PHYS 270 – SUPPL. #3

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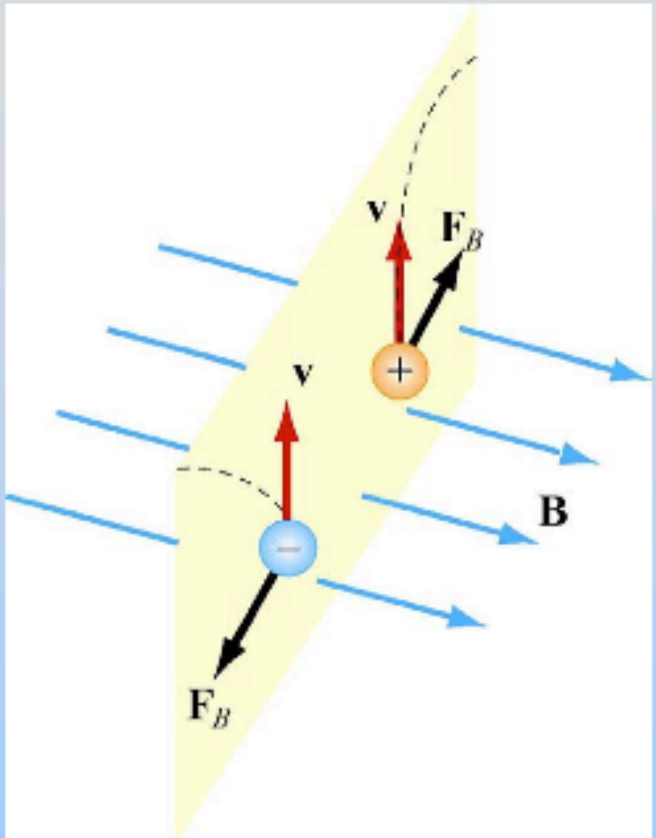
FEBRUARY 1, 2011

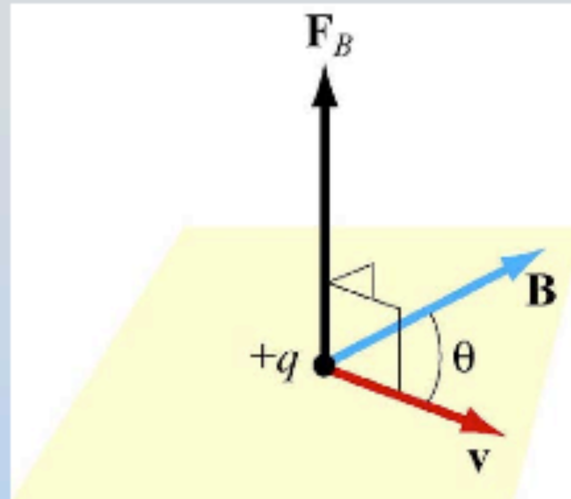
OVERVIEW

- COMPARE E-FIELD TO B-FIELD PROPERTIES
  - FIELDS, FIELD LINES, DIPOLES
- COMPUTE B-FIELDS DUE TO CURRENTS
  - BIOT-SAVART, LONG WIRES, CURRENT LOOPS, SOLENOIDS
  - AMPERE's LAW – EQUIVALENCE TO GAUSS's LAW
- MAGNETIC FORCES ON MOVING CHARGES
- MAGNETIC PROPERTIES OF MATTER - MRI

Magnetic Forces

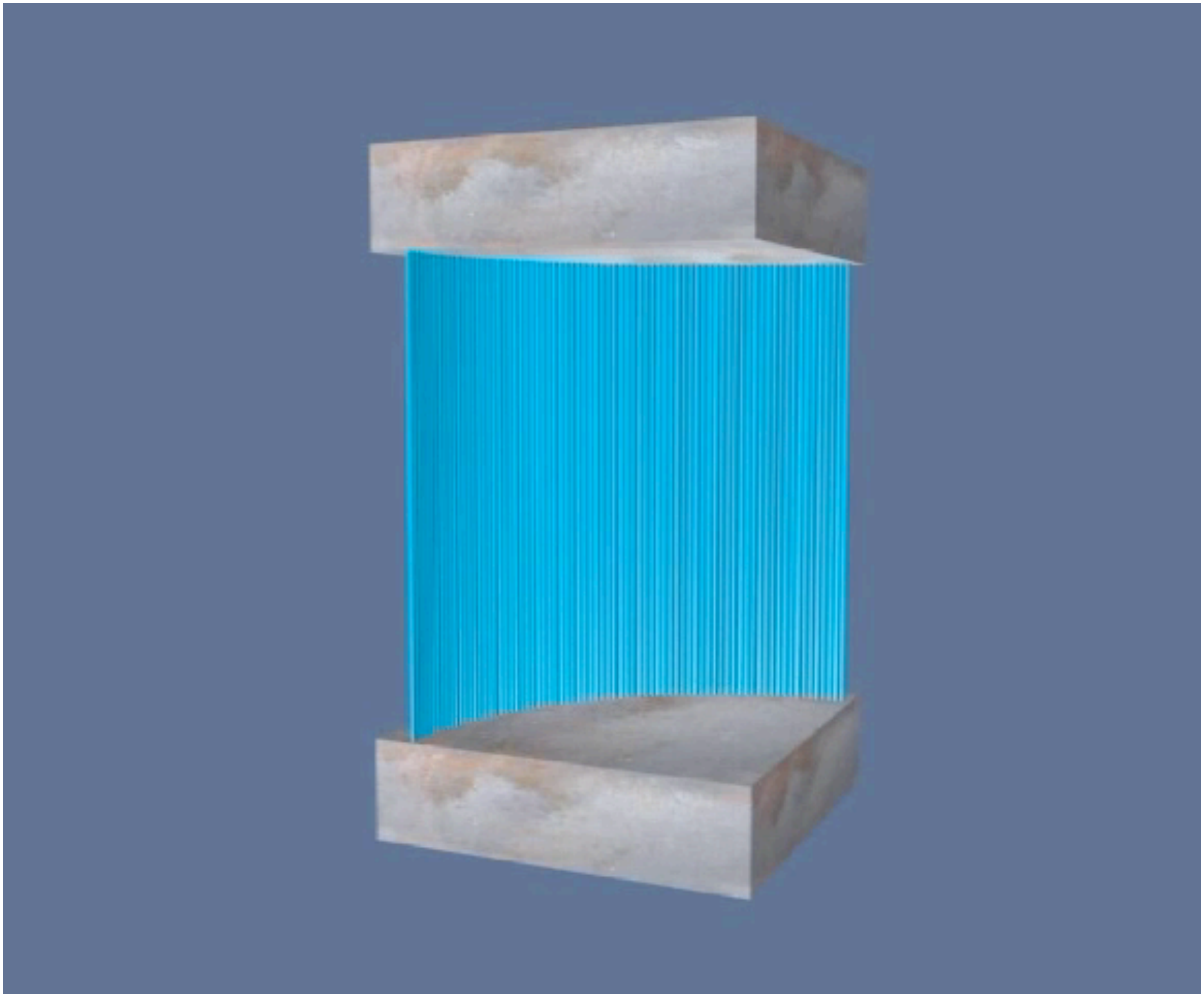
Moving Charges Feel Magnetic Force



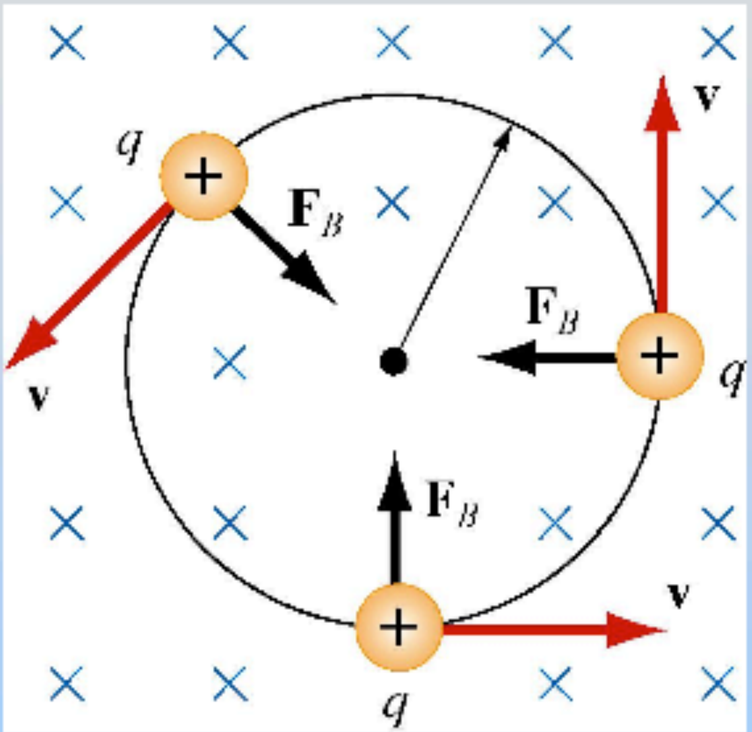


$$\vec{\mathbf{F}}_B = q\,\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

Magnetic force perpendicular both to:
Velocity **v** of charge and magnetic field **B**



Cyclotron Motion



(1) r : radius of the circle

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

(2) T : period of the motion

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

(3) ω : cyclotron frequency

$$\boxed{\omega = 2\pi f = \frac{v}{r} = \frac{qB}{m}}$$

What is the *shape* of the trajectory that a charged particle follows in a uniform magnetic field?

