PHYS 270 – SUPPL. #2

DENNIS PAPADOPOULOS

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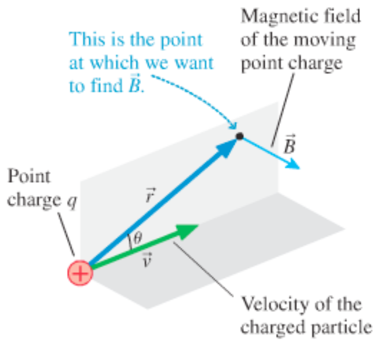
OVERVIEW

- COMPARE E-FIELD TO B-FIELD PROPERTIES
  - FIELDS, FIELD LINES, DIPOLES
- COMPUTE B-FIELDS DUE TO CURRENTS
  - BIOT-SAVART, LONG WIRES, CURRENT LOOPS, SOLENOIDS
  - AMPERE’s LAW – EQUIVALENCE TO GAUSS’s LAW
- MAGNETIC FORCES ON MOVING CHARGES
- MAGNETIC PROPERTIES OF MATTER - MRI

LAST TIME

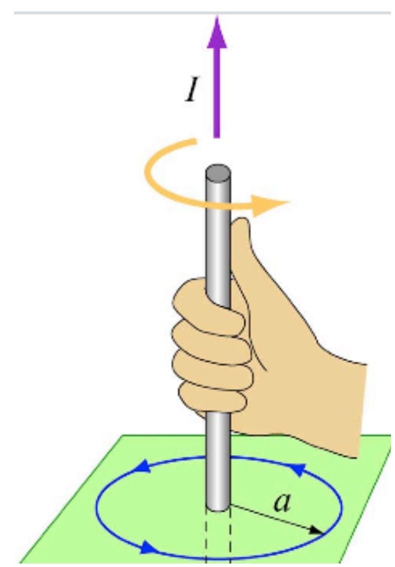
BIOT-SAVART LAW

FIGURE 33.6 The magnetic field of a moving point charge.

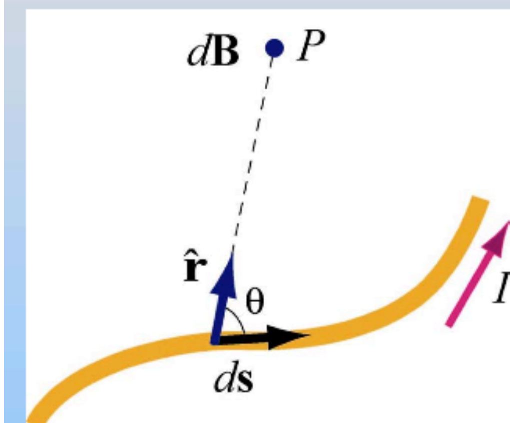


$$\vec{B}_{\text{point charge}} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} \qquad \text{(magnetic field of a point charge)}$$

Direction by RHS rule

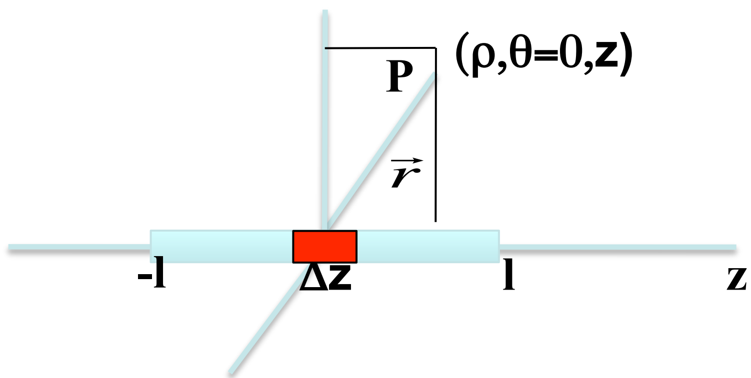


Current element of length ds carrying current I produces a magnetic field:



$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

Field due to a long wire



$$\Delta\vec{B}_{\Delta z}(\vec{r}) = k_M \frac{I\Delta z(\hat{e}_z \times \vec{r})}{r^3}$$

$$\vec{r} = \rho\hat{e}_\rho + z\hat{e}_z$$

$$r^2 = \rho^2 + z^2$$

$$\Delta\vec{B}_{\Delta z}(\vec{r}) = k_M \frac{I\Delta z(\hat{e}_z \times \vec{r})}{r^3} = k_M \frac{I\Delta z[\hat{e}_z \times (\hat{e}_\rho\rho + \hat{e}_z z)}{r^3} = k_M \frac{I\Delta z\rho(\hat{e}_z \times \hat{e}_\rho)]}{r^3} = k_M \frac{I\Delta z\rho\hat{e}_\theta}{r^3}$$

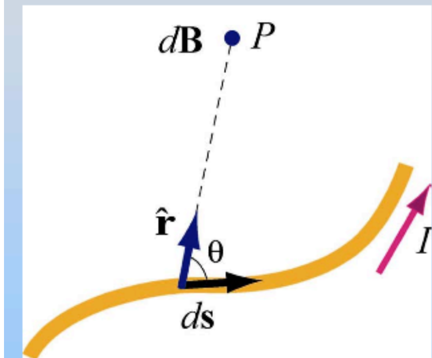
$$\vec{B}(\vec{r}) = \hat{e}_\theta(k_M I\rho)\int_{-l}^{l} \frac{dz}{(\rho^2 + z^2)^{3/2}} = \hat{e}_\theta(k_M I\rho)[\frac{z}{\rho^2(\rho^2 + z^2)^{1/2}}]_{-l}^{l}$$

$$l \rightarrow \infty, \rho << l$$

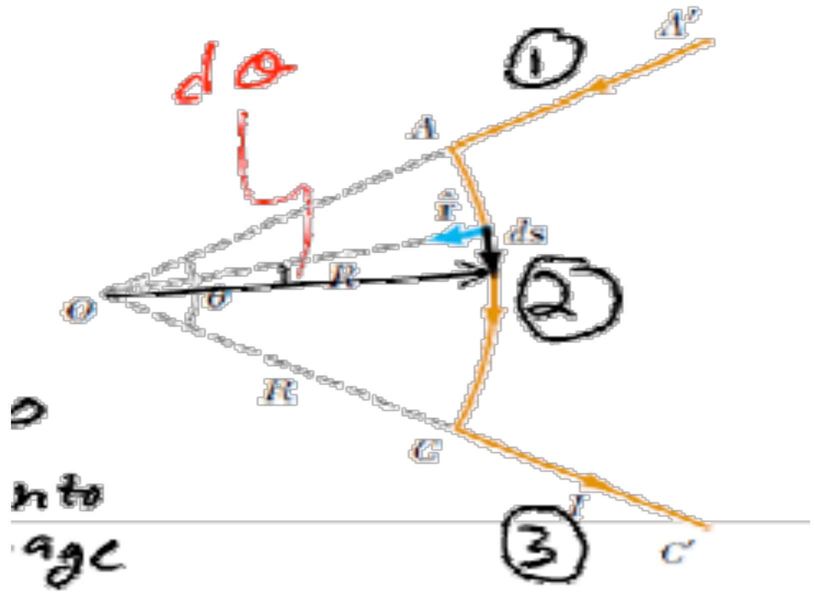
$$\vec{B}(\vec{r}) = \hat{e}_\theta k_M (2I/\rho)$$

Biot-Savart: Circular Arc-> Circle

Current element of length ds carrying current I produces a magnetic field:



$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$



$For\ 1\ and\ 3$

$d\vec{s}\ parallel\ \text{to}\ \hat{r} \rightarrow d\vec{s} \times \hat{r} = 0$

$For 2 \rightarrow d\vec{s} \times \hat{r} = ds\ \text{direction into the}\ page$

$ds = Rd\theta$

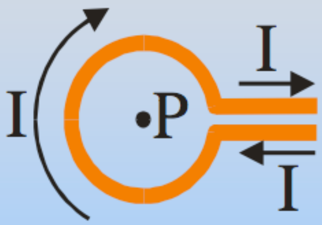
$$\left|\vec{B}\right| = \frac{\mu_o}{4\pi} I \int_0^{\theta} \frac{Rd\theta}{R^2} = \frac{\mu_o}{4\pi}\frac{I}{R}\theta$$

$Circle : \theta \rightarrow 2\pi$

$$\left|\vec{B}\right| = \frac{\mu_o}{2R} I$$

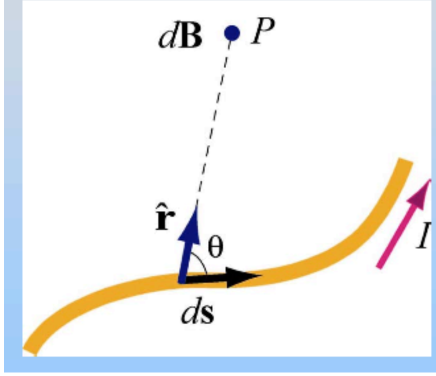
B field of a current loop

Consider a coil with radius *R* and current *I*



Find the magnetic field B at the center (P)

Current element of length ds carrying current I produces a magnetic field:

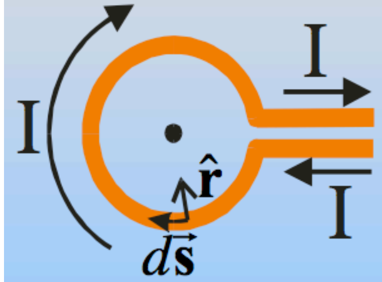


$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\mathbf{s}}\times\hat{\mathbf{r}}}{r^2}$$

1) Think about it:
   - Legs contribute nothing
     I parallel to *r*
   - Ring makes field into page
2) Choose a ds
3) Pick your coordinates
4) Write Biot-Savart

In the circular part of the coil…

$$d\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad\rightarrow\quad |d\vec{\mathbf{s}}\times\hat{\mathbf{r}}| = ds$$



