✔ **E. $\boldsymbol{B_c > B_a > B_d > B_b}$**

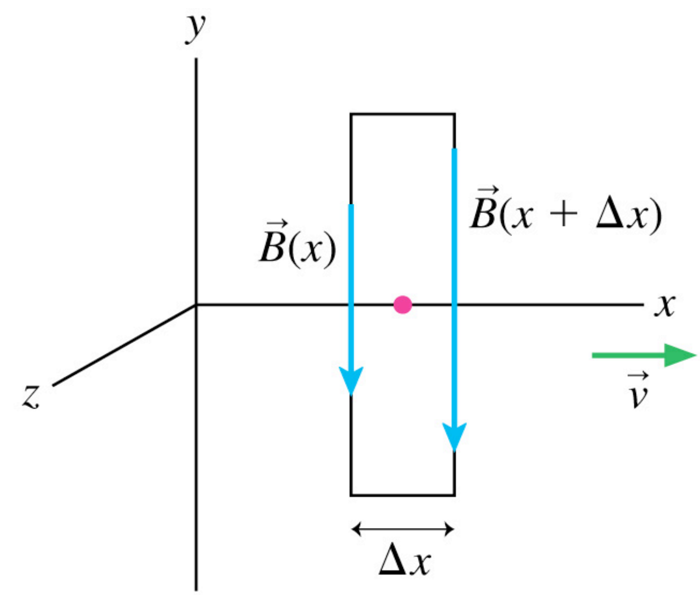
Fastest change of E with time corresponds to biggest slope

An electromagnetic wave is propagating in the positive x-direction. At this instant of time, what is the direction of $\vec{E}$ at the center of the rectangle?



A. In the positive x-direction
B. In the negative x-direction
C. In the positive z-direction
D. In the negative z-direction
E. In the positive y-direction

An electromagnetic wave is propagating in the positive *x*-direction. At this instant of time, what is the direction of $\vec{E}$ at the center of the rectangle?



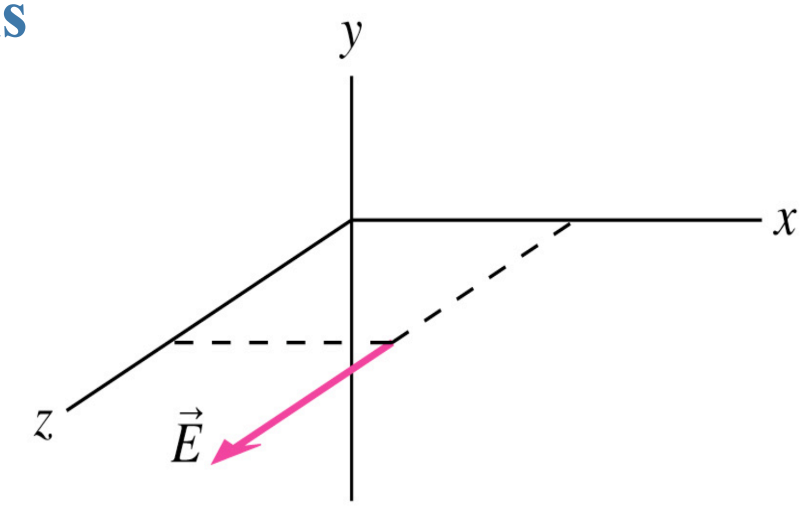
A. In the positive *x*-direction
B. In the negative *x*-direction
✔ **C. In the positive *z*-direction**
D. In the negative *z*-direction
E. In the positive *y*-direction

$$\vec{v} = \vec{E} \times \vec{B}$$

$$\vec{v} = (v,0,0)$$

$$\vec{B} = (0,-B,0)$$

An electromagnetic wave is traveling in the positive *y*-direction. The electric field at one instant of time is shown at one position. The magnetic field at this position points



A. In the positive *y*-direction.
B. In the negative *y*-direction.
C. In the positive *x*-direction.
D. In the negative *x*-direction.
E. Away from the origin.

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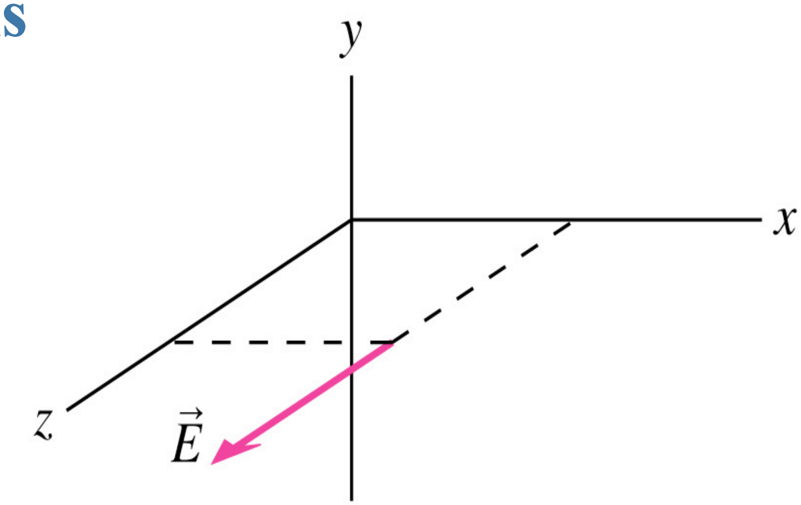

A. In the positive *y*-direction.
B. In the negative *y*-direction.
✔ **C. In the positive *x*-direction.**
D. In the negative *x*-direction.
E. Away from the origin.

$\vec{v} = \vec{E} \times \vec{B}$

$\vec{B} = \vec{v} \times \vec{E}$

$\vec{v} = (0, v, 0)$

$\vec{E} = (0, 0, E)$

The amplitude of the oscillating electric field at your cell phone is 4.0 μV/m when you are 10 km east of the broadcast antenna. What is the electric field amplitude when you are 20 km east of the antenna?

A. 4.0 μV/m
B. 2.0 μV/m
C. 1.0 μV/m
D. There’s not enough information to tell.

The amplitude of the oscillating electric field at your cell phone is 4.0 μV/m when you are 10 km east of the broadcast antenna. What is the electric field amplitude when you are 20 km east of the antenna?

A. 4.0 μV/m

✔ **B. 2.0 μV/m**

C. 1.0 μV/m

D. There's not enough information to tell.

$$P \propto 1/R^2$$

$$E \propto 1/R$$