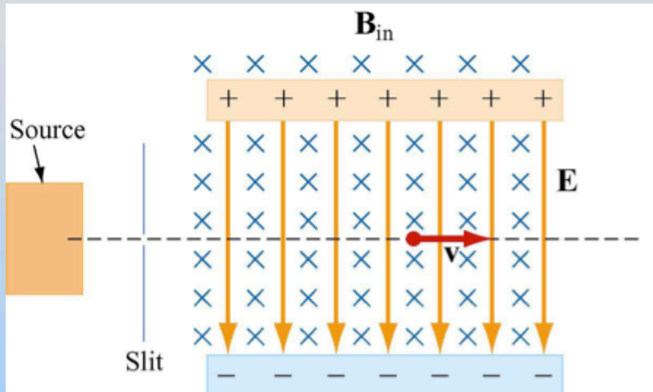
A. Helix
B. Parabola
C. Circle
D. Ellipse
E. Hyperbola

Work due to magnetic force on a charged particle

$$W = \int \vec{F} \bullet \vec{v} dt =$$

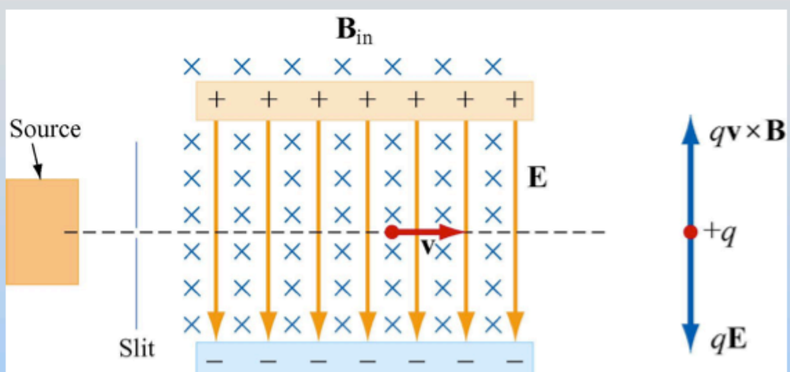
$$= q \int (\vec{v} \times \vec{B}) \bullet \vec{v} dt = 0$$

Application: Velocity Selector



What happens here?

Velocity Selector



Particle moves in a straight line when

$$\vec{\mathbf{F}}_{net} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}) = 0 \Rightarrow v = \frac{E}{B}$$

MAGNETIC FORCES ON CHARGES

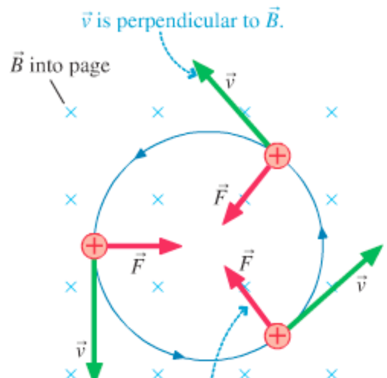
$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$qvB = ma_r = m\frac{v^2}{r}$$

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi r}{v} = 2\pi\frac{m}{qB}, f_c = \frac{1}{2\pi}\frac{qB}{m}, \omega_c = \frac{qB}{m}$$

FIGURE 33.37 Cyclotron motion of a charged particle moving in a magnetic field.



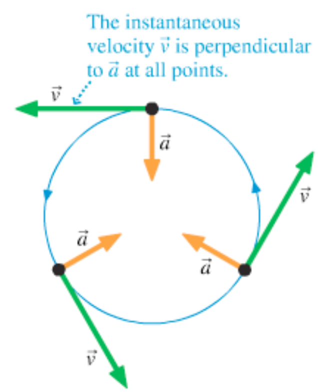
Do the B forces do work? e.g. $q\Phi = 1/2\ (mv^2)$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = m\vec{v} \bullet \frac{d\vec{v}}{dt}$$

$$m\vec{v} \bullet \frac{d\vec{v}}{dt} = q\vec{v} \bullet (\vec{v} \times \vec{B}) = 0$$

Magnetic force acts to change the direction of v but not the magnitude or the value of the force

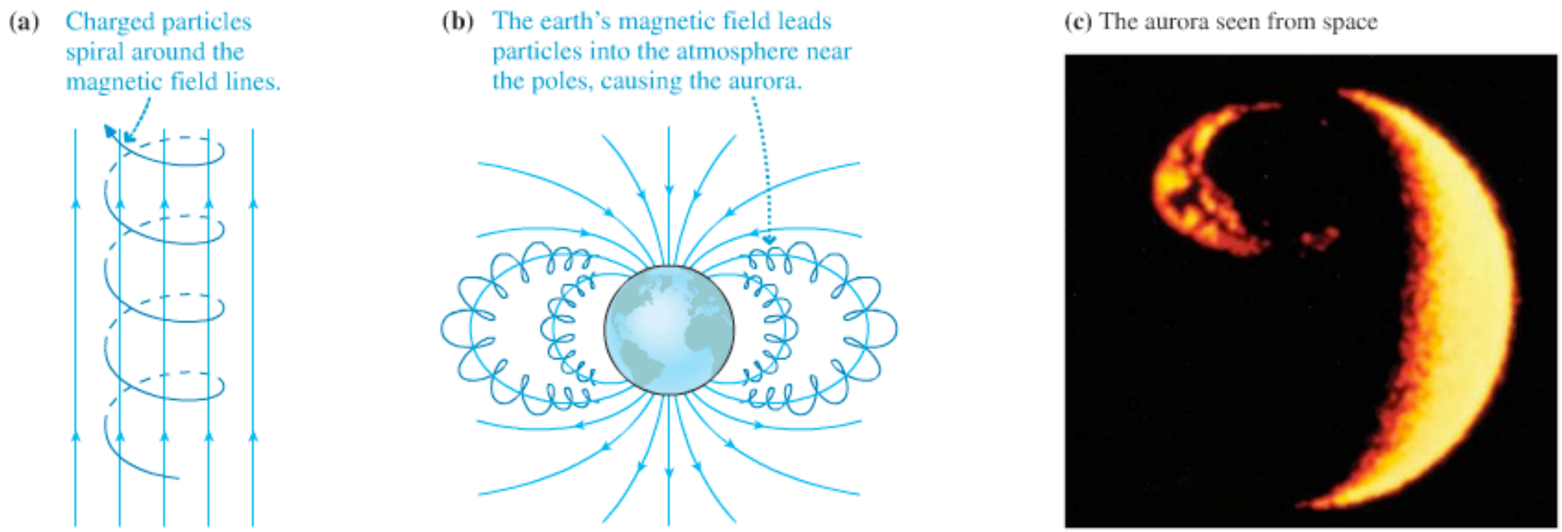
FIGURE 4.40 For uniform circular motion, the acceleration $\vec{a}$ always points to the center.



Centripetal acceleration

PARTICLE ORBITS

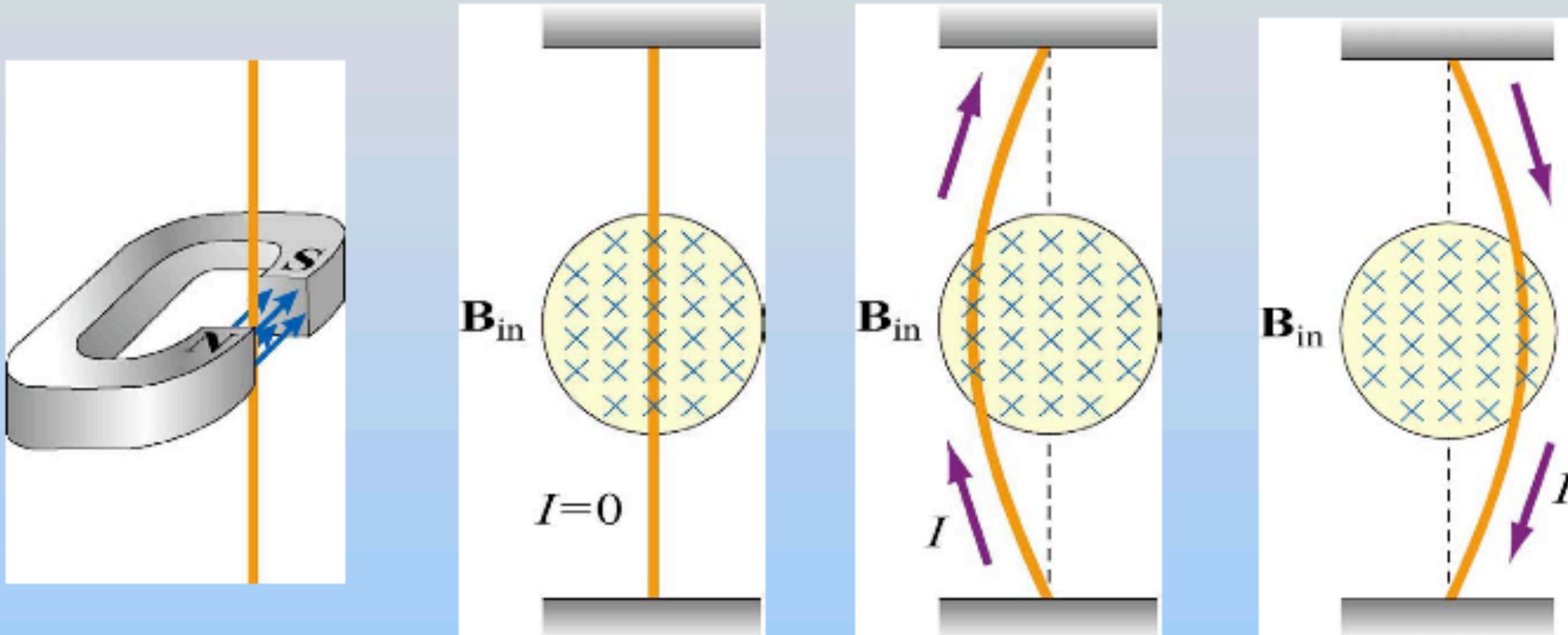
FIGURE 33.39 In general, charged particles spiral along helical trajectories around the magnetic field lines. This motion is responsible for the earth's aurora.