Biot-Savart:

$$dB = \frac{\mu_0 I}{4\pi}\frac{|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\frac{ds}{r^2}$$

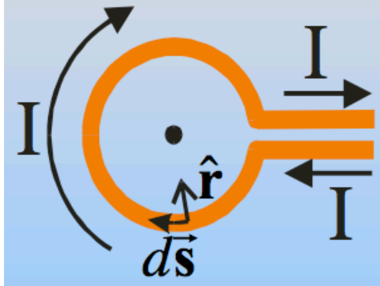
$$= \frac{\mu_0 I}{4\pi}\frac{R\,d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi}\frac{d\theta}{R}$$

B field of a current loop

In the circular part of the coil…

$$d\vec{\mathbf{s}} \perp \hat{\mathbf{r}} \quad \rightarrow \quad |d\vec{\mathbf{s}} \times \hat{\mathbf{r}}| = ds$$



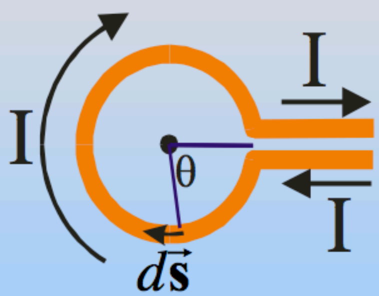
Biot-Savart:

$$dB = \frac{\mu_0 I}{4\pi} \frac{|d\vec{\mathbf{s}} \times \hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi} \frac{ds}{r^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{R\,d\theta}{R^2}$$

$$= \frac{\mu_0 I}{4\pi} \frac{d\theta}{R}$$

Consider a coil with radius *R* and current *I*



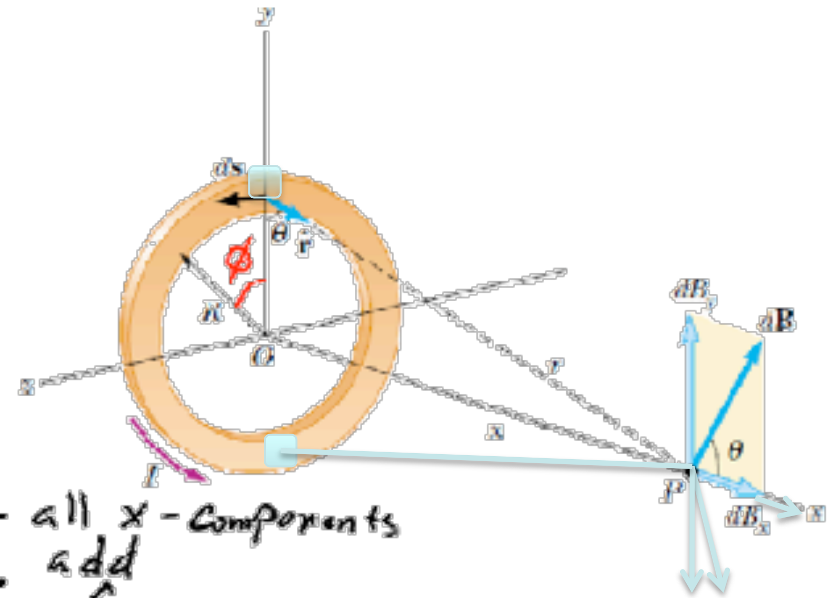
$$dB = \frac{\mu_0 I}{4\pi} \frac{d\theta}{R}$$

$$B = \int dB = \int_0^{2\pi} \frac{\mu_0 I}{4\pi} \frac{d\theta}{R}$$

$$= \frac{\mu_0 I}{4\pi R} \int_0^{2\pi} d\theta = \frac{\mu_0 I}{4\pi R}(2\pi)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2R} \text{ into page}$$

Applying the Biot-Savart Law: On-axis of circular loop



$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{s} \times \hat{r}}{r^2}$$

When we integrate, $B_y \to 0$ & all x-components add

Let θ be angle between $d\vec{B}$ & $\hat{x}$:

$$|d\vec{B}| = d\vec{B} \cdot \hat{x} = |dB| \cos\theta$$

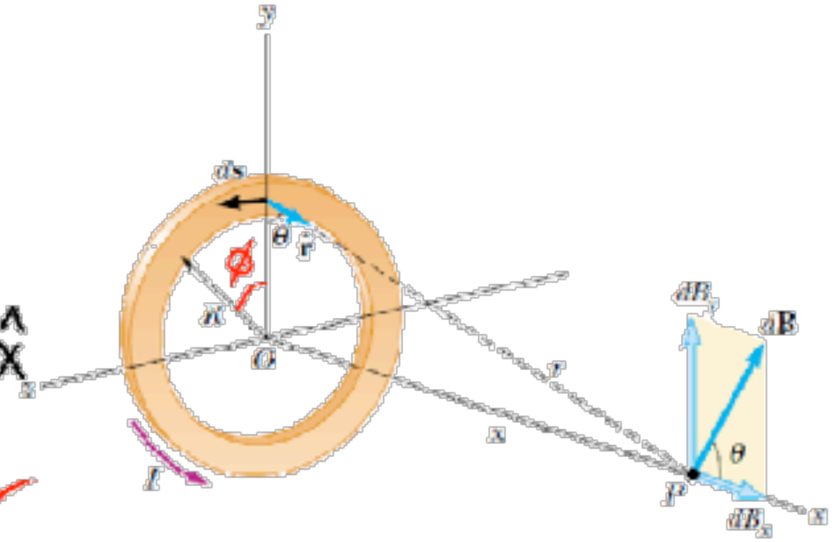
$$\cos\theta = \frac{R}{r}, \quad r = \sqrt{x^2 + R^2}, \quad d\vec{s} \perp \hat{r}$$

$$\Rightarrow |d\vec{s}| = R\, d\phi$$

$$|\vec{B}| = \int |dB| = \frac{\mu_0}{4\pi} I \int \frac{R\, d\phi}{r^2} \frac{R}{r} \quad (R\, d\phi \to 2\pi)$$

$$\vec{B} = \frac{\mu_0}{2} I \frac{R^2}{(x^2 + R^2)^{3/2}} \hat{x}$$

Applying the Biot-Savart Law: On-axis of circular loop



$$\vec{B} = \frac{\mu_0}{2} I \frac{R^2}{(x^2+R^2)^{3/2}} \hat{x}$$

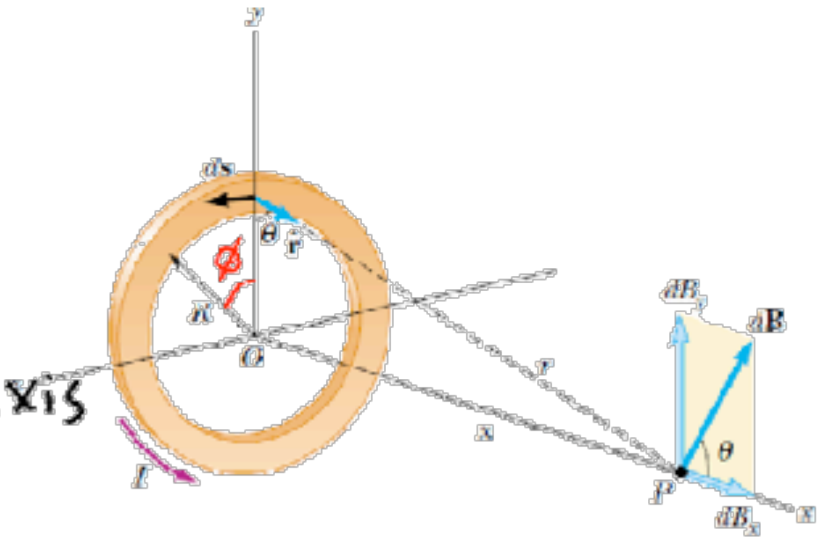
$$\lim_{x \to 0} \vec{B} = \frac{\mu_0 I}{2R} \hat{x} \quad ✓$$

Let $x \gg R$, & $(x^2+R^2)^{3/2} = x^3 \underbrace{\left(1+\frac{R^2}{x^2}\right)^{3/2}}_{\to 1}$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} 2 \frac{\overbrace{I \pi R^2 \hat{x}}^{\equiv \vec{\mu} = I\vec{A}}}{x^3}$$

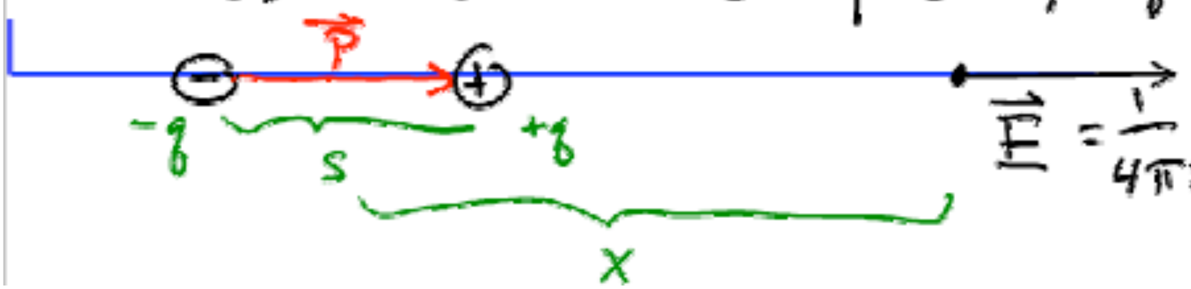
$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{x^3} \quad \text{on axis}$$

Applying the Biot-Savart Law: On-axis of circular loop



$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{x^3} \text{ on axis}$$

Looks Like Electric dipole: $\vec{p} \equiv q\vec{s}$



$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{x^3}$$

Remember from Electrostatics

$$U_E = -\vec{p}\cdot\vec{E}$$

$$U_M = -\vec{\mu}\cdot\vec{B}\,?$$

$$\tau_E = \vec{p}\times\vec{E}$$

$$\tau_B = \vec{\mu}\times\vec{B}\,?$$

Magnetic Dipole - Example 33.5

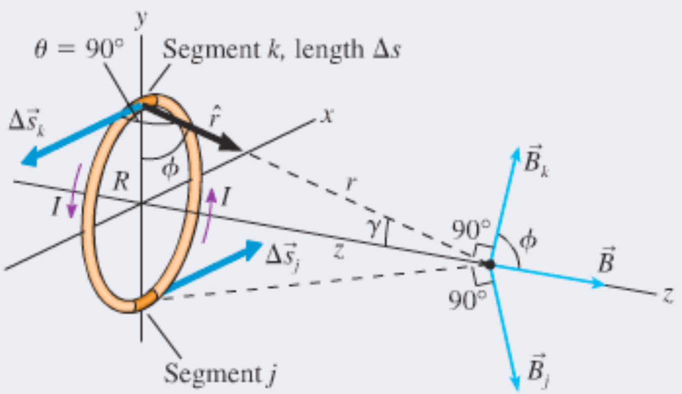


FIGURE 33.16 Calculating the magnetic field of a current loop.

