PHYS 270-SPRING 2011

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LECTURE # 24

WAVE PACKETS

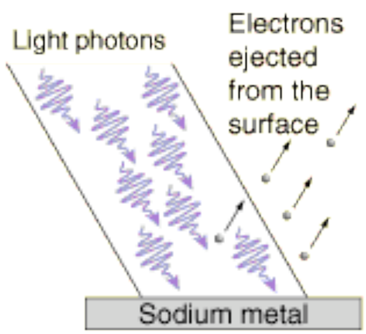
UNCERTAINTY

CHAPTER 40

MAY 5 2011

WAVE PACKETS

The word "particle" in the phrase "wave-particle duality" suggests that this wave is somewhat localized.



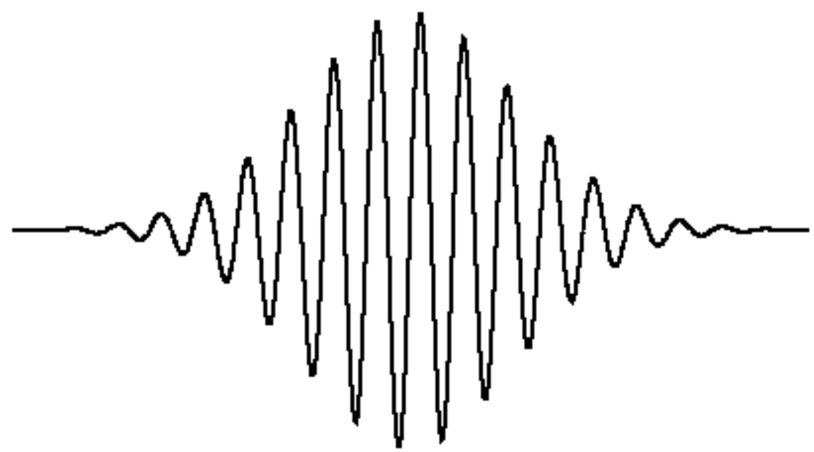
Photon energy

$E = h\nu$

explains the experiment and shows that light behaves like particles.

How do we describe this mathematically?

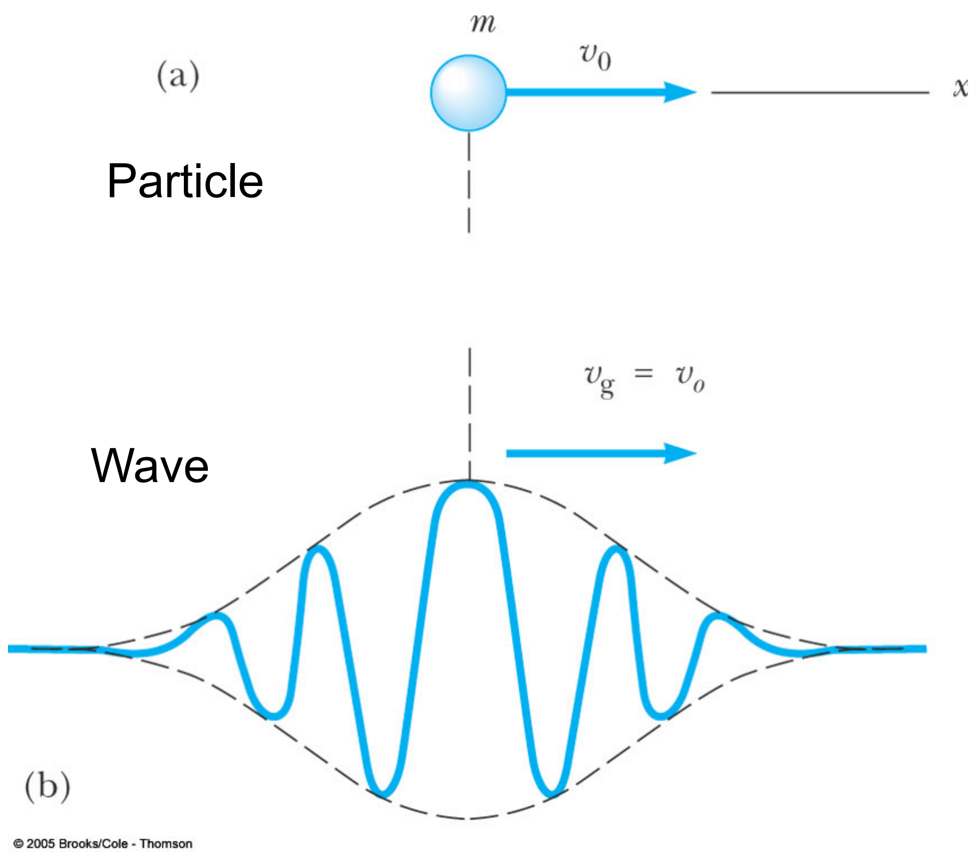
...or this

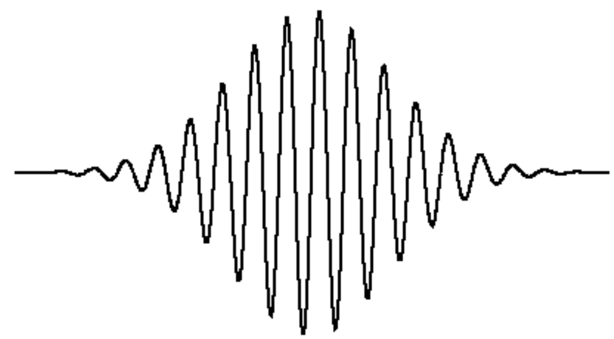


...or this

WAVE PACKETS

How do we describe this mathematically?



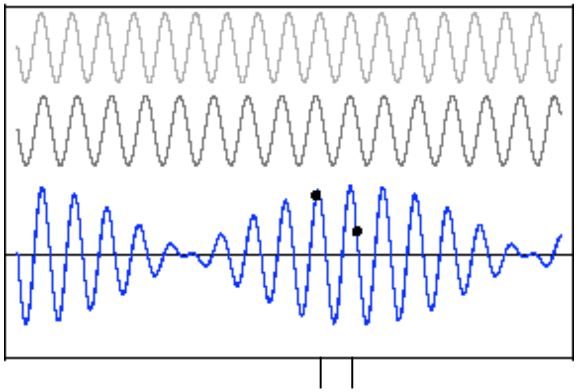


Interefering waves, generally…

$$y = y_1 + y_2 = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$\Downarrow$$

$$y = 2A\cos\frac{1}{2}\{(k_2 - k_1)x - (\omega_2 - \omega_1)t\} \bullet \cos\frac{1}{2}\{(k_1 + k_2)x - (\omega_1 + \omega_2)t\}$$



"Beats" occur when you add two waves of slightly different frequency. They will interfere constructively in some areas and destructively in others.

Can be interpreted as a sinusoidal envelope: $2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$

Modulating a high frequency wave within the envelope: $\cos\left[\frac{1}{2}(k_1 + k_2)x - \frac{1}{2}(\omega_1 + \omega_2)t\right]$

$$y = y_1 + y_2 = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$\Downarrow$$

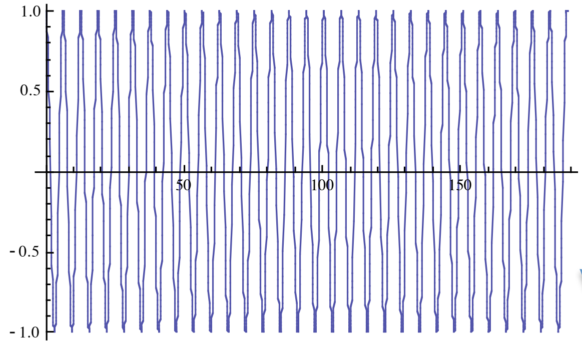
$$y = 2A\cos\frac{1}{2}\{(k_2 - k_1)x - (\omega_2 - \omega_1)t\} \bullet \cos\frac{1}{2}\{(k_1 + k_2)x - (\omega_1 + \omega_2)t\}$$

$$y = 2A\cos\frac{1}{2}\{(\omega_2 - \omega_1)t\} \bullet \cos\frac{1}{2}\{(\omega_1 + \omega_2)t\}$$

