PHYS 270 – SUPPL. #8

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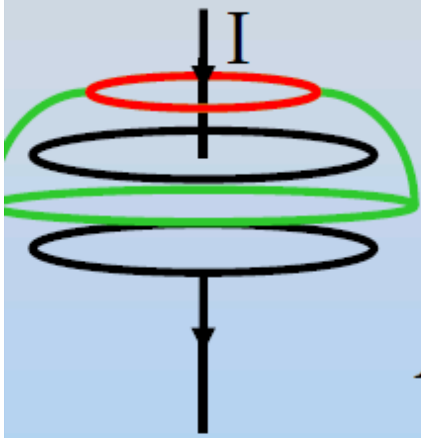
CHAPTER 35

Ampere's Law: Capacitor

Consider a charging capacitor:



Use Ampere's Law to calculate the magnetic field just above the top plate

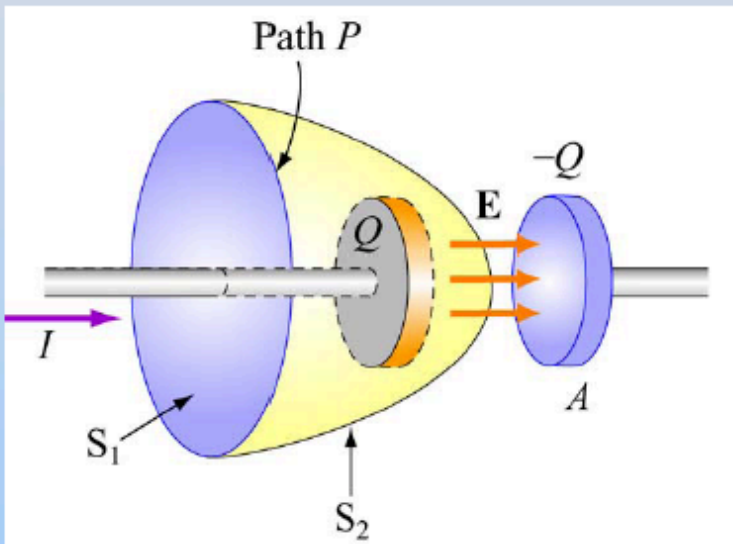
$$\text{Ampere's law: } \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

1) Red Amperian Area, $I_{enc} = I$
2) Green Amperian Area, $I = 0$

What's Going On?

Displacement Current

We don't have current between the capacitor plates but we do have a changing E field. Can we "make" a current out of that?



$$E = \frac{Q}{\varepsilon_0 A} \Rightarrow Q = \varepsilon_0 EA = \varepsilon_0 \Phi_E$$

$$\boxed{\frac{dQ}{dt} = \varepsilon_0 \frac{d\Phi_E}{dt} \equiv I_d}$$

This is called (for historic reasons) the **Displacement Current**

Maxwell-Ampere's Law

$$\oint_C \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 (I_{encl} + I_d)$$

$$= \mu_0 I_{encl} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

Electromagnetism Review

- E fields are created by:
  - (1) electric charges — Gauss's Law
  - (2) time changing B fields — Faraday's Law
- B fields are created by
  - (1) moving electric charges (*NOT* magnetic charges) — Ampere's Law
  - (2) time changing E fields — Maxwell's Addition
- E (B) fields exert forces on (moving) electric charges

Lorentz Force

Maxwell's Equations

$$\oiint_S \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{Q_{in}}{\varepsilon_0}$$ (Gauss's Law)

$$\oint_C \vec{\mathbf{E}} \cdot d\,\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}$$ (Faraday's Law)

$$\oiint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$ (Magnetic Gauss's Law)

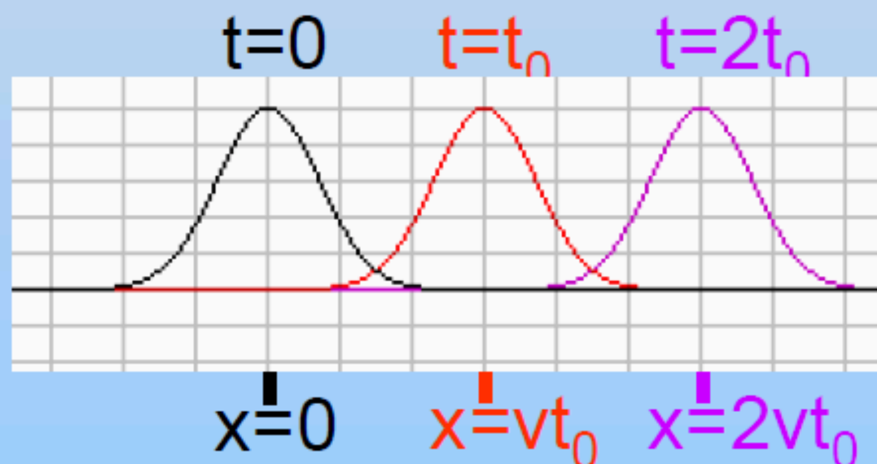
$$\oint_C \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$ (Ampere-Maxwell Law)

$$\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$$ (Lorentz force Law)