VAN ALLEN BELTS

The beautiful aurora borealis, the northern lights, is due to the earth's magnetic field.

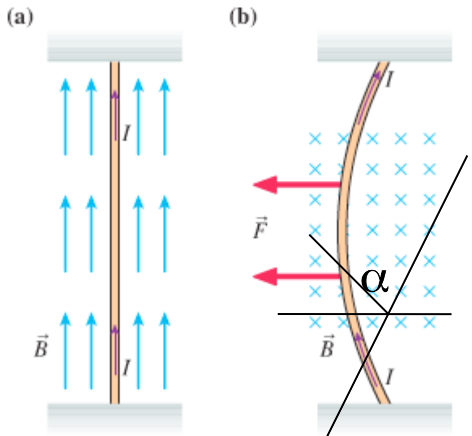
Magnetic Force on Current-Carrying Wire



Current is moving charges, and we know that moving charges **feel** a force in a magnetic field

FORCES ON WIRES

FIGURE 33.43 Magnetic force on a current-carrying wire.



(a) There's no force on a current parallel to a magnetic field.

(b) A current perpendicular to the field experiences a force in the direction of the right-hand rule.

$$\vec{F} = \Delta Q(\vec{v} \times \vec{B})$$

$$I = \frac{\Delta Q}{\Delta t}, \vec{v} = \frac{\vec{l}}{\Delta t}$$

$$\vec{F} = I\Delta t(\vec{v} \times \vec{B}) = I(\vec{l} \times \vec{B})$$

$$|F| = Il\sin\alpha$$

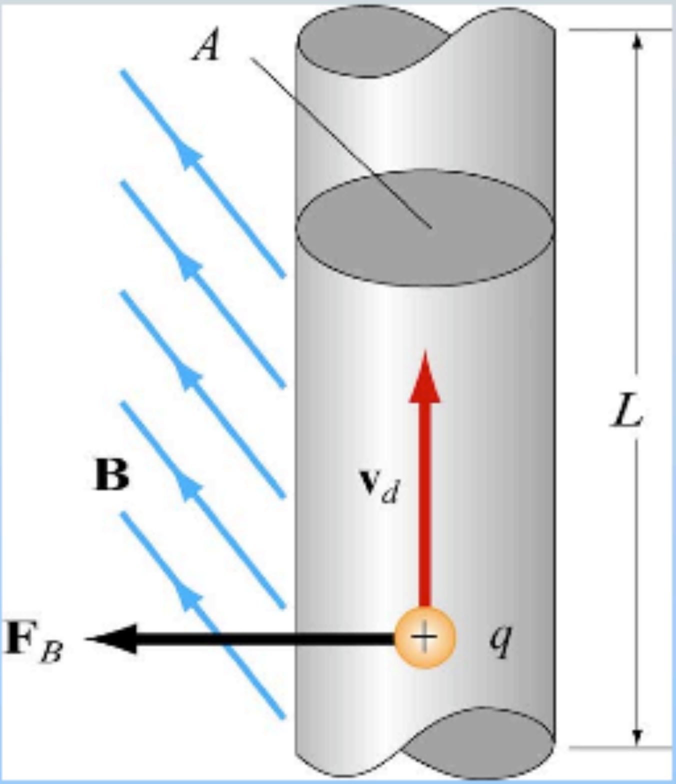
Magnetic Force on Current-Carrying Wire

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$= (\text{charge})\frac{\text{m}}{\text{s}} \times \vec{\mathbf{B}}$$

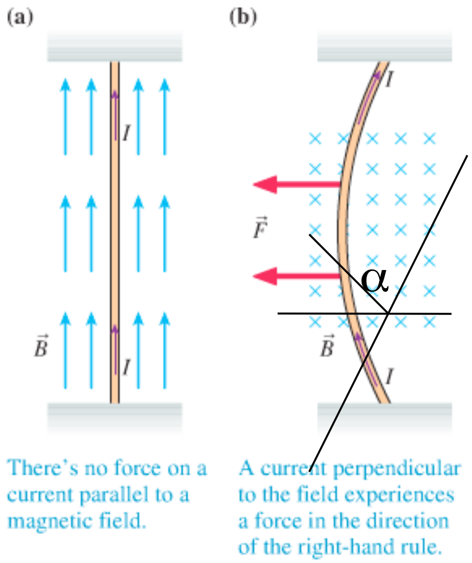
$$= \frac{\text{charge}}{\text{s}}\text{m} \times \vec{\mathbf{B}}$$

$$\boxed{\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right)}$$



FORCES ON WIRES

FIGURE 33.43 Magnetic force on a current-carrying wire.



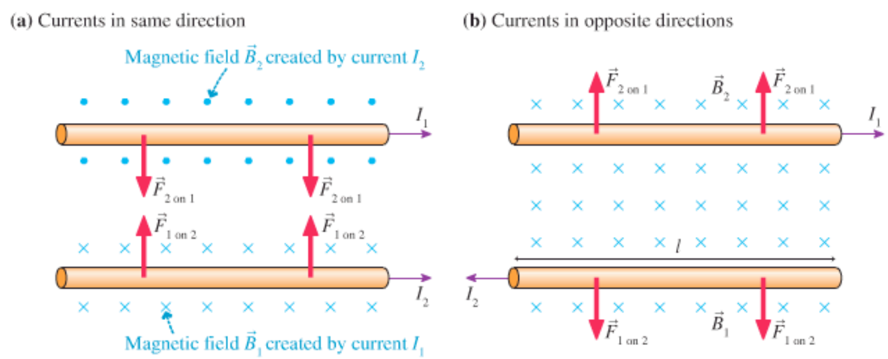
$$\vec{F} = \Delta Q(\vec{v} \times \vec{B})$$

$$I = \frac{\Delta Q}{\Delta t}, \vec{v} = \frac{\vec{l}}{\Delta t}$$

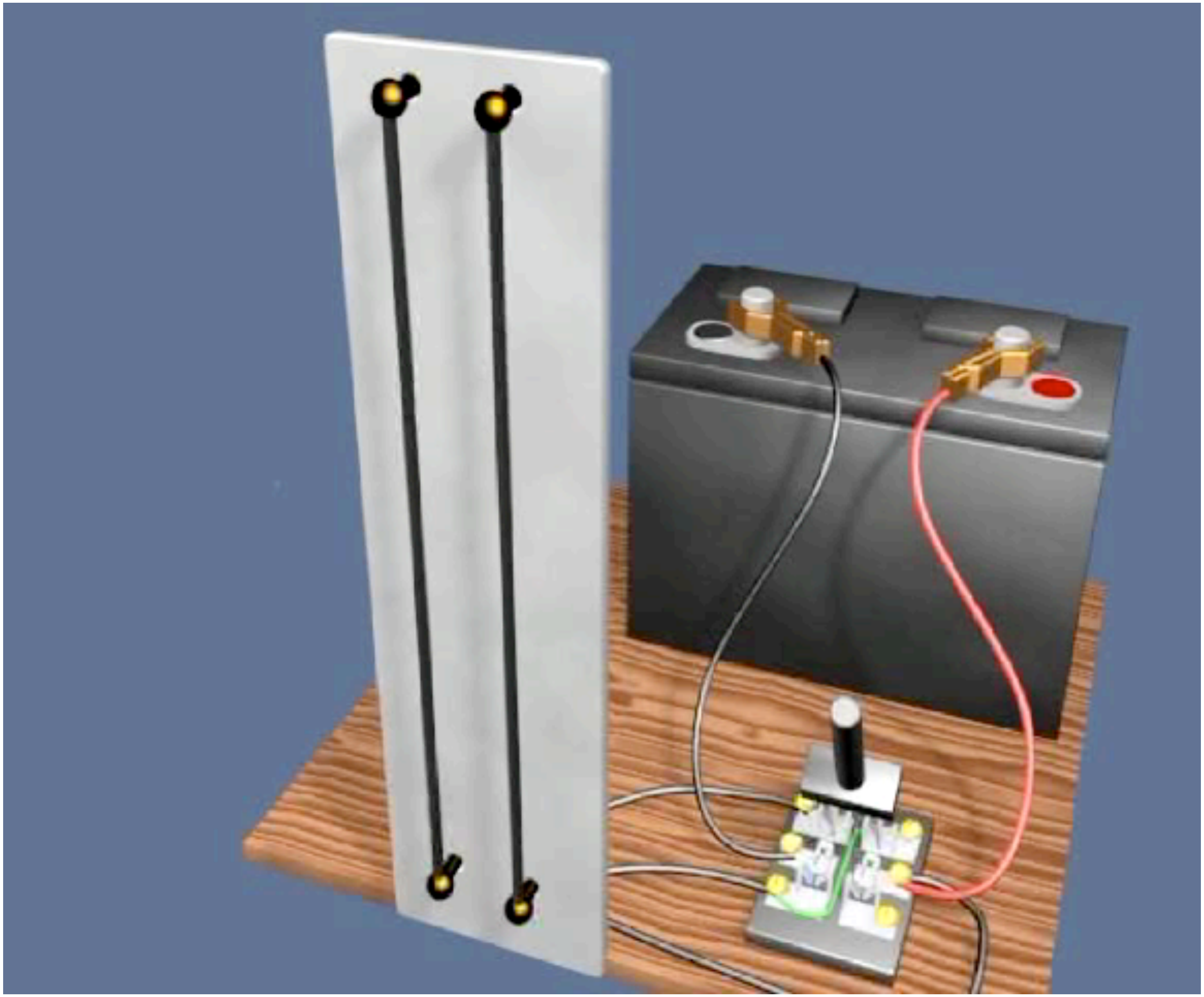
$$\vec{F} = I\Delta t(\vec{v} \times \vec{B}) = I(\vec{l} \times \vec{B})$$

