PHYS 270 – SUPPL. #5

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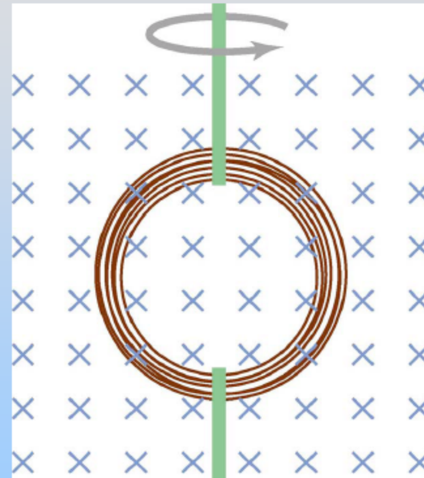
Ways to Induce EMF

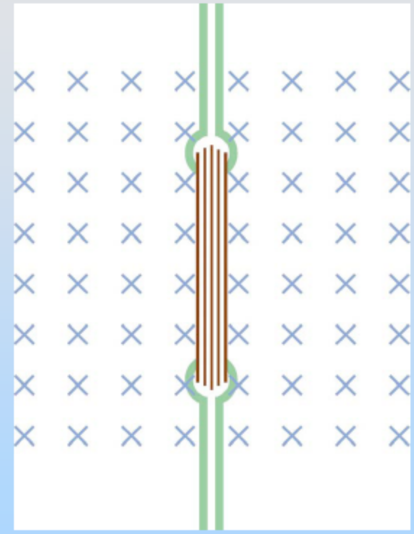
$$\mathcal{E} = -N\frac{d}{dt}\left(BA\cos\theta\right)$$

Quantities which can vary with time:

- Magnitude of B **e.g. Falling Magnet**
- **Area A enclosed by the loop**
- Angle θ between B and loop normal

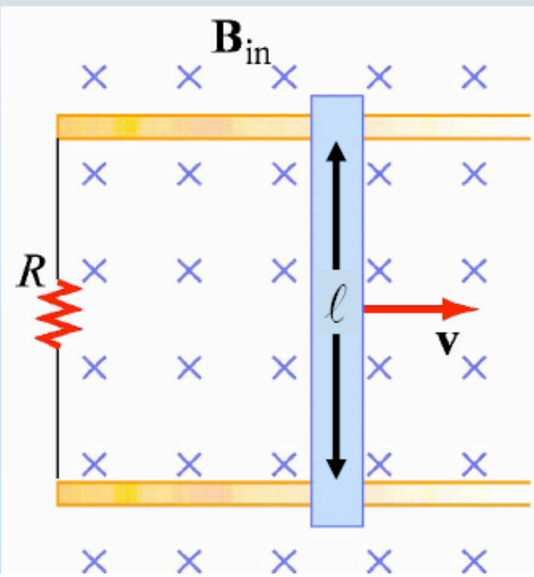


$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA$$



$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = 0$$

Conducting rod pulled along two conducting rails in a uniform magnetic field B at constant velocity v

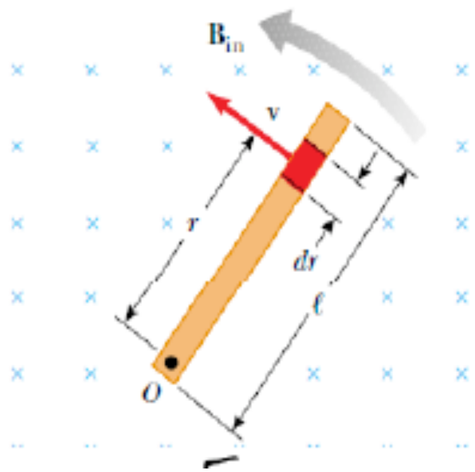


1. Direction of induced current?
2. Direction of resultant force?
3. Magnitude of EMF?
4. Magnitude of current?
5. Power externally supplied to move at constant v?

Will be discussed in detail during recitation

From last lecture – Example of calculation

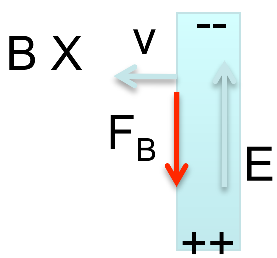
Rotating conducting bar in magnetic field



A conducting bar of length ℓ rotates with a constant angular speed ω about a pivot at one end. A uniform magnetic field **B** is directed perpendicular to the plane of rotation, as shown in Figure . Find the motional emf induced between the ends of the bar.

Let dq be charge within element dr

In equilibrium:



$$F_E = F_B \rightarrow dqdE = dqvB$$

dE parallel to the bar and across dE

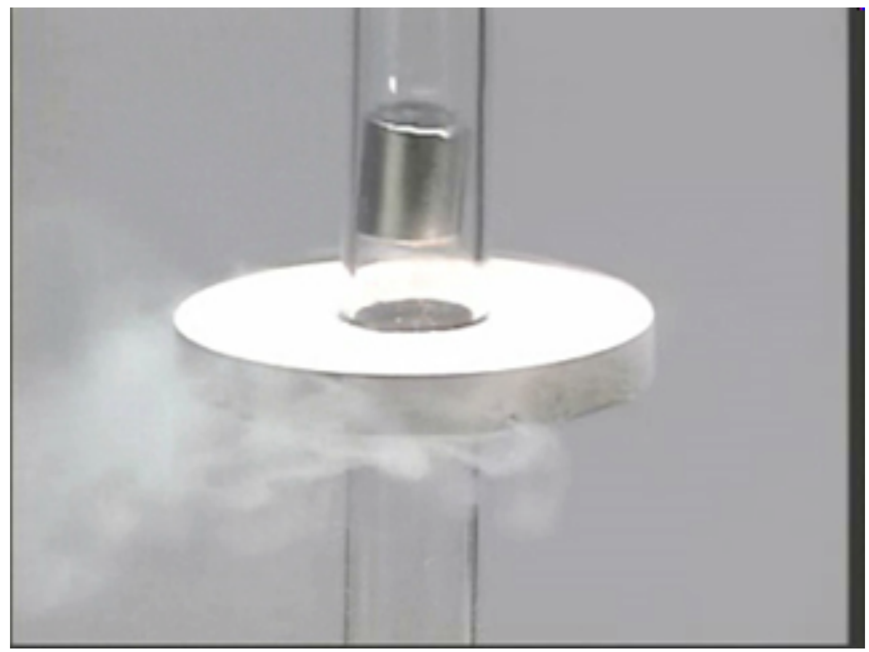
$$\mathrm{v} = \omega \mathrm{r} \rightarrow \mathrm{dE} = \omega \mathrm{rB}$$

$$\text{For uniform E-field} \rightarrow |\Delta V| = \mathrm{Ed}$$

$$\mathrm{dV} = \mathrm{dEdr} = \omega \mathrm{Brdr}$$

$$\Delta V = \omega B \int_0^l r dr = \frac{l^2}{2} \omega B$$

Falling magnet slows down as it approaches a copper ring immersed in liquid Nitrogen



Why ? and what is the use of liquid Nitrogen ?