Traveling Waves

Consider f(x) =

What is g(x,t) = f(x-vt)?



f(x-vt) is traveling wave moving to the right!

$$g(x,t) = f(x - vt)$$

$$\frac{\partial g(x,t)}{\partial x} = \frac{\partial f(x - vt)}{\partial (x - vt)} \frac{\partial (x - vt)}{\partial x} = f'(x - vt)$$

$$\frac{\partial^2 g(x,t)}{\partial^2 x} = f''(x - vt)$$

$$\frac{\partial g(x,t)}{\partial t} = -vf'(x - vt)$$

$$\frac{\partial^2 g(x,t)}{\partial^2 t} = v^2 f''(x - vt)$$

$$\frac{\partial^2 g(x,t)}{\partial^2 t} = v^2 \frac{\partial^2 g(x,t)}{\partial^2 x}$$

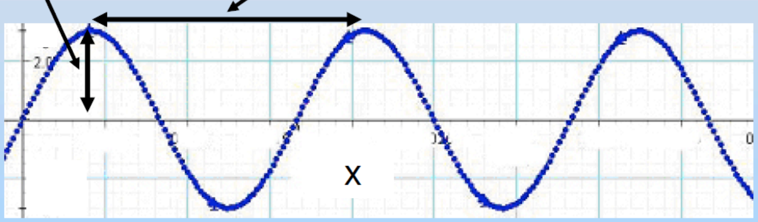
The wave equation

Traveling Sine Wave

Now consider f(x) = y = $y_0$sin(kx):

Amplitude (y_0)

$\text{Wavelength}\ (\lambda) = \dfrac{2\pi}{\text{wavenumber}\ (k)}$



What is g(x,t) = f(x+vt)? Travels to left at velocity v

y = $y_0$sin(k(x+vt)) = $y_0$sin(kx+kvt)

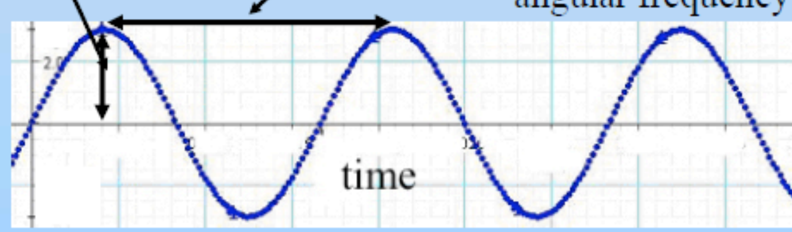
Traveling Sine Wave

$$y = y_0 \sin(kx + kvt)$$

At x=0, just a function of time: $y = y_0 \sin(kvt) \equiv y_0 \sin(\omega t)$

$\text{Period}\ (T) = \dfrac{1}{\text{frequency}\ (f)} = \dfrac{2\pi}{\text{angular frequency}\ (\omega)}$

Amplitude (y_0)