Compute on axis B for z>>R

B_k perpendicular to Δs_k and r. The ρ-component of of B_k is cancelled by the ρ component of B_j. Same for all anti-diametric segments. We are thus left by only a z-component. Need to compute only $(B_k)_z = B_k \cos\phi$.

$$\left|\Delta\vec{s}_k \times \hat{r}\right| = \Delta s$$

$$(B_k)_z = \frac{\mu_o}{4\pi}\frac{I\Delta s}{r^2}\cos\phi$$

$$\cos\phi = R/r$$

$$(B_k)_z = \frac{\mu_o}{4\pi}\frac{IR\Delta s}{(z^2+R^2)^{3/2}}$$

For z>>R

$$(B_k)_z \approx \frac{\mu_o}{4\pi}\frac{IR\Delta s}{z^3}$$

$$B_{loop} \approx \frac{\mu_o}{4\pi}\frac{IR\Delta s}{z^3} = \frac{\mu_o}{2}\frac{IR^2}{z^3}$$

Magnetic Dipole moment

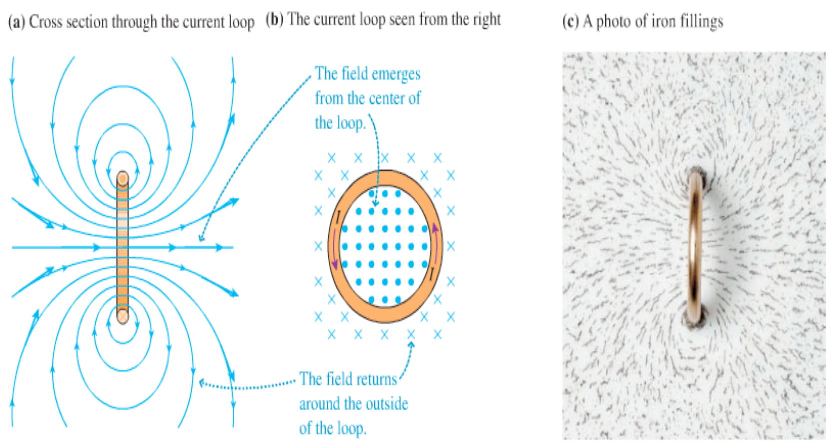
$$B_{loop} \approx \frac{\mu_o}{4\pi}\frac{2I\pi R^2}{z^3} = \frac{\mu_o}{4\pi}\frac{2IA}{z^3} = \frac{\mu_o}{4\pi}\frac{2I\mu}{z^3}$$

$$\vec{\mu} \equiv \vec{A}I$$

$$\vec{E}_{dipole} = \frac{1}{4\pi\varepsilon_o}\frac{2\vec{p}}{z^3}$$

MAGNETIC DIPOLE

FIGURE 33.18 The magnetic field of a current loop.



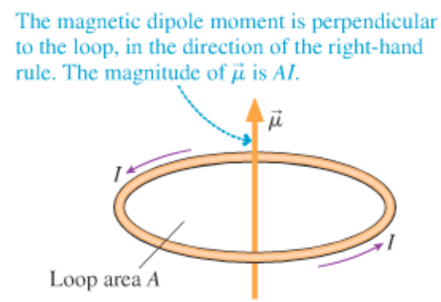


FIGURE 33.19 A current loop has magnetic poles and generates the same magnetic field as a flat permanent magnet.

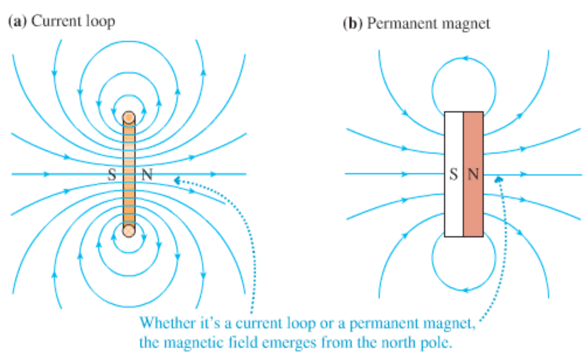
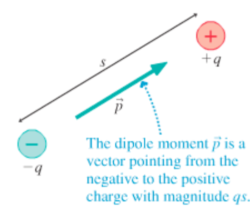


FIGURE 27.8 The dipole moment.



$$\vec{E}(0,0,z) = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{p}}{z^3},\ \vec{p} = e_z p, z >> l$$

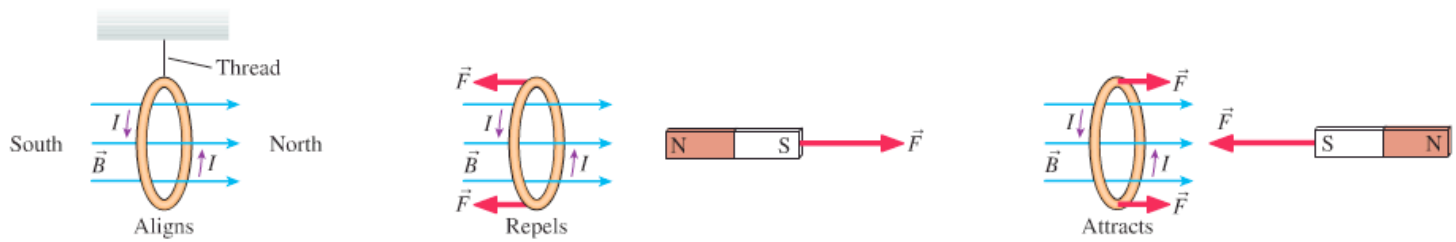
$$\vec{B}(0,0,z) = \frac{\mu_o}{4\pi}\frac{2\vec{\mu}}{z^3}, z >> R$$

$$\mu = I\pi R^2 = I(Area)$$

NOTE ▶ The magnetic field *inside* a permanent magnet differs from the magnetic field at the center of a current loop. Only the exterior field of a magnet matches the field of a current loop. ◀

LOOP – FLAT MAGNET EQUIVALENCE

Investigating current loops



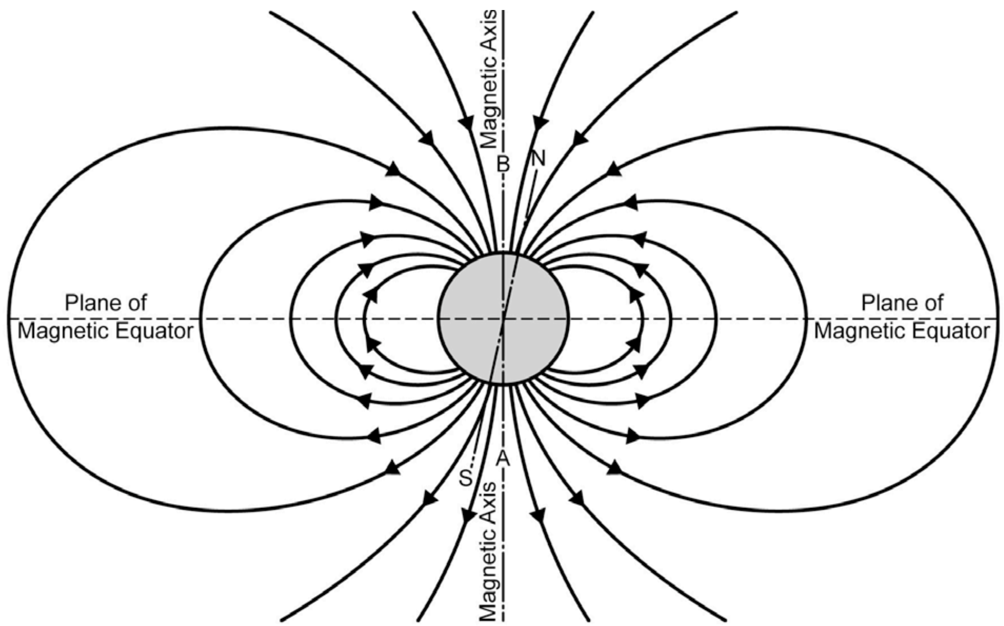
A current loop hung by a thread aligns itself with the magnetic field pointing north.

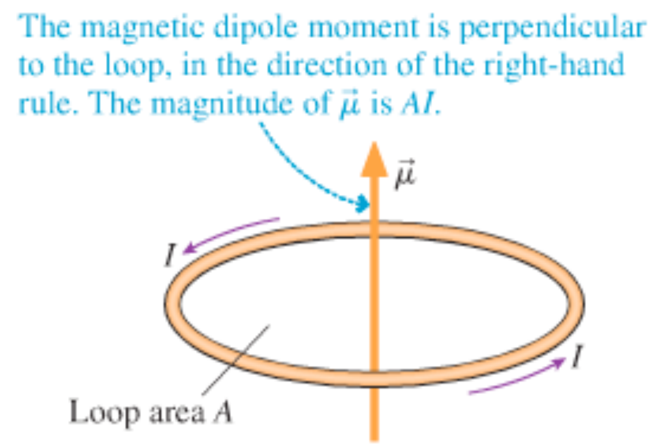
The north pole of a permanent magnet repels the side of a current loop from which the magnetic field is emerging.