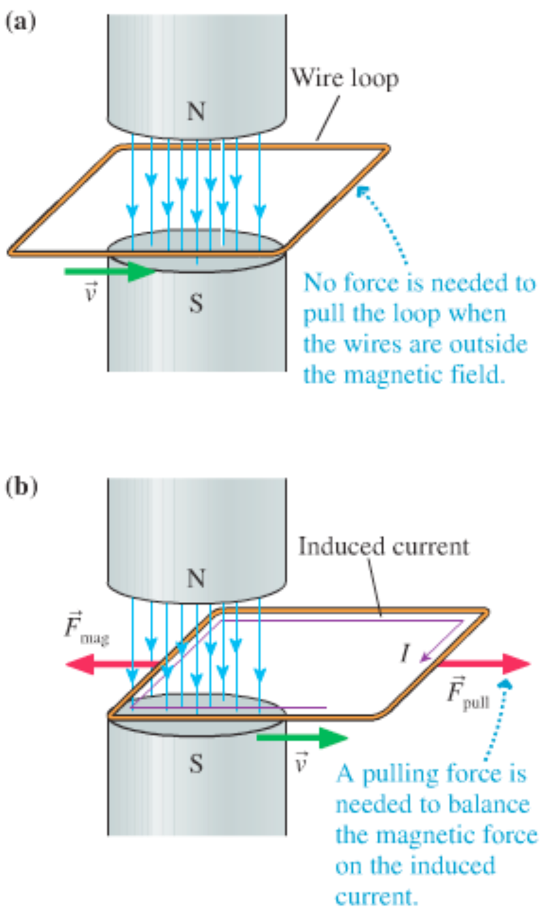
Jumping Ring

movie

An aluminum ring jumps into the air when the solenoid beneath it is energized

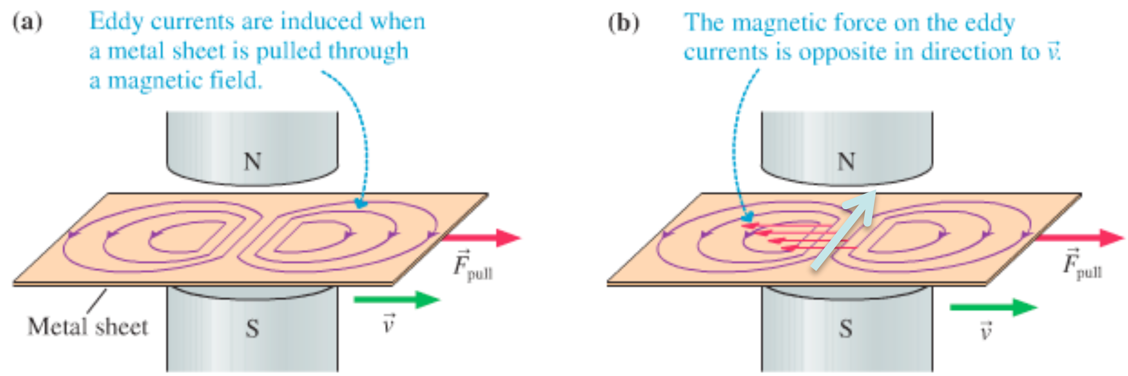
EDDY CURRENTS

FIGURE 34.9 Pulling a loop of wire out of a magnetic field.



What happens if I have no wires to define current path in a metal sheet. Two whirlpools of current circulate.

FIGURE 34.10 Eddy currents.



Power dissipated in moving the metal slab results in braking the motion of the slab

$$P_{diss} = I^2 R$$

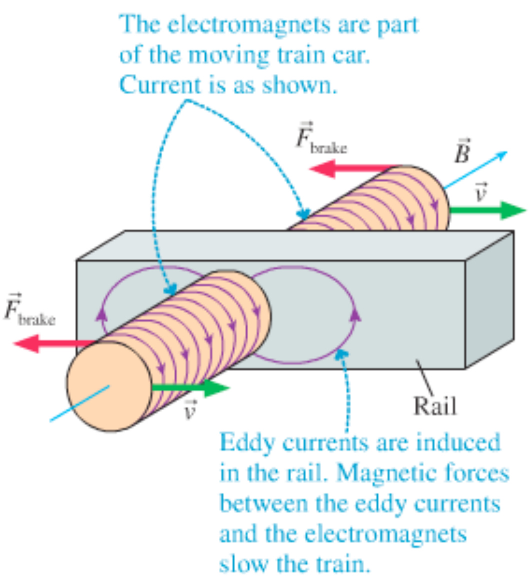
$$I = \varepsilon / R$$

$$P_{diss} = \varepsilon^2 / R$$

Can I reduce the braking action?

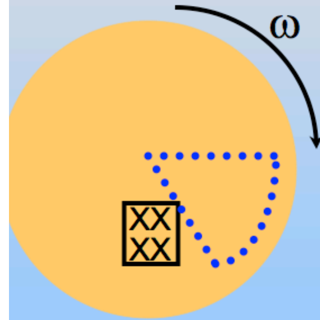
Induced current in complete loop
Causes retarding force on wire

FIGURE 34.11 Magnetic braking systems are an application of eddy currents.



Eddy Current Braking

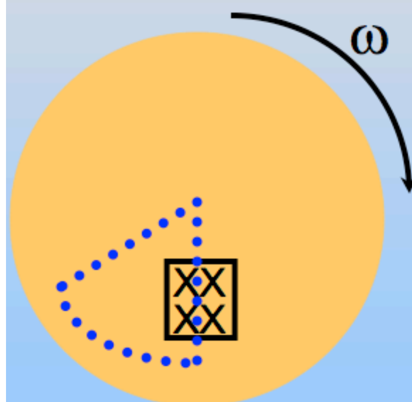
The magnet induces currents in the metal that dissipate the energy through Joule heating:



1. Current is induced counter-clockwise (out from center)
2. Force is opposing motion (creates slowing torque)

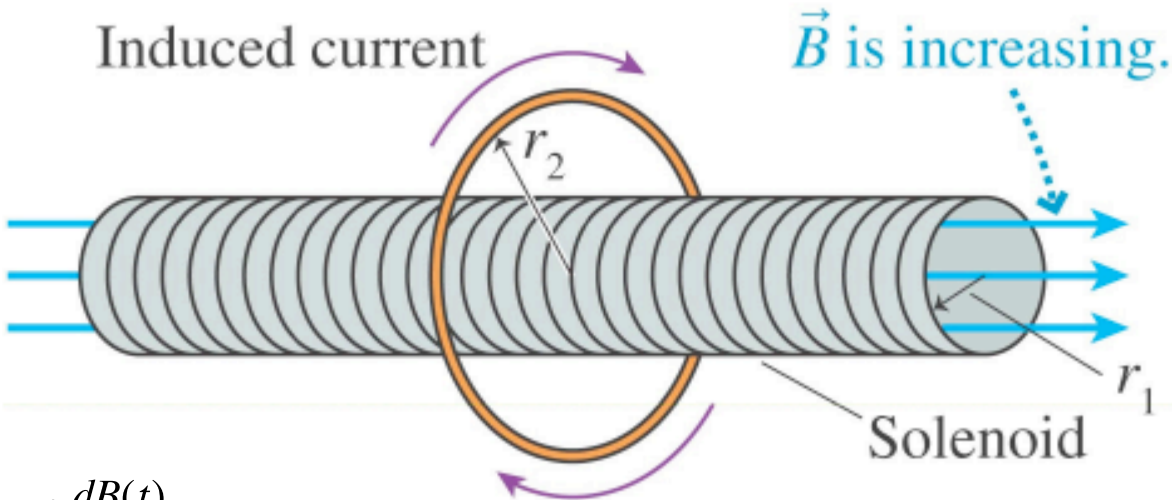
Eddy Current Braking

The magnet induces currents in the metal that dissipate the energy through Joule heating:



1. Current is induced clockwise (out from center)
2. Force is opposing motion (creates slowing torque)
3. EMF proportional to ω
4. $F \propto \frac{\mathcal{E}^2}{R}$

Faraday's Law – New Physics



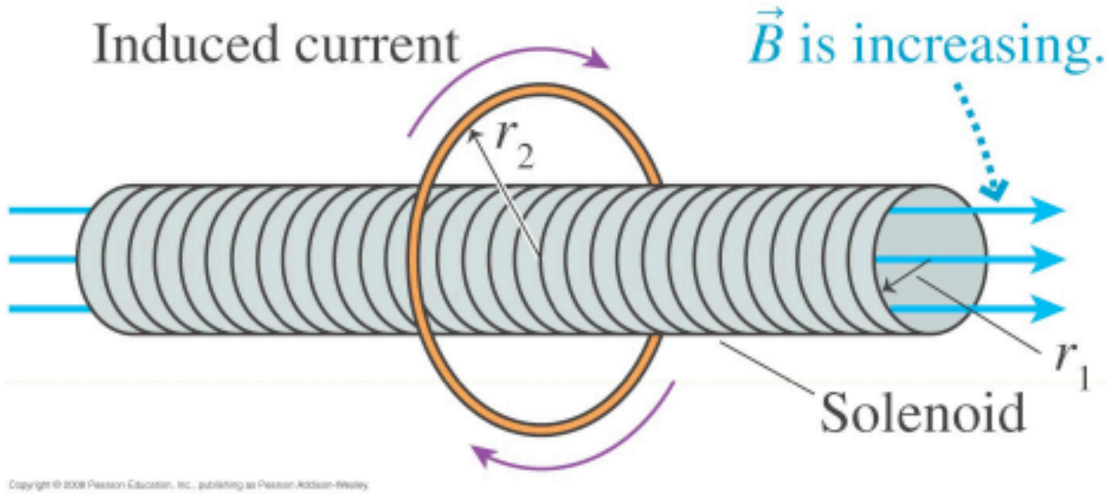
$$\Phi_m = \pi r_2^2 B(t)$$

$$\varepsilon = -\frac{d\Phi_m}{dt} = -\pi r_2^2 \frac{dB(t)}{dt}$$

B=0 outside the infinite solenoid. Faraday's law states that a current will flow in the hoop if the B-field is changing in solenoid. How do the charges in the hoop 'know' the flux is changing? There MUST be something causing the charges to move, and it is NOT directly related to the B-field like motional emf since the charges are initially stationary in the loop.

Wherever there is voltage (emf), there is an E-field. A time varying B-field evidently causes an E-field (even outside the solenoid) which push the charge in the hoop. However, the E-field still exists even outside the hoop!

