PHYS 270-SPRING 2011

Dennis Papadopoulos

LECTURE # 23

WAVE FUNCTIONS

PROBABILITY

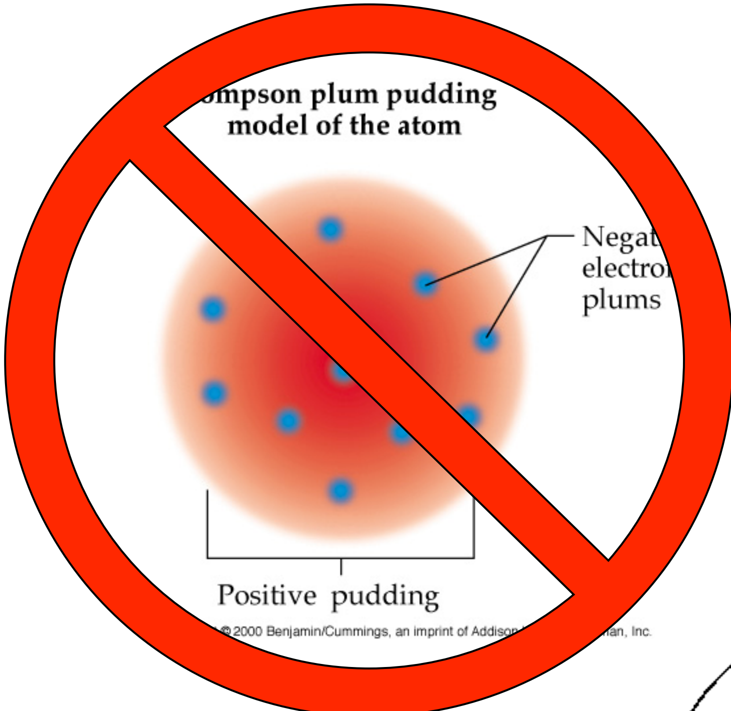
WAVE PACKETS

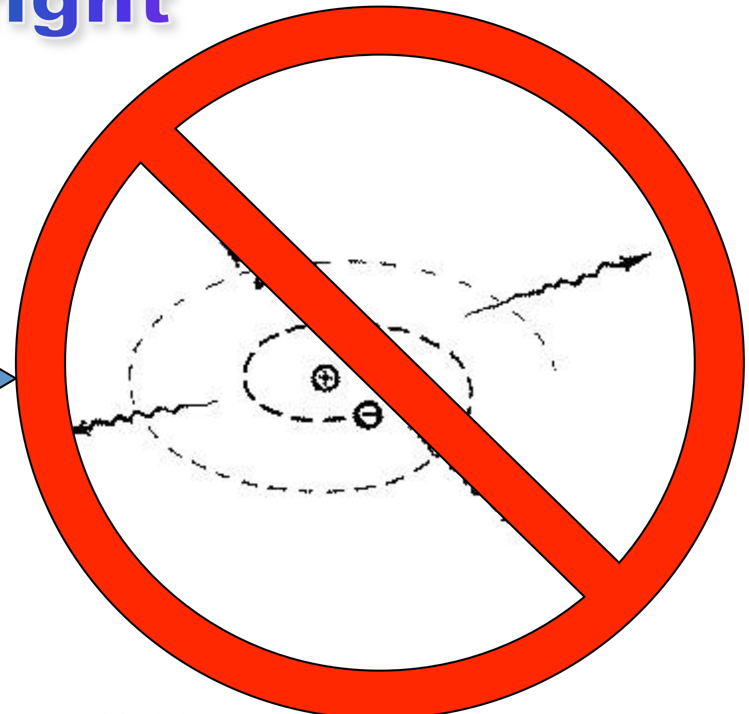
UNCERTAINTY

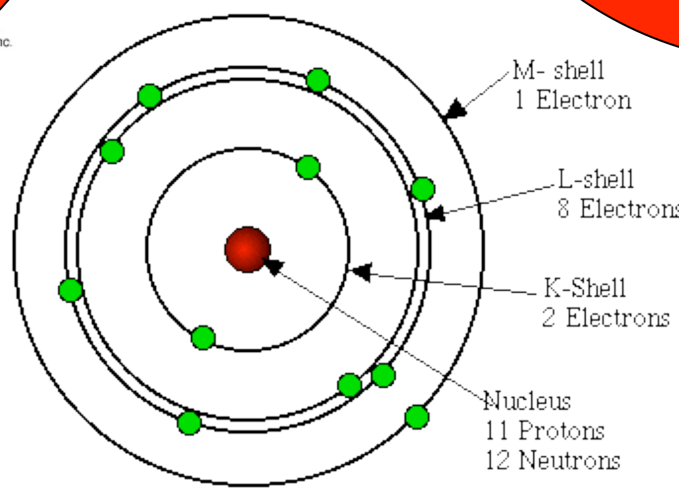
CHAPTER 40

MAY 3 2011