Traveling Sine Wave

$$y = y_0 \sin(kx - \omega t)$$

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

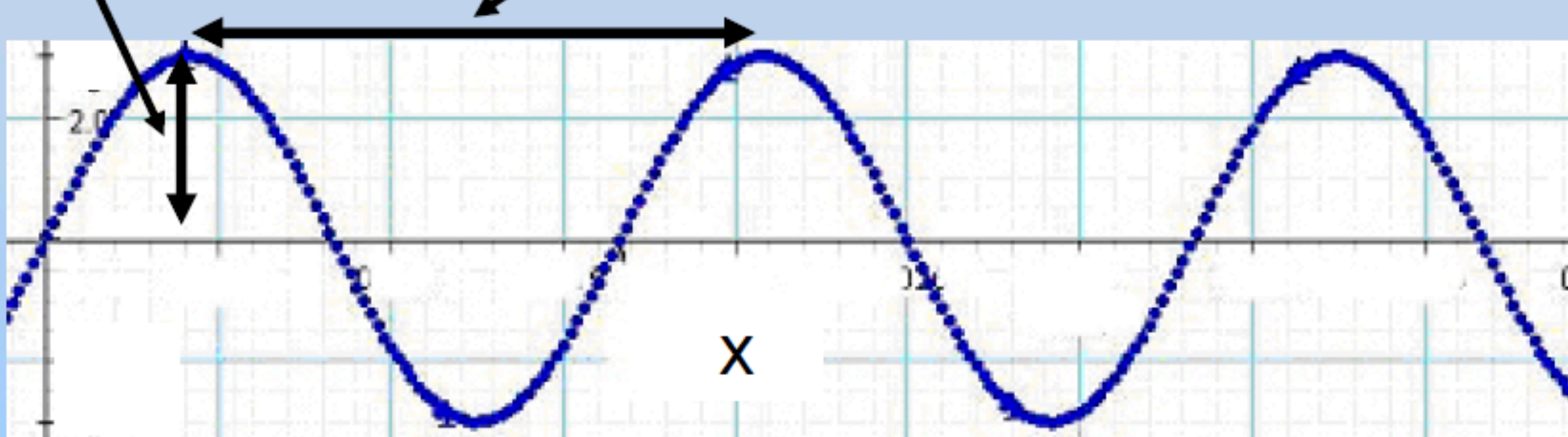
From Chapter 20

Traveling Sine Wave

Now consider $f(x) = y = y_0\sin(kx)$:

Amplitude (y_0)

$$\text{Wavelength } (\lambda) = \frac{2\pi}{\text{wavenumber } (k)}$$



What is $g(x,t) = f(x+vt)$? Travels to left at velocity v

$$y = y_0\sin(k(x+vt)) = y_0\sin(kx+kvt)$$

Traveling Sine Wave

$$y = y_0 \sin\left(kx + kvt\right)$$

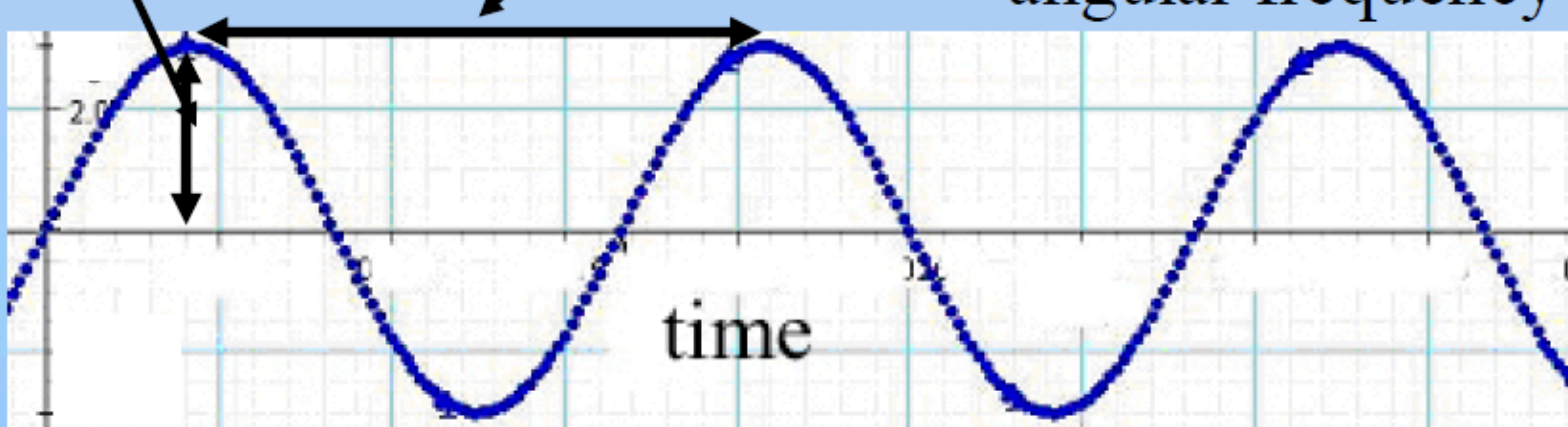
Introduce frequency

At x=0, just a function of time: $y = y_0 \sin(kvt) \equiv y_0 \sin(\omega t)$

$$\text{Period}\ (T) = \frac{1}{\text{frequency}\ (f)}$$

$$= \frac{2\pi}{\text{angular frequency}\ (\omega)}$$

Amplitude (y_0)



Traveling Sine Wave

$$y = y_0 \sin(kx - \omega t)$$

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \dfrac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \dfrac{1}{f} = \dfrac{2\pi}{\omega}$
- Speed of Propagation: $v = \dfrac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