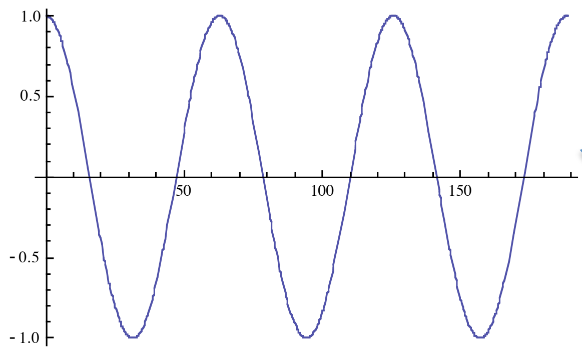
A=1/2, $y=y_1 y_2$

$\omega_1 = 1.1$

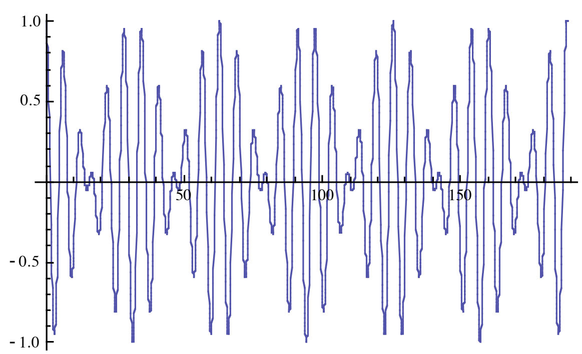
$\omega_2 = 1$



$$y_1 = \cos\frac{1}{2}\{(\omega_1 + \omega_2)t\}$$



$$y_2 = \cos\frac{1}{2}\{(\omega_1 - \omega_2)t\}$$

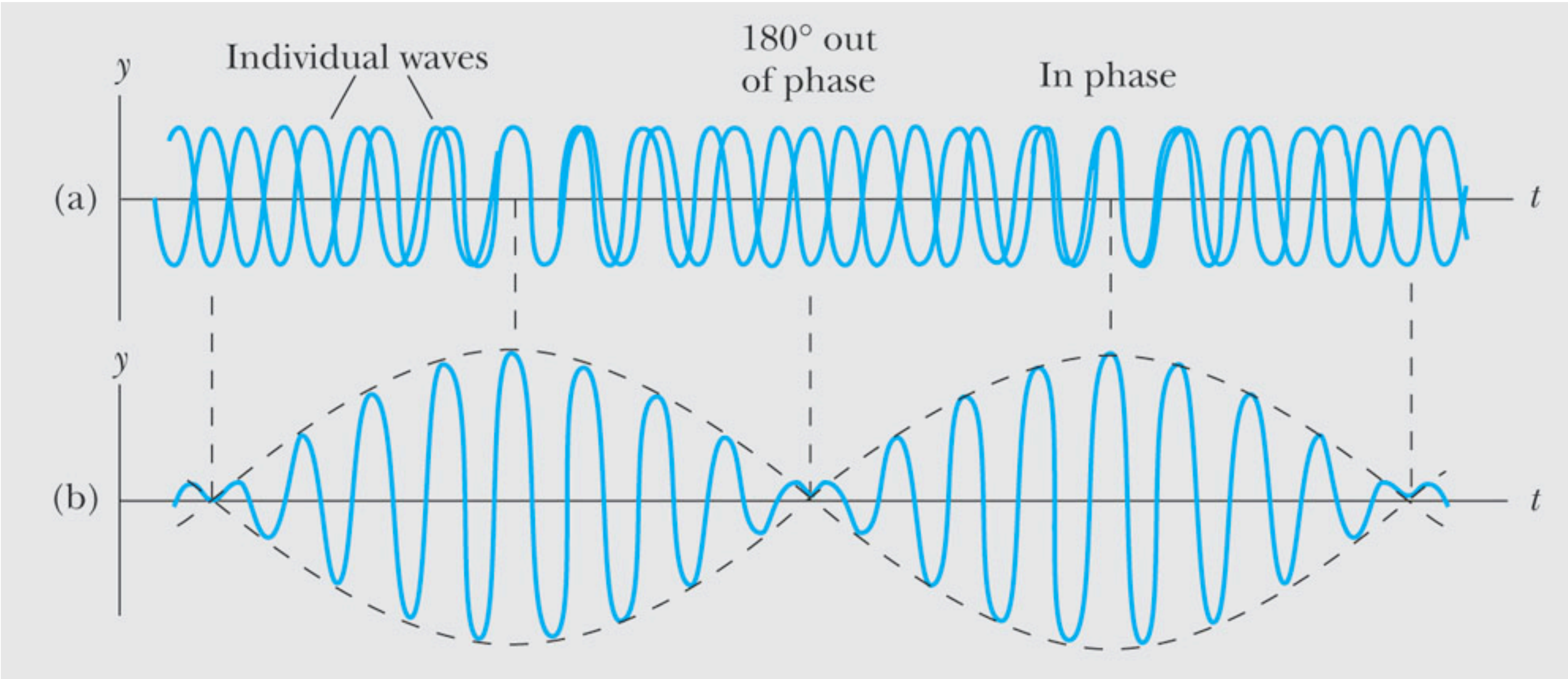


$y=y_1 y_2$

$$\Delta\omega\Delta t/2 \approx \pi$$

$$2\pi\Delta f\Delta t \approx 2\pi$$

$$\Delta f\Delta t \approx 1$$



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At constant value of x

$$2A\cos(\Delta\omega t/2) =$$

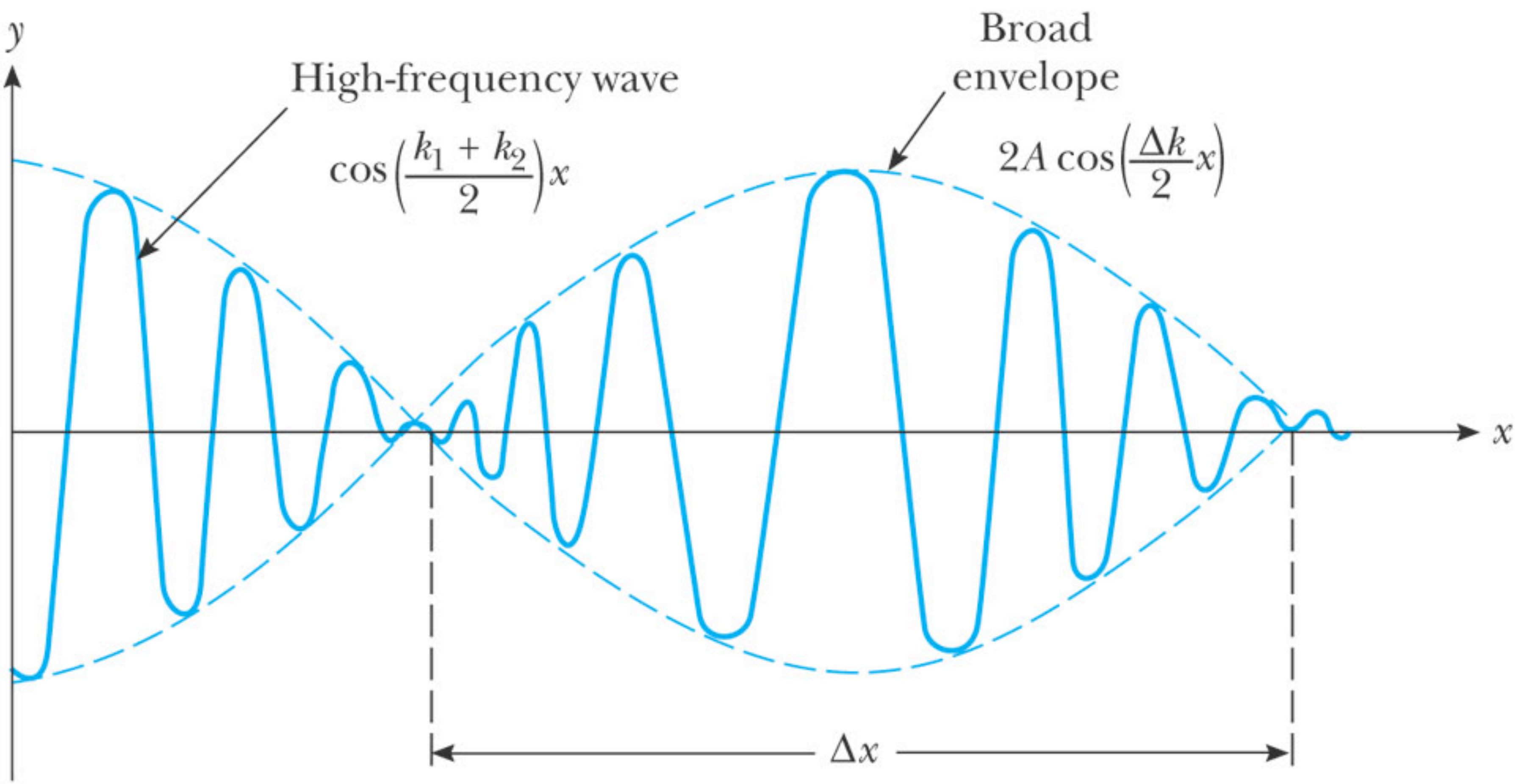
$$2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$$

$$= 2A\cos(\pi\Delta f t)$$

$$\pi\Delta f\Delta t = \pi$$

$$\boxed{\Delta f\Delta t = 1}$$

Fig. 5-18, p. 165



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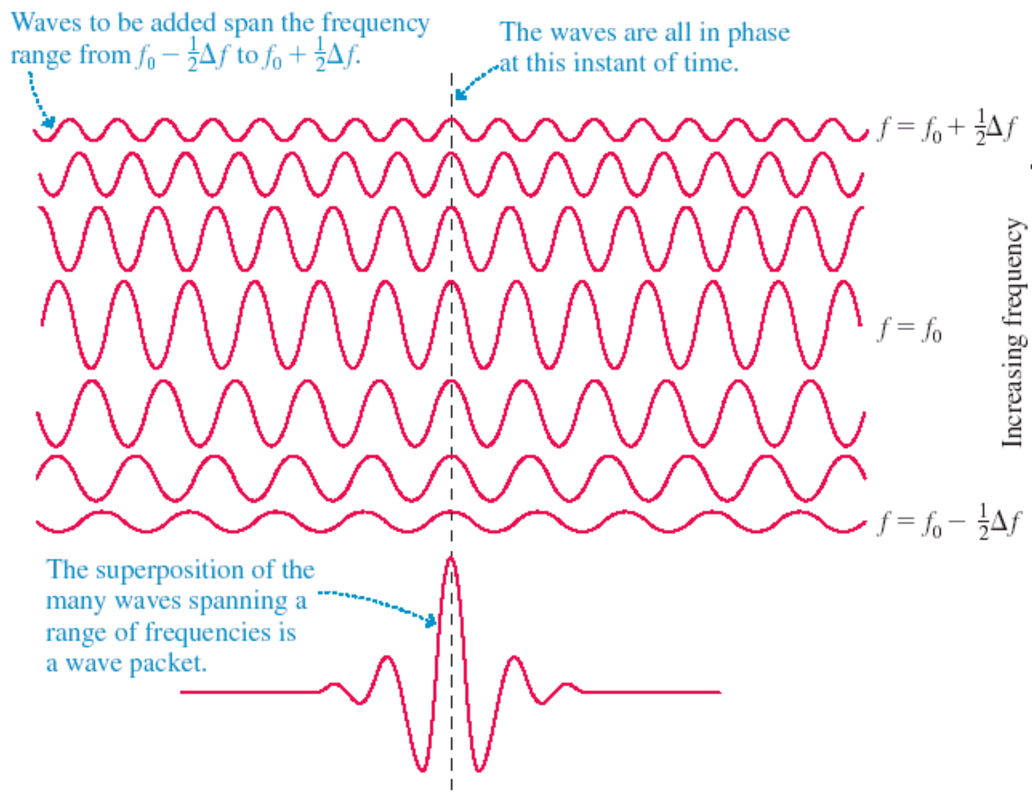
Look in space at any instant of time