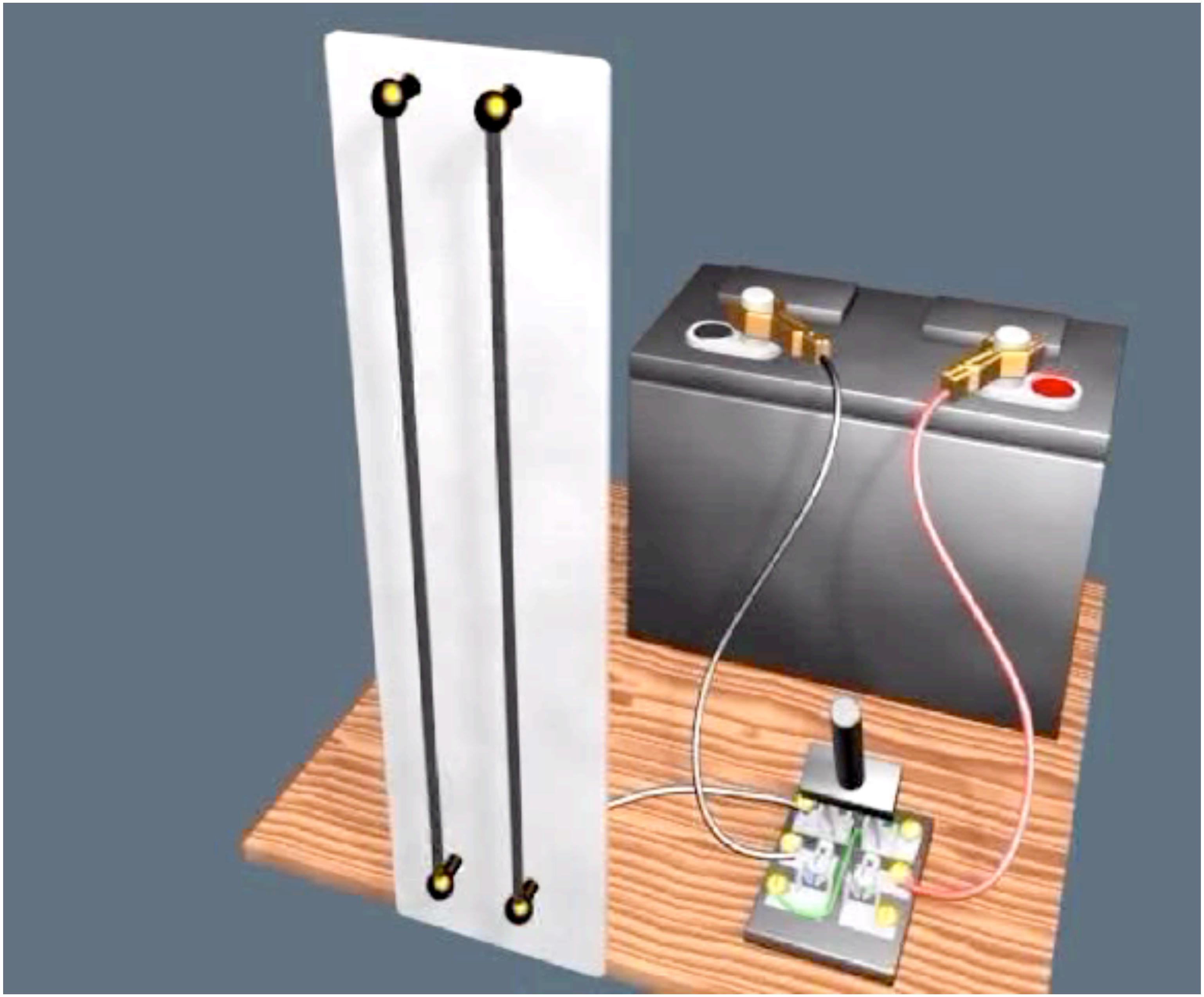
$$|F| = I l \sin\alpha$$

FIGURE 33.46 Magnetic forces between parallel current-carrying wires.



$$F_{2,1} = I_1 l B_2 = I_1 l \frac{\mu_o I_2}{2\pi d}$$



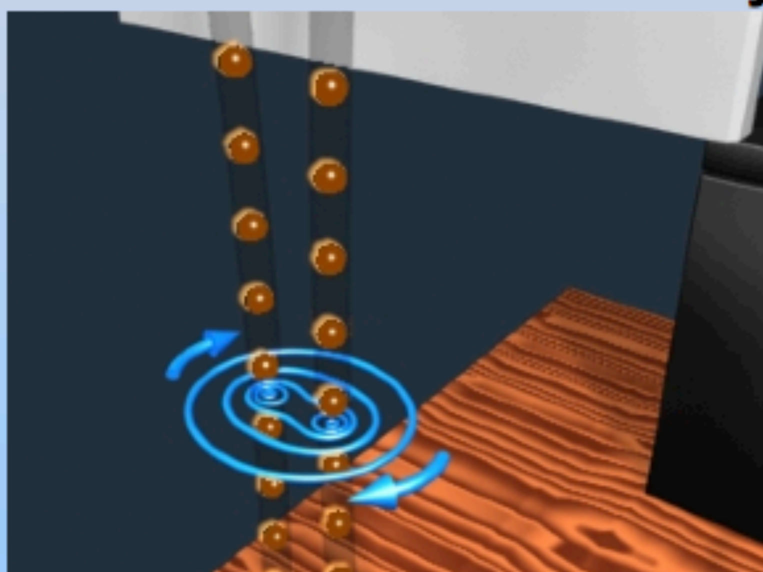


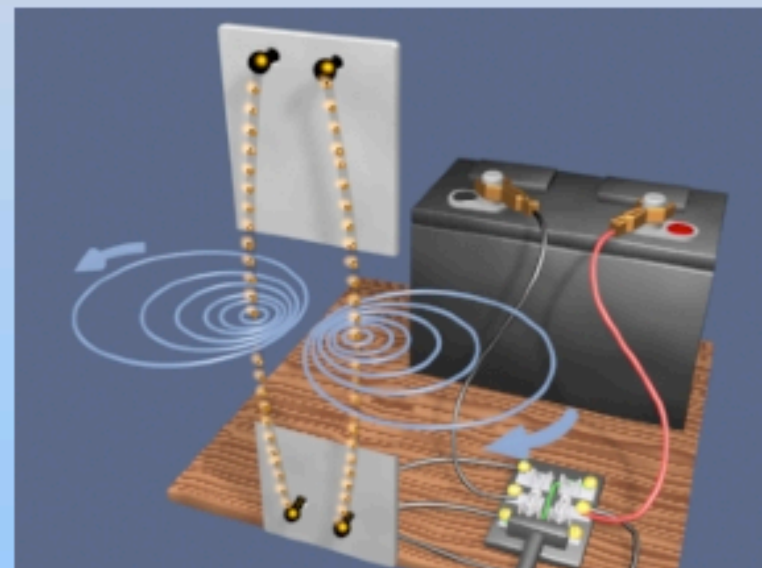
How Do They Interact?

Moving charges also **create** magnetic fields!

The current in one wire *creates* a magnetic field that is *felt* by the other wire.

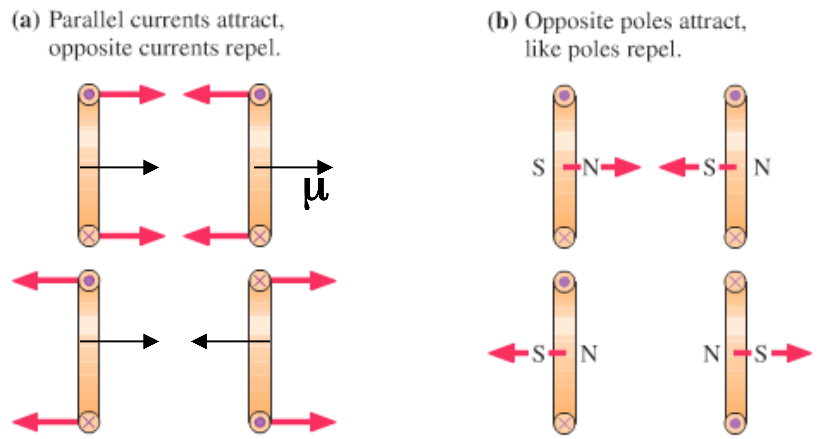
This is the rest of today's focus





MUTUAL LOOP FORCES

FIGURE 33.48 Two alternative but equivalent ways to view magnetic forces.



Attraction

Repulsion

Summary Forces on Charges and Currents

FIGURE 33.33 The relationship among $\vec{v}$, $\vec{B}$, and $\vec{F}$.