The south pole of a permanent magnet attracts the side of a current loop from which the magnetic field is emerging.

A FLAT PERMANENT PERMANENT MAGNET AND A CURRENT LOOP (ELECTROMAGNET) GENERATE THE SAME MAGNETIC FIELD
THE FIELD OF A DIPOLE

EARTH's FIELD – LOOP OR NOT ?



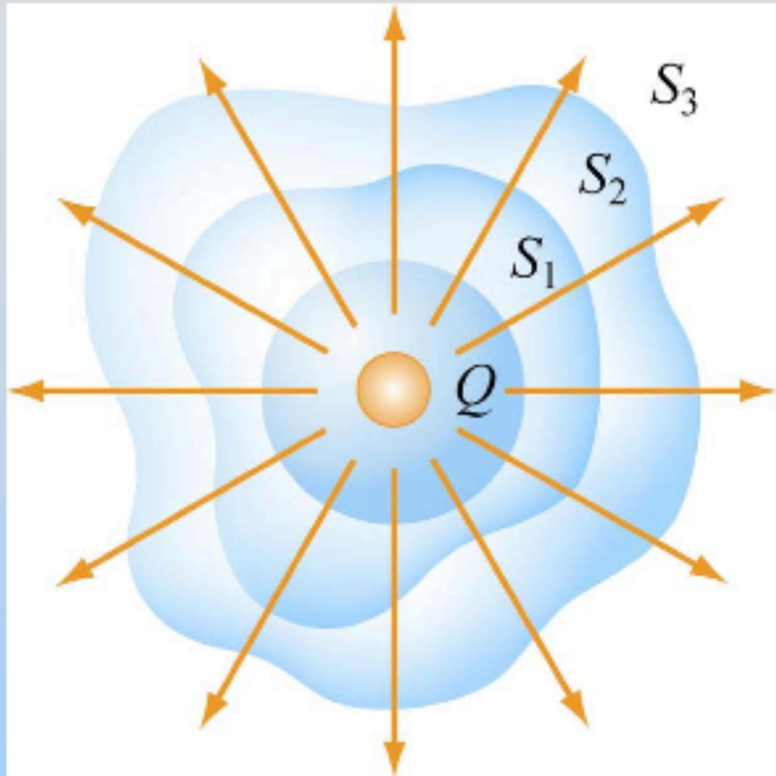


$$\vec{\mu}_{earth} = e_z 6.4 \times 10^{21}\, A - m^2$$

Ampere’s Law

Gauss's Law – The Idea

$$Flux = \int \vec{E} \bullet d\vec{S}$$



The total "flux" of field lines penetrating any of these surfaces is the same and depends only on the amount of charge inside

GAUSS's LAW

FIGURE 28.23 A spherical Gaussian surface surrounding a sphere of charge.

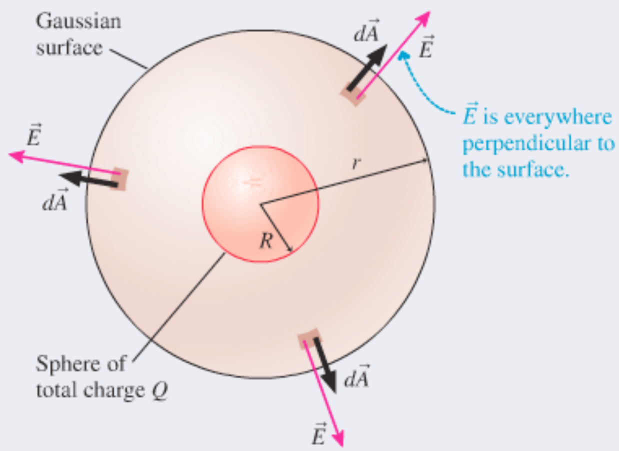
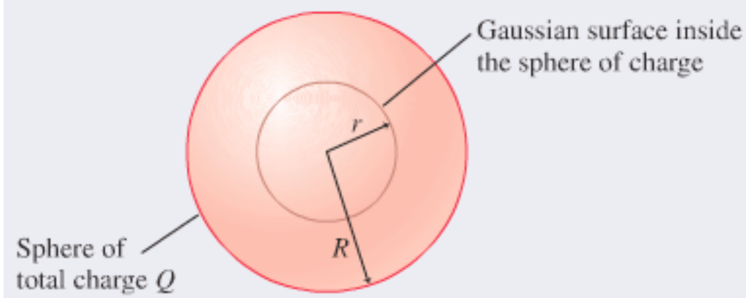
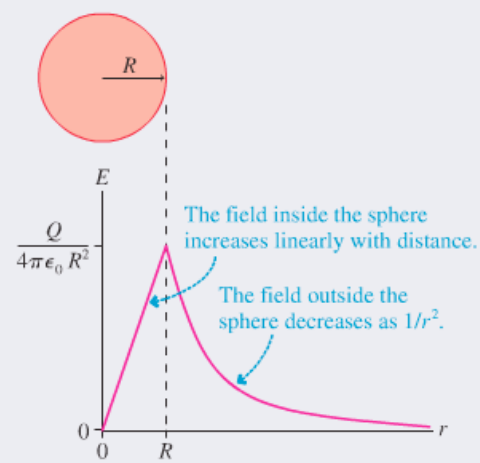


FIGURE 28.24 A spherical Gaussian surface inside a uniform sphere of charge.

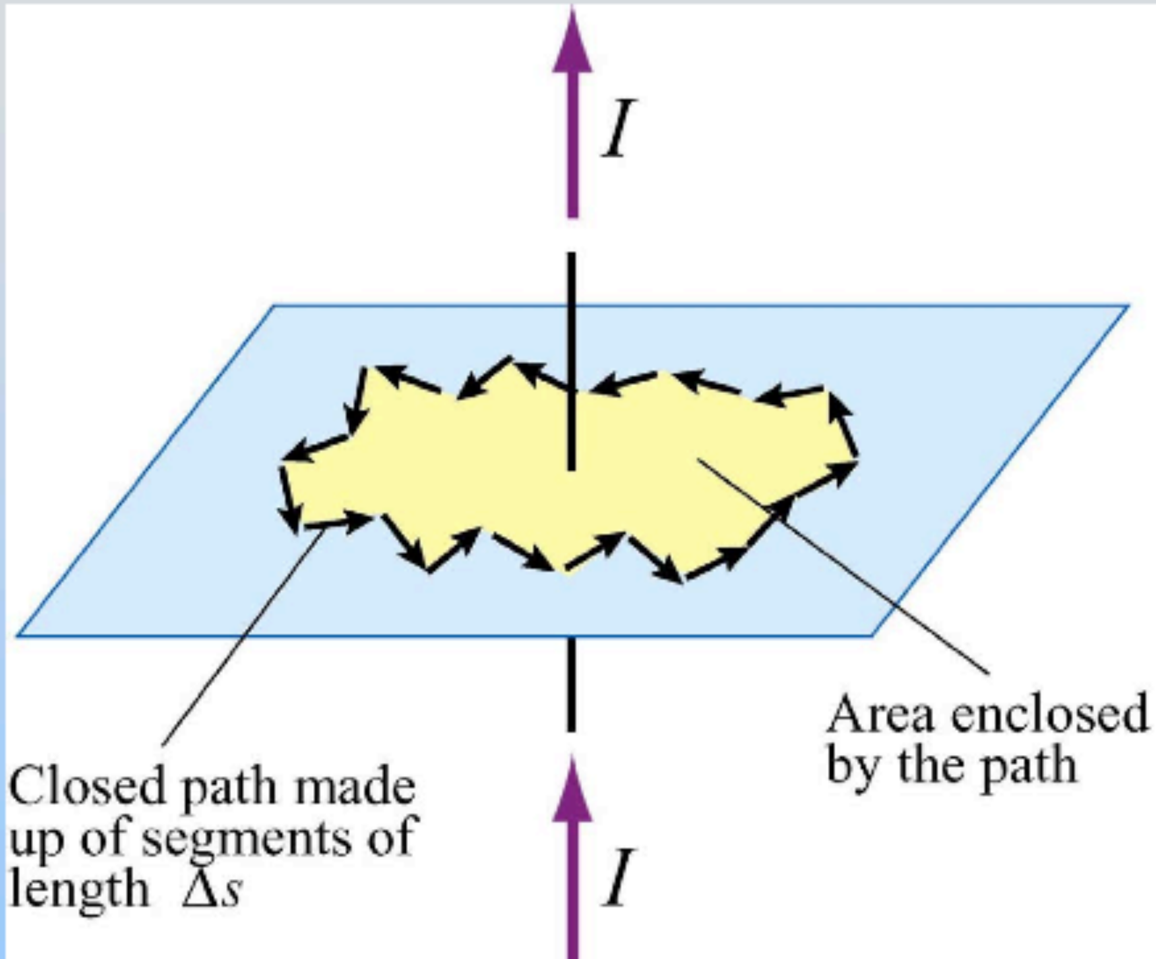


$$\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0}$$

FIGURE 28.25 The electric field strength of a uniform sphere of charge of radius R.