At constant x

$$\vec{E} = e_y E_o \sin(\omega t + \phi)$$

$$\vec{B} = e_x B_o \sin(\omega t + \phi)$$

$$E / B = c$$

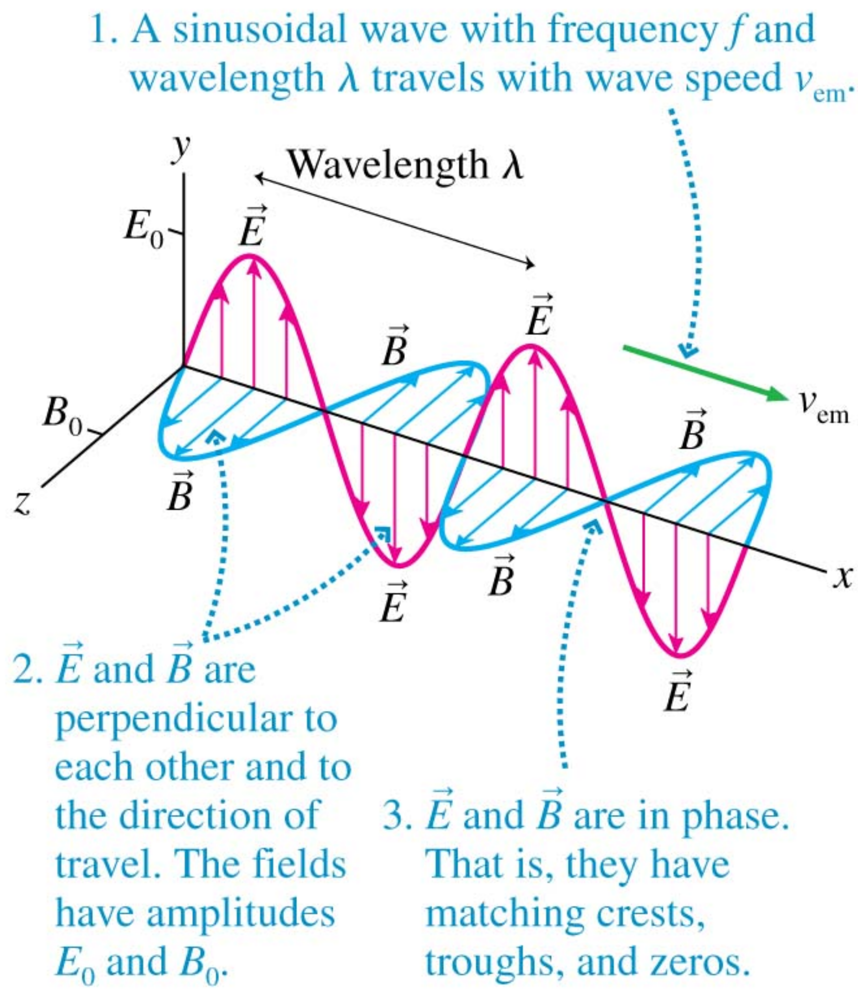
Traveling E & B Waves

- Wavelength: λ
- Frequency : f
- Wave Number: $k = \frac{2\pi}{\lambda}$
- Angular Frequency: $\omega = 2\pi f$
- Period: $T = \frac{1}{f} = \frac{2\pi}{\omega}$
- Speed of Propagation: $v = \frac{\omega}{k} = \lambda f$
- Direction of Propagation: $+x$