The Atom According to Bohr, who was (mostly) right



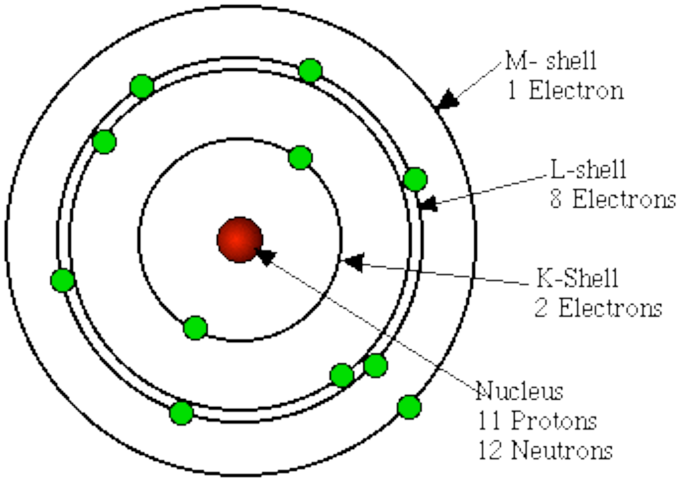




The Bohr Picture of the Sodium (Na 11) Atom

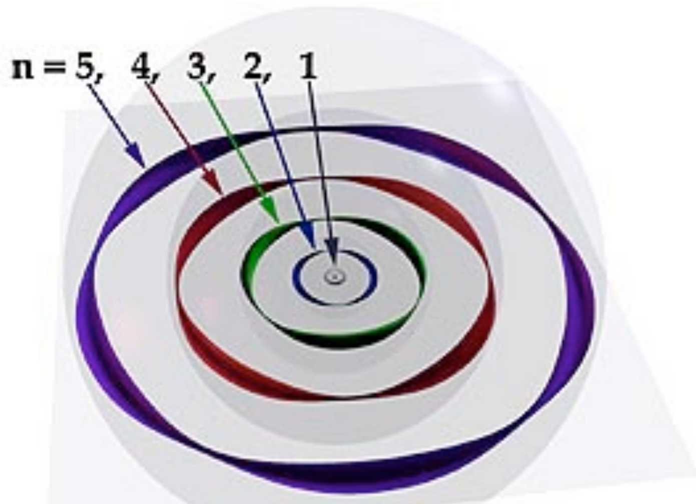
$$mvr = n\hbar$$

$$\lambda = h / mv$$



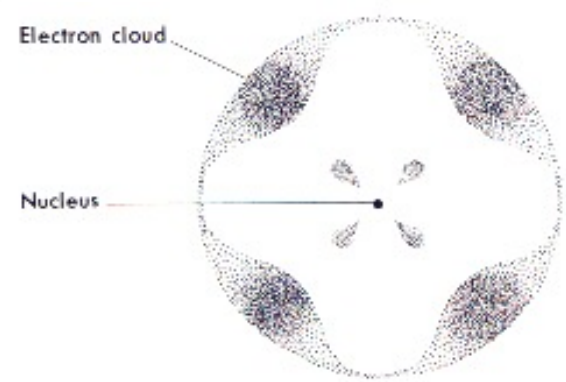
The Bohr Picture of the Sodium (Na 11) Atom

