PHYS 270 – SUPPL. #6

DENNIS PAPADOPOULOS

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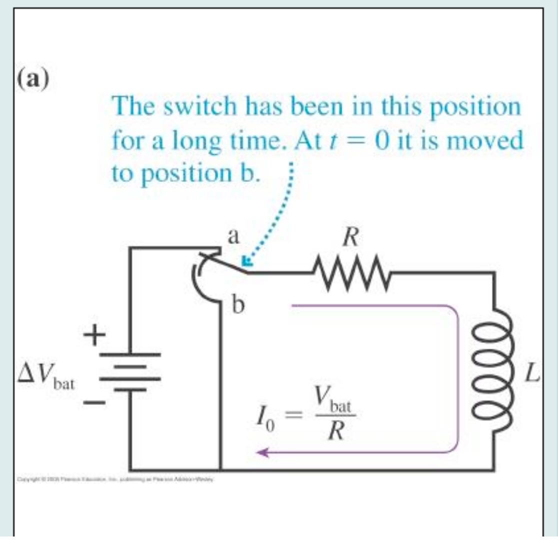
CHAPTER 35

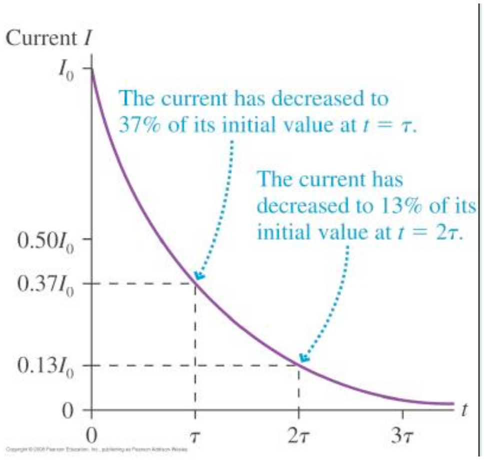
REVIEW FROM CHAPTER 34

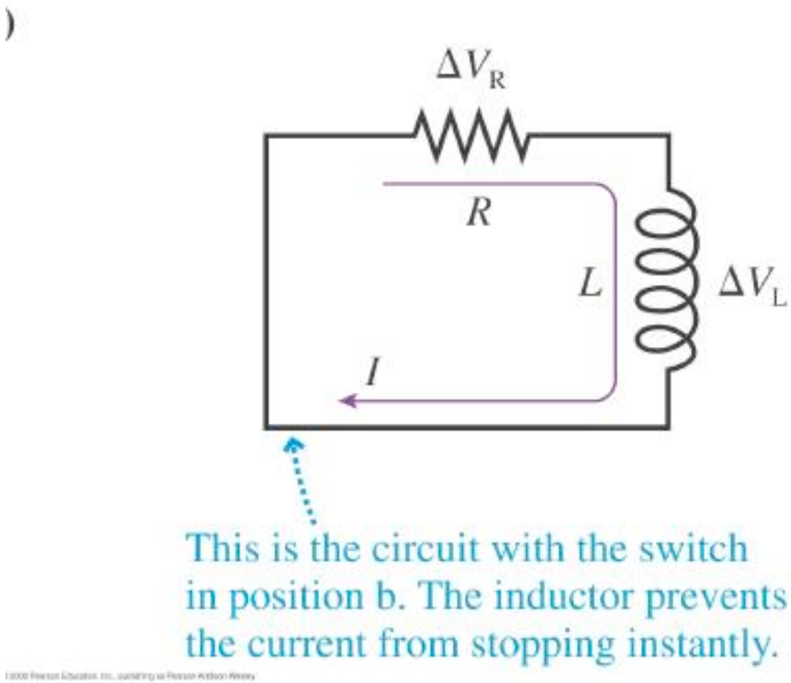


$$P \equiv \frac{dU_L}{dt} = I\Delta V_L = -IL\frac{dI}{dt}$$

$$U_L = L\int_0^I I\,dI = \frac{1}{2}LI^2$$

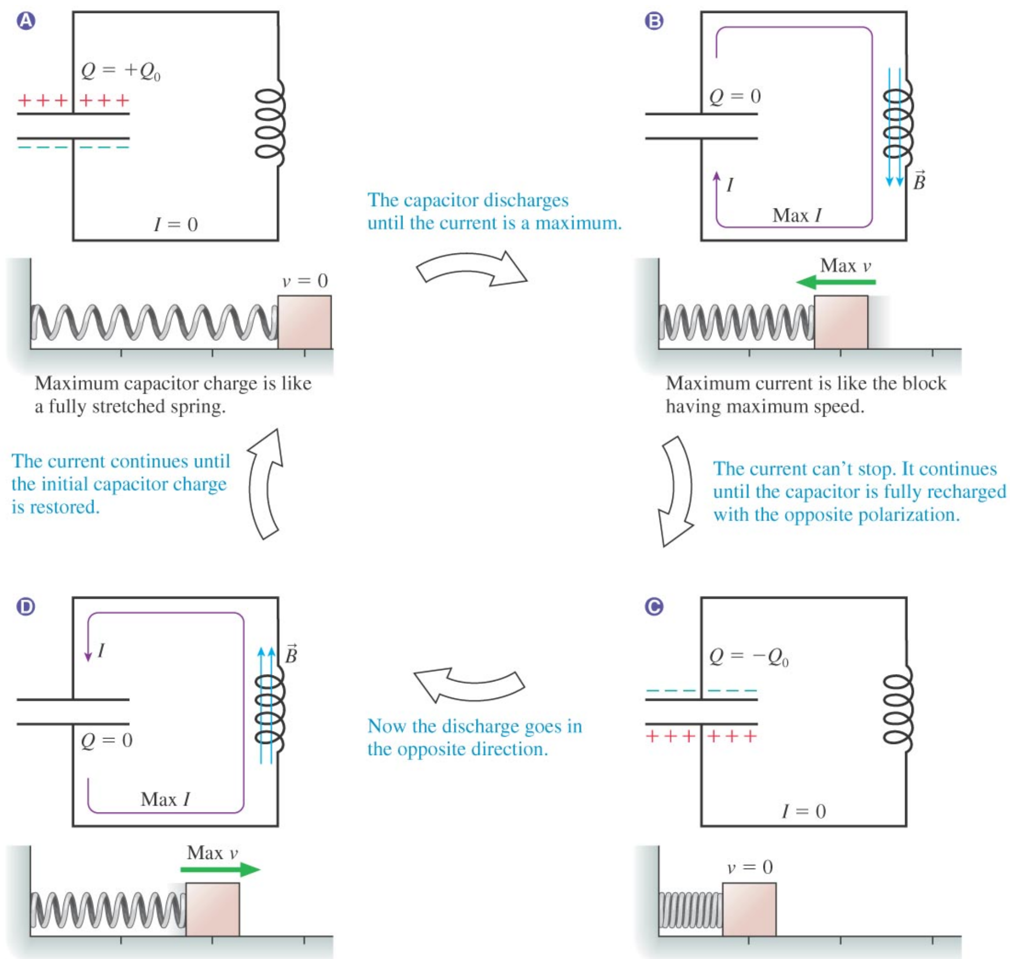
$$U_L = (Al)(B^2/2\mu_o)$$





$$I = I_o \exp[-t/(L/R)]$$

LC CIRCUIT



$$\Delta V_C + \Delta V_L = 0$$

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

$$I = -\frac{dQ}{dt}$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

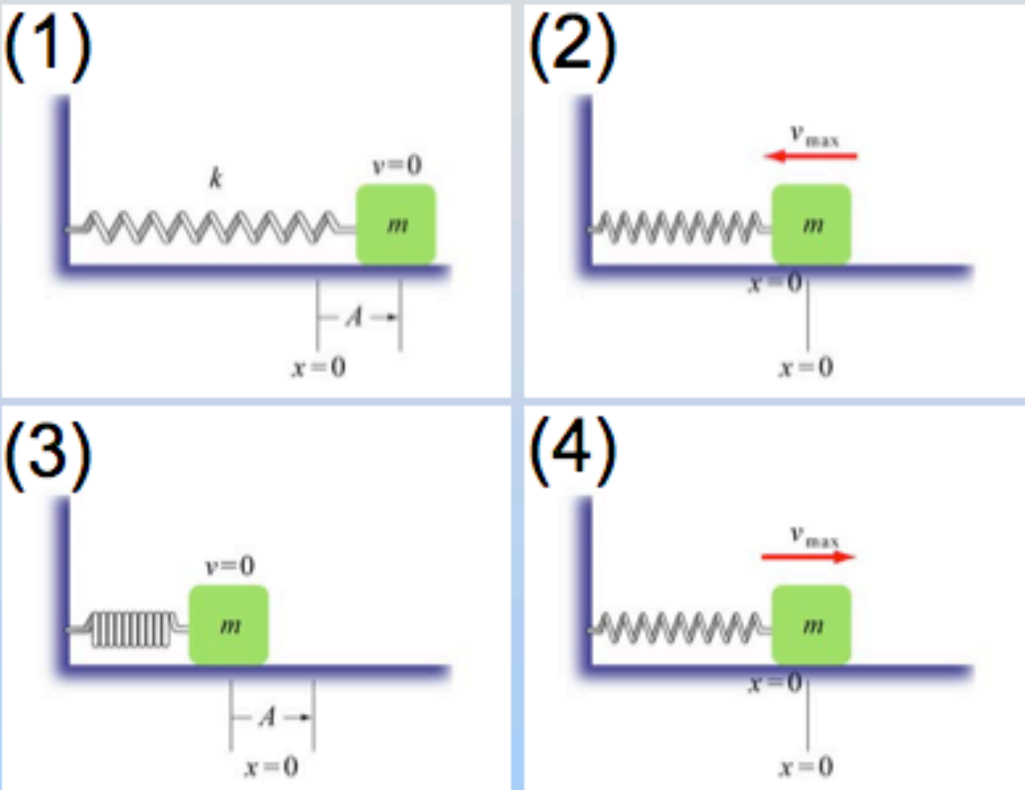