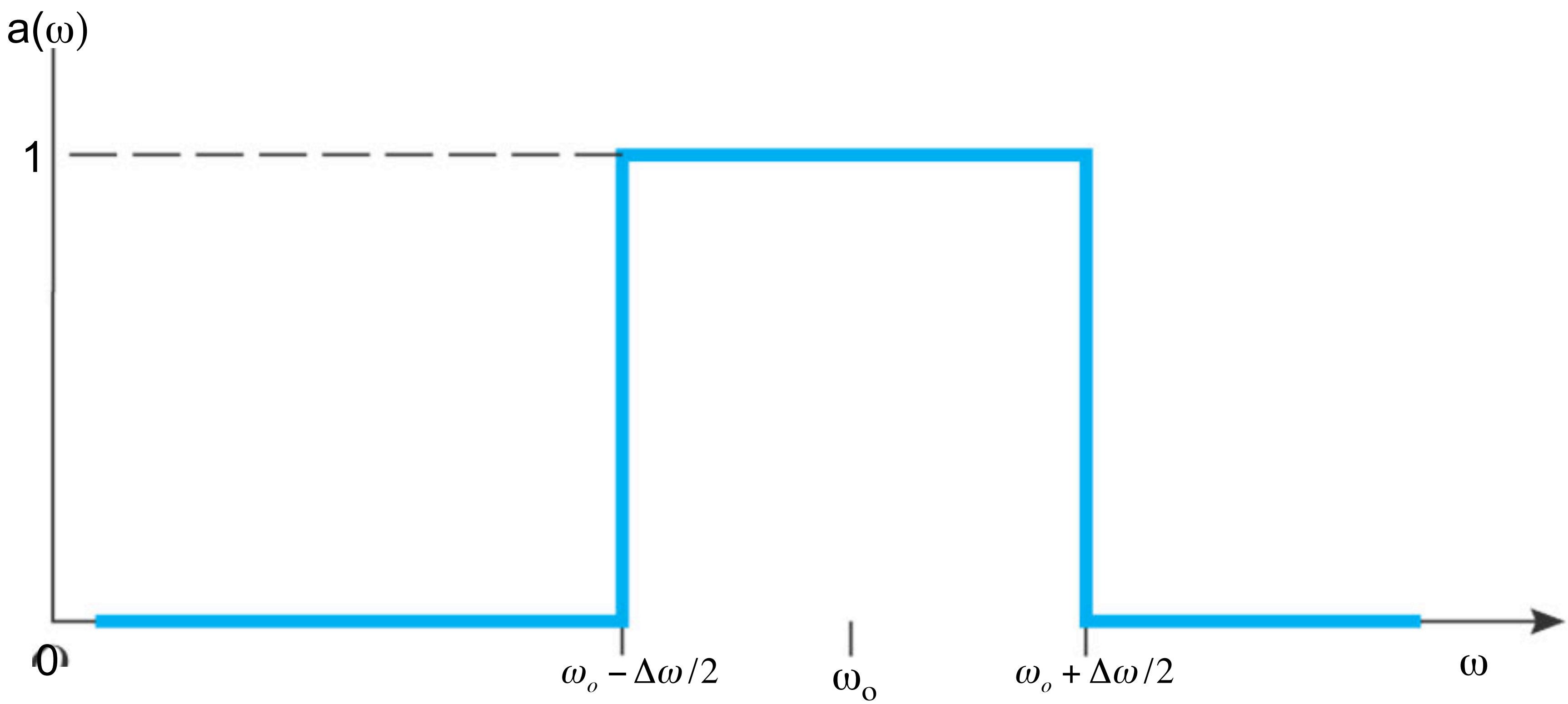
$$(1/2)\Delta k \Delta x = \pi$$

Fig. 5-19, p. 166

FIGURE 40.14 A single wave packet is the superposition of many component waves of similar wavelength and frequency.



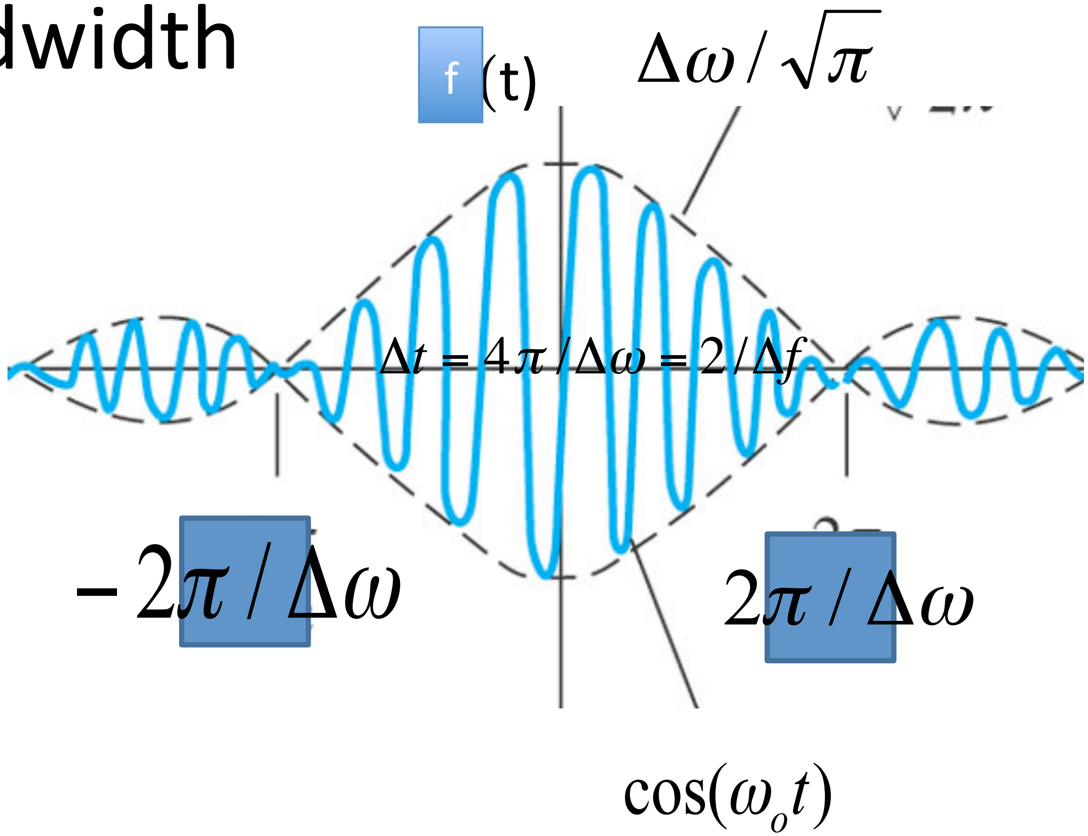
Take many close frequencies instead of two close frequencies



$$f(t) = A\int_0^\infty a(\omega)\cos\omega t\,d\omega$$

Fig. 5-23, p. 172

Bandwidth



$$\Delta f \Delta t \approx 1$$

Wave Packets

Suppose a single nonrepeating wave packet of duration Δt is created by the superposition of *many* waves that span a range of frequencies Δf.

Fourier analysis shows that for *any* wave packet

$$\Delta f \Delta t \approx 1$$

We have not given a precise definition of Δt and Δf for a general wave packet.

The quantity Δt is "about how long the wave packet lasts," while Δf is "about the range of frequencies needing to be superimposed to produce this wave packet."