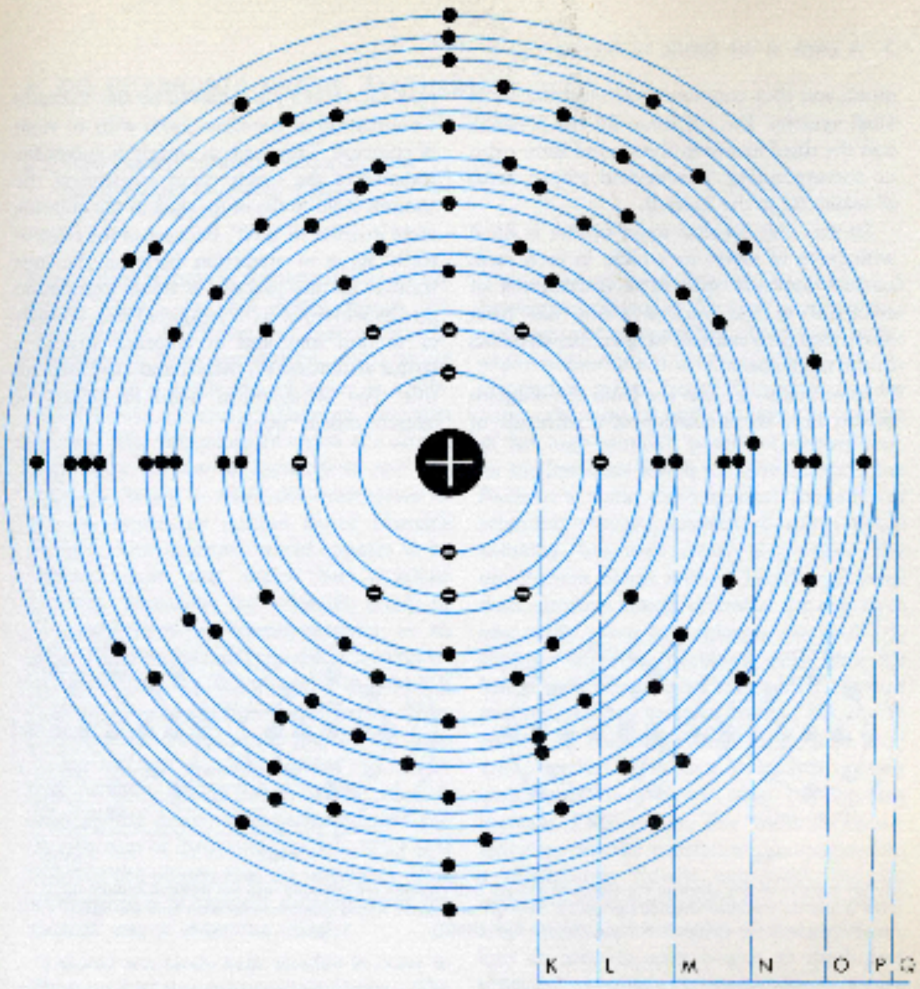
We said earlier that Bohr was mostly right…so where did he go wrong?



Niels Bohr - Louis de Broglie atom, 1924

- Failed to account for why some spectral lines are stronger than others. (To determine transition probabilities, you need QUANTUM MECHANICS!) Augh!
- Treats an electron like a miniature planet…but is an electron a particle…or a wave?