E-field driving the current in hoop – induced E-field

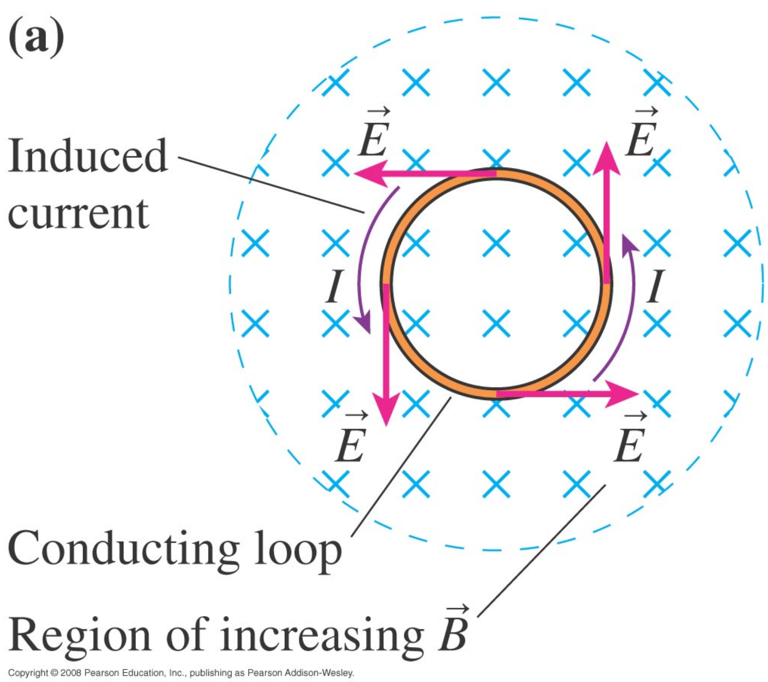


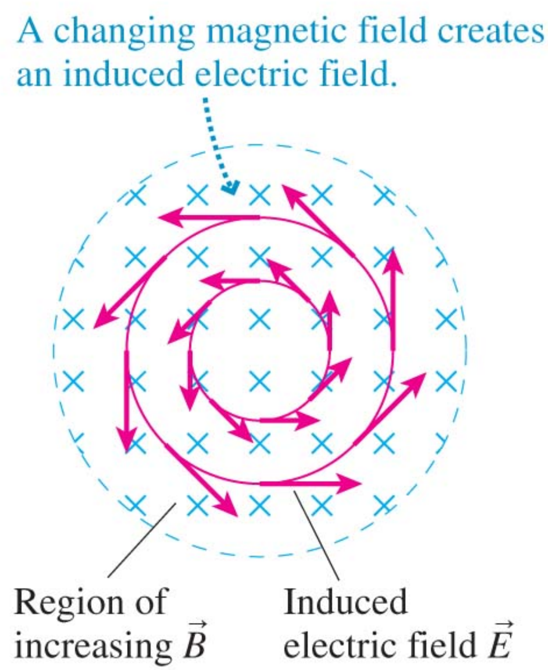
Radial E has to be zero everywhere:

> If we reverse current, we expect E radial to change direction. However, we must end up with the above picture if we reverse current AND flip the solenoid over 180 degrees. Therefore, E radial must be zero.

Since charge flows circumferentially, there must be a tangential component of E-field pushing the charge around the ring. **This E-field exists even without the ring.**

INDUCED FIELDS

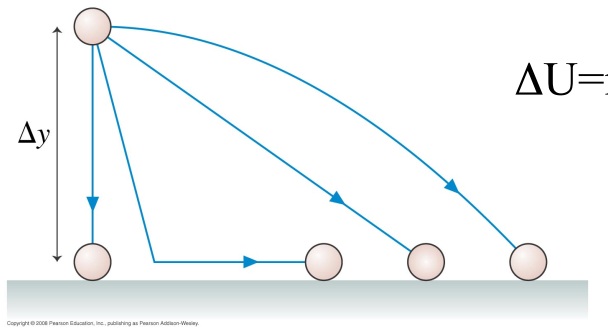




- COULOMB E FIELD CREATED BY POSITIVE AND NEGATIVE CHARGES - ES
- NON-COULOMB E FIELD CREATED BY A CHANGING B - EM

INDUCED FIELDS NON-CONSERVATIVE

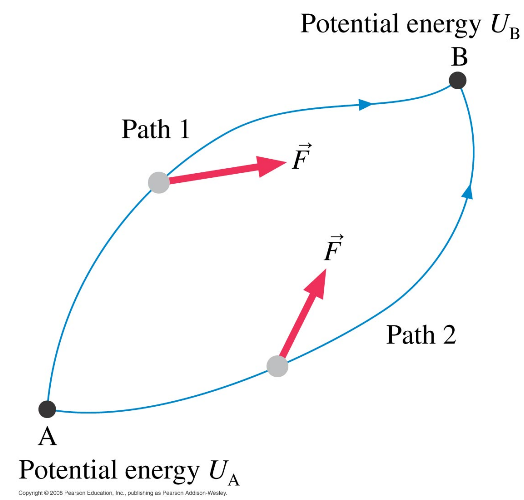
CONSERVATIVE FORCES



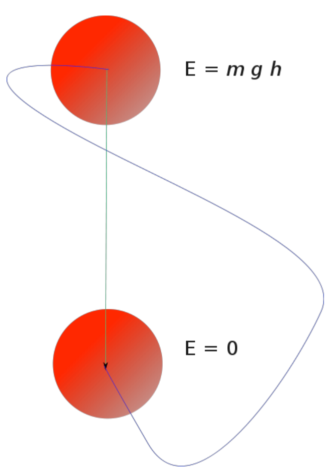
$\Delta U = mg\Delta y$ for all paths

$$\int_A^B \vec{F} \cdot d\vec{s} = U_B - U_A$$

$$\oint_c \vec{F} \cdot d\vec{s} = 0$$



Uphill balanced by downhill



Coulomb Force

$$\nabla \times \vec{E} = 0$$

$$\oint \vec{E} \cdot d\vec{s} = 0$$

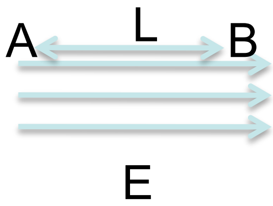
$$\vec{E} = -\nabla \Phi$$

A conservative force can be thought of as a force that *conserves* energy. Suppose a particle starts at point A, and there is a constant force *F* acting on it. Then the particle is moved around by other forces, and eventually ends up at A again. Though the particle may still be moving, at that instant when it passes point A again, it has traveled a closed path. If the net work done by *F* at this point is 0, then *F* passes the closed path test. Any force that passes the closed path test for all possible closed paths is classified as a conservative force.

Is friction a conservative force?