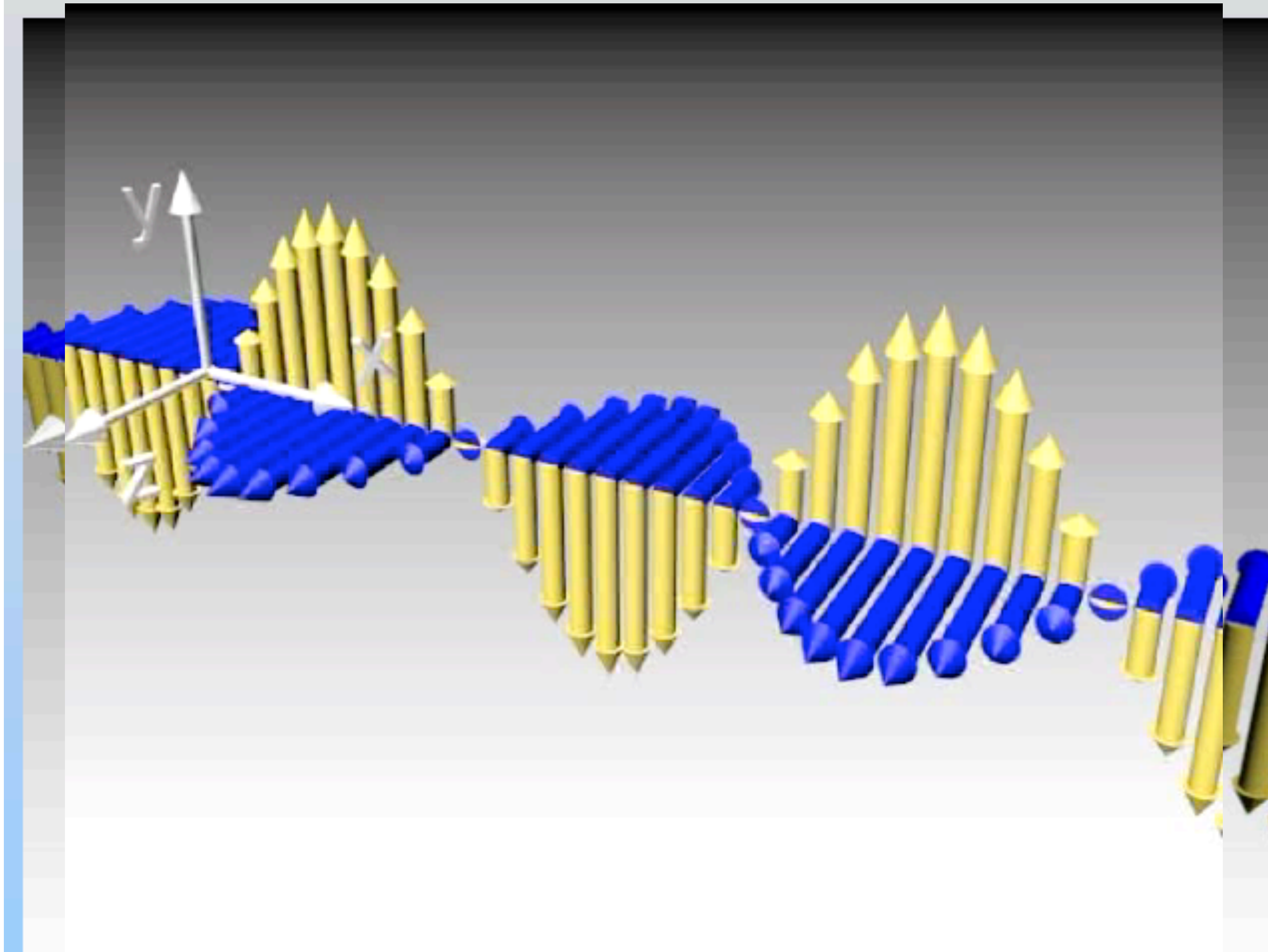
$$\boxed{\vec{\mathbf{E}} = \hat{\mathbf{E}} E_0 \sin(kx - \omega t)}$$

P30-15

FIGURE 35.19 A sinusoidal electromagnetic wave.



EM WAVES – PLANE WAVES



Watch 2 Ways:

1) Sine wave traveling to right (+x)
2) Collection of out of phase oscillators (watch one position)

Don't confuse vectors with heights – they are magnitudes of E (gold) and B (blue)

P28-20

Wave movie

Derivation of Wave Equation

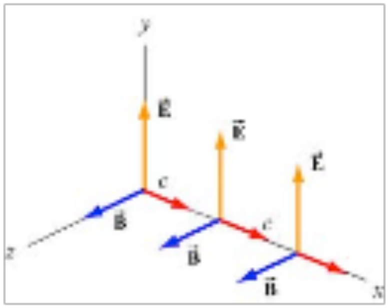


Figure 13.4.1 A plane electromagnetic wave

Loop in
(x,y) plane

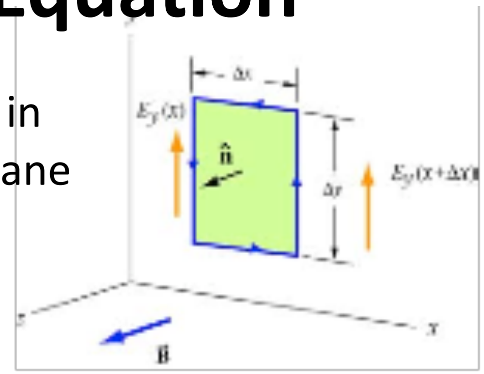


Figure 13.4.2 Spatial variation of the electric field $\vec{\mathbf{E}}$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

$$-\frac{d}{dt}\iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = -\left(\frac{\partial B_z}{\partial t}\right)(\Delta x\, \Delta y)$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E_y(x+\Delta x)\Delta y - E_y(x)\Delta y = [E_y(x+\Delta x) - E_y(x)]\Delta y = \frac{\partial E_y}{\partial x}(\Delta x\, \Delta y)$$

