Chapter 34. Electromagnetic Induction

Electromagnetic induction is the scientific principle that underlies many modern technologies, from the generation of electricity to communications and data storage.
Induction

• E-fields generate B fields (E field in a conductor drives current that creates B)
• Can B fields generate E-fields?
• Yes if the B fields vary in time -Faraday

Current flows only if B through the loop is changing
• Faster movement more current
• Current direction changes when either motion or magnet polarity is reversed
• Current linear function of number of turns

Coil behaves as connected to a battery source. EMF

\[ \text{EMF} \to \varepsilon = -N \frac{d\Phi_m}{dt} \]
K2-04: FARADAY'S EXPERIMENT - EME SET - 20, 40, 80 TURN COILS
MAGNETIC FLUX CONVENTIONS

UNIFORM B

\[ \Phi = \vec{A} \cdot \vec{B} \]

Units: T-m² = Wb (Weber)

NON-UNIFORM B

\[ d\Phi = \vec{B} \cdot d\vec{A} \]

\[ \Phi = \int \vec{B} \cdot d\vec{A} \]
Magnetic Flux Thru Wire Loop

Analogous to Electric Flux (Gauss’ Law)

(1) Uniform $B$

$$\Phi_B = B \cdot A = BA\cos\theta = \vec{B} \cdot \vec{A}$$

(2) Non-Uniform $B$

$$\Phi_B = \int_{S} \vec{B} \cdot d\vec{A}$$
Faraday’s Law of Induction

\[ \mathcal{E} = -N \frac{d\Phi_B}{dt} \]

A changing magnetic flux induces an EMF

What is EMF?

\[ \mathcal{E} = \int \vec{E} \cdot d\vec{s} \]

Looks like potential. It’s a “driving force” for current

N number of loops in a coil

What about the minus sign?
How can induce a emf:
1. Change magnitude of $B$
2. Change area $A$
3. Change angle $\theta$

\[ \varepsilon = -\frac{d}{dt}(BA\cos\theta) = -\left(\frac{dB}{dt}\right)A\cos\theta - B\left(\frac{dA}{dt}\right)\cos\theta + B\sin\theta \frac{d\theta}{dt} \]  

(10.1.5)

Thus, we see that an emf may be induced in the following ways:

(i) by varying the magnitude of $\vec{B}$ with time (illustrated in Figure 10.1.3.)

![Figure 10.1.3 Inducing emf by varying the magnetic field strength](image)

(ii) by varying the magnitude of $\vec{A}$, i.e., the area enclosed by the loop with time (illustrated in Figure 10.1.4.)

![Figure 10.1.4 Inducing emf by changing the area of the loop](image)

(iii) varying the angle between $\vec{B}$ and the area vector $\vec{A}$ with time (illustrated in Figure 10.1.5.)

![Figure 10.1.5 Inducing emf by varying the angle between $\vec{B}$ and $\vec{A}$](image)
Minus Sign? Lenz’s Law

Induced EMF is in direction that opposes the change in flux that caused it.
To illustrate how Lenz’s law works, let’s consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\mathbf{A}$.

2. Assuming that $\mathbf{B}$ is uniform, take the dot product of $\mathbf{B}$ and $\mathbf{A}$. This allows for the determination of the sign of the magnetic flux $\Phi_B$.

3. Obtain the rate of flux change $d\Phi_B / dt$ by differentiation. There are three possibilities:

$$
\frac{d\Phi_B}{dt} = \begin{cases} 
> 0 & \Rightarrow \text{induced emf } \varepsilon < 0 \\
< 0 & \Rightarrow \text{induced emf } \varepsilon > 0 \\
= 0 & \Rightarrow \text{induced emf } \varepsilon = 0
\end{cases}
$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of $\mathbf{A}$, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure 10.1.6.

![Figure 10.1.6 Determination of the direction of induced current by the right-hand rule](image)
LENZ’s LAW APPLICATION

**FIGURE 34.22** The induced current for six different situations.

- **$\vec{B}$ up and steady**
  - No change in flux
  - No induced field
  - No induced current

- **$\vec{B}$ down and steady**
  - No change in flux
  - No induced field
  - No induced current

- **$\vec{B}$ up and increasing**
  - Change in flux $\uparrow$
  - Induced field $\downarrow$
  - Induced current $\text{cw}$

- **$\vec{B}$ down and increasing**
  - Change in flux $\downarrow$
  - Induced field $\uparrow$
  - Induced current $\text{ccw}$

- **$\vec{B}$ up and decreasing**
  - Change in flux $\downarrow$
  - Induced field $\uparrow$
  - Induced current $\text{ccw}$

- **$\vec{B}$ down and decreasing**
  - Change in flux $\uparrow$
  - Induced field $\downarrow$
  - Induced current $\text{cw}$