$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

$$\omega \equiv \sqrt{1/LC},\ \sqrt{\mathrm{g/l}}$$

$$Q(t) = Q_o\cos(\omega t)$$

$$I = \omega Q_o \sin(\omega t) \equiv I_{\max}\sin(\omega t)$$

Mass on a Spring



What is Motion?

$$F = -kx = ma = m\frac{d^2x}{dt^2}$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

Simple Harmonic Motion

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

x_0: Amplitude of Motion

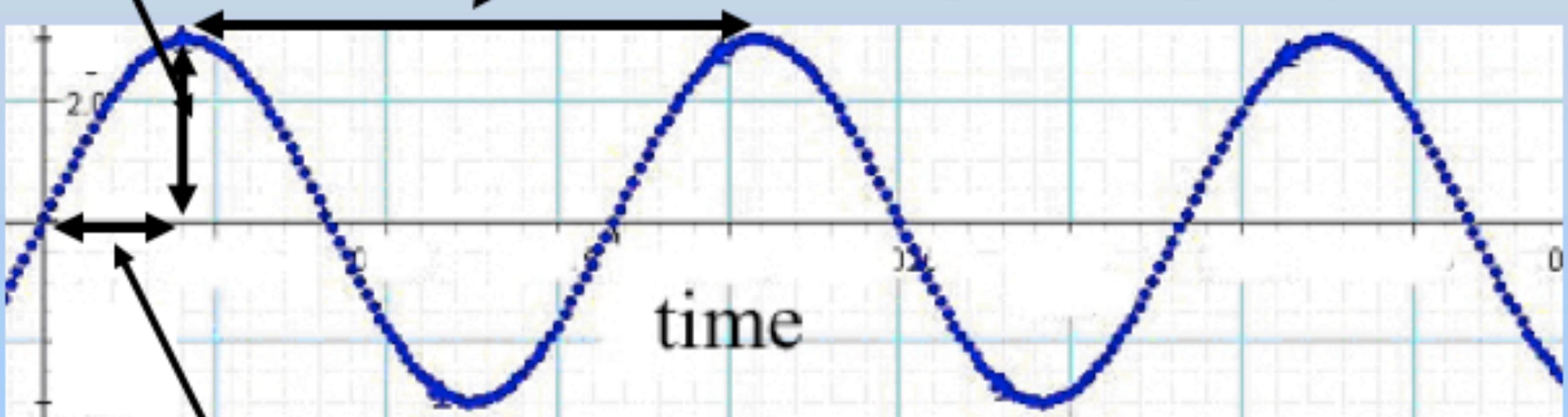
ϕ: Phase (time offset)

$$\omega_0 = \sqrt{\frac{k}{m}} = \text{Angular frequency}$$

Simple Harmonic Motion

$$\text{Period}\ (T) = \frac{1}{\text{frequency}\ (f)} = \frac{2\pi}{\text{angular frequency}\ (\omega)}$$