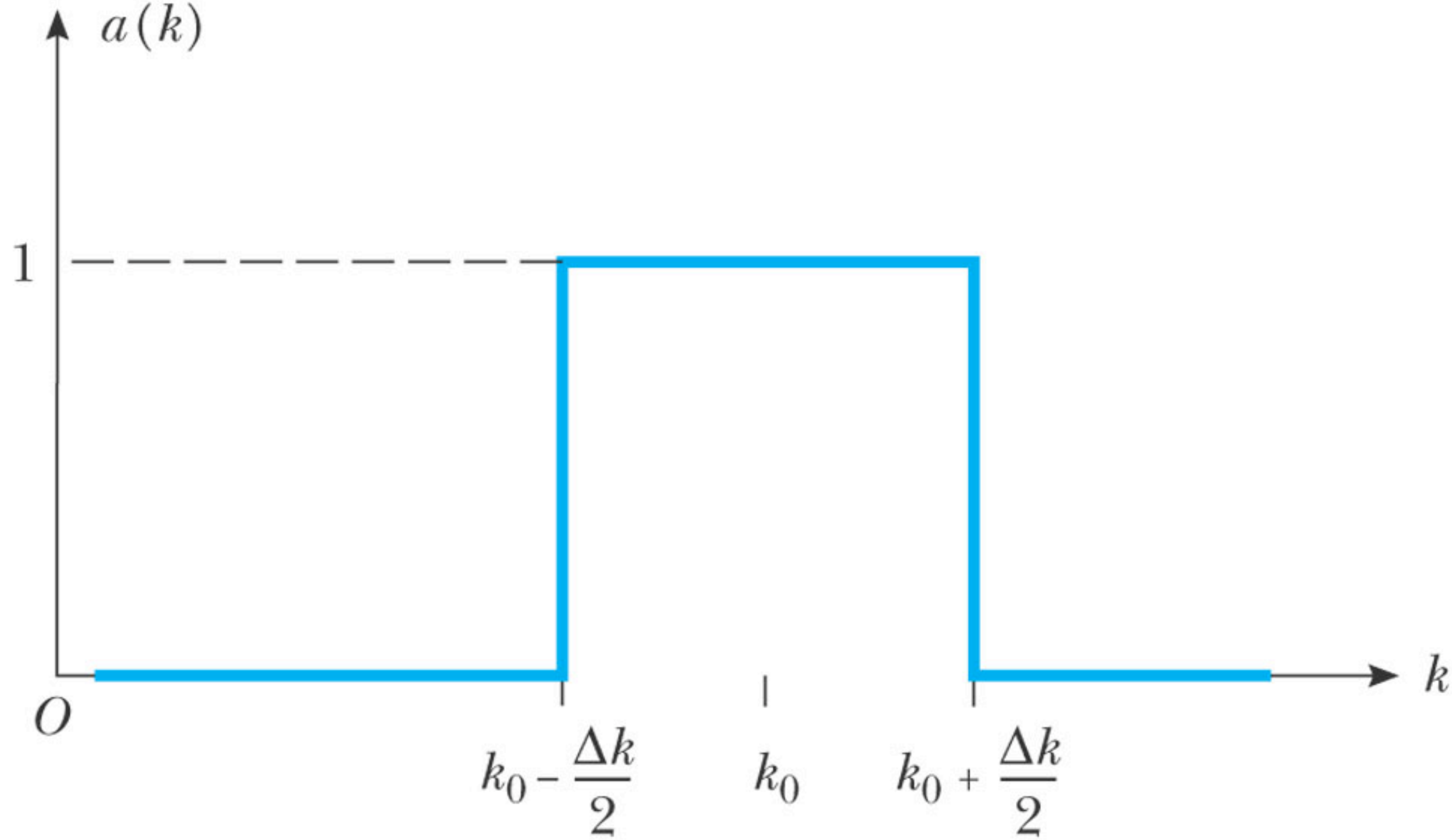


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Fig. 5-23, p. 172

$$\lambda = 2\pi / k$$

$$\lambda = h / p$$

$$h / p = 2\pi / k$$

$$p = \hbar k$$

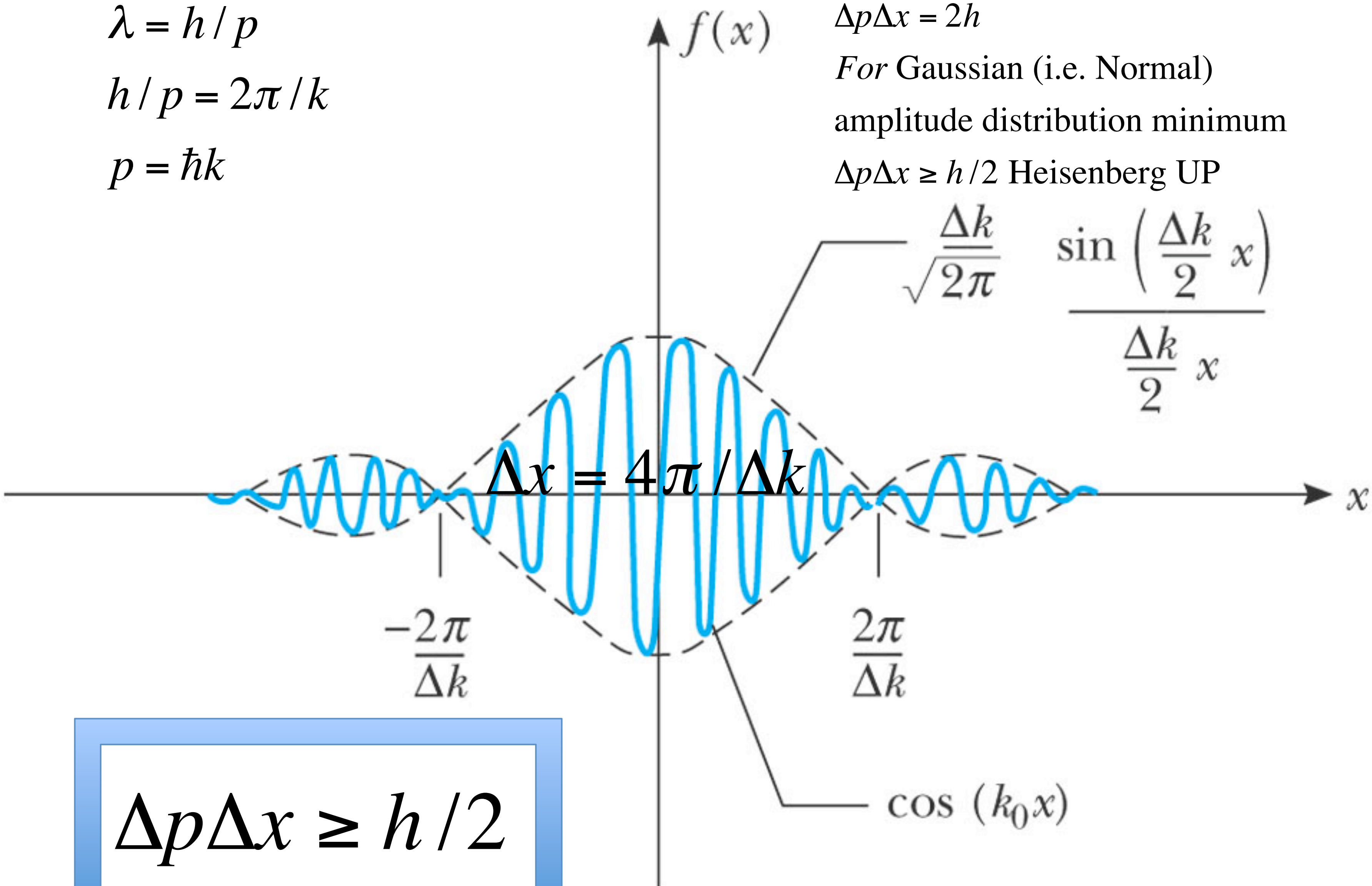
$$\Delta k \Delta x = 4\pi$$

2

$$\Delta p = \hbar \Delta k$$

$$\Delta p \Delta x = 2h$$

For Gaussian (i.e. Normal) amplitude distribution minimum $\Delta p \Delta x \geq h/2$ Heisenberg UP



$$\Delta x = 4\pi / \Delta k$$

$$\Delta p \Delta x \geq h / 2$$

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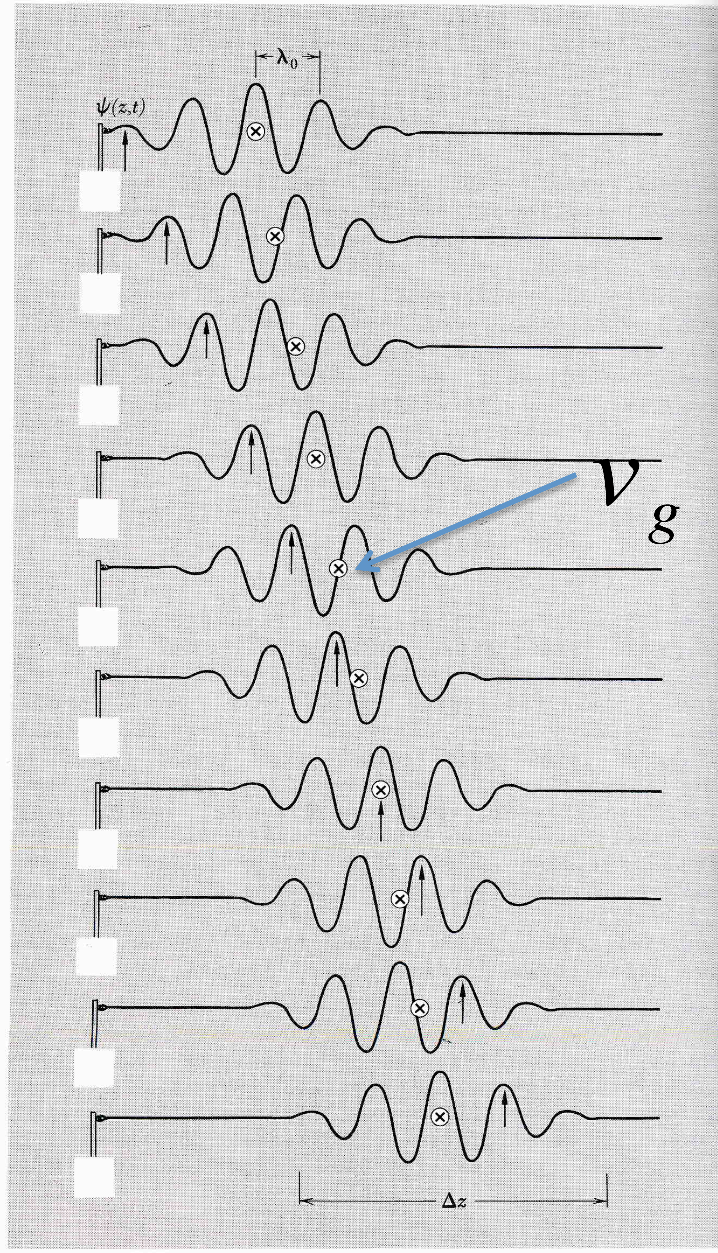
Group velocity

Fig. 6.7 Wave packet with phase velocity twice the group velocity. The arrow travels at the phase velocity, following a point of constant phase for the dominant wavelength. The cross travels at the group velocity with the packet as a whole.



$$v_g = \frac{\partial \omega}{\partial k}$$

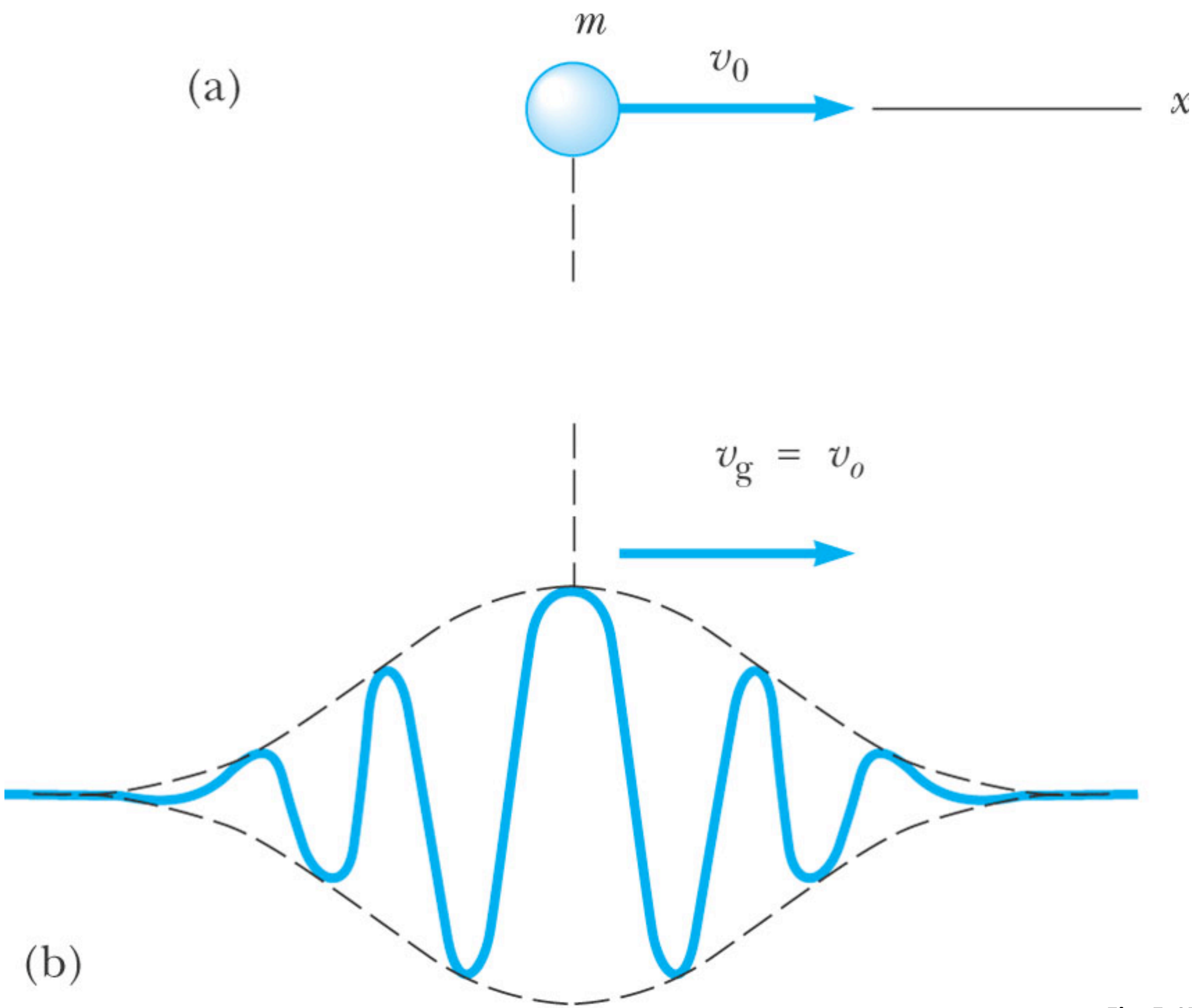


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Fig. 5-17, p. 164

FIGURE 40.12 History graph of a wave packet with duration Δt.