$$E_y(x+\Delta x) = E_y(x) + \frac{\partial E_y}{\partial x}\Delta x + \ldots$$

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

Derivation of Wave Equation

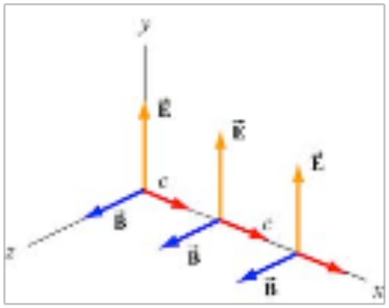


Figure 13.4.1 A plane electromagnetic wave

Loop in
(y,z) plane

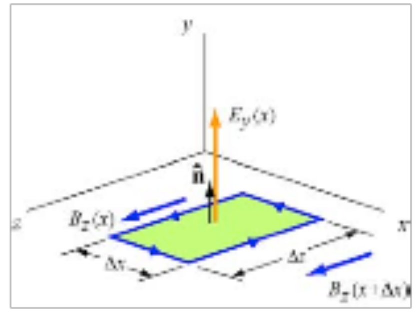


Figure 13.4.3 Spatial variation of the magnetic field $\vec{\mathbf{B}}$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \varepsilon_0 \frac{d}{dt} \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\mu_0 \varepsilon_0 \frac{d}{dt} \iint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \mu_0 \varepsilon_0 \left(\frac{\partial E_y}{\partial t} \right) (\Delta x \, \Delta z)$$

$$\oint \vec{\mathbf{B}} \cdot d\,\vec{\mathbf{s}} = B_z(x)\Delta z - B_z(x+\Delta x)\Delta z = [B_z(x) - B_z(x+\Delta x)]\Delta z$$

$$= -\left(\frac{\partial B_z}{\partial x} \right) (\Delta x \, \Delta z)$$

$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \left(\frac{\partial E_y}{\partial t} \right)$$

Derivation of Wave Equation

$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

$$-\frac{\partial B_z}{\partial x} = \mu_0 \varepsilon_0 \left(\frac{\partial E_y}{\partial t} \right)$$

$$\frac{\partial}{\partial x}$$

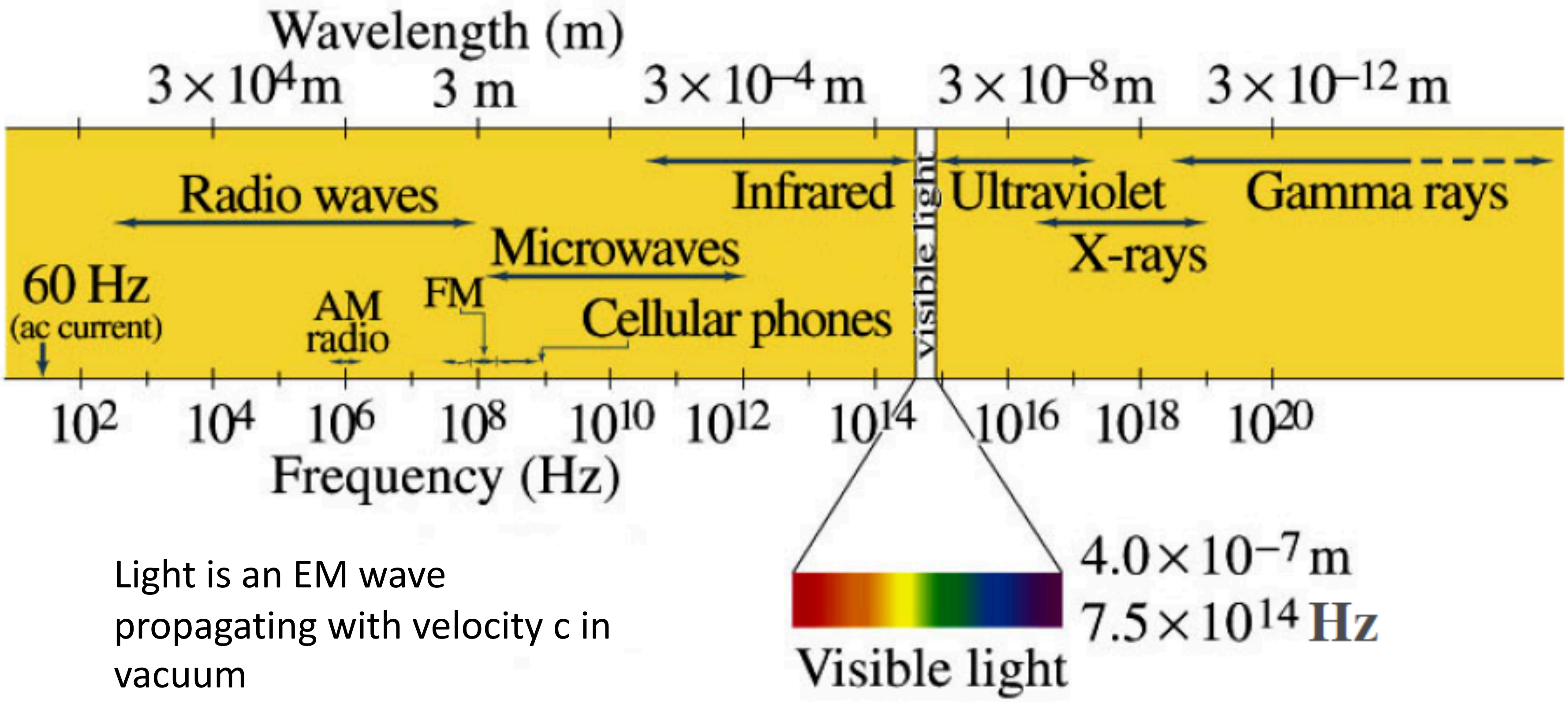
$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial}{\partial x}\left(\frac{\partial B_z}{\partial t}\right) = -\frac{\partial}{\partial t}\left(\frac{\partial B_z}{\partial x}\right) = -\frac{\partial}{\partial t}\left(-\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}\right) = \mu_0 \varepsilon_0 \frac{\partial^2 E_y}{\partial t^2}$$