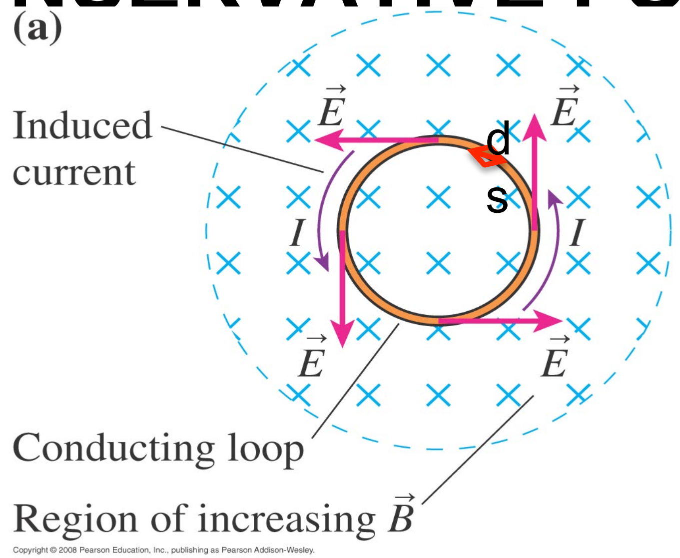
NON-CONSERVATIVE FORCES

Coulomb case



$$\Delta W_{AB} = q\Phi_{BA} = qEL$$

$$\Phi_{BA} = \frac{\Delta W_{AB}}{q} = EL$$

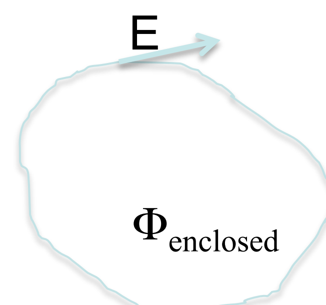


$$dW = q\vec{E}\cdot d\vec{s}$$

Work done by the induced field E as charge q moves around the circle is

$$W_{closed\ curve} = q\oint \vec{E}\cdot d\vec{s}$$

$$\varepsilon = \frac{W_{closed\ curve}}{q} = \oint \vec{E}\cdot d\vec{s}$$

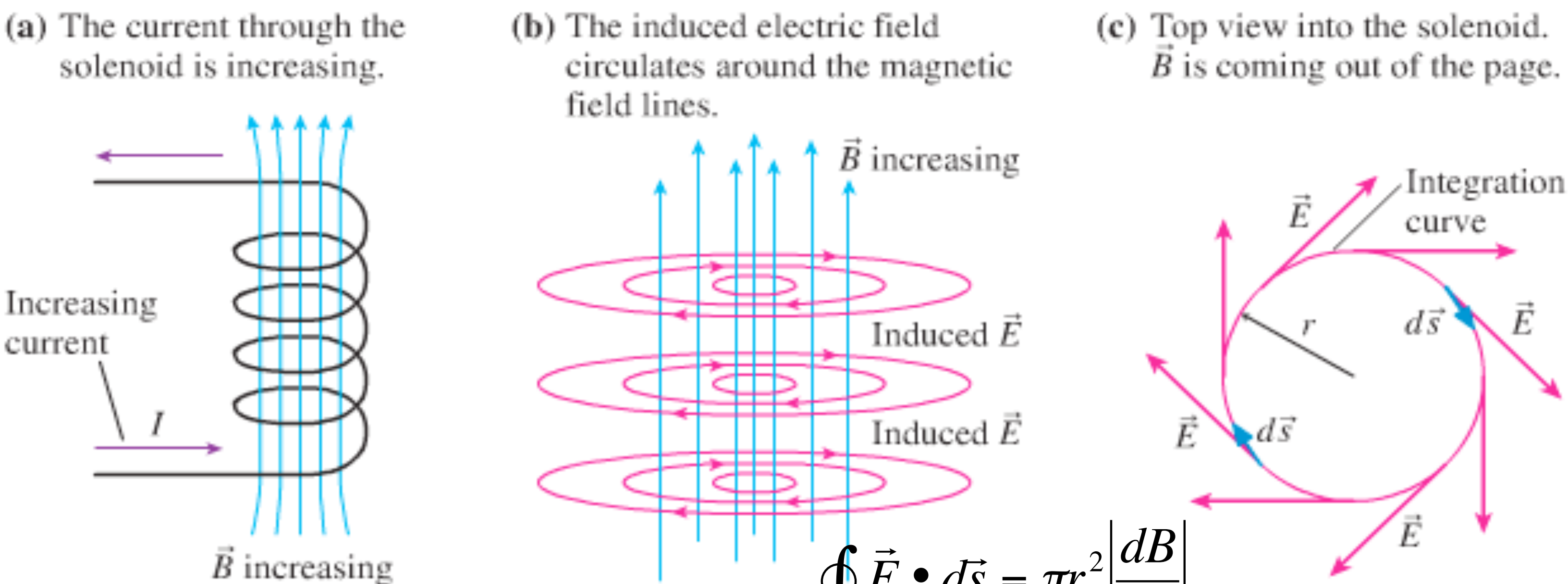


Best form of Faraday's law

$$\oint_C \vec{E}\cdot d\vec{s} = -\frac{d\Phi_{enclosed}}{dt} = -\frac{d}{dt}\oiint_A \vec{B}\cdot d\vec{A}$$

INDUCED FIELD IN FREE SPACE

FIGURE 34.34 The induced electric field circulates around the changing magnetic field inside a solenoid.



$$\oint \vec{E} \bullet d\vec{s} = \pi r^2 \left|\frac{dB}{dt}\right|$$

$$2\pi r E_\theta = \pi r^2 \left|\frac{dB}{dt}\right| \qquad r < R$$

$$E_\theta = \frac{r}{2}\left|\frac{dB}{dt}\right|$$

For practical problems easier to calculate absolute value of E and then use Lenz or other rule

What about outside the coil?

HOW TO FIND THE DIRECTION OF AND EMF

OPTION 1: FARADAY WITH ABSOLUTE SIGN + LENZ's LAW

The direction of I is such as to oppose the change in the flux

$$\varepsilon = \left|\frac{d\Phi}{dt}\right| = \left|\vec{B} \bullet \frac{d\vec{A}}{dt} + \vec{A} \bullet \frac{d\vec{B}}{dt}\right|$$

OPTION 2: EMF OPPOSES CHANGE IN Φ -> OPPOSITE TO THE SIGN OF dB/dt.

$$\varepsilon = \oint \vec{E} \bullet d\vec{s} = -\frac{d\Phi_m}{dt}$$

ENTER MAXWELL

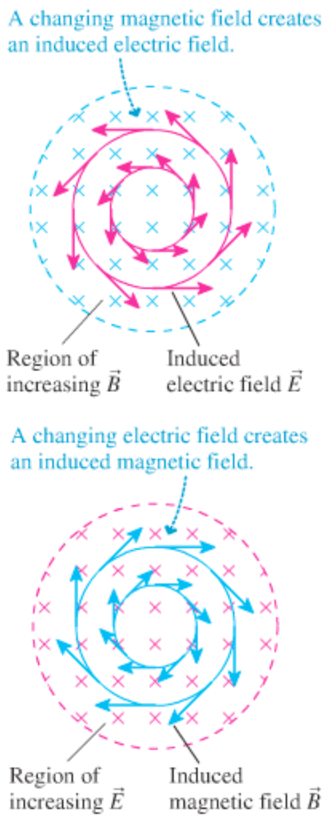


FIGURE 34.35 Maxwell hypothesized the existence of induced magnetic fields.

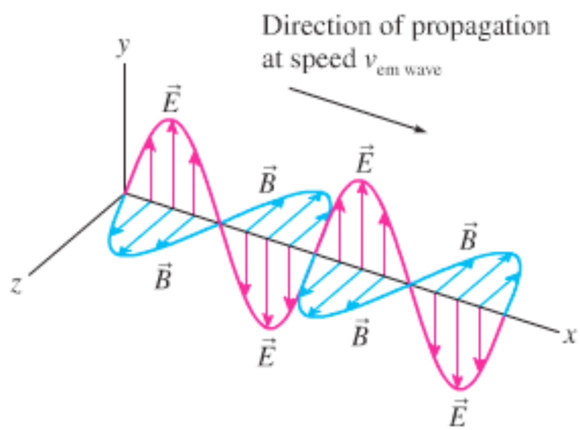
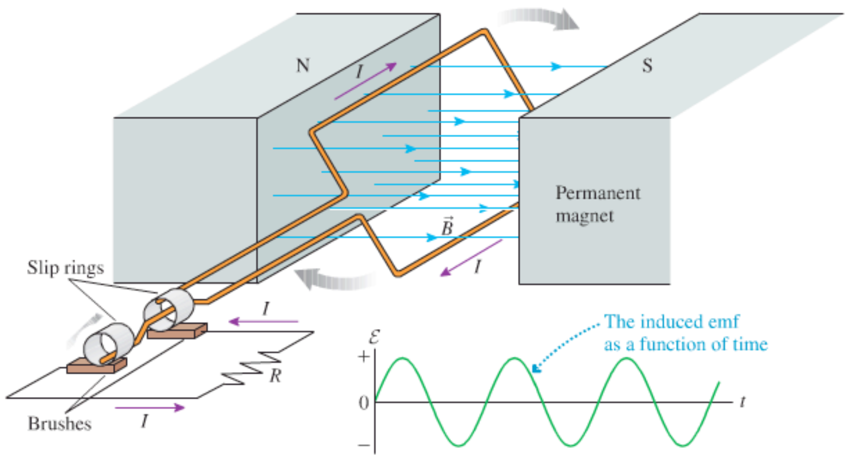
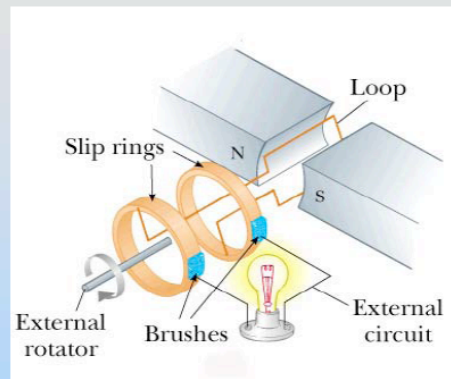


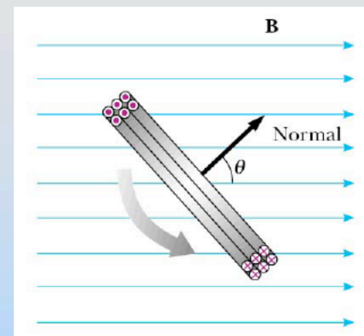
FIGURE 34.36 A self-sustaining electromagnetic wave.