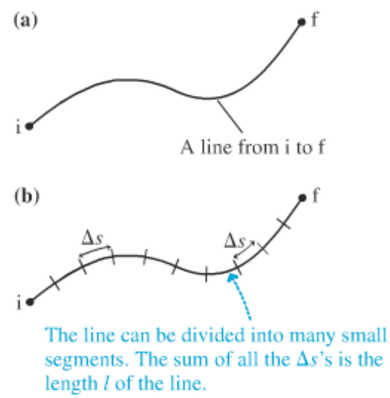
Ampere's Law: The Idea



In order to have a B field around a loop, there must be current punching through the loop

LINE INEGRALS – AMPERE's LAW

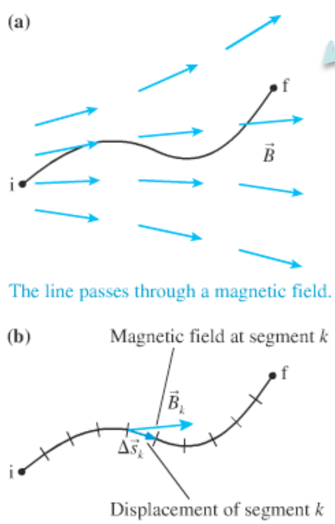
FIGURE 33.21 Integrating along a line from i to f.



$$l = \sum_k \Delta s_k \rightarrow \int_i^f ds$$

$$\sum_k \vec{B}_k \cdot \Delta \vec{s}_k \rightarrow \int_i^f \vec{B} \cdot d\vec{s} = \text{the line integral of } \vec{B} \text{ from i to f}$$

FIGURE 33.22 Integrating $\vec{B}$ along a line from i to f.

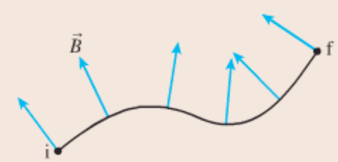


TACTICS BOX 33.3 **Evaluating line integrals**

MP

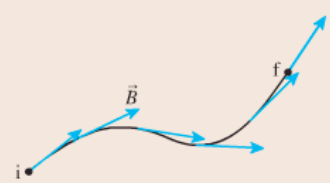
❶ If $\vec{B}$ is everywhere perpendicular to a line, the line integral of $\vec{B}$ is

$$\int_i^f \vec{B} \cdot d\vec{s} = 0$$



❷ If $\vec{B}$ is everywhere tangent to a line of length l *and* has the same magnitude B at every point, then

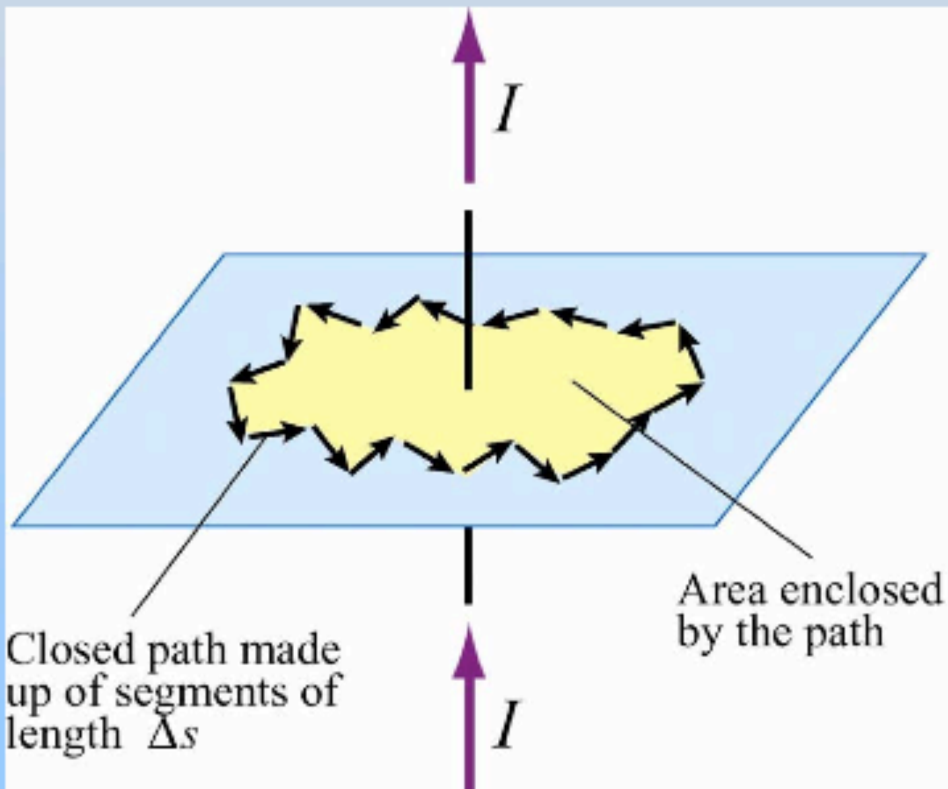
$$\int_i^f \vec{B} \cdot d\vec{s} = Bl$$



Exercises 23–24

Ampere's Law: The Equation

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

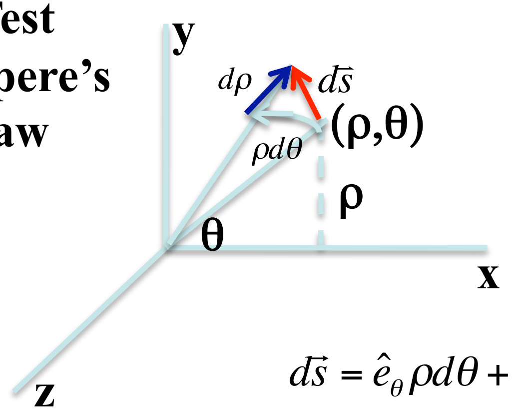


The line integral is around any closed contour bounding an open surface *S*.

I_{enc} is current through S:

$$I_{enc} = \int_S \vec{\mathbf{J}} \cdot d\vec{\mathbf{A}}$$

Test Ampere's law

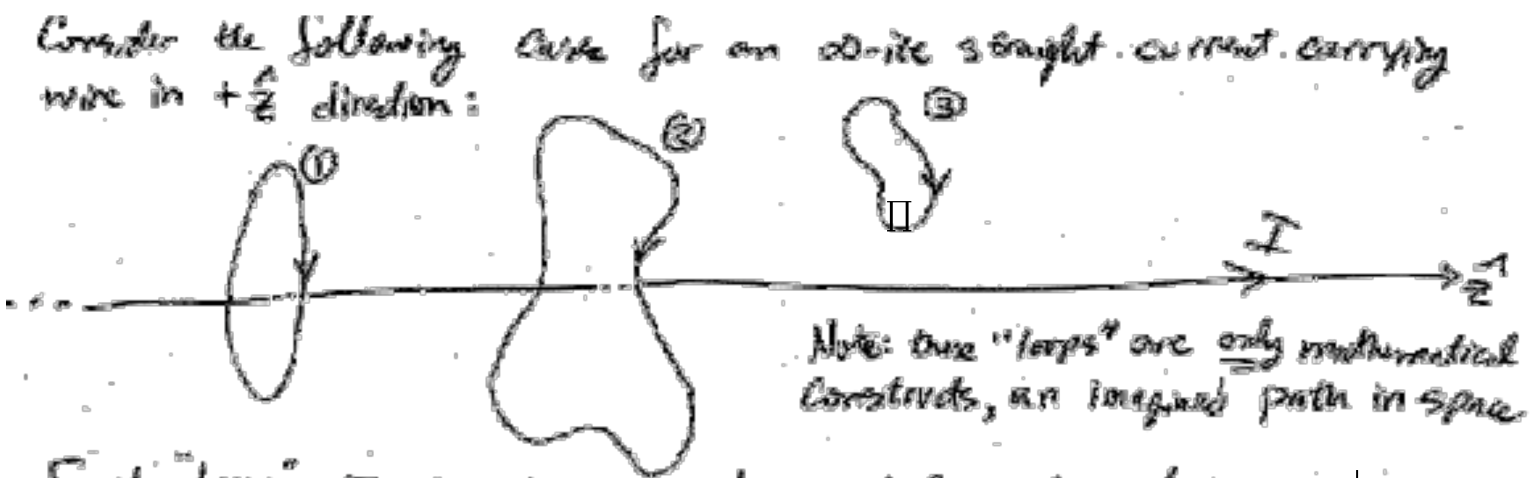


$$d\vec{s} = \hat{e}_\theta \rho d\theta + \hat{e}_\rho d\rho$$

Calculate A for each of the three loops

$$A \equiv \oint \vec{B}(\rho)\cdot d\vec{s}$$

$$\vec{B}(\rho) = \hat{e}_\theta \frac{\mu_o I}{2\pi\rho}$$

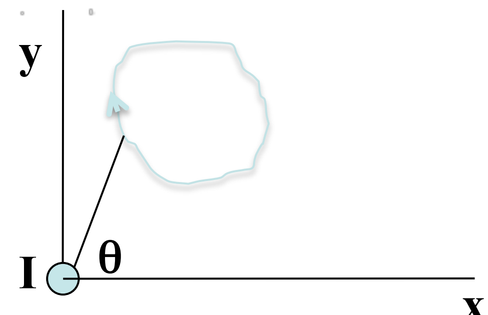


Loop 1 $$A_1 = \oint_1 \frac{\mu_o I}{2\pi\rho}\hat{e}_\theta \cdot (\hat{e}_\theta \rho d\theta + \hat{e}_\rho d\rho) = \frac{\mu_o I}{2\pi}(2\pi + 0) = \mu_o I$$

Loop 2 $$A_2 = \oint_2 \frac{\mu_o I}{2\pi\rho}\hat{e}_\theta \cdot (\hat{e}_\theta \rho d\theta + \hat{e}_\rho d\rho) = \frac{\mu_o I}{2\pi}[\oint_2 d\theta + 0] = \mu_o I$$

Loop 3 $$A_3 = \int_{\theta_i}^{\theta_f} \frac{\mu_o I}{2\pi\rho}\hat{e}_\theta \cdot (\hat{e}_\theta \rho d\theta + \hat{e}_\rho d\rho) = \int_{\theta_i}^{\theta_f} \frac{\mu_o I}{2\pi} d\theta$$

But $\theta_i = \theta_f \rightarrow A_3 = 0$



Biot-Savart vs. Ampere

Biot-Savart Law	$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\,\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$	general current source ex: finite wire wire loop
Ampere's law	$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$	symmetric current source ex: infinite wire infinite current sheet

Applying Ampere's Law

1. Identify regions in which to calculate B field
   Get B direction by right hand rule
2. Choose Amperian Loops S: Symmetry
   B is 0 or constant on the loop!
3. Calculate $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$
4. Calculate current enclosed by loop S
5. Apply Ampere's Law to solve for B

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc}$$

Always True, Occasionally Useful

Like Gauss's Law,

Ampere's Law is **always** true

However, it is only useful for calculation in certain specific situations, involving highly symmetric currents.