Amplitude (x_0)



$$x(t) = x_0 \cos(\omega_0 t - \phi)$$

$$\text{Phase Shift}\ (\varphi) = \frac{\pi}{2}$$

Analog: LC Circuit

Mass doesn't like to accelerate

Kinetic energy associated with motion

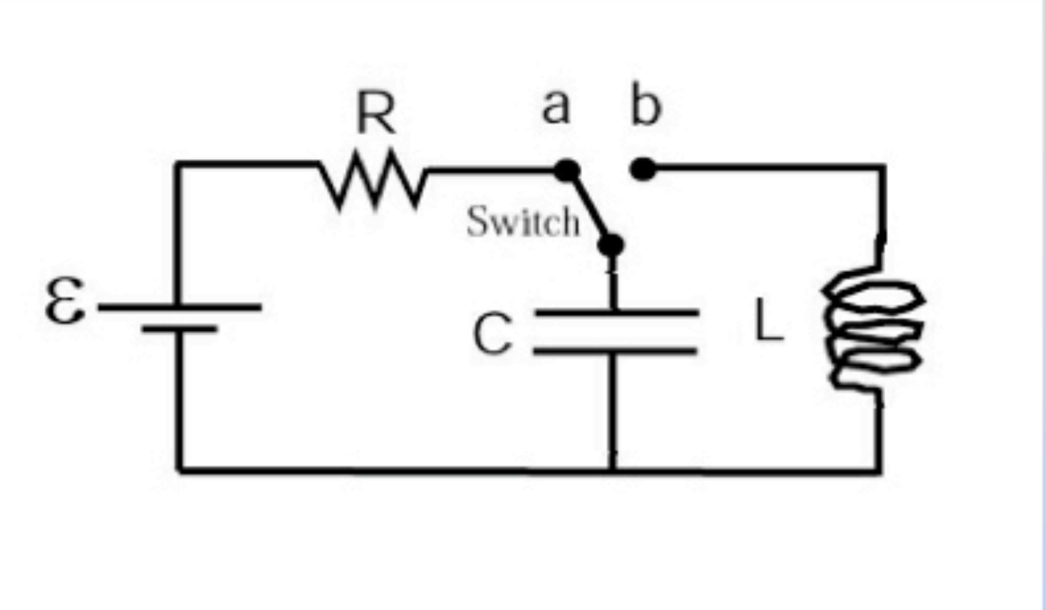
$$F = ma = m\frac{dv}{dt} = m\frac{d^2x}{dt^2}; \quad E = \frac{1}{2}mv^2$$

Inductor doesn't like to have current change

Energy associated with current

$$\varepsilon = -L\frac{dI}{dt} = -L\frac{d^2q}{dt^2}; \quad E = \frac{1}{2}LI^2$$

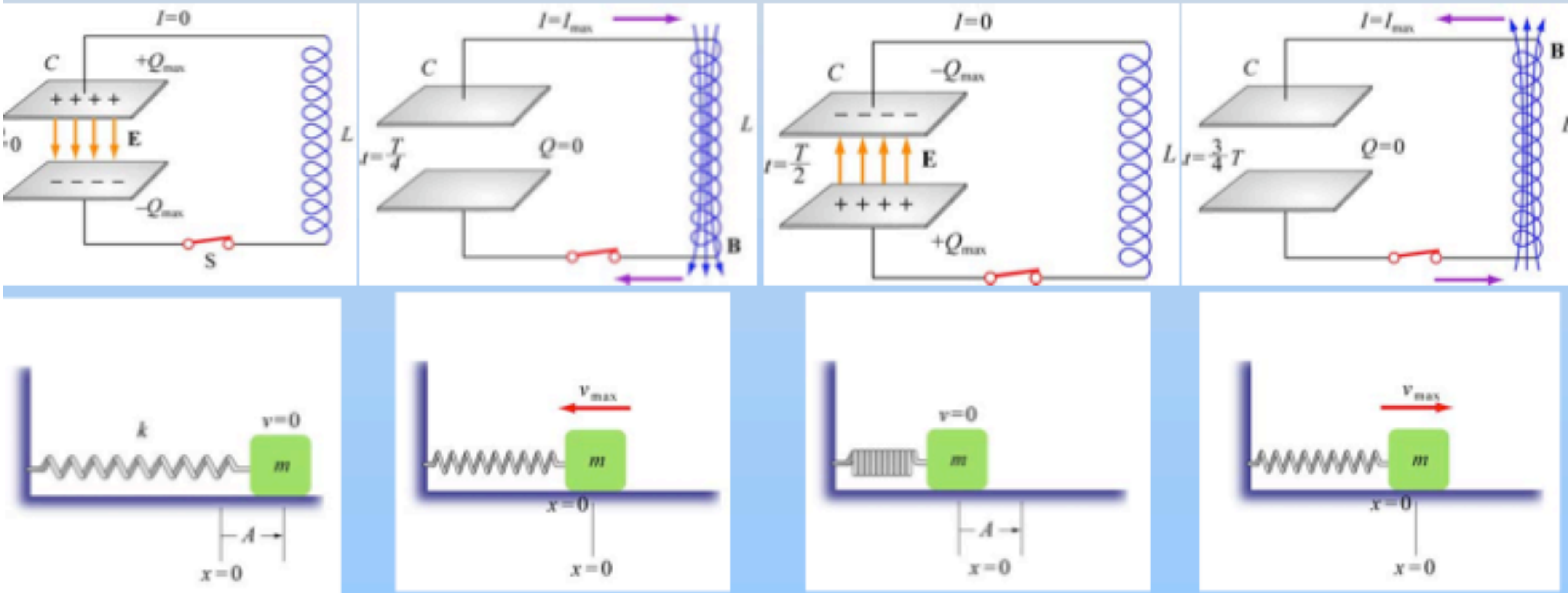
LC Circuit



1. Set up the circuit above with capacitor, inductor, resistor, and battery.
2. Let the capacitor become fully charged.
3. Throw the switch from a to b
4. What happens?

LC Circuit

It undergoes simple harmonic motion, just like a mass on a spring, with trade-off between charge on capacitor (Spring) and current in inductor (Mass)



Analog: LC Circuit

Spring doesn't like to be compressed/extended

Potential energy associated with compression

$$F = -kx; \quad E = \frac{1}{2}kx^2$$

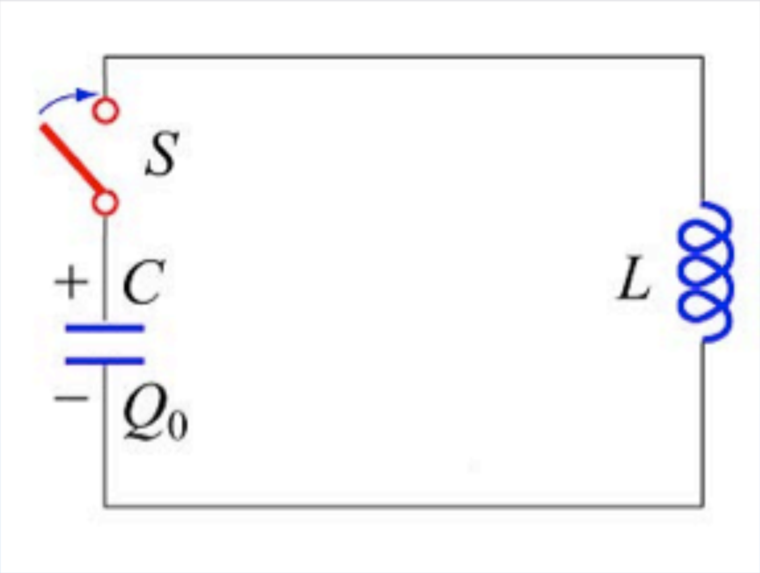
Capacitor doesn't like to be charged (+ or -)