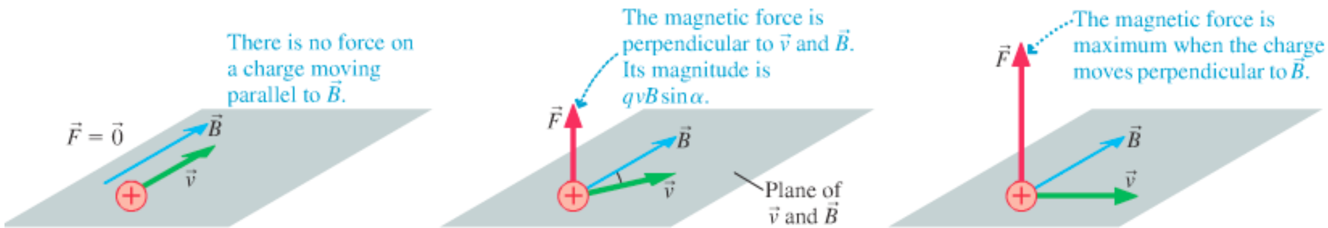


FIGURE 33.35 Magnetic forces on moving charges.

FIGURE 33.32 Ampère's experiment to observe the forces between parallel current-carrying wires.

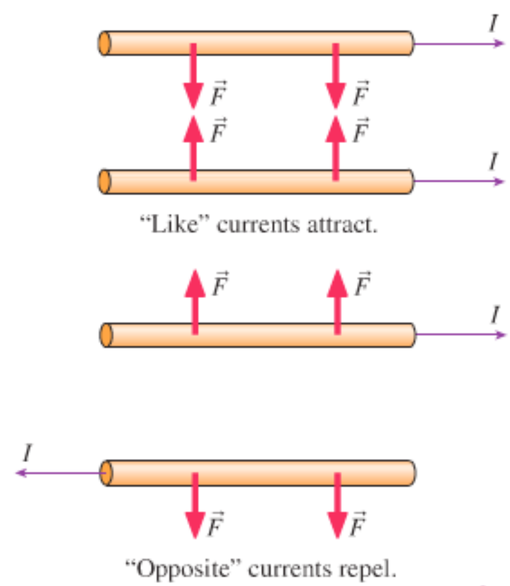
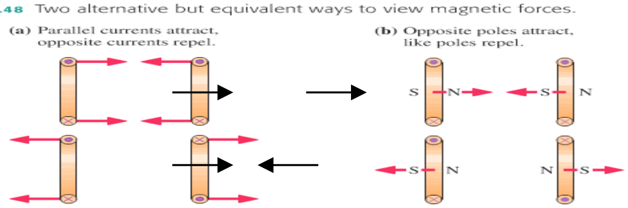


FIGURE 33.48 Two alternative but equivalent ways to view magnetic forces.

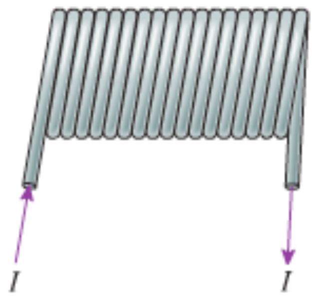


Attraction

Repulsion

SOLENOIDS

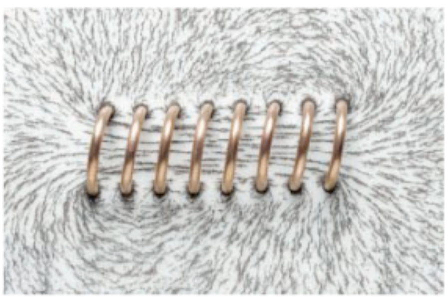
FIGURE 33.27 A solenoid.

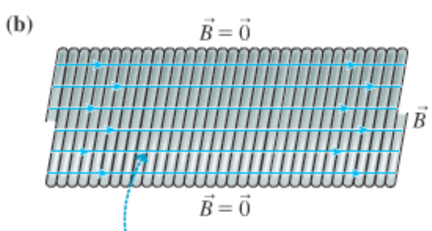


What is a solenoid – A device that creates a uniform magnetic field inside and zero outside (in both cases almost uniform and almost zero)
Who needs it . Electronic devices, MRI machines, Fusion machines etc

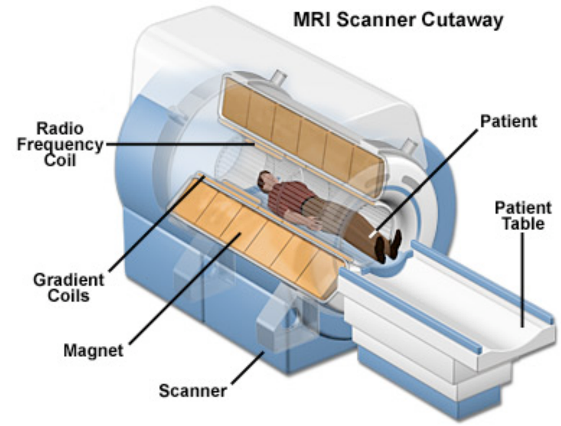
FIGURE 33.29 The magnetic field of a solenoid.

(a) A short solenoid

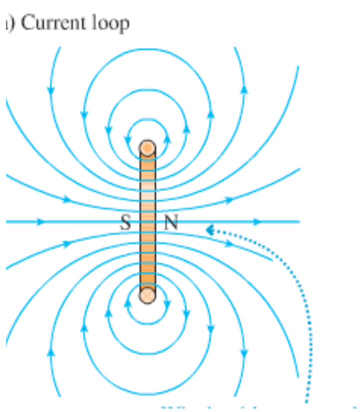


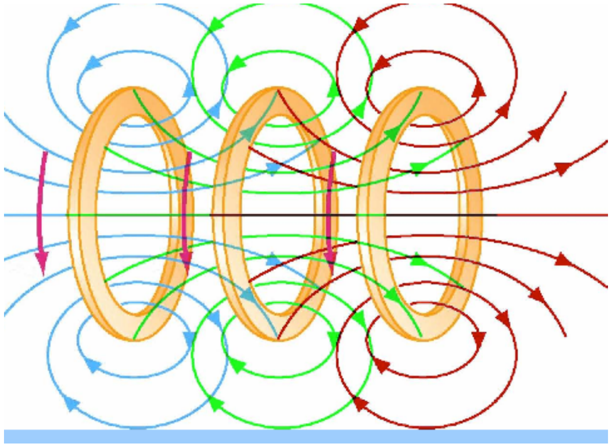


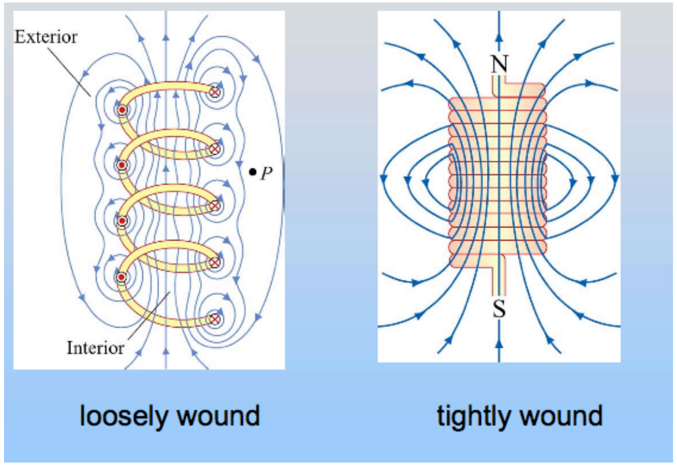
The magnetic field is uniform inside this section of an ideal, infinitely long solenoid. The magnetic field outside the solenoid is zero.