Here are examples…

AMPERE's LAW

FIGURE 33.23 Integrating the magnetic field around a wire.

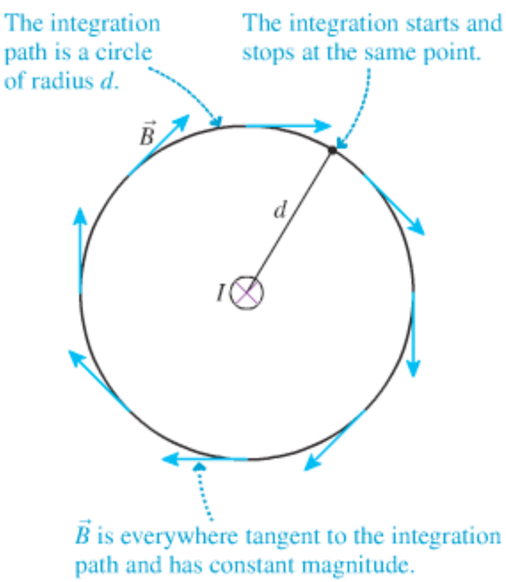
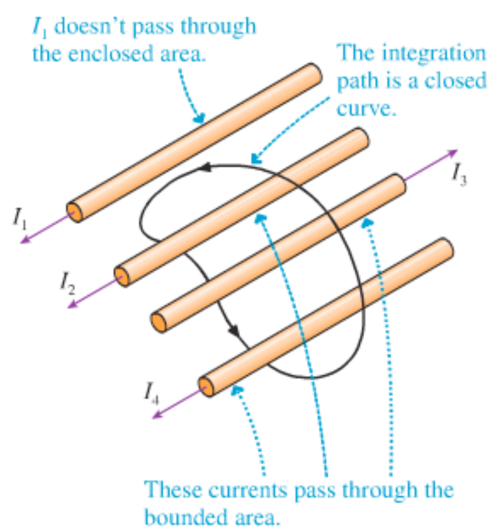


FIGURE 33.24 Using Ampère's law.

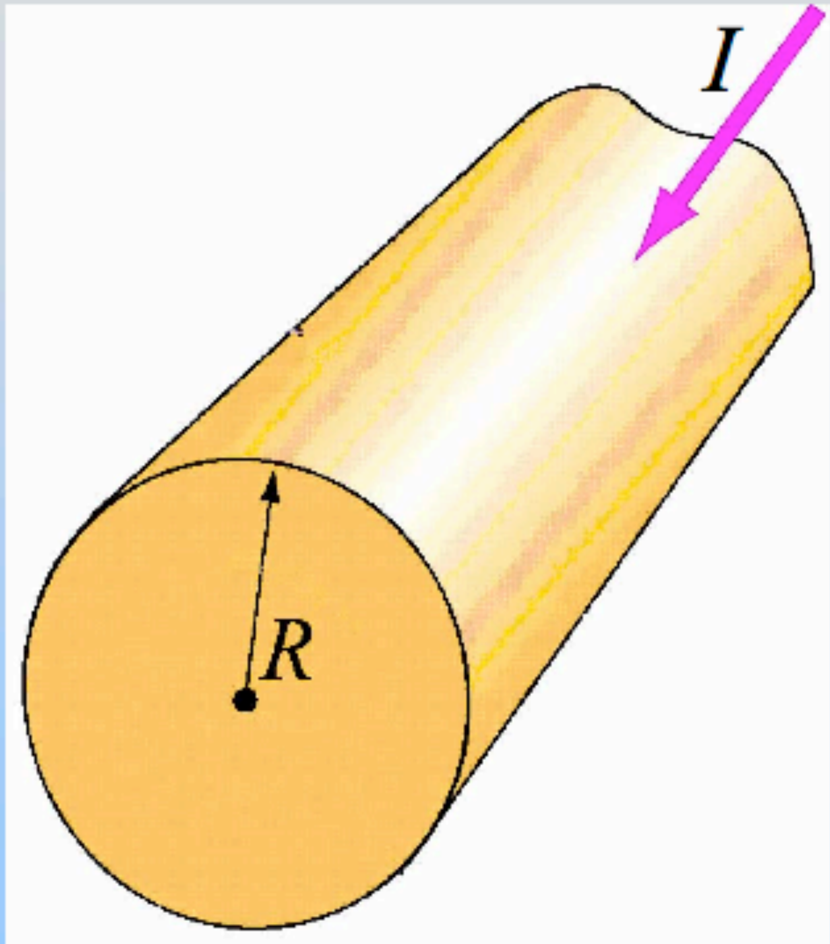


$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{\text{through}}$$

$$(2\pi d)B_\theta = \mu_o I$$

$$B_\theta = \mu_o I / 2\pi d$$

Example: Infinite Wire



A cylindrical conductor has radius R and a uniform current density with total current I

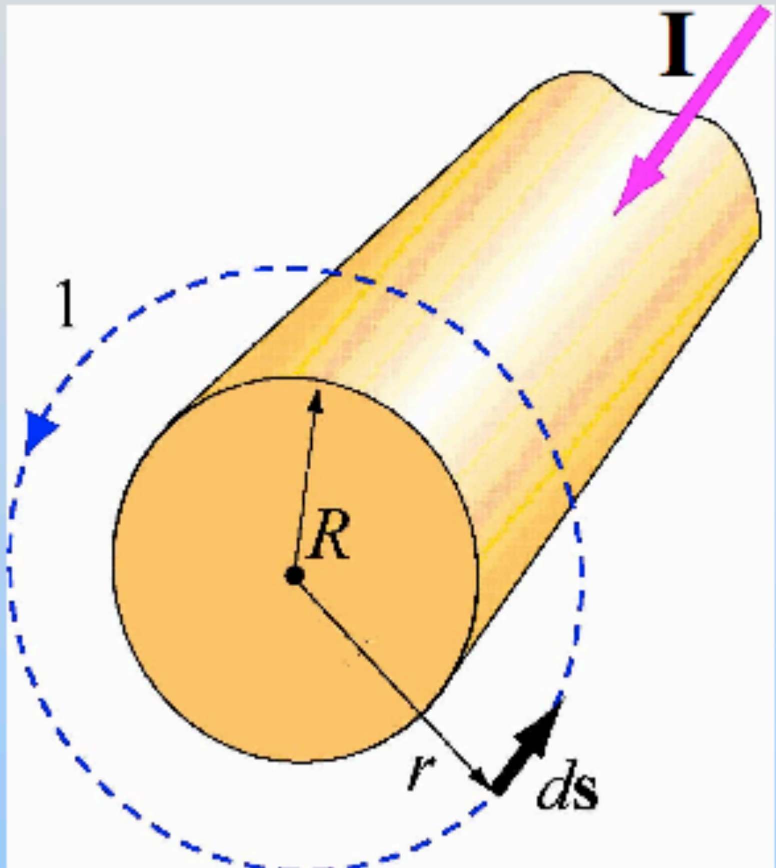
Find B everywhere

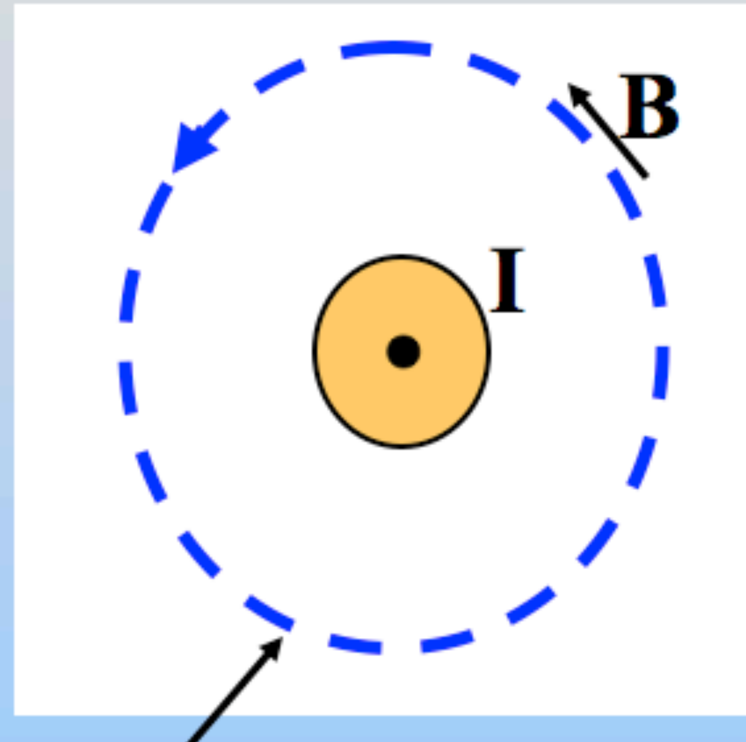
Two regions:

(1) outside wire ($r \geq R$)

(2) inside wire ($r < R$)