GENERATORS – TRANSFORMERS-DETECTORS

FIGURE 34.37 An alternating-current generator.







$$\Phi_B = BA\cos\theta = BA\cos\omega t$$

$$\varepsilon = -N\frac{d\Phi_B}{dt} = -NAB\frac{d}{dt}(\cos\omega t) = NAB\omega\sin\omega t$$

$$\Phi_m = AB\cos\theta = AB\cos\omega t$$

$$\varepsilon_{coil} = -N\frac{d\Phi_m}{dt} = NAB\omega\sin\omega t$$

GENERATORS – TRANSFORMERS-DETECTORS

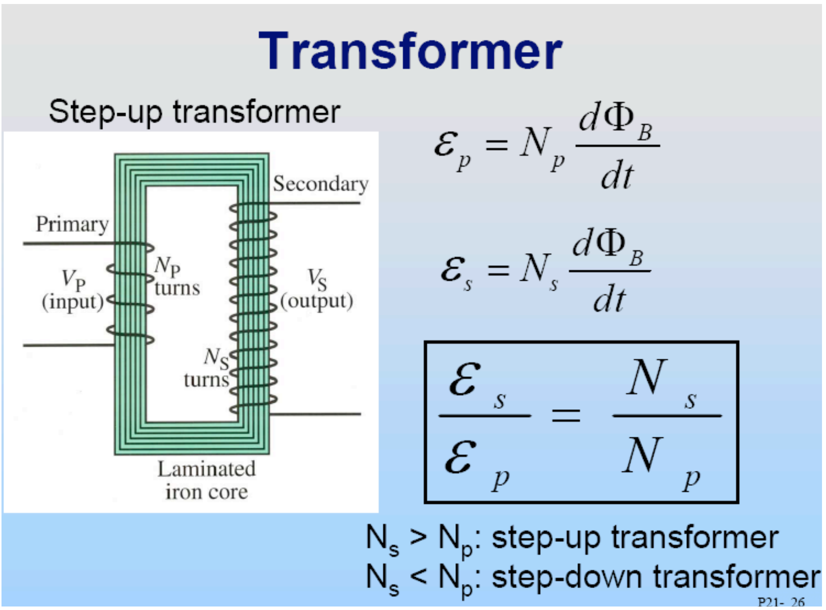


FIGURE 34.38 A transformer.

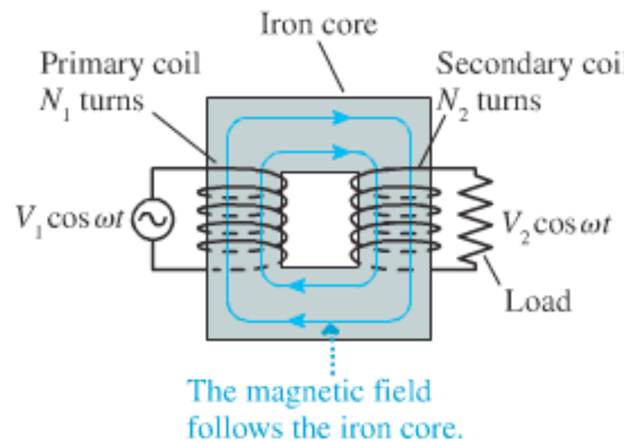
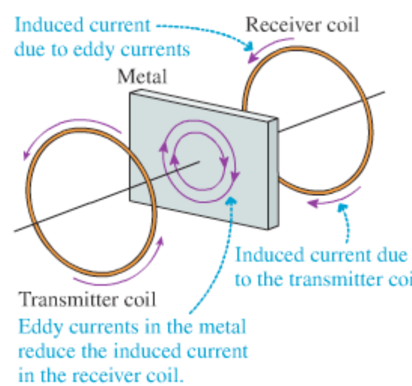
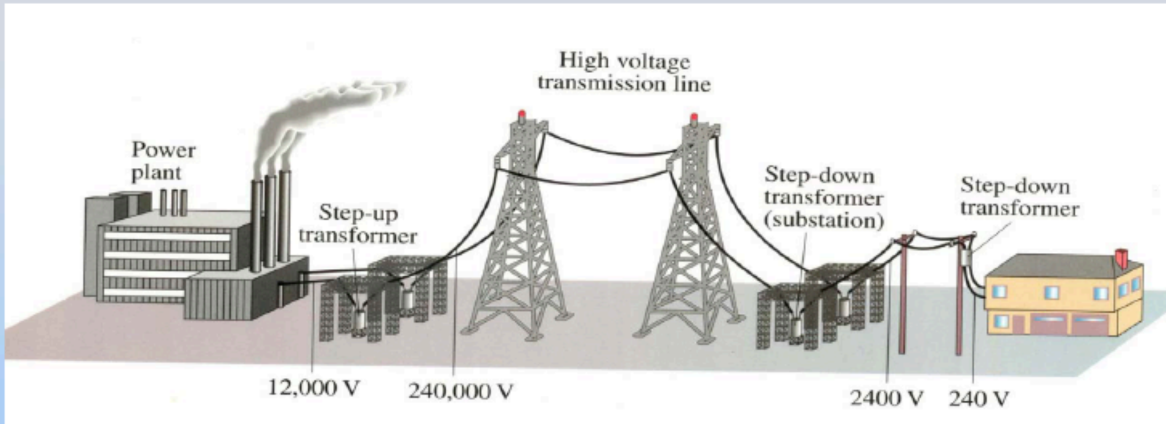


FIGURE 34.39 A metal detector.



Transmission of Electric Power



Power loss can be greatly reduced if transmitted at high voltage

Example: Transmission lines

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V.

(a) $$I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^2 V} = 500A$$

$$P_L = I^2R = (500A)^2(0.40\Omega) = 100kW$$

83% loss!!

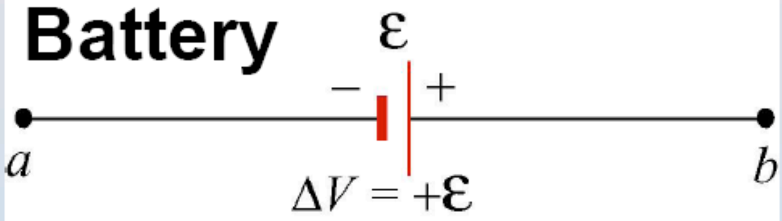
(b) $$I = \frac{P}{V} = \frac{1.2\times10^5 W}{2.4\times10^4 V} = 5.0A$$

$$P_L = I^2R = (5.0A)^2(0.40\Omega) = 10W$$

0.0083% loss

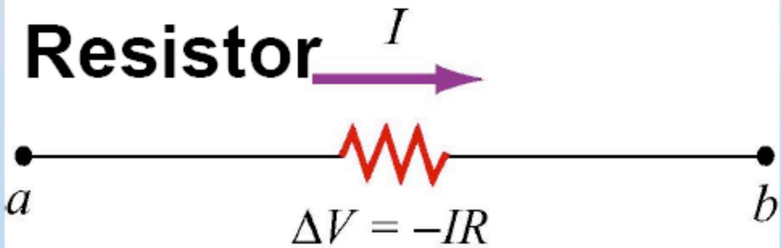
Current, Voltage & Power

Battery



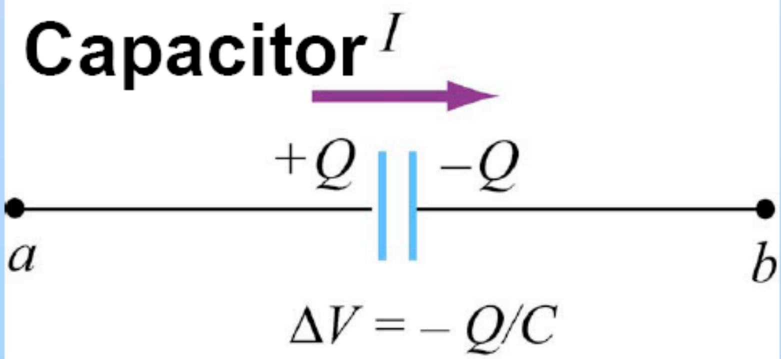
$$P_{\text{supplied}} = I\,\Delta V = I\,\varepsilon$$

Resistor



$$P_{\text{dissipated}} = I\,\Delta V = I^2 R = \frac{\Delta V^2}{R}$$

Capacitor



$$P_{\text{absorbed}} = I\,\Delta V = \frac{dQ}{dt}\frac{Q}{C}$$

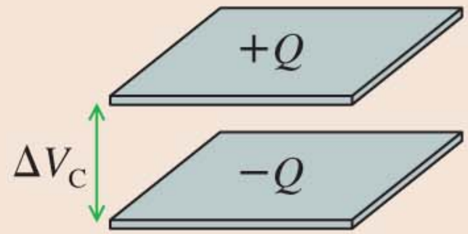
$$= \frac{d}{dt}\frac{Q^2}{2C} = \frac{dU}{dt}$$