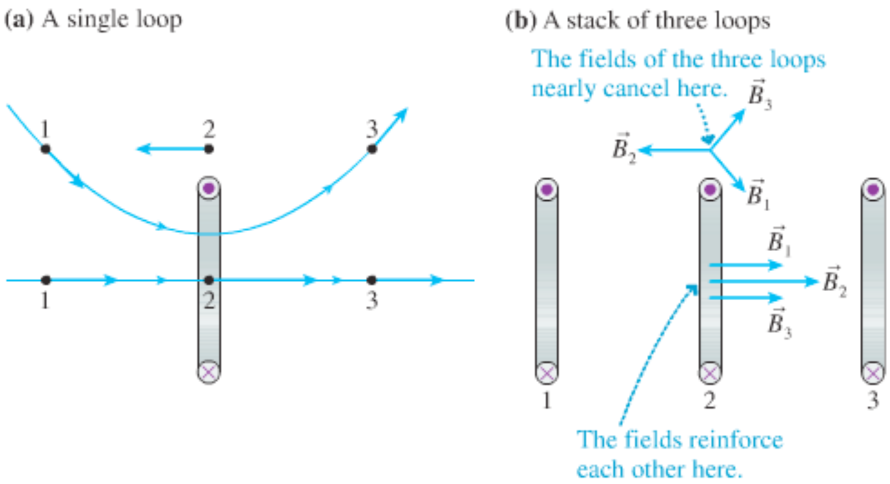


How to make a solenoid

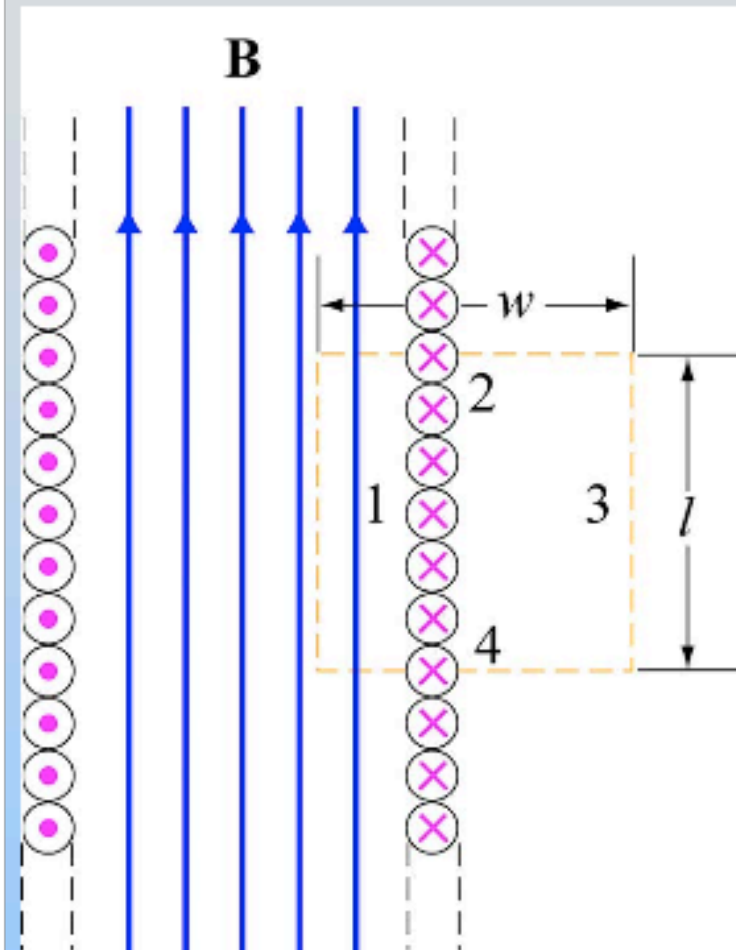








Magnetic Field of Ideal Solenoid



Using Ampere's law: Think!

$$\begin{cases} \vec{\mathbf{B}} \perp d\vec{\mathbf{s}} \text{ along sides 2 and 4} \\ \vec{\mathbf{B}} = 0 \text{ along side 3} \end{cases}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_1 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_2 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_3 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_4 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$$

$$= Bl + 0 + 0 + 0$$

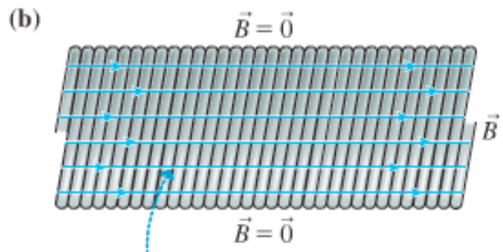
$I_{enc} = nlI$ n: turn density

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl = \mu_0 nlI$$

$n = N / L$: # turns/unit length

$$\boxed{B = \frac{\mu_0 nlI}{l} = \mu_0 nI}$$

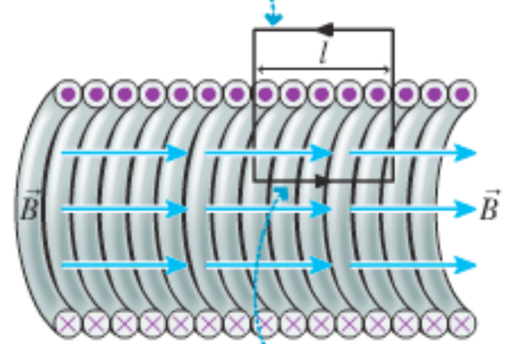
B FIELD INSIDE A SOLENOID



The magnetic field is uniform inside this section of an ideal, infinitely long solenoid. The magnetic field outside the solenoid is zero.

FIGURE 33.30 A closed path inside and outside an ideal solenoid.

This is the integration path for Ampère's law. There are N turns inside.



$\vec{B}$ is tangent to the integration path along the bottom edge.

$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 NI$$

uniform magnetic field inside a solenoid is

$$B_{\text{solenoid}} = \frac{\mu_0 NI}{l} = \mu_0 nI$$

n is turns per unit length (e.g. per meter)

MAGNETIC TORQUE

TORQUE ON CURRENT LOOPS

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\tau = \mu B \sin\theta$$

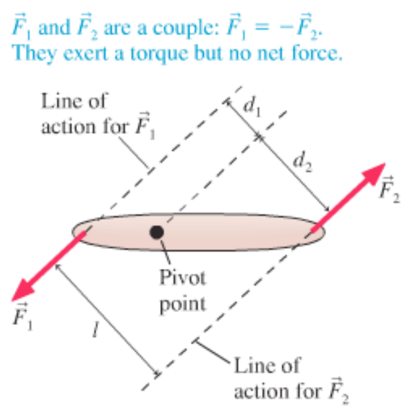
$$\mu = IA$$

$$\vec{F}_{front} = \vec{F}_{back}$$

$$\vec{F}_{top} = IlB\sin\theta = -\vec{F}_{bottom}$$

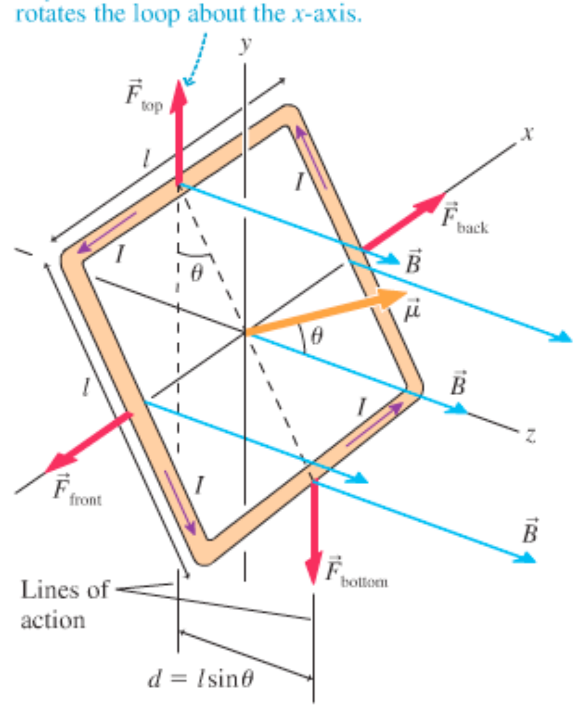
$$\tau = Fd = (IlB\sin\theta)l = (Il^2)B\sin\theta$$

FIGURE 12.27 Two equal but opposite forces form a *couple*.



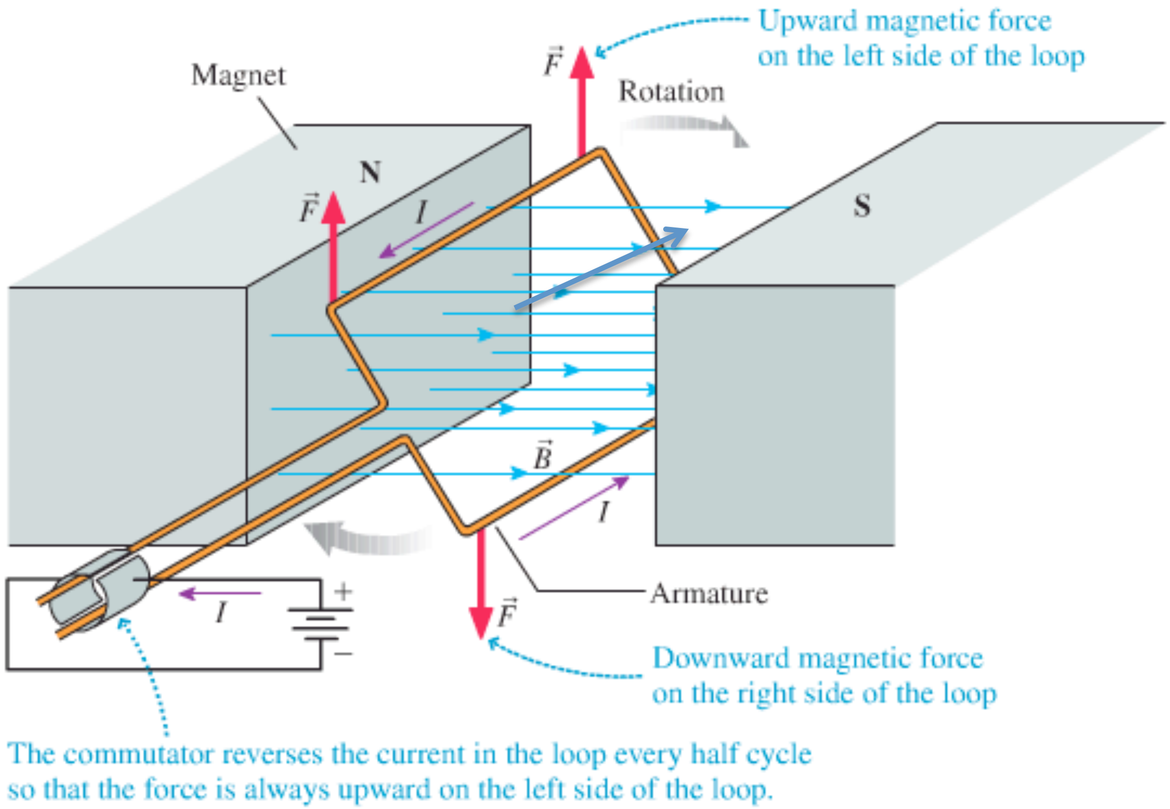
$$I_M \frac{d^2\theta}{dt^2} = T_\theta = \mu B \sin\theta$$

FIGURE 33.49 A uniform magnetic field exerts a torque on a current loop.