Energy associated with stored charge

$$\varepsilon = \frac{1}{C}q; \quad E = \frac{1}{2}\frac{1}{C}q^2$$

$$F \to \varepsilon;\ x \to q;\ v \to I;\ m \to L;\ k \to C^{-1}$$

LC Circuit



$$\frac{Q}{C} - L\frac{dI}{dt} = 0 \;\; ; \;\; I = -\frac{dQ}{dt}$$

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

Simple Harmonic Motion

$$Q(t) = Q_0 \cos(\omega_0 t + \phi) \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Q_0: Amplitude of Charge Oscillation

ϕ: Phase (time offset)

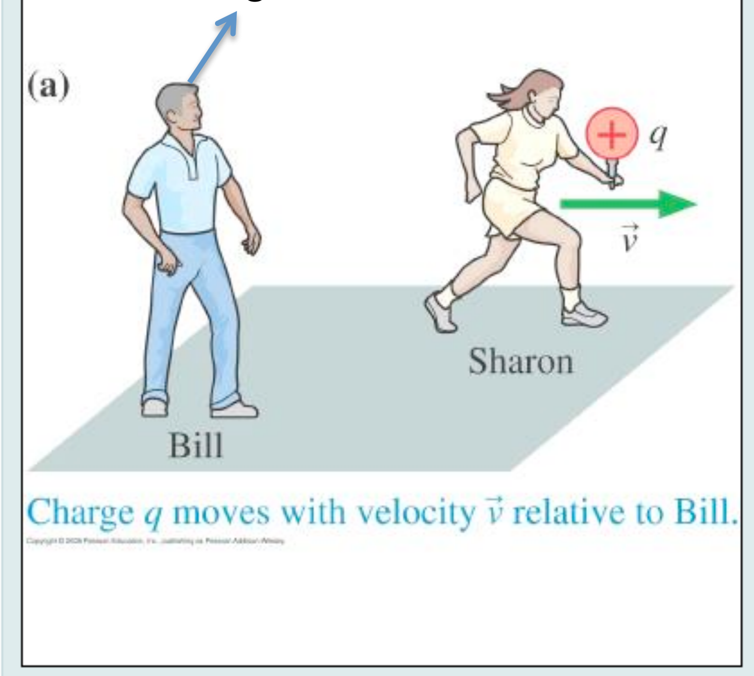
Chapter 35. Electromagnetic Fields and Waves

Topics:

- *E* or *B*? It Depends on Your Perspective
- The Field Laws Thus Far
- The Displacement Current
- Maxwell's Equations
- Electromagnetic Waves
- Properties of Electromagnetic Waves
- Polarization

OBSERVER's RELATIVE STATE OF MOTION

There should be a magnetic field



Charge q moves with velocity $\vec{v}$ relative to Bill.

Sharon:

- static charge
- E field
- No current
- No B field

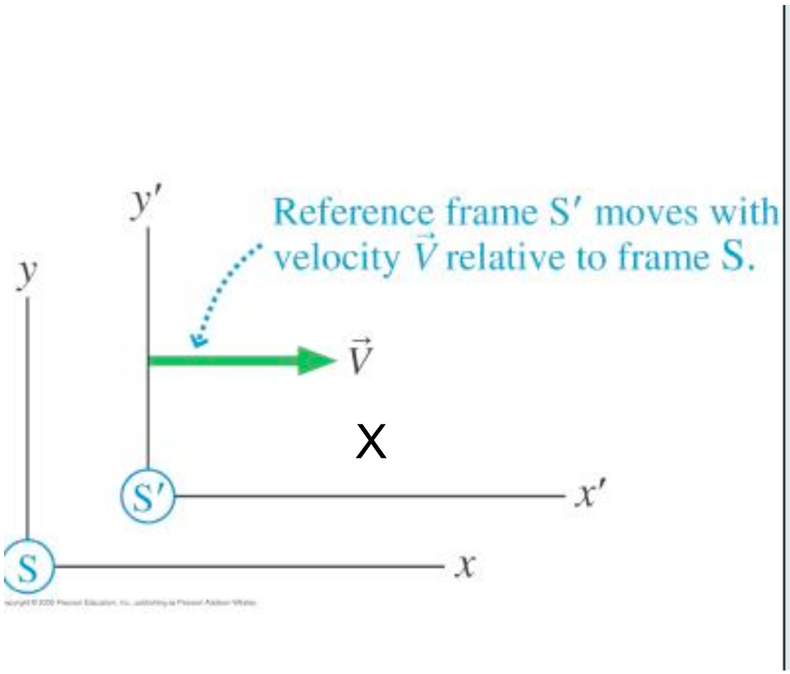
Bill:

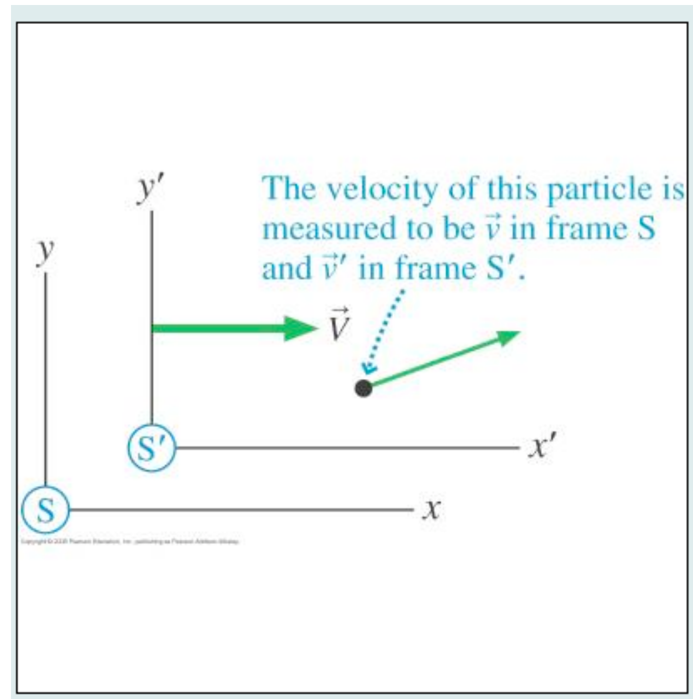
- static charge
- E field
- Current
- B field