(clockwise)
Figure 10.1.7 Direction of the induced current using Lenz’s law

The above situations can be summarized with the following sign convention:

<table>
<thead>
<tr>
<th>$\Phi_B$</th>
<th>$d\Phi_B/dt$</th>
<th>$\mathcal{E}$</th>
<th>$I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
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<tr>
<td>-</td>
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<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

The positive and negative signs of $I$ correspond to a counterclockwise and clockwise currents, respectively.
HOW TO CREATE E-FIELDS

(a) Magnetic forces separate the charges and cause a potential difference between the ends. This is a motional emf.

\[ \Delta V = v l \, B \]

Charge carriers in the wire experience an upward force of magnitude \( F_B = qvB \). Being free to move, positive charges flow upward (or, if you prefer, negative charges downward).

\[ \Delta V = V_{\text{top}} - V_{\text{bottom}} = - \int_0^l E_y \, ds = - \int_0^l (vB) \, ds = vlB \]

The charge separation creates an electric field in the conductor, \( E \) increases as more charge flows.

(b) Chemical reactions separate the charges and cause a potential difference between the ends. This is a chemical emf.

\[ F_E = qE \]
\[ F_B = qvB \]
\[ E = vB \]
\[ \text{EMF, } \varepsilon = vlB \]
Motional emf

The motional emf of a conductor of length $l$ moving with velocity $v$ perpendicular to a magnetic field $B$ is

$$\mathcal{E} = vlB$$
HOW TO CREATE CURRENTS

Who provides the energy? What happens if I give the wire a push?

\[ I = \frac{\varepsilon}{R} = vlB/R \]

\[ \vec{F}_B = I(l\hat{e}_y) \times (-B\hat{e}_z) = -IlB\hat{e}_x = -\hat{e}_x\left(\frac{B^2l^2v}{R}\right) \]

\[ m \frac{dv}{dt} = F_B = -\frac{B^2l^2v}{R} \]

\[ \frac{dv}{v} = -\frac{B^2l^2v}{mR} dt = -dt / \tau \]

\[ v(t) = v(t = 0) \exp(-t / \tau) \]
Energy – Power Issues

\[ dW = \vec{F} \cdot d\vec{s} \]

\[ P = \frac{dW}{dt} = \vec{F} \cdot (d\vec{s} / dt) = \vec{F} \cdot \vec{v} \]

To maintain constant \( \vec{v} \) need to apply a force

\[ \vec{F}_{pull} = e_x l I B = e_x (v l B / R) l B \]

Energy Conservation

\[ P_{inp} = \frac{(v l B)^2}{R} \]

\[ P_{diss} = I^2 R = \frac{(v l B)^2}{R} \]

A simple generator transforms mechanical power to electrical power.
Rotating conducting bar in magnetic field

A conducting bar of length \( \ell \) rotates with a constant angular speed \( \omega \) about a pivot at one end. A uniform magnetic field \( \mathbf{B} \) is directed perpendicular to the plane of rotation, as shown in Figure 31.11. Find the motional emf induced between the ends of the bar.

Let \( dq \) be charge within element \( dr \)

In equilibrium:

\[
F_E = F_B \rightarrow dqdE = dqvB
\]

\( dE \) parallel to the bar and across \( dE \)

\( v = \omega r \rightarrow dE = \omega rB \)

For uniform \( E \) - field \( \rightarrow |\Delta V| = Ed \)

\( dV = dEdr = \omega Brdr \)

\( \Delta V = \omega B \int_0^1 rdr = \frac{1}{2} \omega B \)

Hydroelectric Power
**FIGURE 34.4** Pictorial representation of a metal bar rotating in a magnetic field.

The electric field strength increases with $r$.

The speed at distance $r$ is $v = \omega r$.

**FIGURE 34.6** A pulling force is needed to move the wire to the right.

The induced current flows through the moving wire.

The magnetic force on the current-carrying wire is opposite the motion.

A pulling force to the right must balance the magnetic force to keep the wire moving at constant speed. This force does work on the wire.

**FIGURE 34.7** A pushing force is needed to move the wire to the left.

1. The magnetic force on the charge carriers is down, so the induced current flows clockwise.

2. The magnetic force on the current-carrying wire is to the right.
Hydro-Electric Station

Watch movie at www.opg.com/power/hydro/howitworks.asp
Magnetic data storage encodes information in a pattern of alternating magnetic fields. When these fields move past a small *pick-up coil*, the changing magnetic field creates an induced current in the coil. This current is amplified into a sequence of voltage pulses that represent the 0s and 1s of digital data.
Electric Guitar
EMF = vB = (8 km/sec)(20 km)(0.3 x 10^{-4} T) = 4.8 kV

R = 4.8 kV/1A = 4800 Ohm

P = I(EMF) = 4.8 kW