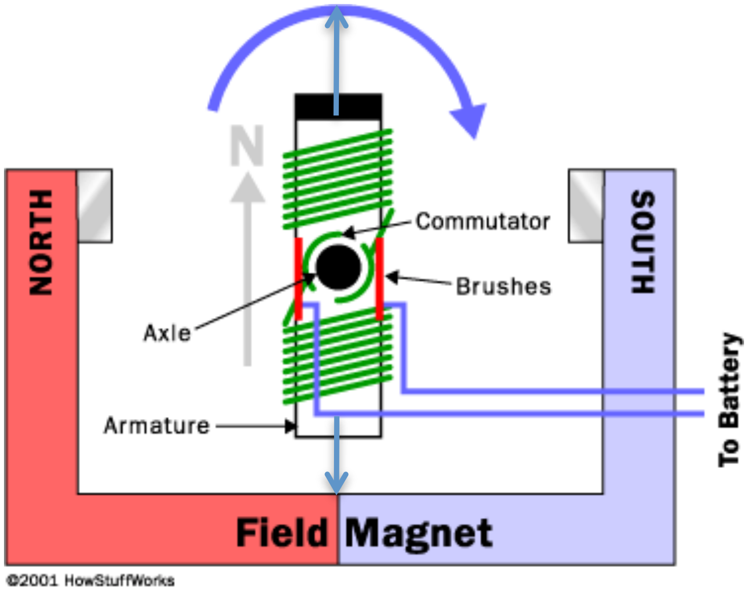


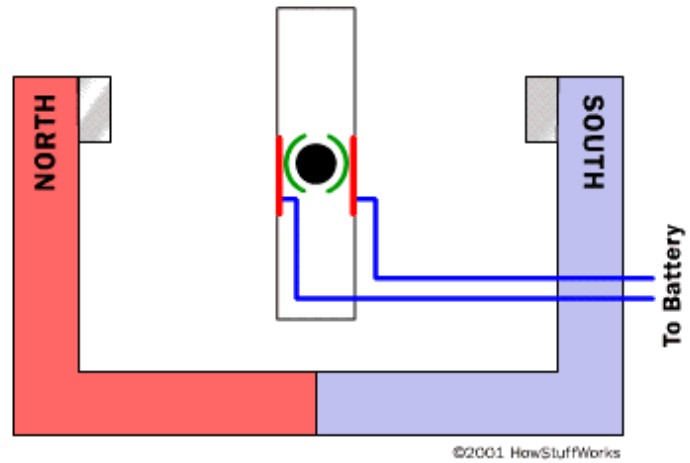
ELECTRIC MOTORS

FIGURE 33.50 A simple electric motor.



Armature is a coil


N
NORTH
SOUTH
Commutator
Brushes
Axle
Armature
To Battery
Field Magnet
©2001 HowStuffWorks


NORTH
SOUTH
To Battery
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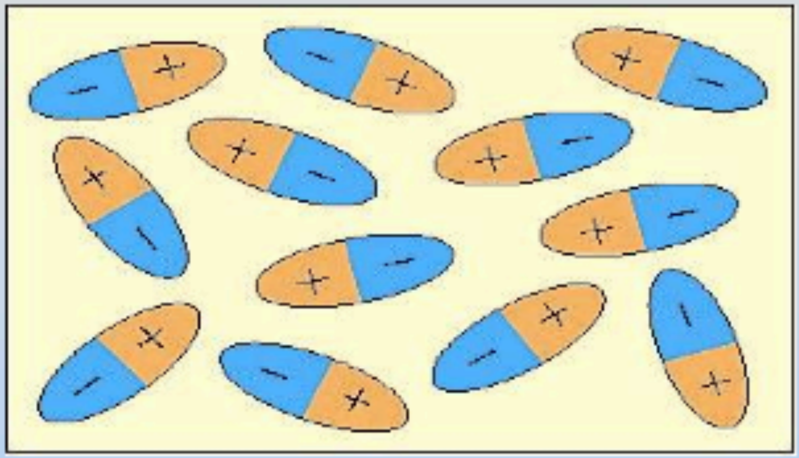
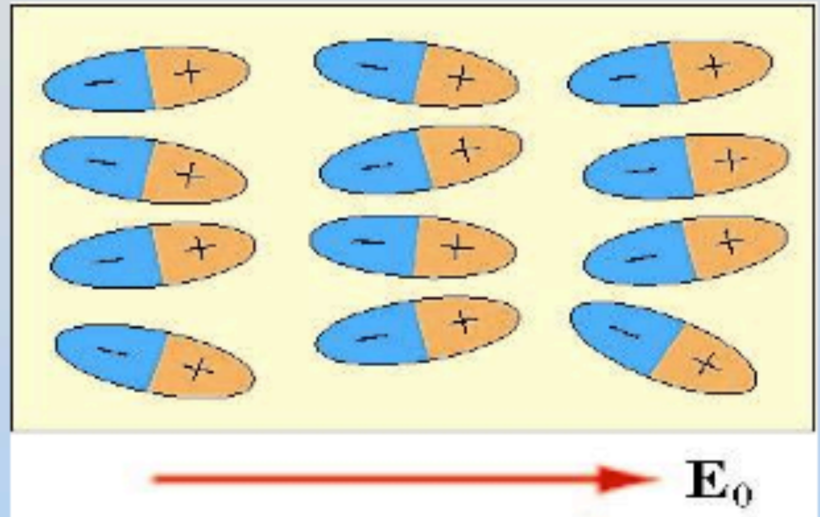
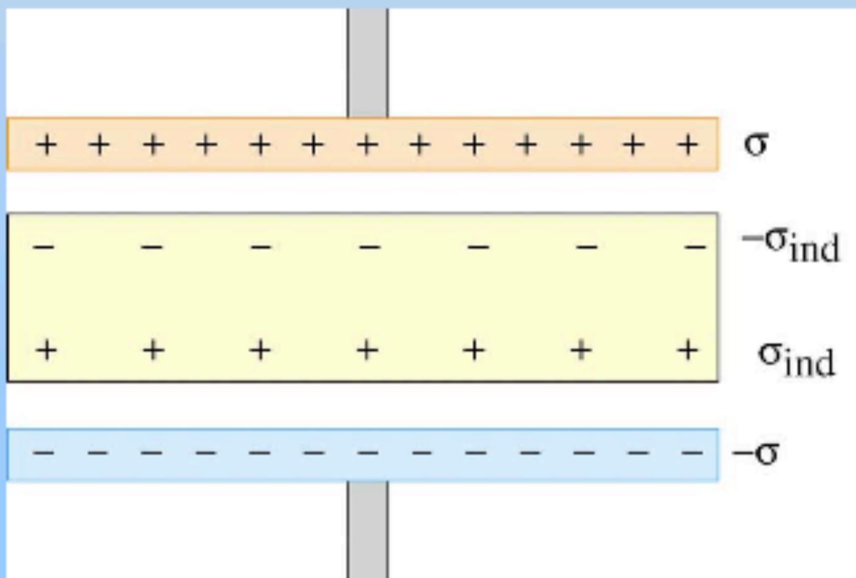
MAGNETIC PROPERTIES OF MATTER

How does a magnet picks up metal paper clips ? Why not plastic ?
How is it different from a charged comb picking up pieces of paper ?

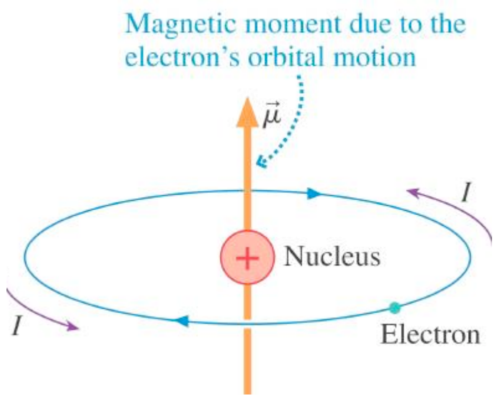
Recall Polar Dielectrics

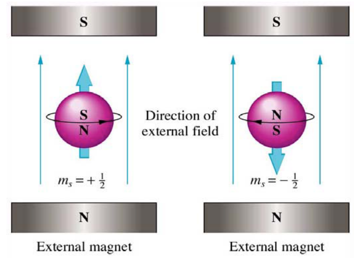




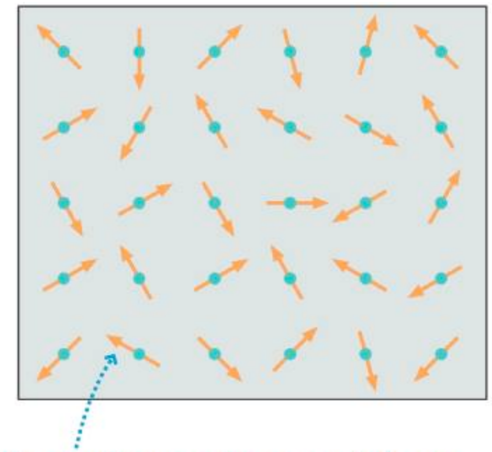


Dielectric polarization ***decreases*** Electric Field!