Both agree on no net force on the particle

How to reconcile observations involving forces that depend on velocity

GALILEAN RELATIVITY





$$x = x' + Vt$$

$$y = y'$$

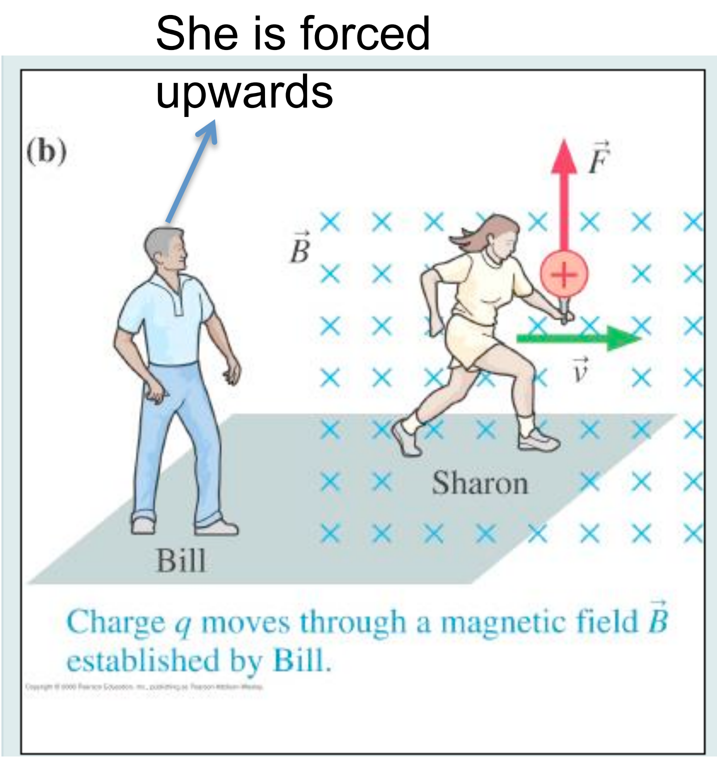
$$v = dx/dt = dx'/dt + V = v' + V$$

$$a = dv/dt = dv'/dt = a'$$

$$F = F'$$

Experimenters in all inertial frames agree about the forces acting on bodies including charges

OBSERVER's RELATIVE STATE OF MOTION



Charge q moves through a magnetic field $\vec{B}$ established by Bill.

Bill sets a B field in his frame and observes the motion (force) on the test charge q given by

$$\vec{F}_B = q\vec{v} \times \vec{B} = qvB \text{ up}$$

However Sharon sees a zero magnetic force since v=0

Physics requires that F=F'

The only option is the presence of an electric field in Sharon's frame

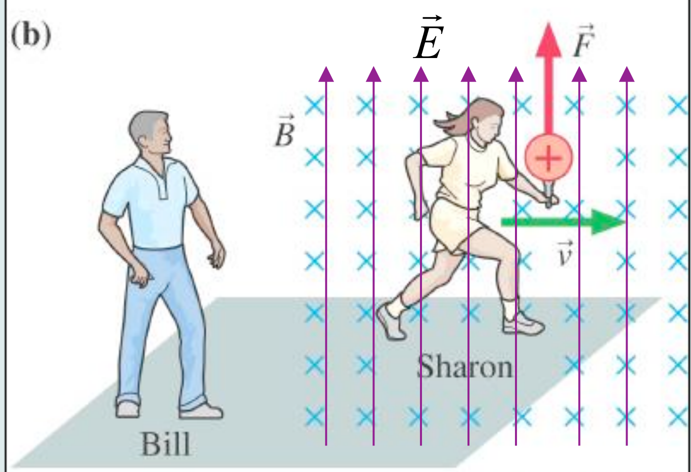
$$q\vec{E}' + q\vec{v}' \times \vec{B}' = q\vec{E} + q\vec{v} \times \vec{B}$$

$$Sharon(S')v' = 0$$

$$Bill(S)E = 0$$

$$\vec{E}' = \vec{v} \times \vec{B}$$

OBSERVER's RELATIVE STATE OF MOTION



Charge q moves through a magnetic field $\vec{B}$ established by Bill.

Add an electric field E in Bill's frame

$$\vec{F}_B = q\vec{E} + q\vec{v} \times \vec{B}$$

Sharon's frame S'

$$\vec{F}' = q\vec{E}'$$

$$\vec{F}' = \vec{F} \Rightarrow \vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

E or *B*? It Depends on Your Perspective

FIGURE 35.4 A charge moves through a magnetic field in frame S and experiences a magnetic force.

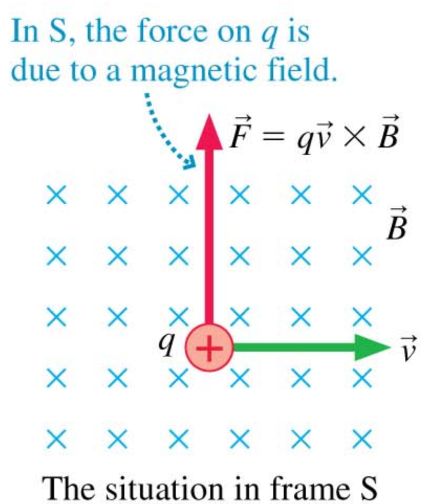
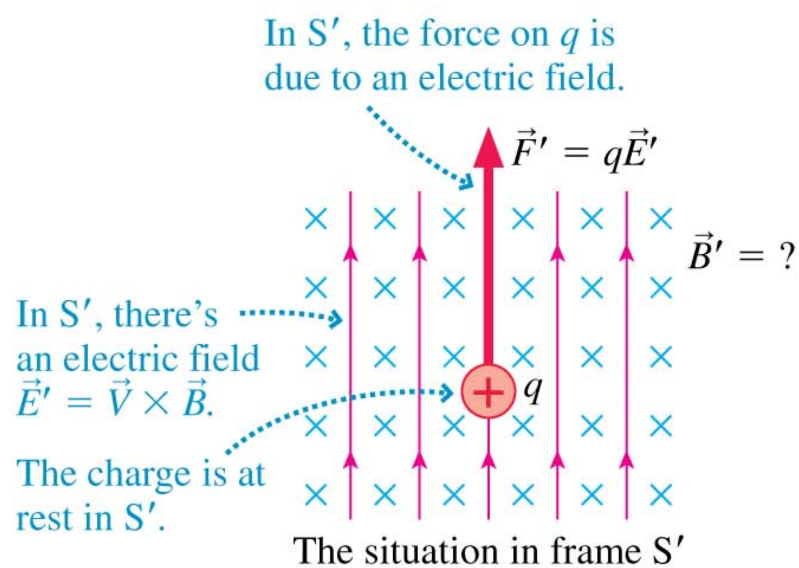