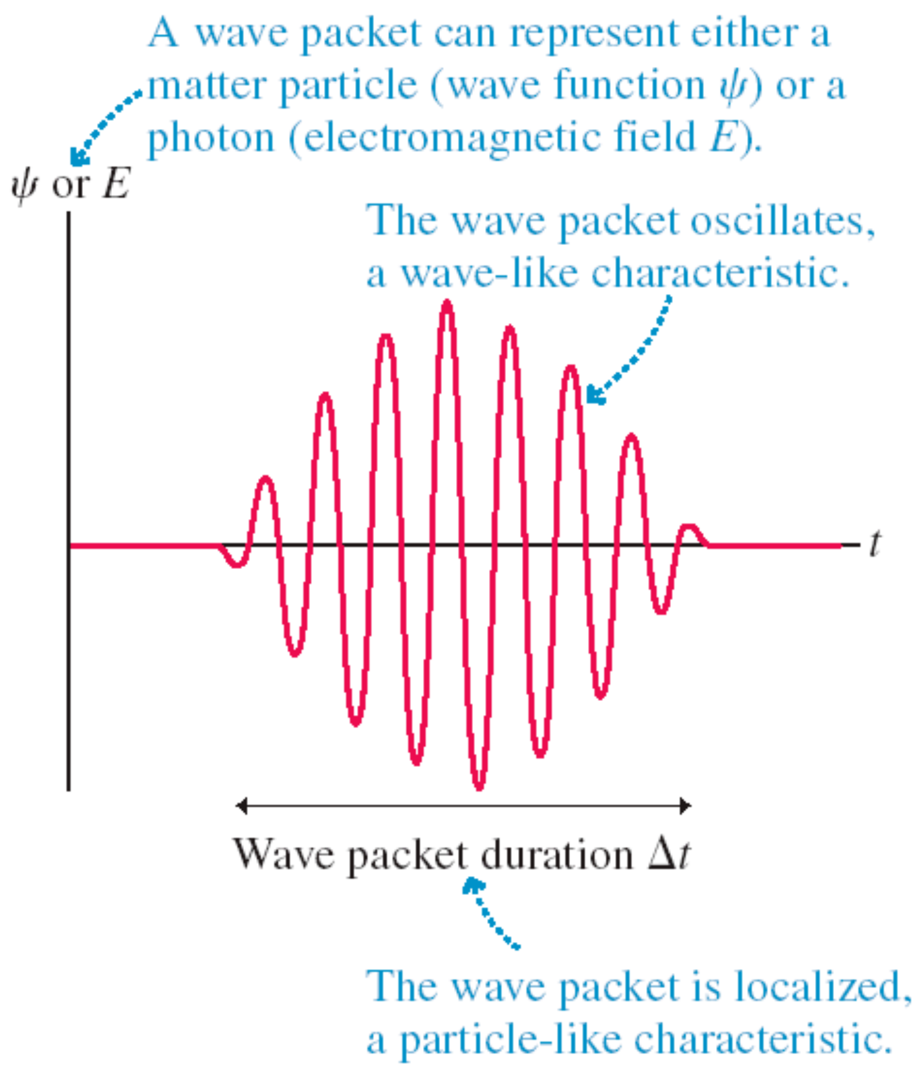
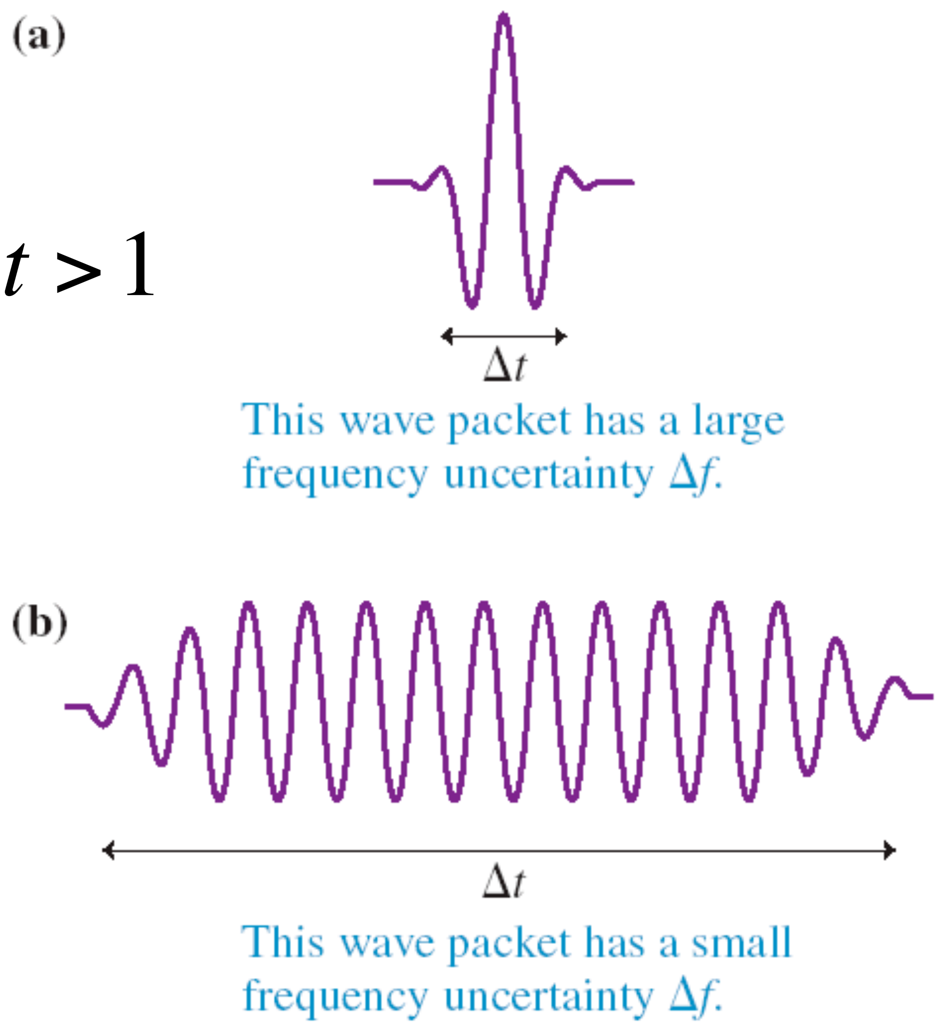
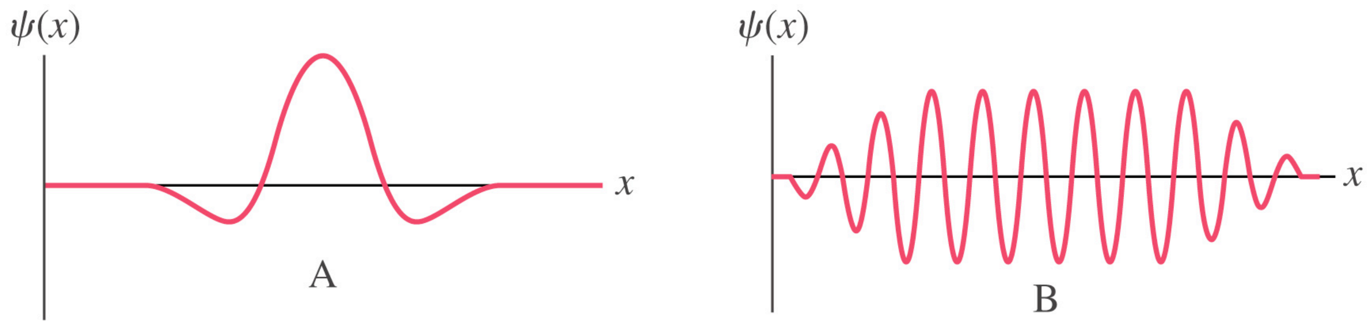


FIGURE 40.16 Two wave packets with different Δt.

$$\Delta f \Delta t > 1$$

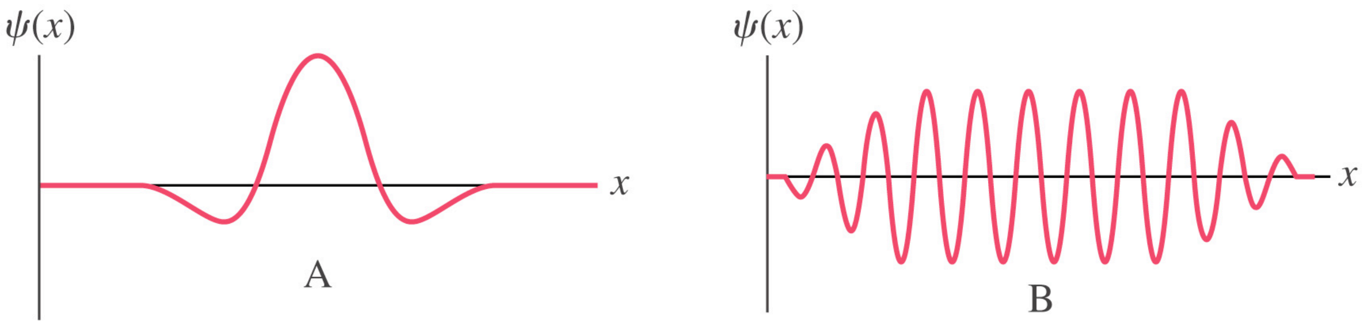


Which of these particles, A or B, can you locate more precisely?



A. A
B. B
C. Both can be located with same precision.

Which of these particles, A or B, can you locate more precisely?