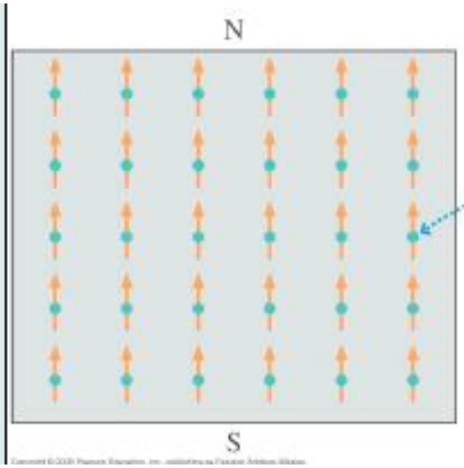
Paired (diamagnetic)
repelled by B



The atomic magnetic moments due to unpaired spins point in random directions. The sample has no net magnetic moment.

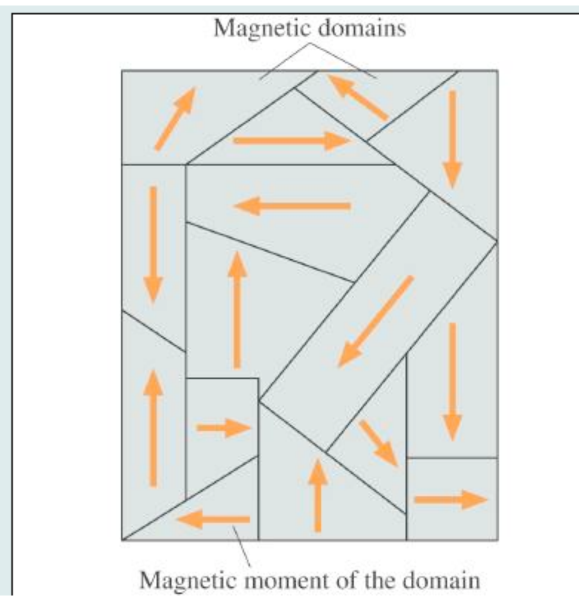
Mutual cancellation
No net μ

Unpaired (paramagnetic)
attracted by B



The atomic magnetic moments are aligned. The sample has north and south magnetic poles.

Ferromagnetic



1. Electrons are microscopic magnets due to their spin
2. Ferromagnetic materials are organized in spin aligned domains
3. In external B produce induced magnetic dipole moment

Hysteresis

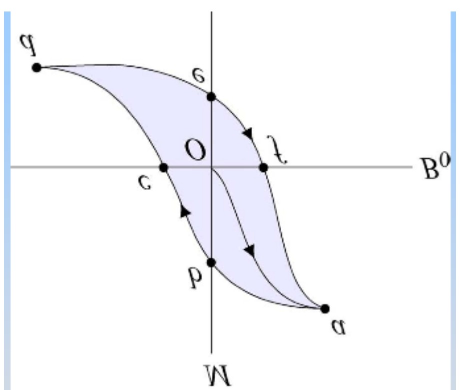


FIGURE 33.55 The magnetic field of the solenoid creates an induced magnetic dipole in the iron.

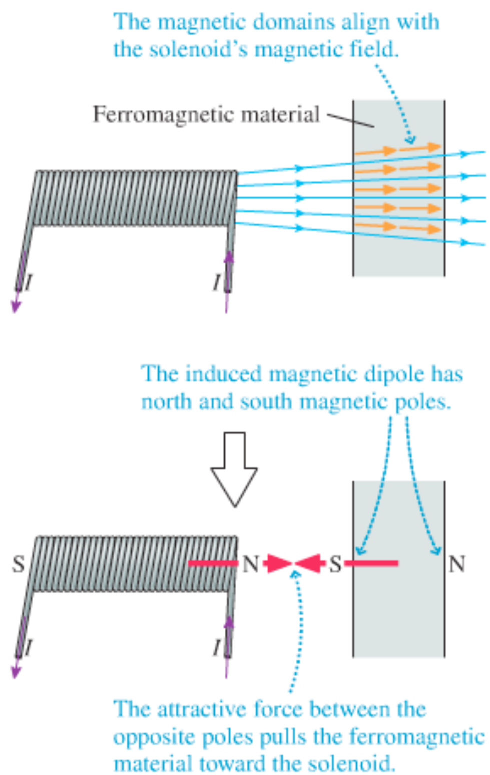
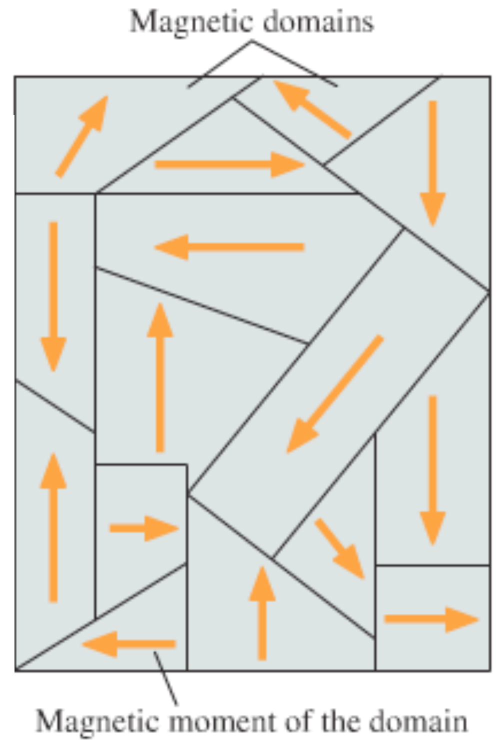
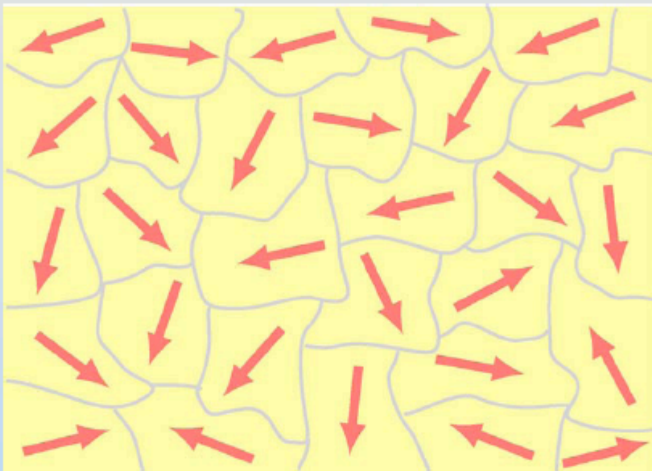
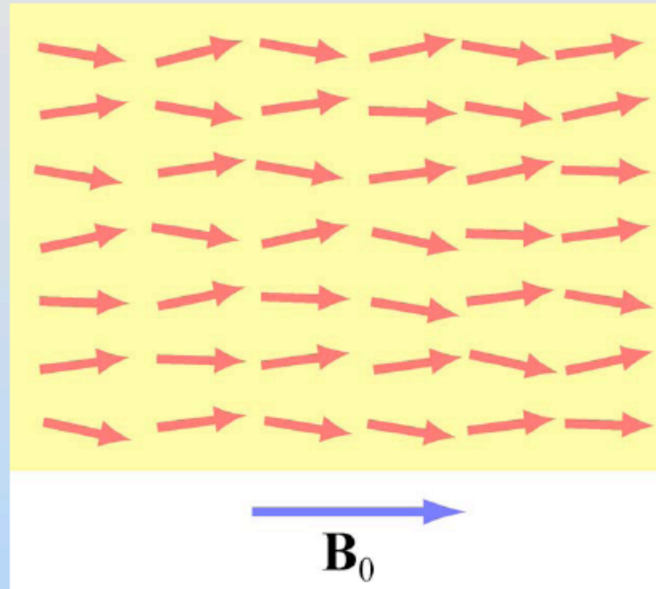


FIGURE 33.54 Magnetic domains in a ferromagnetic material. The net magnetic dipole is nearly zero.



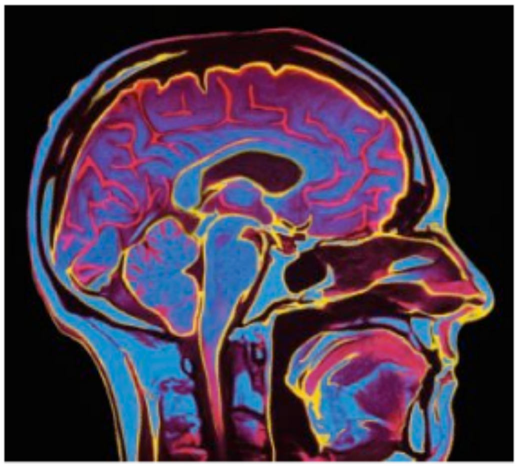
Para/Ferromagnetism





Applied external field B_0 tends to align the atomic magnetic moments (unpaired electrons)

MRI



Magnetic resonance imaging, or MRI, uses the magnetic properties of atoms as a noninvasive probe of the human body.

BODY CONTAINS MAINLY WATER MOLECULES, EACH CONTAINING TWO HYDROGEN ATOMS(TWO PROTONS). IN A STRONG UNIFORM MAGNETIC FIELD THE MAGNETIC MOMENTS OF THE PROTONS ALIGN WITH THE B-FIELD. WHEN A RADIO-FREQUENCY EM FIELD IS TURNED ON THEY ABSORB SOME ENERGY AND GIVE IT BACK WHEN IT IS TURNED-OFF. DESEASED TISSUE, SUCH AS TUMORS, IS DETECTED BECAUSE THE RELAXATION RATE (RETURN TO EQUILIBRIUM) DEPENDS ON THE TISSUE.