INDUCTORS VS. CAPACITORS

Capacitors

The **capacitance** of two conductors charged to $\pm Q$ is

$$C = \frac{Q}{\Delta V_C}$$



A parallel-plate capacitor has

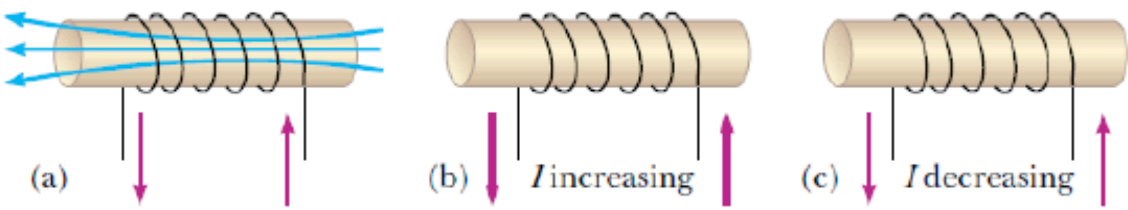
$$C = \frac{\epsilon_0 A}{d}$$

Filling the space between the plates with a dielectric of dielectric constant κ increases the capacitance to $C = \kappa C_0$

The energy stored in a capacitor is $u_C = \frac{1}{2}C(\Delta V_C)^2$

This energy is stored in the electric field at density $u_E = \frac{1}{2}\epsilon_0 E^2$.

Back emf and inductors

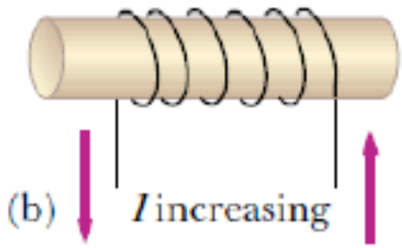


(a) Steady current, magnetic field is to the left
(b) Current increasing, magnetic field increasing to the left. Lenz's law states that an emf is set-up to oppose this increasing flux, thus creating a voltage that opposes the increasing current.

Considering case (b) for the case of argument, the back emf from Faraday's Law (we do not assume an infinite solenoid here!)

$$\varepsilon \equiv -L\frac{dI}{dt} \qquad \Delta V = Q / C$$

Inductors - Inductance



$$|\varepsilon_{back}| = -N\frac{d\Phi}{dt}$$

$$\Phi = \int \vec{B} \bullet d\vec{A} \propto B$$

$$\text{Biot Savart} \rightarrow B \propto I$$

$$|\varepsilon_{back}| \propto \frac{dI}{dt}$$

Define the Self Inductance L as the proportionality constant in

$$\Delta V_L \equiv -L\frac{dI}{dt}$$

Units ?

For infinite solenoid

$$B = \mu_o \frac{NI}{l}$$

$$\frac{d\Phi}{dt} = A\mu_o \frac{N}{l}\frac{dI}{dt}$$

$$L = A\mu_o \frac{N^2}{l}$$

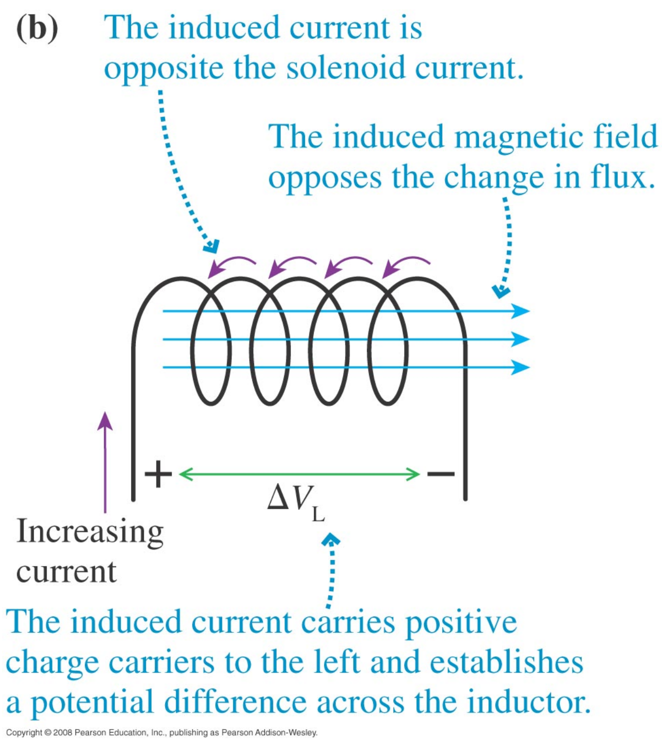
Inductance of a solenoid

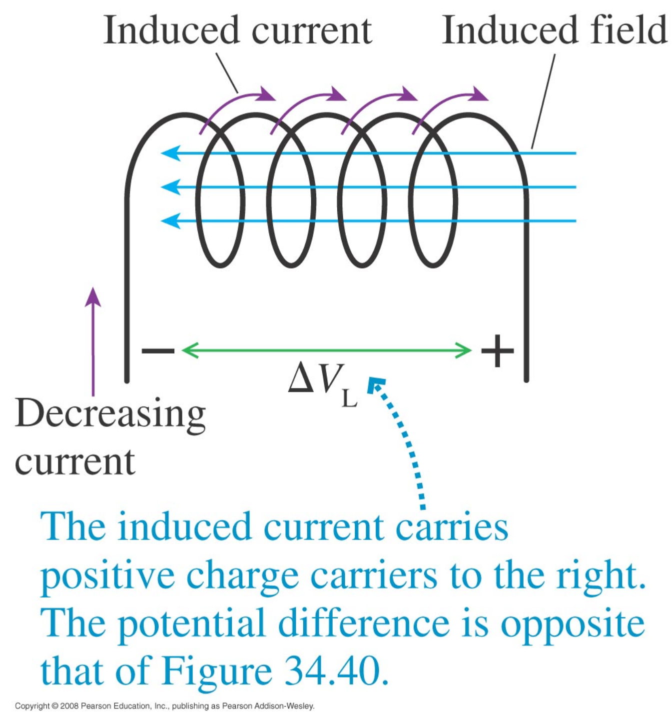
The inductance of a solenoid having *N* turns, length *l* and cross-section area *A* is

$$L_{\text{solenoid}} = \frac{\Phi_m}{I} = \frac{\mu_0 N^2 A}{l}$$

← Depends only on geometry

T-m^2 /A =Henry =H





INDUCTOR VOLTAGE

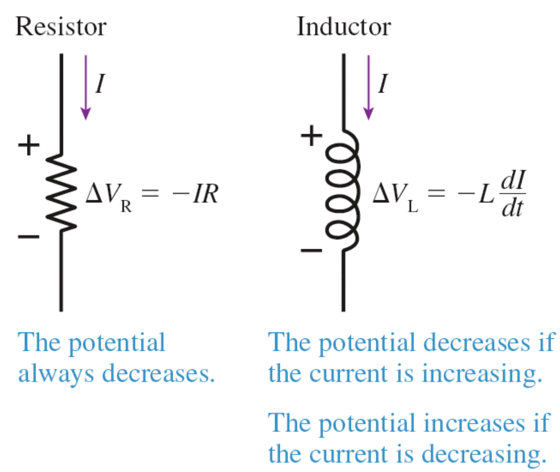
$$L_{\text{solenoid}} = \frac{\Phi_m}{I} = \frac{\mu_0 N^2 A}{l}$$

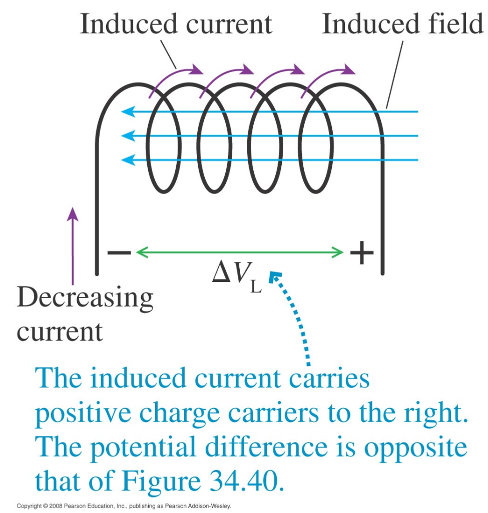
$$\varepsilon = \left|\frac{d\Phi_m}{dt}\right| = L\left|\frac{dI}{dt}\right|$$

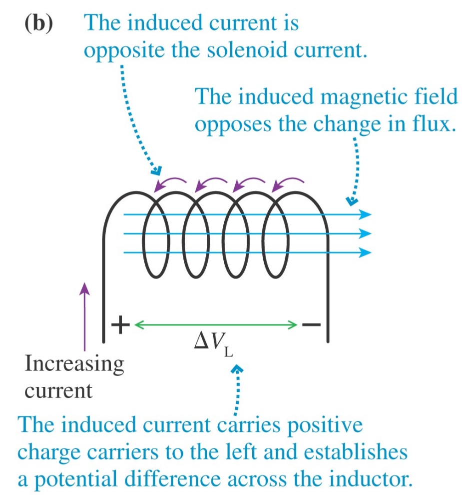
$$\Delta V_R = -IR$$

$$\Delta V_L = -L\frac{dI}{dt}$$

FIGURE 34.42 The potential difference across a resistor and an inductor.