$$\frac{\partial}{\partial x}$$

$$\frac{\partial^2 B_z}{\partial x^2} = -\frac{\partial}{\partial x}\left(\mu_0 \varepsilon_0 \frac{\partial E_y}{\partial t}\right) = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E_y}{\partial x}\right) = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial B_z}{\partial t}\right) = \mu_0 \varepsilon_0 \frac{\partial^2 B_z}{\partial t^2}$$

$$\left(\frac{\partial^2}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \right) \begin{Bmatrix} E_y(x,t) \\ B_z(x,t) \end{Bmatrix} = 0 \qquad \mu_o \varepsilon_o = 1/c^2$$

Electromagnetic Waves



Light is an EM wave propagating with velocity c in vacuum

EM Wave Properties

$$\left(\frac{\partial^2}{\partial x^2} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2}\right)\begin{Bmatrix} E_y(x,t) \\ B_z(x,t) \end{Bmatrix} = 0$$

Possible solution of WE

$$\vec{\mathbf{E}} = E_y(x,t)\hat{\mathbf{j}} = E_0 \cos k(x - vt)\hat{\mathbf{j}} = E_0 \cos(kx - \omega t)\hat{\mathbf{j}}$$

$$\vec{\mathbf{B}} = B_z(x,t)\hat{\mathbf{k}} = B_0 \cos k(x - vt)\hat{\mathbf{k}} = B_0 \cos(kx - \omega t)\hat{\mathbf{k}}$$

$$\frac{\partial E_y}{\partial x} = -kE_0 \sin(kx - \omega t)$$

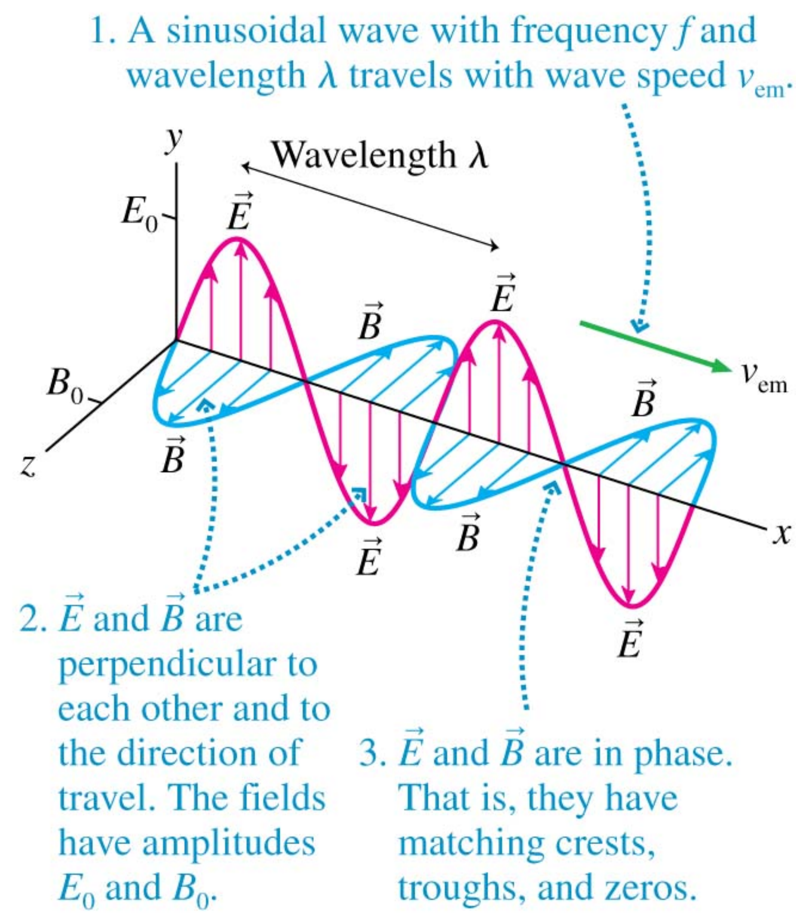
$$\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$$