Ampere's Law Example: Infinite Wire



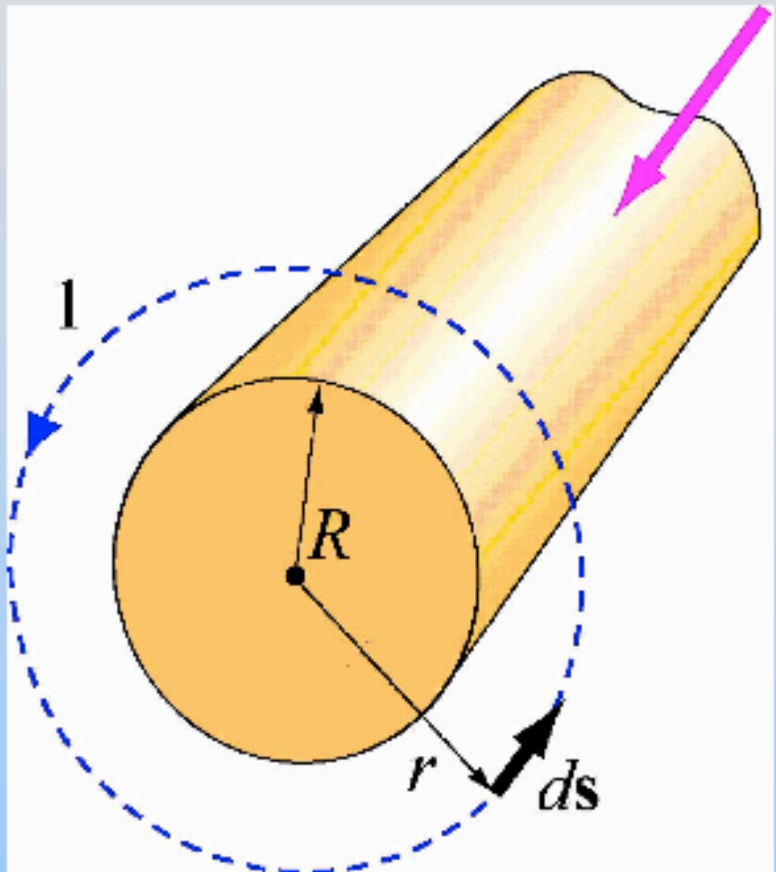


Amperian Loop:

B is Constant & Parallel

I Penetrates

Example: Wire of Radius *R*



Region 1: Outside wire (r ≥ R)

Cylindrical symmetry →

Amperian Circle

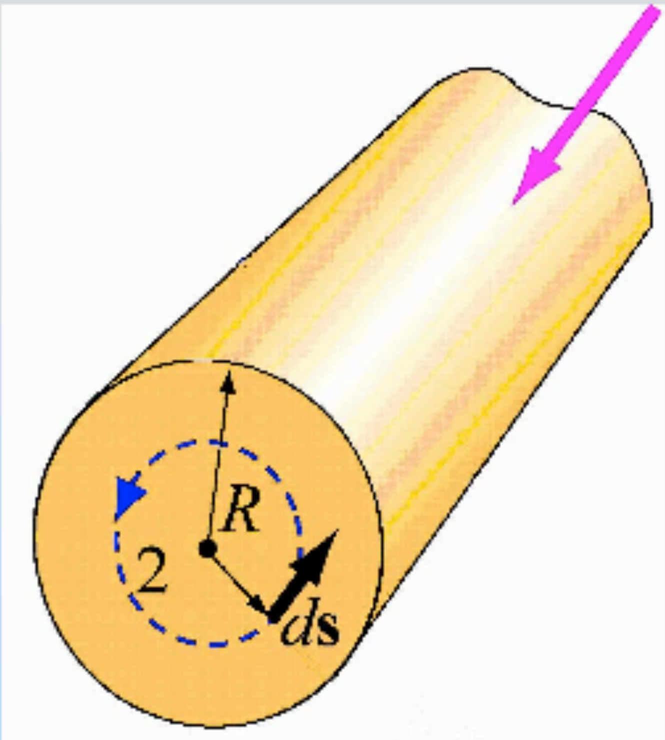
B-field counterclockwise

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint d\vec{\mathbf{s}} = B(2\pi r)$$

$$= \mu_0 I_{enc} = \mu_0 I$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r} \text{counterclockwise}$$

Example: Wire of Radius *R*



Region 2: Inside wire (r < R)

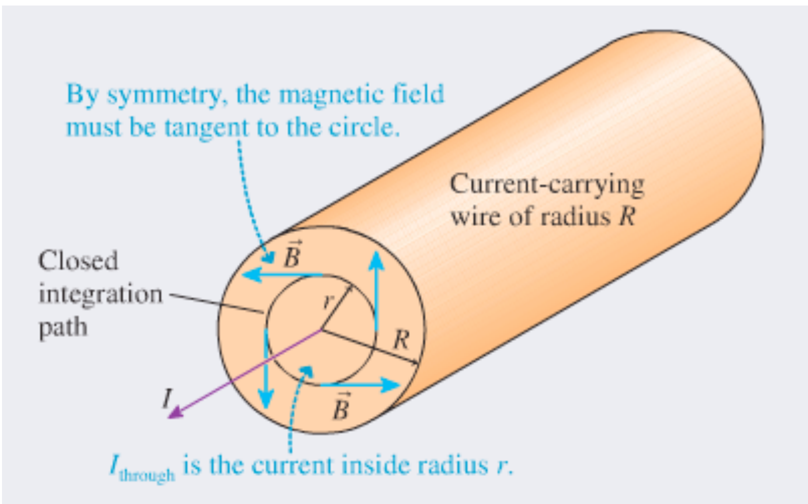
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \oint d\vec{\mathbf{s}} = B\left(2\pi r\right)$$

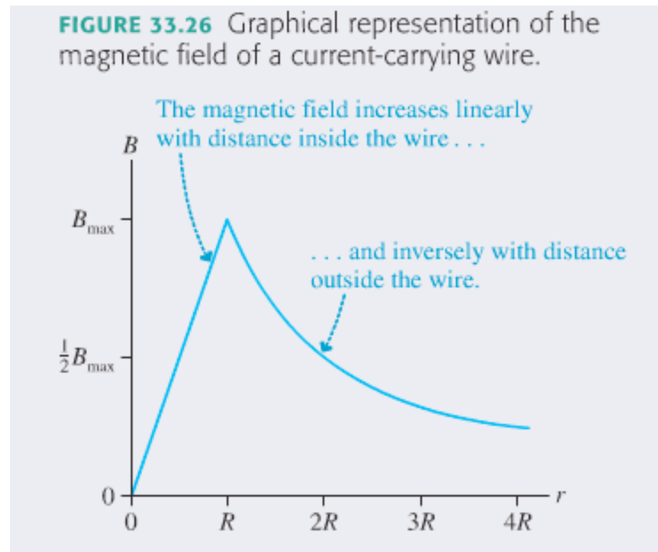
$$= \mu_0 I_{enc} = \mu_0 I \left(\frac{\pi r^2}{\pi R^2}\right)$$

$$\vec{\mathbf{B}} = \frac{\mu_0 I r}{2\pi R^2} \text{ counterclockwise}$$

Could also say: $J = \frac{I}{A} = \frac{I}{\pi R^2}; \; I_{enc} = J A_{enc} = \frac{I}{\pi R^2}\left(\pi r^2\right)$

OVERALL FIELD





r ≤ R

$$2\pi r B(r) = \mu_o I(r^2 / R^2)$$

$$B(r) = (\mu_o / 2\pi) I(r / R^2)$$

$r \geq R$

$$2\pi r B(r) = \mu_o I$$

$$B(r) = (\mu_o / 2\pi)(I / r)$$

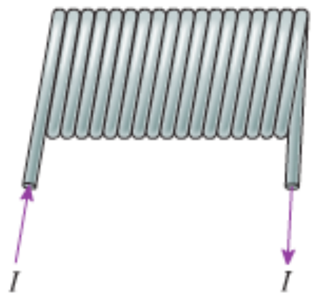
Applying Ampere's Law

In Choosing Amperian Loop:

- Study & Follow Symmetry
- Determine Field Directions First
- Think About Where Field is Zero
- Loop Must
  - Be Parallel to (Constant) Desired Field
  - Be Perpendicular to Unknown Fields
  - Or Be Located in Zero Field

SOLENOIDS

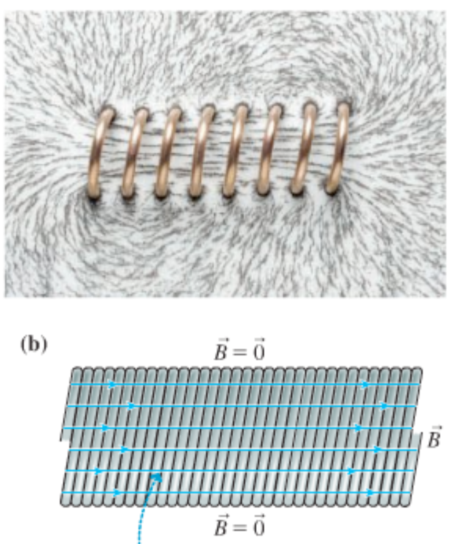
FIGURE 33.27 A solenoid.



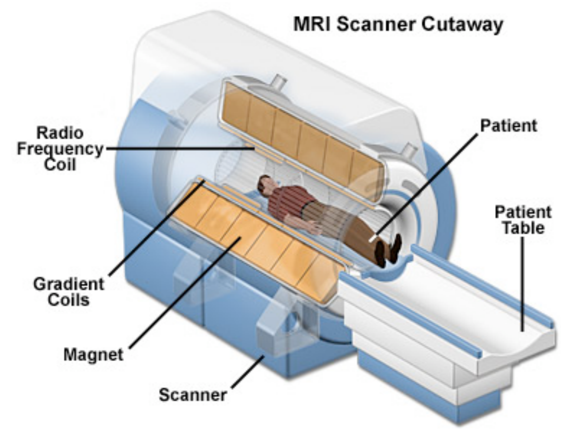
What is a solenoid – A device that creates a uniform magnetic field inside and zero outside (in both cases almost uniform and almost zero)
Who needs it . Electronic devices, MRI machines, Fusion machines etc

FIGURE 33.29 The magnetic field of a solenoid.