ENERGY STORAGE IN INDUCTOR

WHERE?

$$u_E = \frac{1}{2}\varepsilon_o E^2$$

$$u_B = \frac{B^2}{2\mu_o}$$

$$U_c = \frac{Q^2}{2C}$$

$$U_c = \frac{1}{2}LI^2$$

WHY?

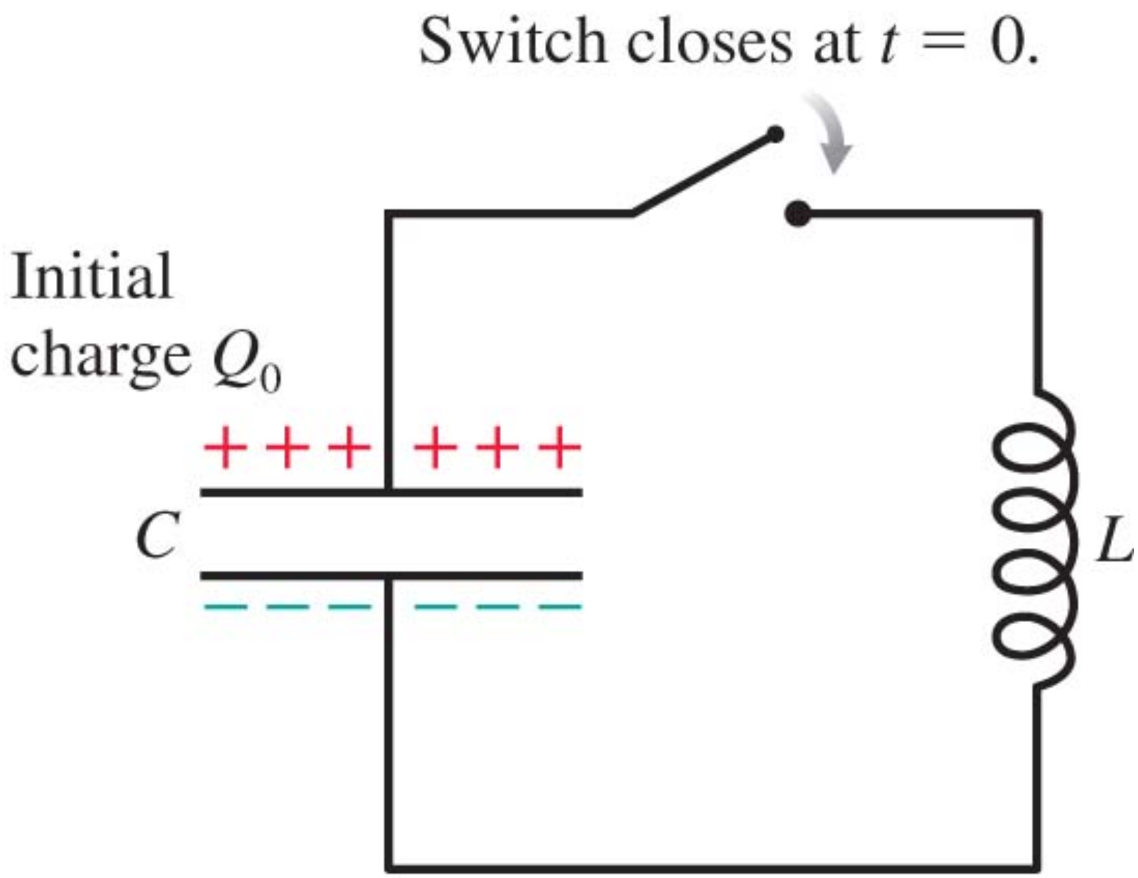
$$P \equiv \frac{dU_L}{dt} = I\Delta V_L = -IL\frac{dI}{dt}$$

$$U_L = L\int_0^I I\,dI = \frac{1}{2}LI^2$$

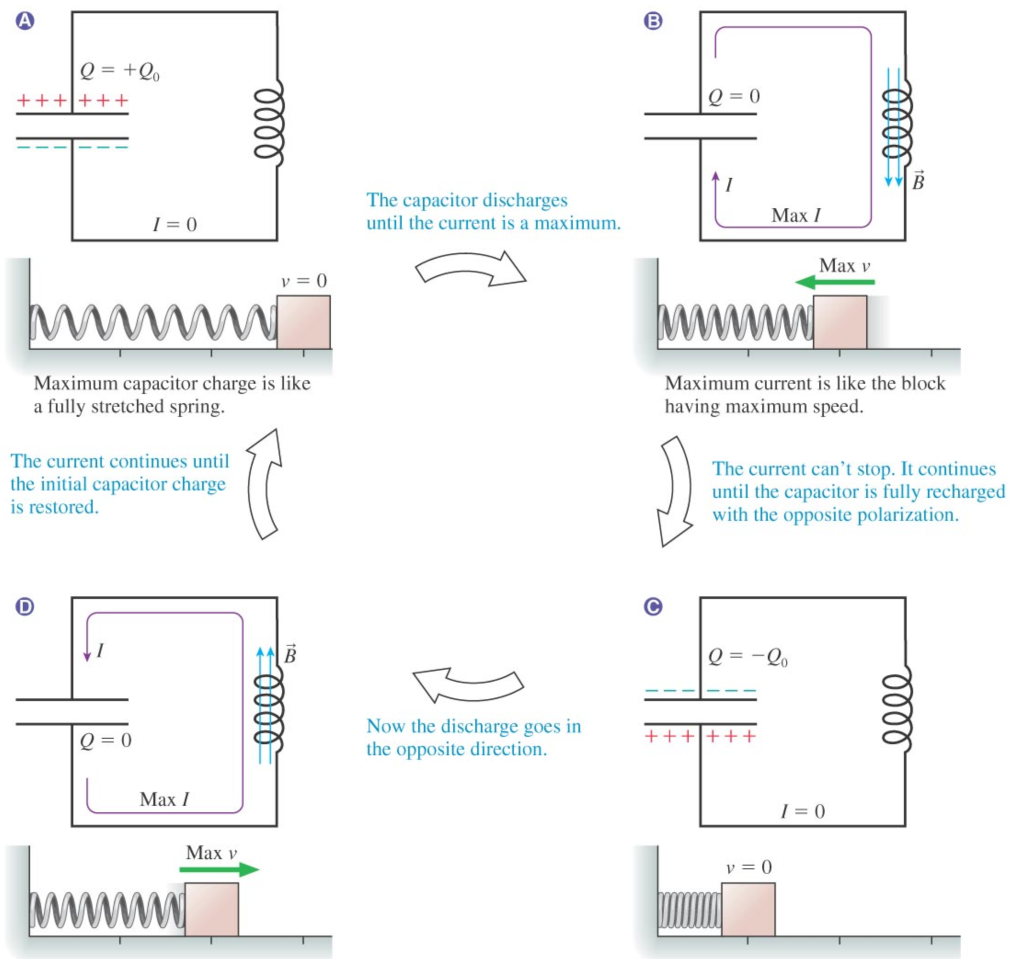
$SHOW$ THAT

$$U_L = (Al)(B^2/2\mu_o)$$

FIGURE 34.44 An *LC* circuit.