A. A

B. B

C. Both can be located with same precision.

$$\Delta x = v\Delta t = (p/m)\Delta t$$
$$\Delta t = (m/p)\Delta x$$

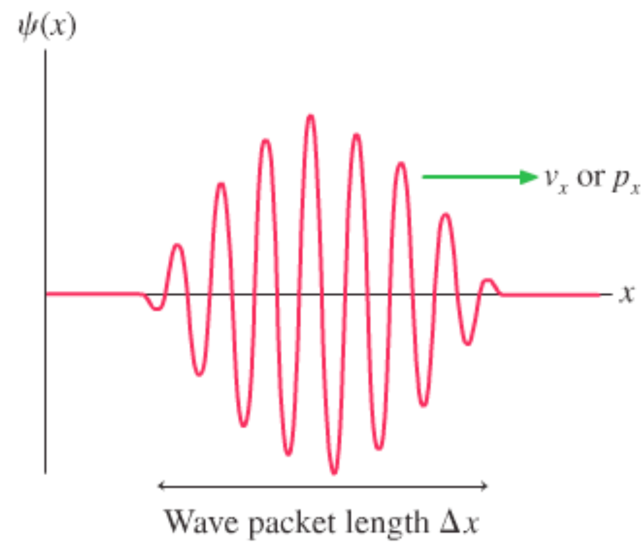
$$f = v/\lambda = \frac{p/m}{h/p} = \frac{p^2}{hm}$$
$$\Delta f = 2p\Delta p/hm$$

$$\Delta f\Delta t = (2p\Delta p/hm)(m\Delta x/p) = \frac{2}{h}\Delta x\Delta o$$

$$\Delta f\Delta t > 1$$

$$\Delta x\Delta p \geq h/2$$

FIGURE 40.17 A snapshot graph of a wave packet.



UP Our knowledge about particle properties in *inherently uncertain*

Electron localized in the hole and indivisible.
Clearly goes through slit #1 or slit #2

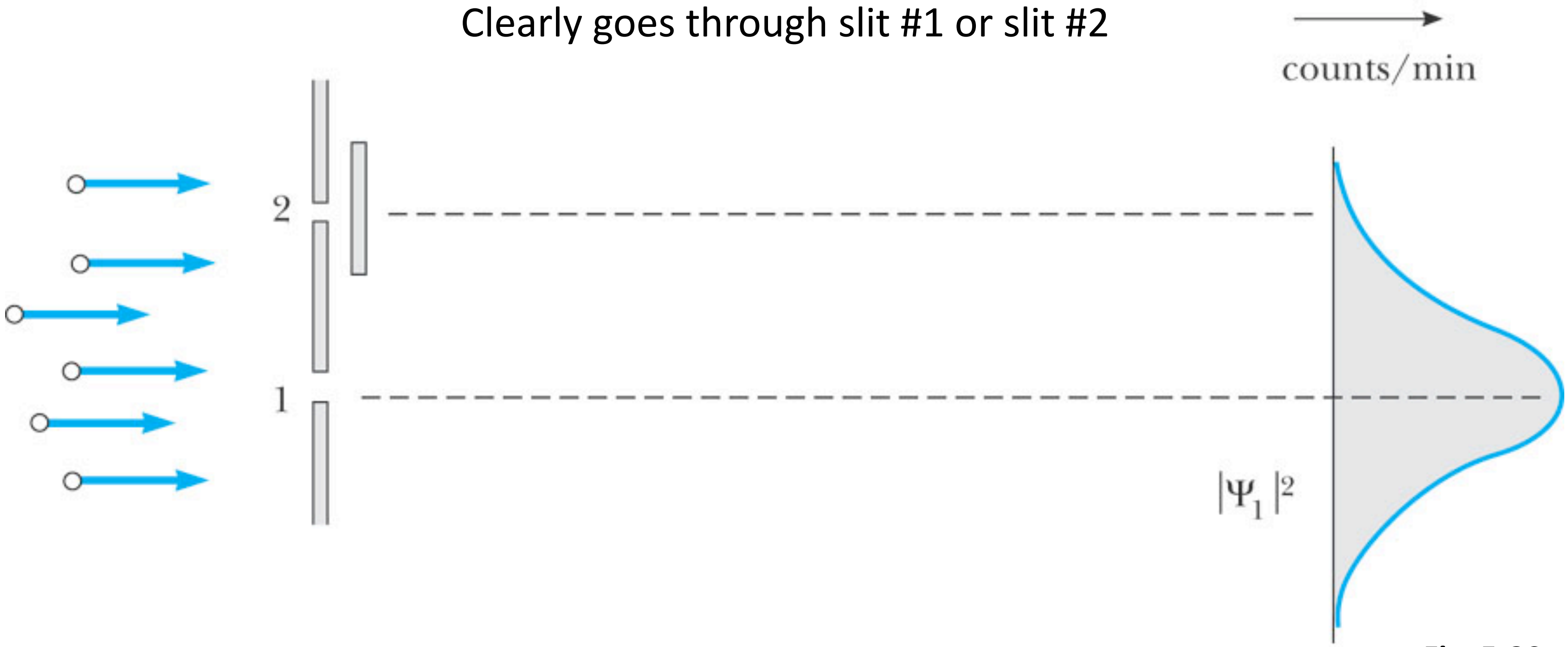


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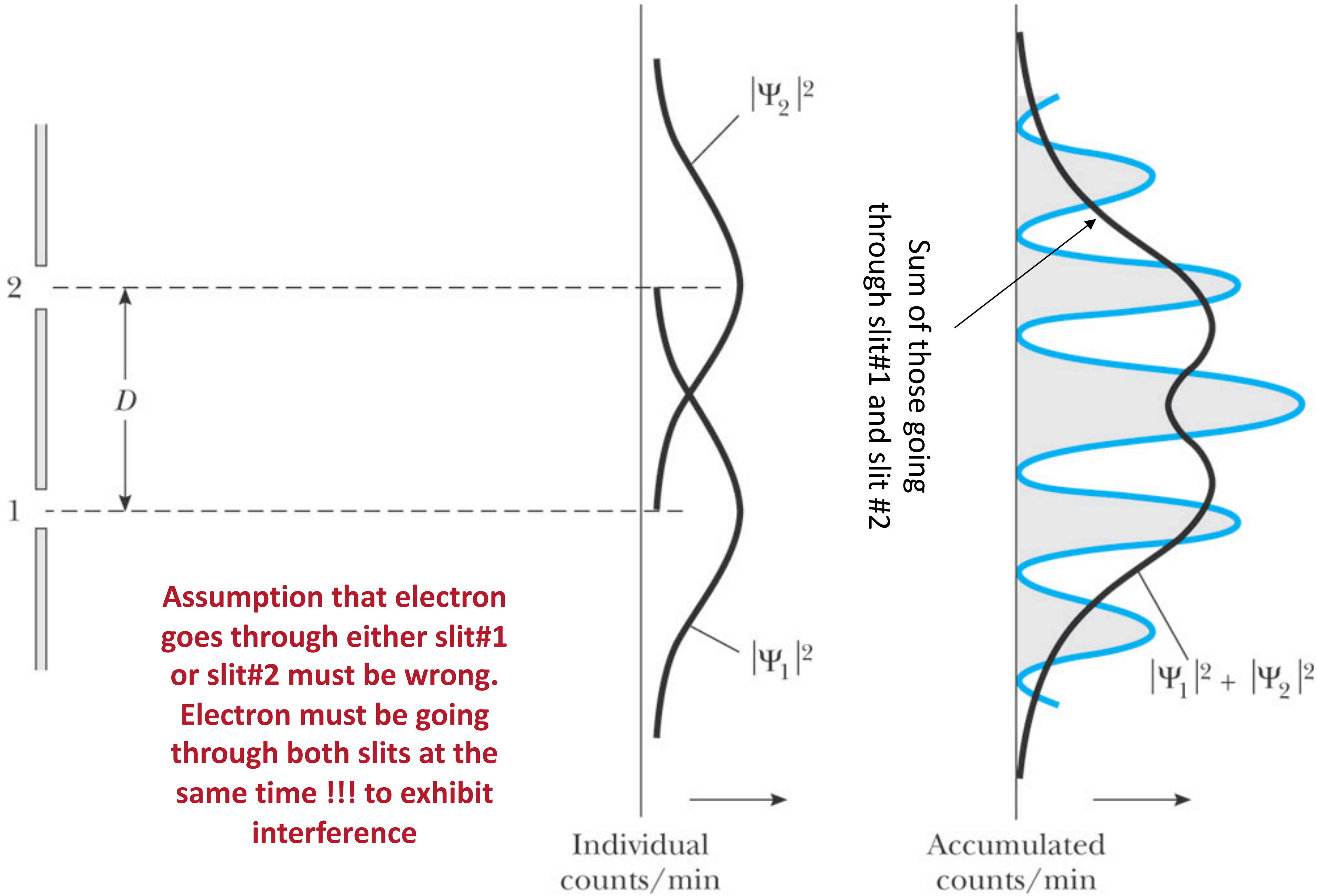
Fig. 5-30, p. 182



Assumption that electron goes through either slit#1 or slit#2 must be wrong. Electron must be going through both slits at the same time !!! to exhibit interference