No.	Symbol	Element	K	L		M			N				O				P			Q
8	O	Oxygen	2	2	4															
29	Cu	Copper	2	2	6	2	6	10	1											
56	Ba	Barium	2	2	6	2	6	10	2	6	10	–	2	6	–	–	2			
80	Hg	Mercury	2	2	6	2	6	10	2	6	10	14	2	6	10	–	2			
88	Ra	Radium	2	2	6	2	6	10	2	6	10	14	2	6	10	–	2	6	–	2
92	U	Uranium	2	2	6	2	6	10	2	6	10	14	2	6	10	3	2	6	1	2

The energy levels of the orbits in which the electrons can move without suffering radiative energy loss, are called electron shells. The electrons of all the chemical elements can be accounted for in this way. The table gives the distribution of electrons in the different shells for the atoms of several chemical elements.

Are electrons waves or particles?
Wave particle duality
Quantum Mechanics

$$\frac{d^2\vec{r}}{dt^2} = \vec{F}/m$$

$$\vec{r}(t)$$

Descriptor is particle orbit. Law that controls it is Newton's law

$$\Psi(\vec{r},t)$$

- To introduce the *wave function* as the descriptor of particles in quantum mechanics.
- To provide the wave function with a probabilistic interpretation.
- To understand the wave function through pictorial and graphical exercises.
- To introduce the idea of normalization.
- To recognize the limitations on knowledge imposed by the Heisenberg uncertainty principle.

$$D_1 = a\sin(kr_1 - \omega t)$$
$$D_2 = a\sin(kr_1 - \omega t)$$
$$A(x) = D_1 + D_2$$

$$A(x) = 2a\cos\left(\frac{\pi d x}{\lambda L}\right)$$

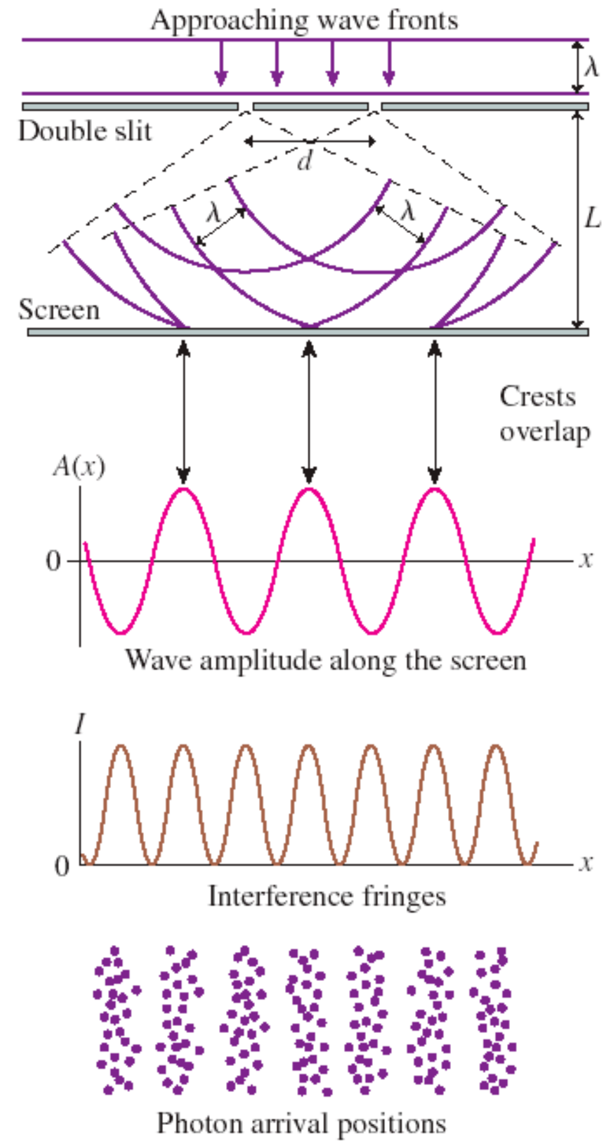
$$I(x) = C\cos^2\left(\frac{\pi d x}{\lambda L}\right)$$