LC CIRCUIT



$$\Delta V_C + \Delta V_L = 0$$

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

$$I = -\frac{dQ}{dt}$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

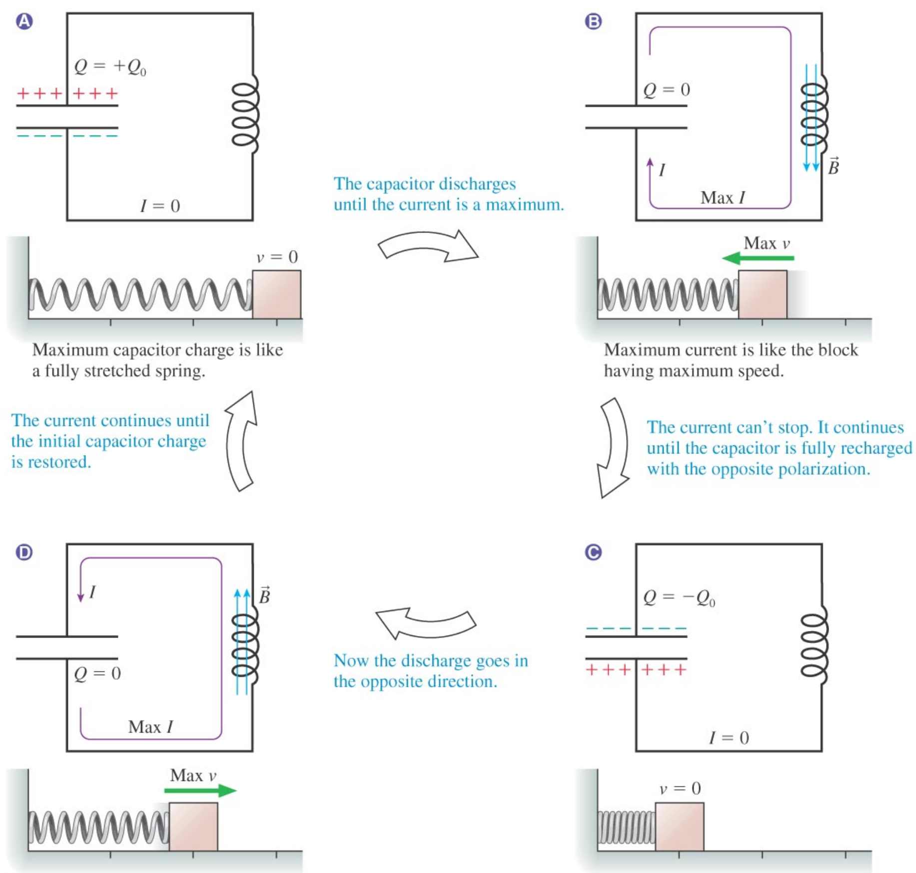
$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$$

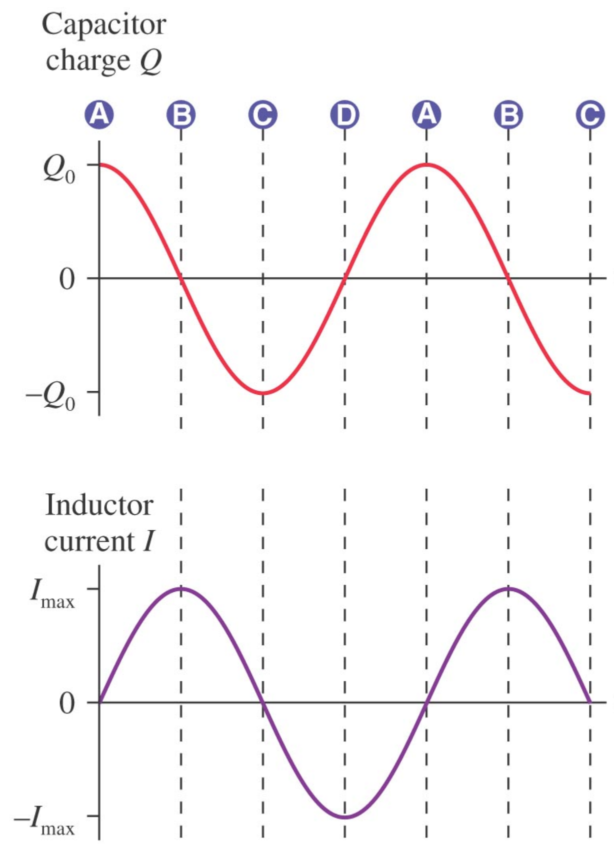
$$\omega \equiv \sqrt{1/LC}$$

$$Q(t) = Q_o\cos(\omega t)$$

$$I = \omega Q_o \sin(\omega t) \equiv I_{max}\sin(\omega t)$$

ENERGY STORAGE OSCILLATION





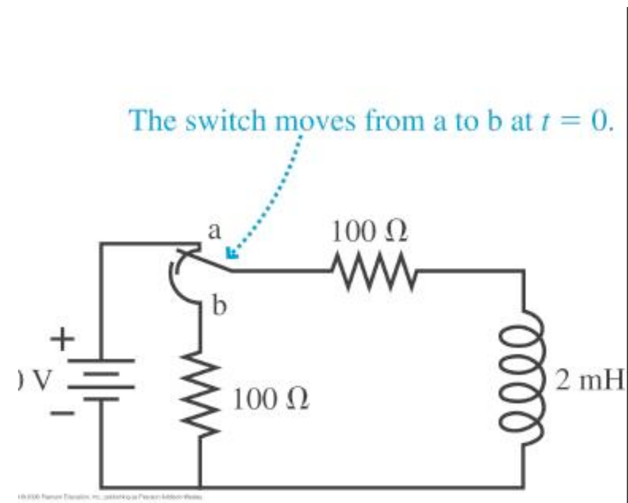
The current in an LC circuit

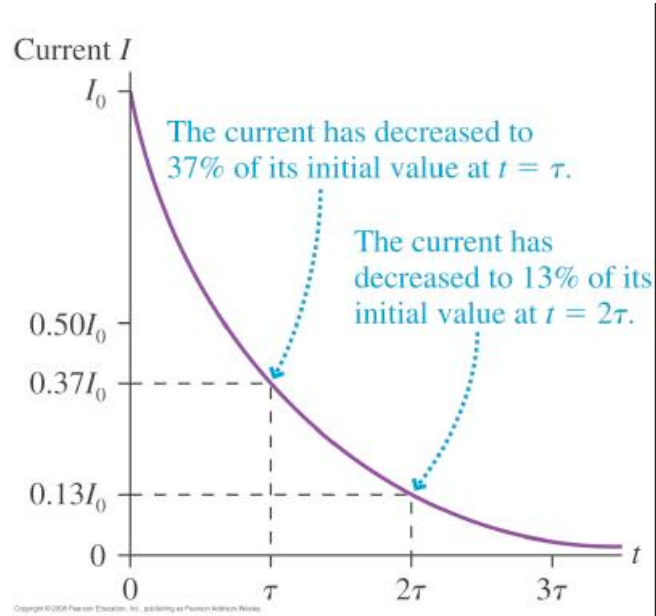
The current in an *LC* circuit where the initial charge on the capacitor is Q_0 is

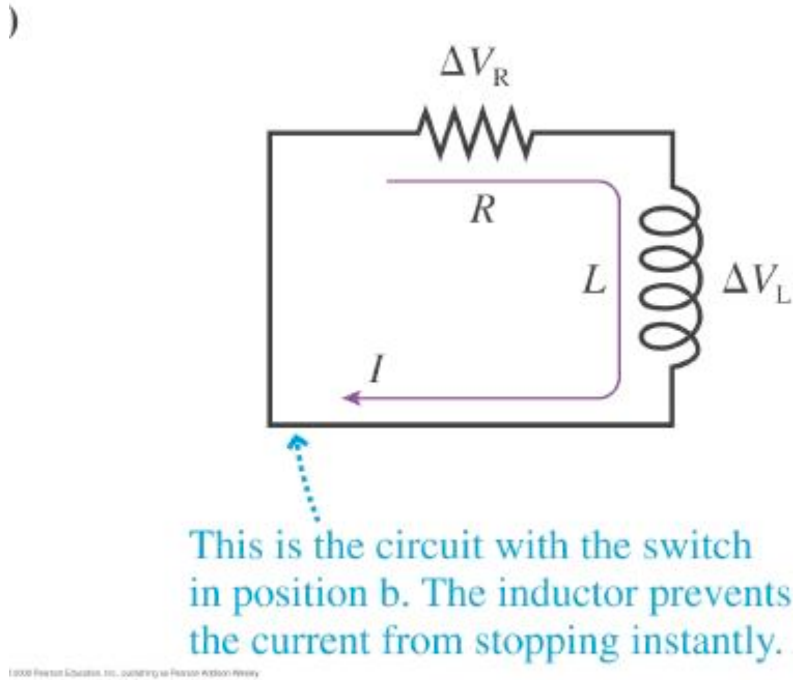
$$I = -\frac{dQ}{dt} = \omega Q_0 \sin \omega t = I_{\max} \sin \omega t$$

The oscillation frequency is given by

$$\omega = \sqrt{\frac{1}{LC}}$$


The switch moves from a to b at $t = 0$.
a
b
100 Ω
100 Ω
2 mH
+
−


Current I
I_0
$0.50I_0$
$0.37I_0$
$0.13I_0$
0
The current has decreased to 37% of its initial value at $t = \tau$.
The current has decreased to 13% of its initial value at $t = 2\tau$.
0
τ
2τ
3τ
t


ΔV_R
R
L
ΔV_L
I
This is the circuit with the switch in position b. The inductor prevents the current from stopping instantly.

$$I = I_o \exp[-t/(L/R)]$$