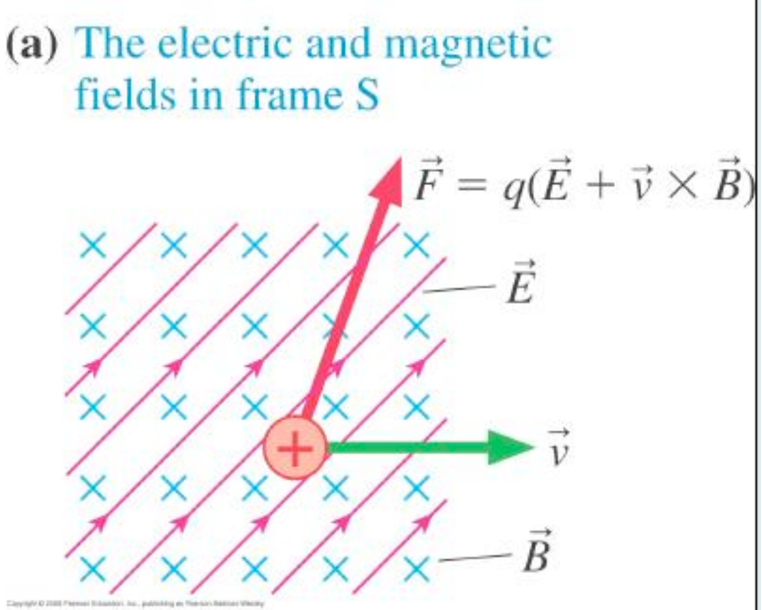
FIGURE 35.5 In frame S′ the charge experiences an electric force.

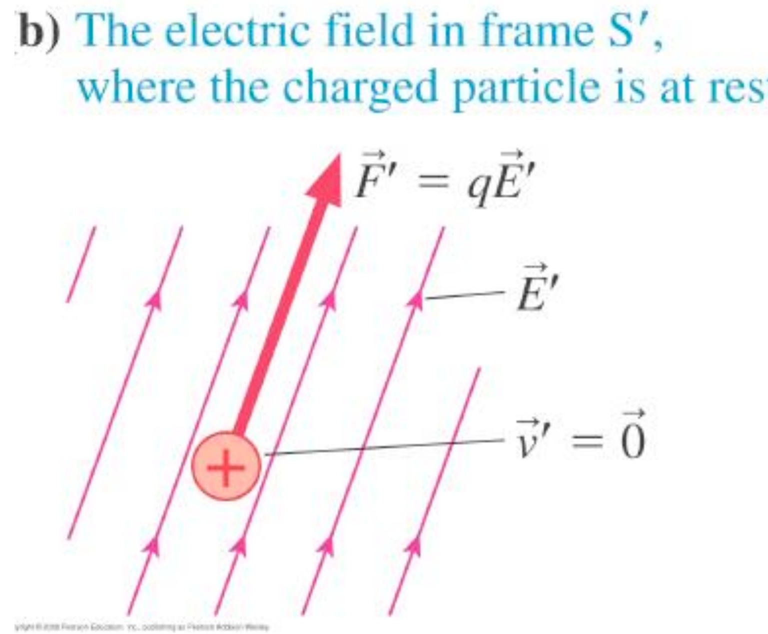


Whether a field is seen as "electric" or "magnetic" depends on the motion of the reference frame relative to the sources of the field.

FROM S TO S’

Bill creates an E and B

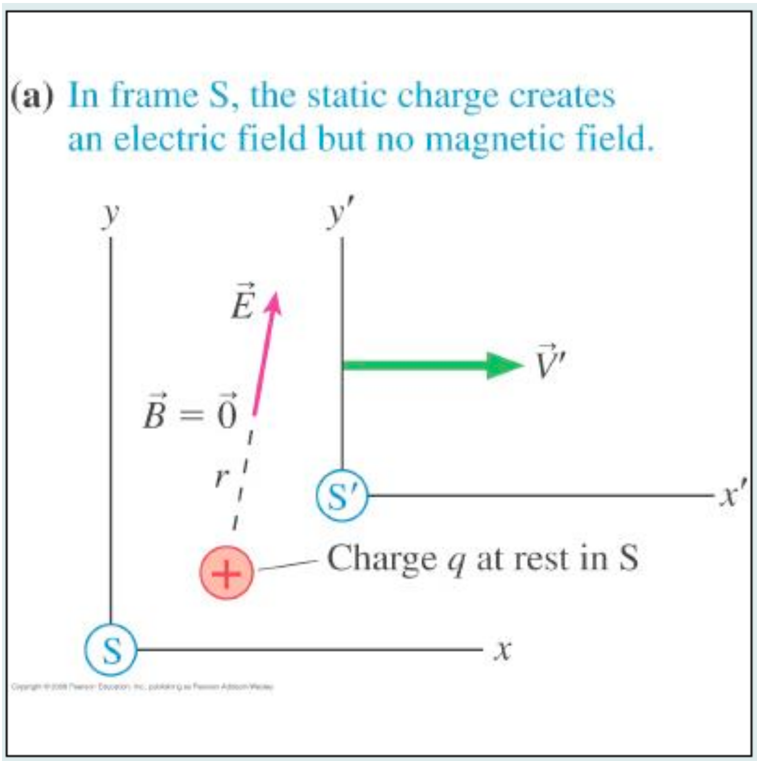




$$\vec{F}' = \vec{F}, \vec{E}' = \vec{E} + \vec{V} \times \vec{B}$$

E FIELD IN FRAME S’ FROM E AND B FIELDS IN FRAME S

HOW DO B FIELDS TRANSFORM



$$\vec{B}' = -\varepsilon_o \mu_o (\vec{V} \times \vec{E})$$

$$\vec{B}' = \vec{B} - \varepsilon_o \mu_o (\vec{V} \times \vec{E})$$

$$c = 1/\sqrt{\varepsilon_o \mu_o}$$

Bill's frame S at point r

$$\vec{E}(r) = (1/4\pi\varepsilon_0)(q/r^2)\hat{r}$$

$$B(r) = 0$$

Sharon's frame S': Charge moves and creates a B-field as well as an E-field.