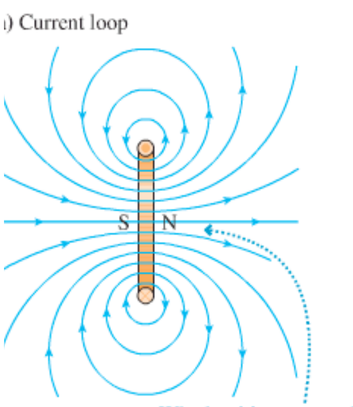
(a) A short solenoid

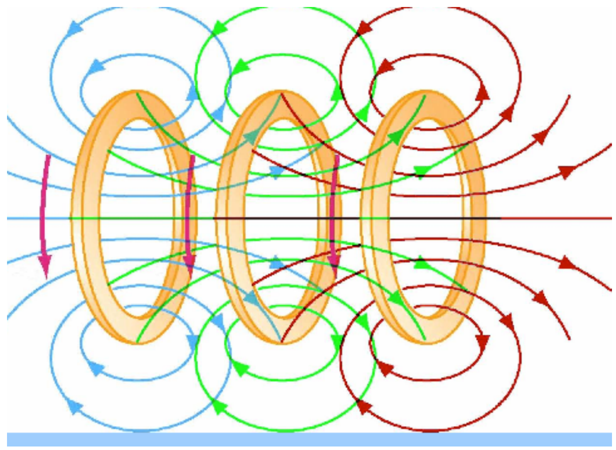


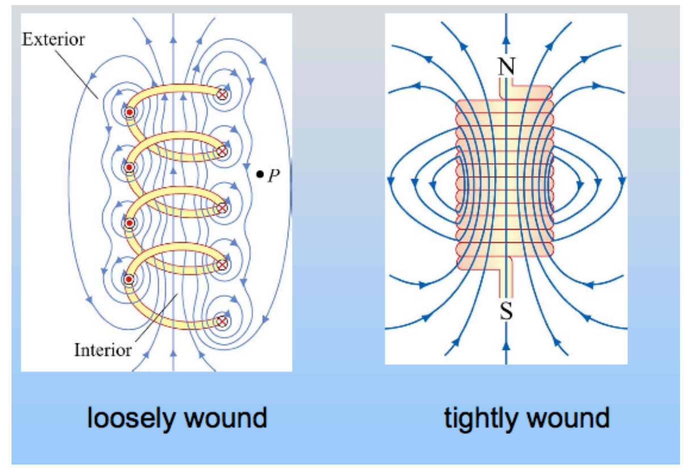
The magnetic field is uniform inside this section of an ideal, infinitely long solenoid. The magnetic field outside the solenoid is zero.

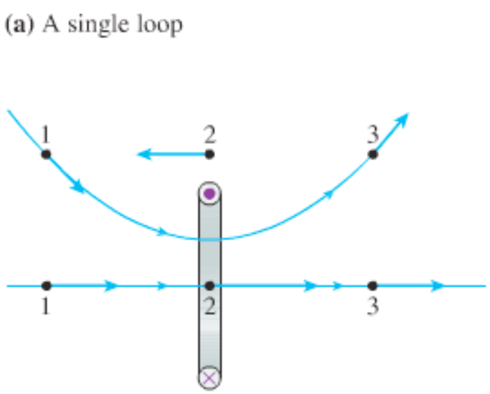


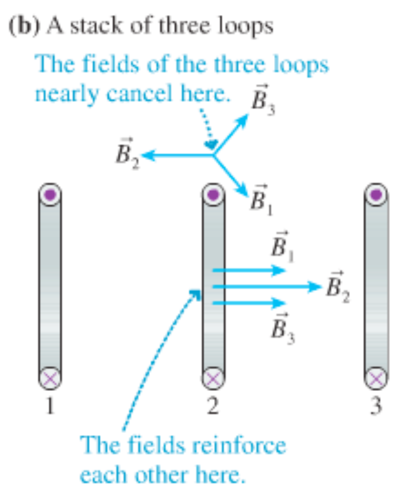
How to make a solenoid

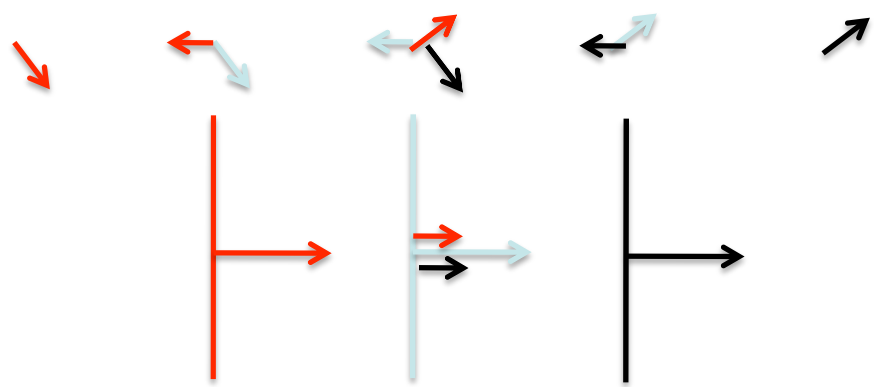




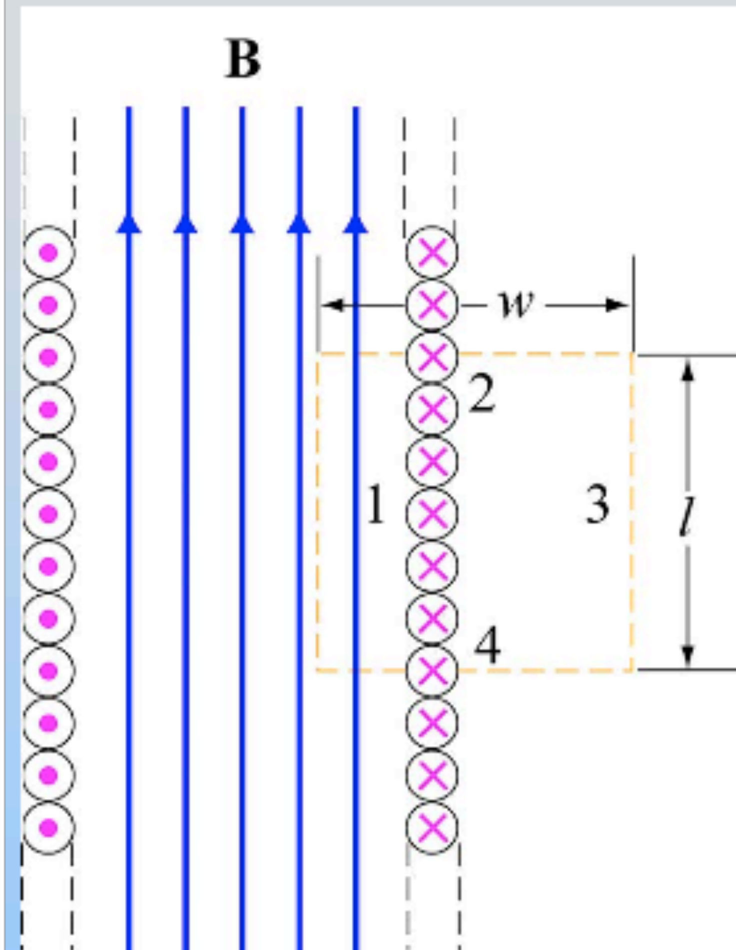








Magnetic Field of Ideal Solenoid



Using Ampere's law: Think!

$$\begin{cases} \vec{\mathbf{B}} \perp d\vec{\mathbf{s}} \text{ along sides 2 and 4} \\ \vec{\mathbf{B}} = 0 \text{ along side 3} \end{cases}$$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_1 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_2 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_3 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} + \int_4 \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$$

$$= Bl + 0 + 0 + 0$$

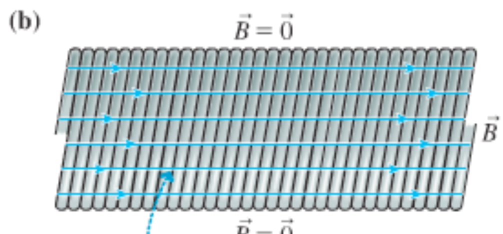
$I_{enc} = nlI$ n: turn density

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = Bl = \mu_0 nlI$$

$n = N / L$: # turns/unit length

$$\boxed{B = \frac{\mu_0 nlI}{l} = \mu_0 nI}$$

B FIELD INSIDE A SOLENOID



The magnetic field is uniform inside this section of an ideal, infinitely long solenoid. The magnetic field outside the solenoid is zero.

$$\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 NI$$

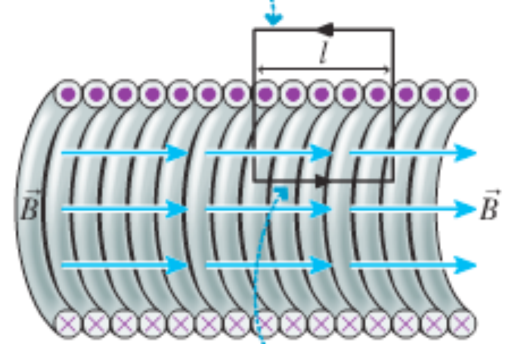
uniform magnetic field inside a solenoid is

$$B_{\text{solenoid}} = \frac{\mu_0 NI}{l} = \mu_0 nI$$

n is turns per unit length (e.g. per meter)

FIGURE 33.30 A closed path inside and outside an ideal solenoid.

This is the integration path for Ampère's law. There are N turns inside.



$\vec{B}$ is tangent to the integration path along the bottom edge.