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Watching Electrons

Need to measure the position of the electron with accuracy $\Delta y < D$

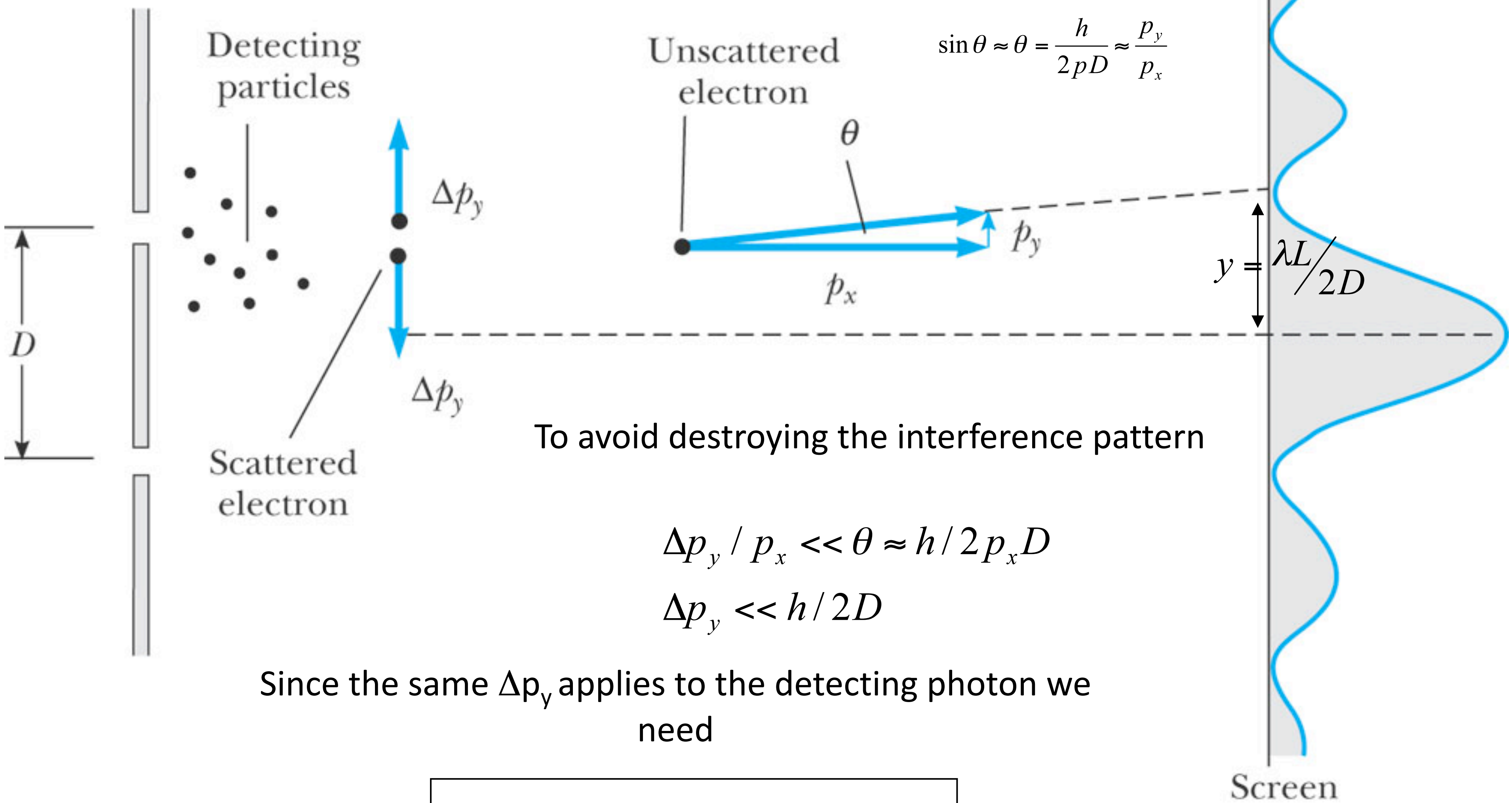
minima occur at

$D\sin\theta = \lambda/2$

electron wavelength : $\lambda = h/p$

for small θ :

$\sin\theta \approx \theta = \dfrac{h}{2pD} \approx \dfrac{p_y}{p_x}$



To avoid destroying the interference pattern

$$\Delta p_y / p_x << \theta \approx h/2p_x D$$

$$\Delta p_y << h/2D$$

Since the same Δp_y applies to the detecting photon we need

$\Delta p_y \Delta y < h/2$

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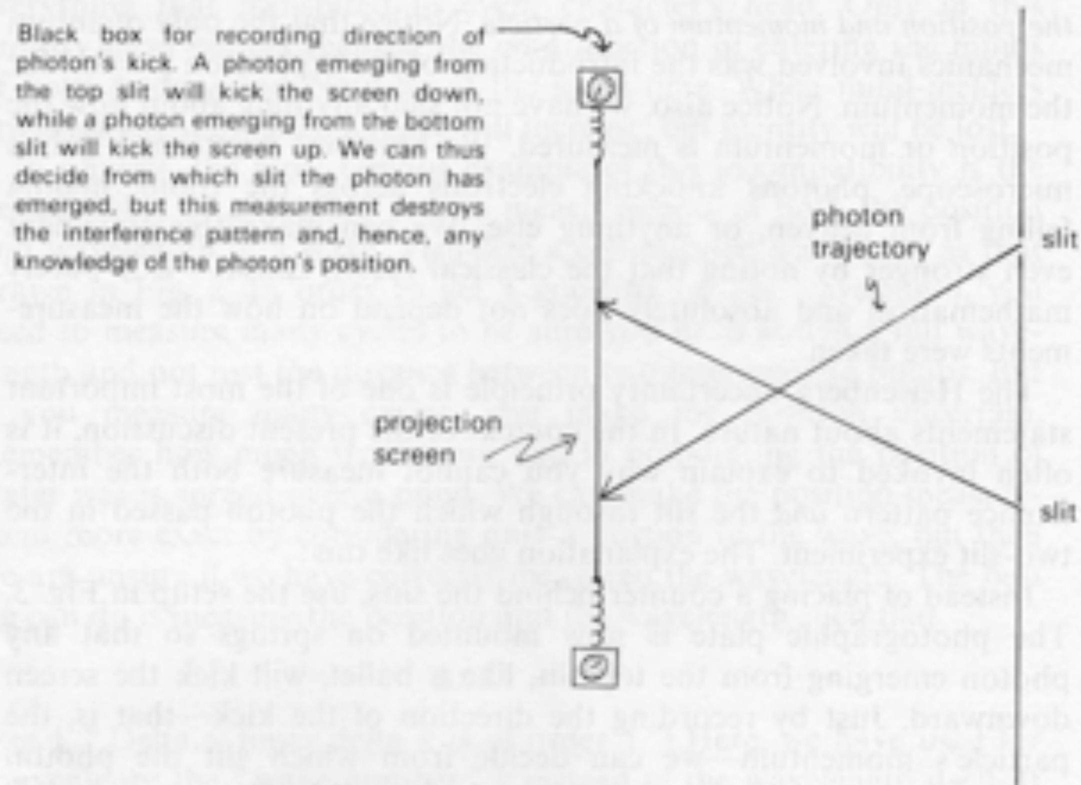

FIGURE 3 Two-slit experiment with springs.

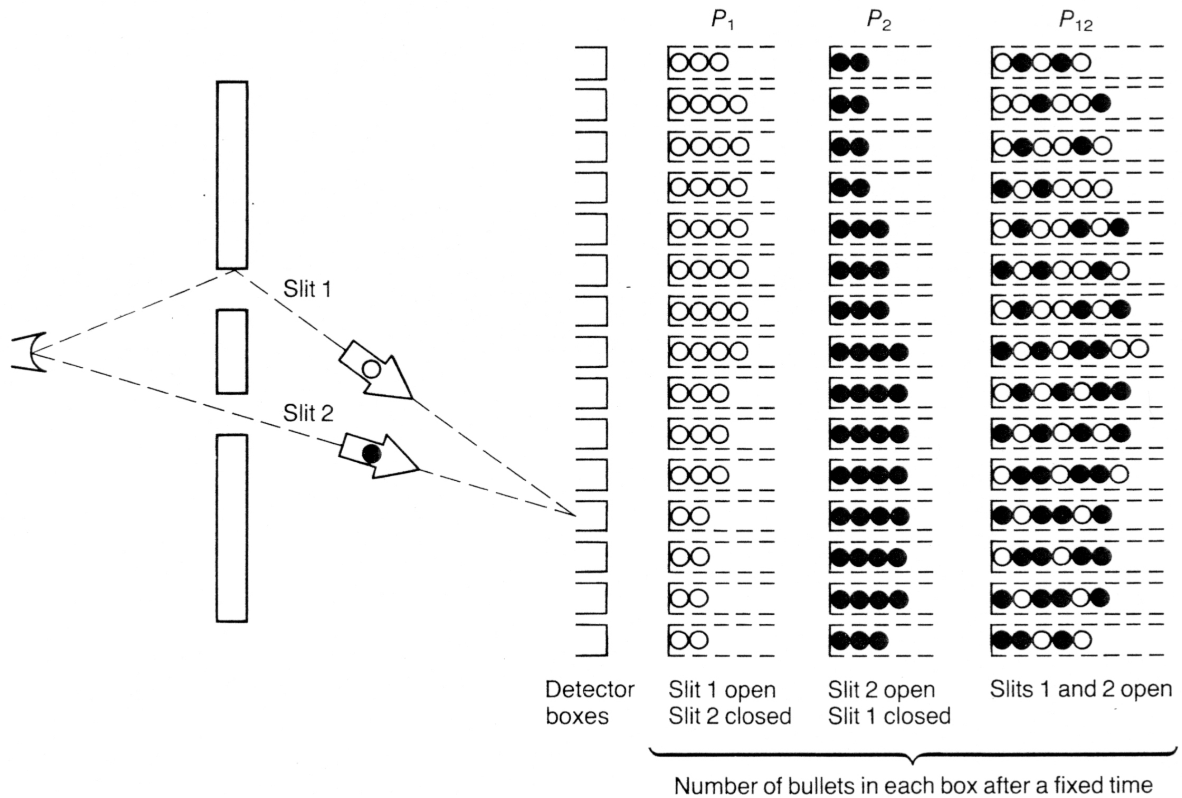

Fig. 1.7 A diagram of a double-slit experiment with bullets. The experimental set-up is shown on the left of the figure and the results of three different experiments indicated on the right. We have shown bullets that pass through slit 1 as open circles and bullets through slit 2 as black circles. The column labelled P_1 *shows the distribution of bullets arriving at the detector boxes when slit 2 is closed and only slit 1 is open. Column* P_2 *shows a similar distribution obtained with slit 1 closed and slit 2 open. As can be seen, the maximum number of bullets appears in the boxes directly in line with the slit that is left open. The result obtained with both slits open is shown in the column labelled* P_{12}*. It is now a matter of chance through which slit a bullet will come and this is shown by the scrambled mixture of black and white bullets collected in each box. The important point to notice is that the total obtained in each box when both slits are open is just the sum of the numbers obtained when only one or other of the slits is open. This is obvious in the case of bullets since we know that bullets must pass through one of the slits to reach the detector boxes.*