$$\vec{E}' = \vec{E}$$

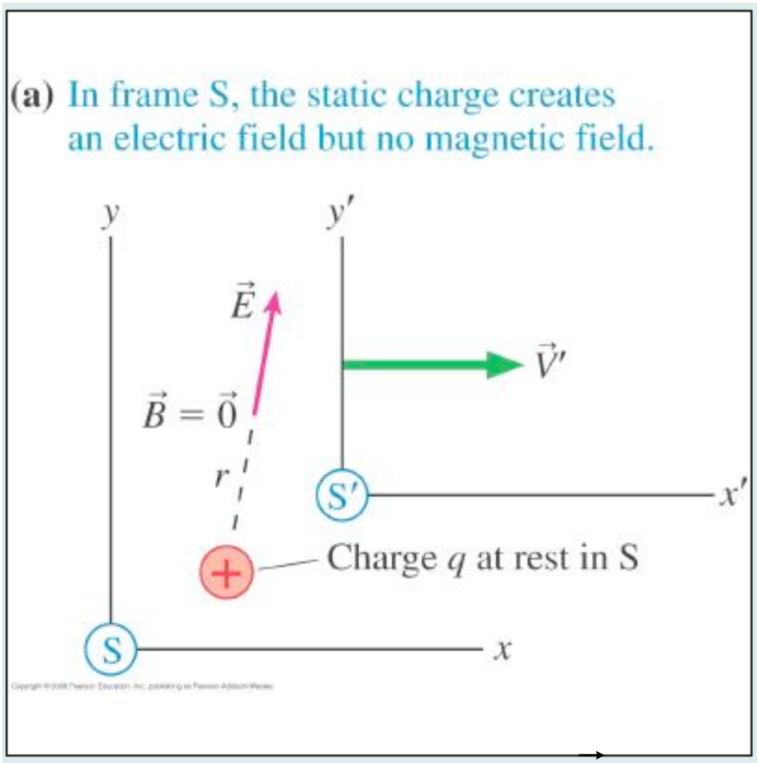
$$Biot - Savart$$

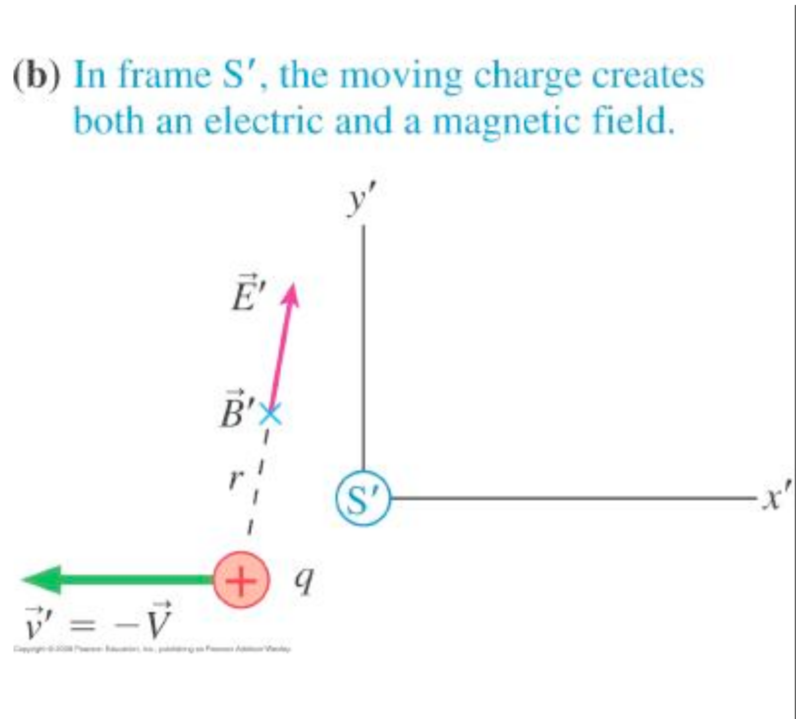
$$\vec{B}'(r) = (\mu_o / 4\pi) q \frac{-\vec{v} \times \hat{r}}{r^2} = -(\mu_o \varepsilon_o)\vec{v} \times [(1/4\pi\varepsilon_o)\frac{q}{r^2}\hat{r}] =$$

$$= -(\mu_o \varepsilon_o)\vec{v} \times \vec{E}$$

Biot-Savart is nothing more than the electric field of a stationary charge transformed into a moving frame

FROM S' TO S





$$\vec{B}' = -\varepsilon_o \mu_o (\vec{V} \times \vec{E})$$

$$\vec{B}' = \vec{B} - \varepsilon_o \mu_o (\vec{V} \times \vec{E})$$

$$c = 1/\sqrt{\varepsilon_o \mu_o}$$

E or *B*? It Depends on Your Perspective

The **Galilean field transformation** equations are

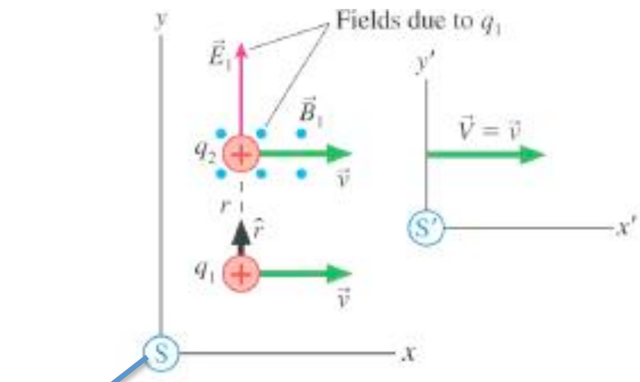
$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B} \qquad \text{or} \qquad \vec{E} = \vec{E}' - \vec{V} \times \vec{B}'$$

$$\vec{B}' = \vec{B} - \frac{1}{c^2}\vec{V} \times \vec{E} \qquad \qquad \vec{B} = \vec{B}' + \frac{1}{c^2}\vec{V} \times \vec{E}'$$

where V is the velocity of frame S′ relative to frame S and where the fields are measured *at the same point in space* by experimenters *at rest* in each reference frame.

NOTE: These equations are only valid if $V \ll c$.