With photons. Hear random clicks.
But after many of them you see a
clear interference pattern

$$\text{Prob}(\text{in } \delta x \text{ at } x) \propto |A(x)|^2 \delta x$$

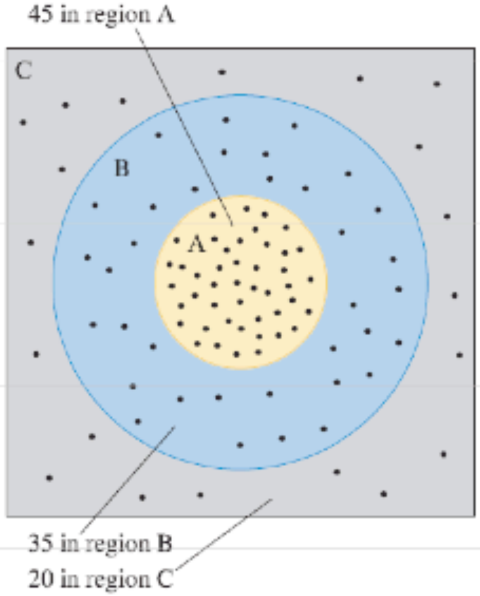
Light travels as a wave and interacts as a particle

FIGURE 40.1 The double-slit experiment with light.



Blindfolded dart throwing

FIGURE 40.2 One hundred throws at a dart board.



$$P_A = \lim_{N_{tot}\to\infty} \frac{N_A}{N_{tot}}$$

$$P_A \approx 45\%,\ P_B \approx 35\%,\ \text{and } P_C \approx 20\%$$

$$P_{A \text{ or } B} = \lim_{N_{tot}\to\infty} \frac{N_{A \text{ or } B}}{N_{tot}} = \lim_{N_{tot}\to\infty} \frac{N_A + N_B}{N_{tot}}$$