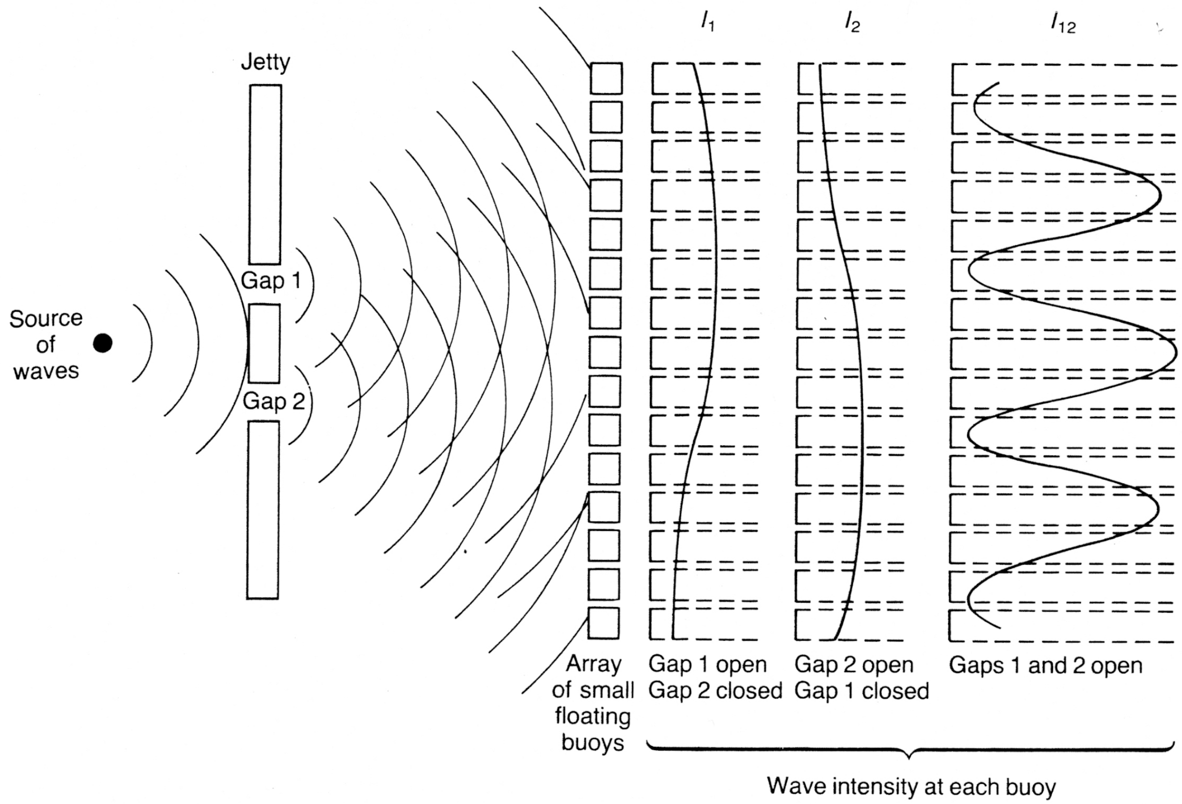

Fig. 1.9 A diagram of a double-slit experiment with water waves. The detectors are a line of small floating buoys whose jiggling up and down provides a measure of the wave energy. The wave crests spreading out from each slit are shown in the figure and can be compared with fig. 1.8. The column labelled I_1 shows the smoothly varying wave intensity obtained when only gap 1 is open. Notice that this is very similar to the pattern P_1 obtained with bullets in fig. 1.7 with only slit 1 open. Again it is largest at the detector directly in line with gap 1 and the source. The second column shows that a similar pattern, I_2, is obtained when gap 1 is closed and gap 2 is open. The final column, I_{12}, shows the wave intensity pattern obtained with both slits open. It is dramatically different from the pattern obtained for bullets with both slits open. It is not equal to the sum of the patterns I_1 and I_2 obtained with one of the gaps closed. This rapidly varying intensity curve is called an interference pattern.

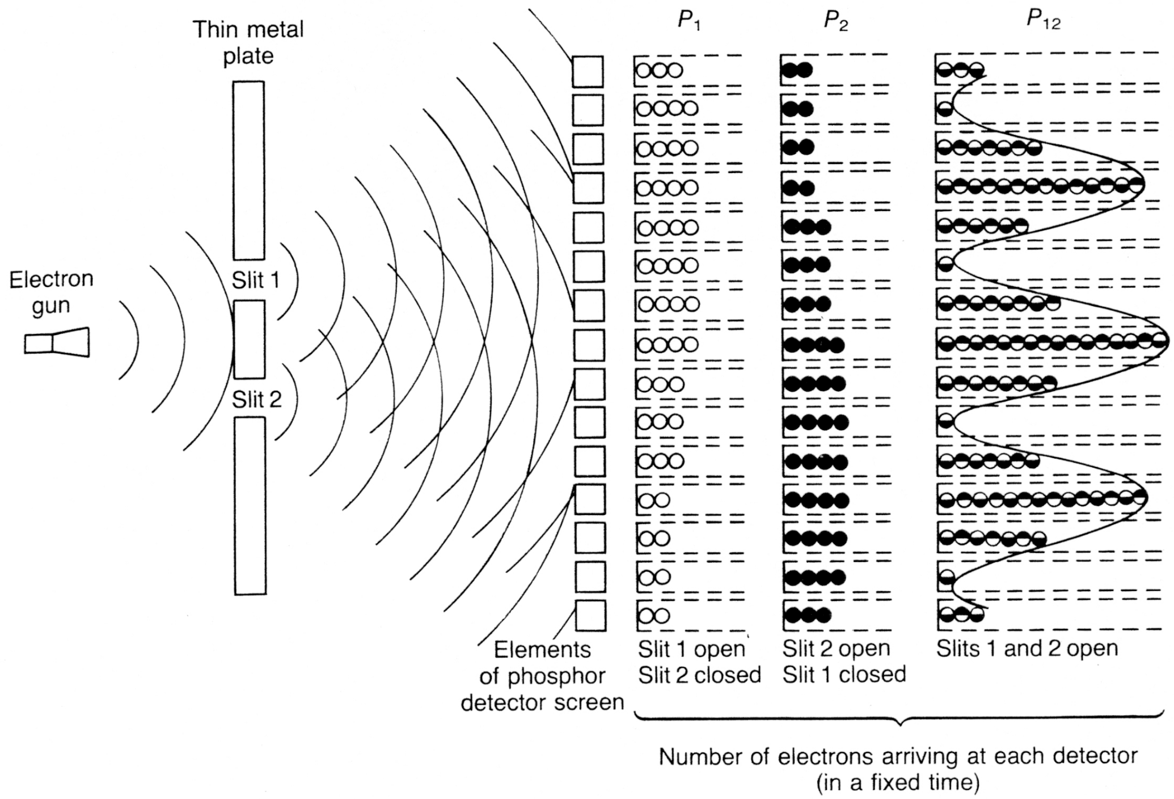

Fig. 1.11 A diagram of a double-slit experiment with electrons. Electrons always arrive with a flash at the phosphor detector at one point, in the same way that bullets always end up in just one of the detector boxes rather than the energy being spread out, as in a wave. The column marked P_1 *shows the pattern obtained with only slit 1 open. Electrons that have gone through slit 1 are represented as open circles, like the bullets of fig. 1.7. Column* P_2 *shows the same thing with only slit 2 open and the electrons that have gone through slit 2 indicated by black circles. These two patterns are exactly the same as those obtained with bullets. The difference lies in the column headed* P_{12}*, which shows the pattern obtained for electrons when both slits are open. This is just the interference pattern obtained with water waves and requires some kind of wave motion arising from each slit as indicated on the figure. It is not the sum of* P_1 *and* P_2 *and so we cannot say which slit any electron goes through. We have indicated this lack of knowledge by drawing the electrons, which still arrive like bullets, as half white and half black circles. This fact, that quantum objects such as electrons possess attributes of both wave and particle motion but behave like neither, is the central mystery of quantum mechanics.*

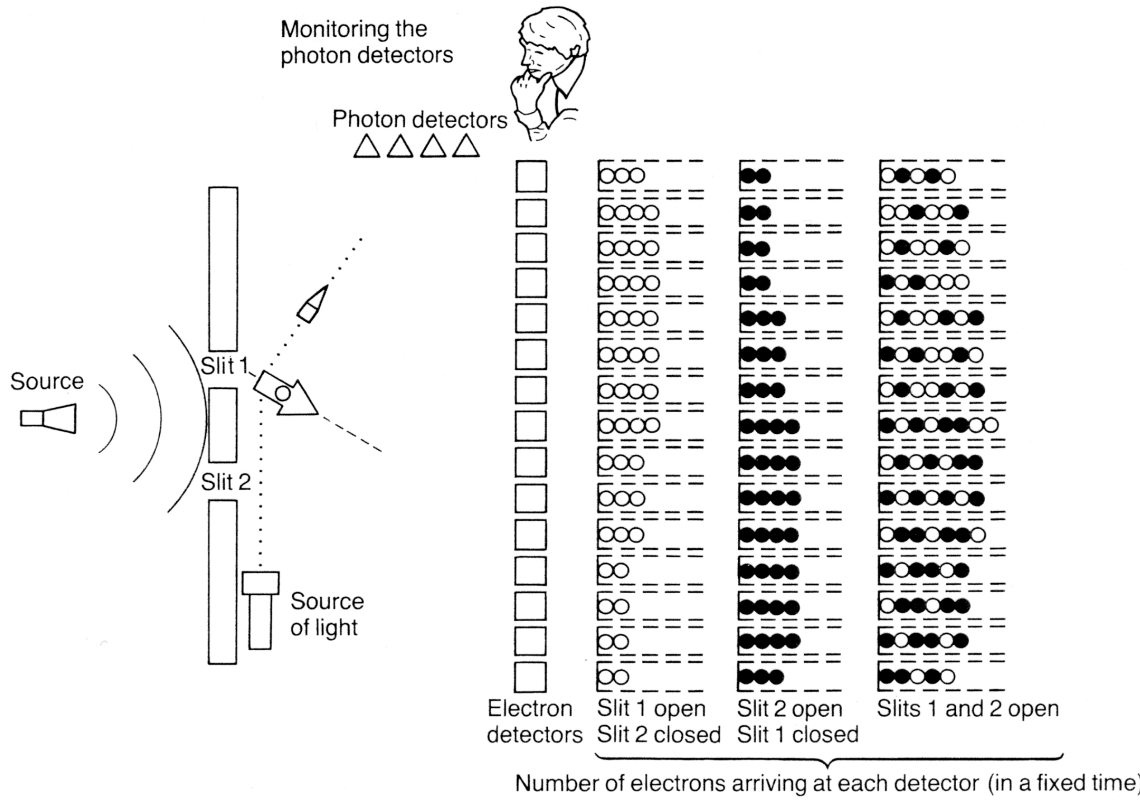

Fig. 2.3 Sketch of the experimental set-up required to observe through which slit the electron passes in a double-slit experiment. Light, in the form of photons, is directed at the slits. In the figure a photon, represented as a small bullet, has hit an electron behind slit 1. The electron is disturbed slightly in its motion and the scattered photon is observed at the photon detectors. The electron patterns obtained with only one of the slits open are almost the same as before, when we did not observe the electron behind the slits. The surprise occurs with both slits open: there is no interference pattern. The small nudges given to the electrons in their collisions with the photons are always sufficient to wash out the interference pattern completely! We can now say with certainty through which slit the electron went but now the electrons are behaving just like bullets. The observed pattern is just the sum of the patterns for slit 1 and slit 2 separately.