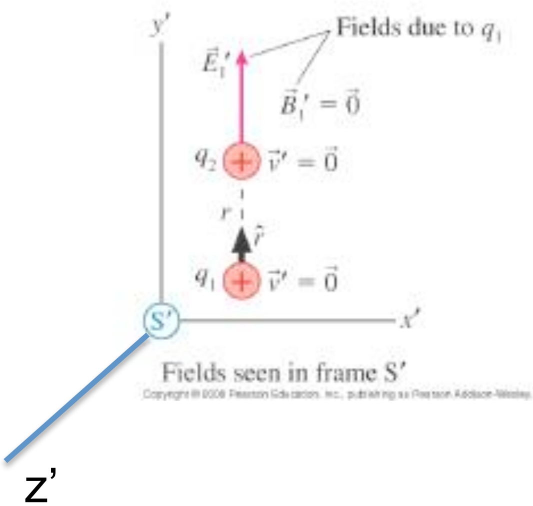
How correct are the Galilean Transformations



Fields seen in frame S

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B} \qquad \text{or} \qquad \vec{E} = \vec{E}' - \vec{V} \times \vec{B}'$$

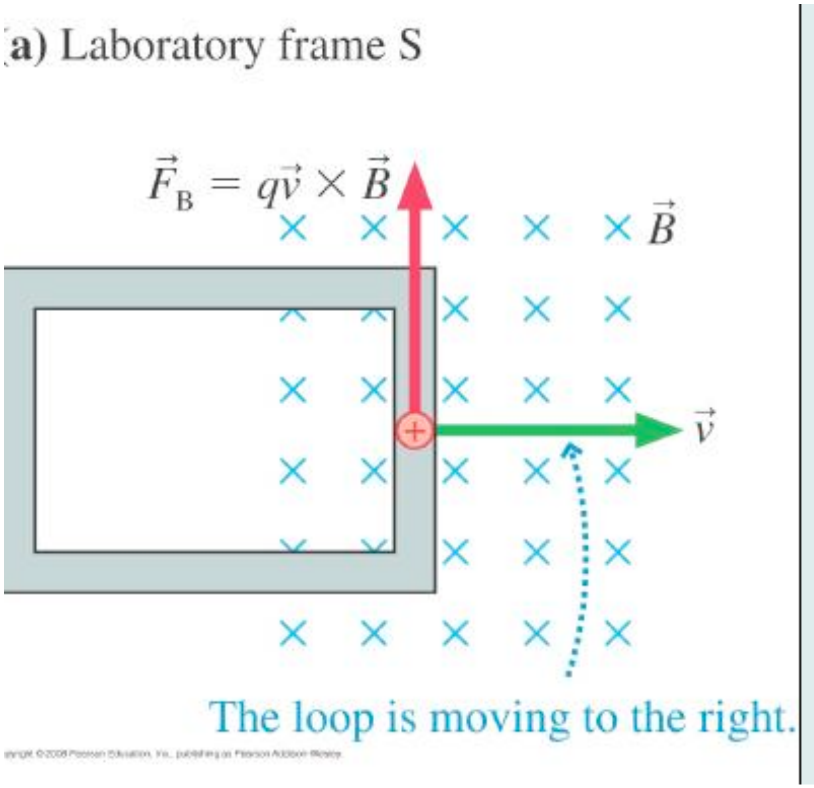
$$\vec{B}' = \vec{B} - \frac{1}{c^2}\vec{V} \times \vec{E} \qquad \qquad \vec{B} = \vec{B}' + \frac{1}{c^2}\vec{V} \times \vec{E}'$$

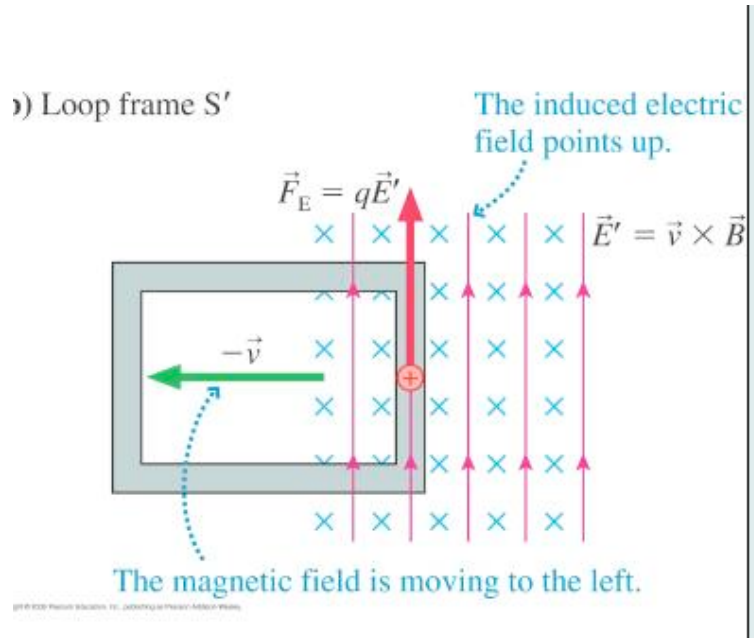


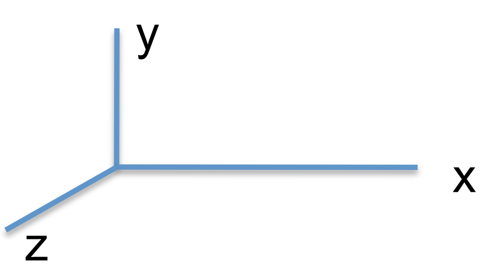
Fields seen in frame S'

Step 1. Find fields (E_1, B_1) due to charge q_1 at the position of q_2 in S-frame.
Step 2: Use Galilean transformations to find fields (E_1',B_1') in S'-frame.
Step 3 : Compute directly by Cb-law E_1' in S'-frame.
Step 4 : Compare the results and show that they are consistent only if $(V/c)^2 \ll 1$

Faraday's Law Using Transformaions



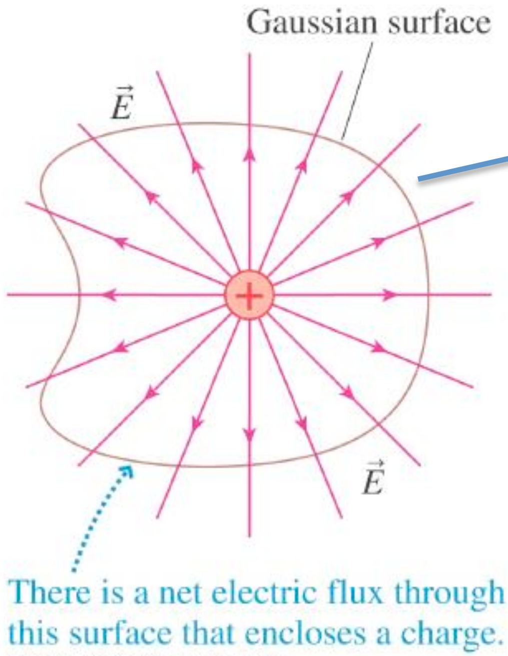




$$\vec{E}' = e_x(-v) \times e_z(-B) = e_x(vB)$$

$$B' = B$$

$$\vec{F}' = q\vec{E}' = e_y qvB$$



(Flux of E through a closed surface) = (Charge inside)/ϵ_0

(Line integral of E around a loop) $= -\dfrac{d}{dt}$(Flux of B through the loop)

(Flux of B through a closed surface) = 0

(Integral of B around a loop) = (Current through the loop)μ_0