$$\frac{\partial B_z}{\partial t} = \omega B_0 \sin(kx - \omega t)$$

$$\frac{E_o}{B_o} = \frac{\omega}{k} = c$$

$$E / B = c$$

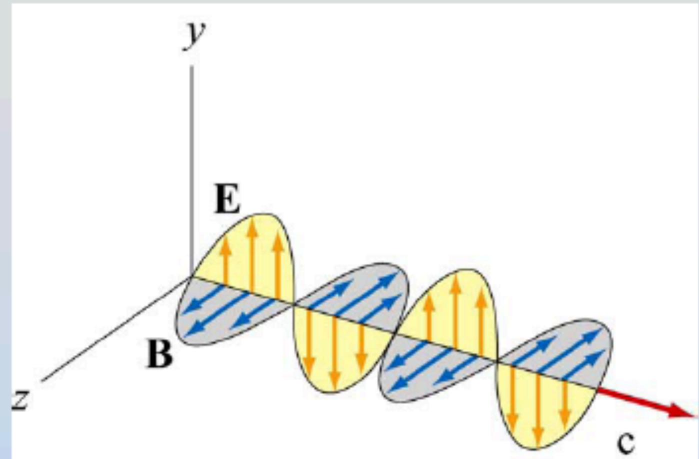
FIGURE 35.19 A sinusoidal electromagnetic wave.



Properties of EM Waves

Travel (through vacuum) with speed of light

$$v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times 10^8 \frac{m}{s}$$



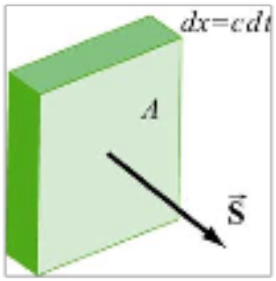
At every point in the wave and any instant of time, E and B are in phase with one another, with

$$\frac{E}{B} = \frac{E_0}{B_0} = c$$

E and B fields perpendicular to one another, and to the direction of propagation (they are **transverse**):

$$\text{Direction of propagation} = \text{Direction of } \vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

Poynting Vector



Electromagnetic wave passing through a volume element

$$dU = uA\,dx = (u_E + u_B)A\,dx = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right)A\,dx$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2, \qquad u_B = \frac{B^2}{2\mu_0}$$

Rate of energy flow per unit area

$$S = \frac{dU}{Adt} = \frac{uAdx}{Adt} = uc = c(u_E + u_B)$$

$$S = \frac{c}{2}\left(\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right) = \frac{cB^2}{\mu_0} = c\varepsilon_0 E^2 = \frac{EB}{\mu_0}$$

$$|\vec{\mathbf{S}}| = \frac{\left|\vec{\mathbf{E}}\times\vec{\mathbf{B}}\right|}{\mu_0} = \frac{EB}{\mu_0} = S$$

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$$

Wave Intensity

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$$

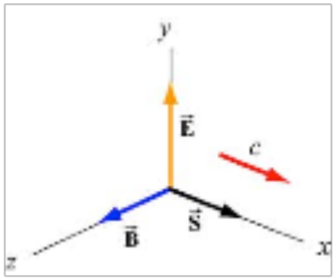


Figure 13.6.2 Poynting vector for a plane wave

Consider Sinusoidal waves

$$\vec{\mathbf{S}} = \frac{1}{\mu_0}\left(E_0 \cos(kx-\omega t)\hat{\mathbf{j}}\right) \times \left(B_0 \cos(kx-\omega t)\hat{\mathbf{k}}\right) = \frac{E_0 B_0}{\mu_0}\cos^2(kx-\omega t)\hat{\mathbf{i}}$$

Watts/m^2

The intensity of the wave, I, defined as the time average of S, is given by

$$I = \langle S \rangle = \frac{E_0 B_0}{\mu_0}\langle \cos^2(kx-\omega t)\rangle = \frac{E_0 B_0}{2\mu_0} = \frac{E_0^2}{2c\mu_0} = \frac{cB_0^2}{2\mu_0}$$

where we have used

$$\langle \cos^2(kx-\omega t)\rangle = \frac{1}{2}$$

Wave Momentum and Radiation Pressure

Maxwell showed that EM waves carry in addition to energy momentum.
They push a surface that impact upon it

$$\Delta p = \Delta U / c \text{ for complete absorption}$$

$$\Delta p = 2\Delta U / c \text{ for complete reflection}$$

Average Radiation Pressure (Force per unit area) P

$$P = \frac{\langle F \rangle}{A} = \frac{1}{A}\left\langle \frac{dp}{dt} \right\rangle = \frac{1}{Ac}\left\langle \frac{dU}{dt} \right\rangle \qquad \left\langle \frac{dU}{dt} \right\rangle = \langle S \rangle A = IA$$

$$P = \frac{I}{c} \quad \text{(complete absorption)} \qquad P = \frac{2I}{c} \quad \text{(complete reflection)}$$