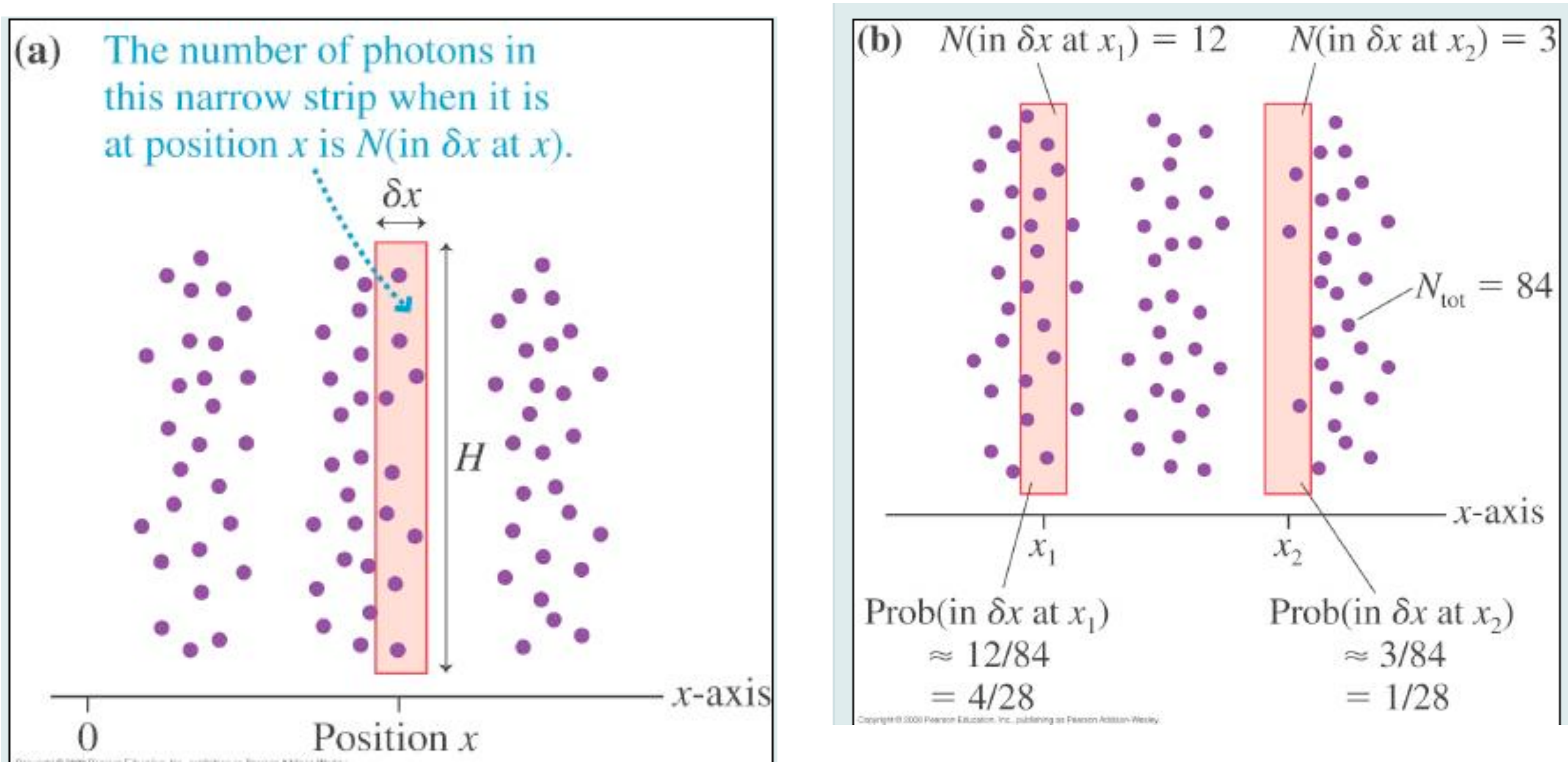
$$= \lim_{N_{tot}\to\infty} \frac{N_A}{N_{tot}} + \lim_{N_{tot}\to\infty} \frac{N_B}{N_{tot}} = P_A + P_B$$

$$P_{somewhere} = P_{A \text{ or } B \text{ or } C} = P_A + P_B + P_C = 0.45 + 0.35 + 0.20 = 1.00$$

If you throw N darts, **expected value** $N_{A\ expected} = NP_A$

Expected value of throwing a dice 99 times to get 1's and 4's?

P(in δx at x) is Probability density



$$P(in\ \delta x \text{ at x}) = \lim \frac{\text{N(in } \delta x \text{ at x)}}{\text{N}_{tot}},\ \text{N}_{tot} \to \infty$$

$$N(in\ \delta x \text{ at x}) = \text{N} \times \text{P(x,}\delta\text{x)}$$

$$\text{Prob(in } \delta x \text{ at } x) = P(x)\,\delta x$$

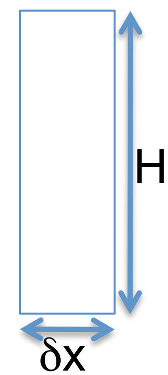
Probability is dimensionless. In one dimension probability density has dimensions of of 1/length. E.g. per cm or per mm or per nm)

Energy E intercepted by an area Hδx

WAVE PICTURE

W/m^2

$$E(x,\delta x) = I(x)H\delta x \propto |A(x)|^2 H\delta x$$



PHOTON PICTURE

$$N(\delta x, x) = \frac{E(x,\delta x)}{hf} = \frac{H}{hf} I(x)\delta x$$

but

$$P(\delta x, x) = \frac{N(\delta x, x)}{N_{tot}} = (H / hfN_{tot}) I(x)\delta x$$

Probability of detecting a photon at a particular point is directly proportional to the light square amplitude at that point

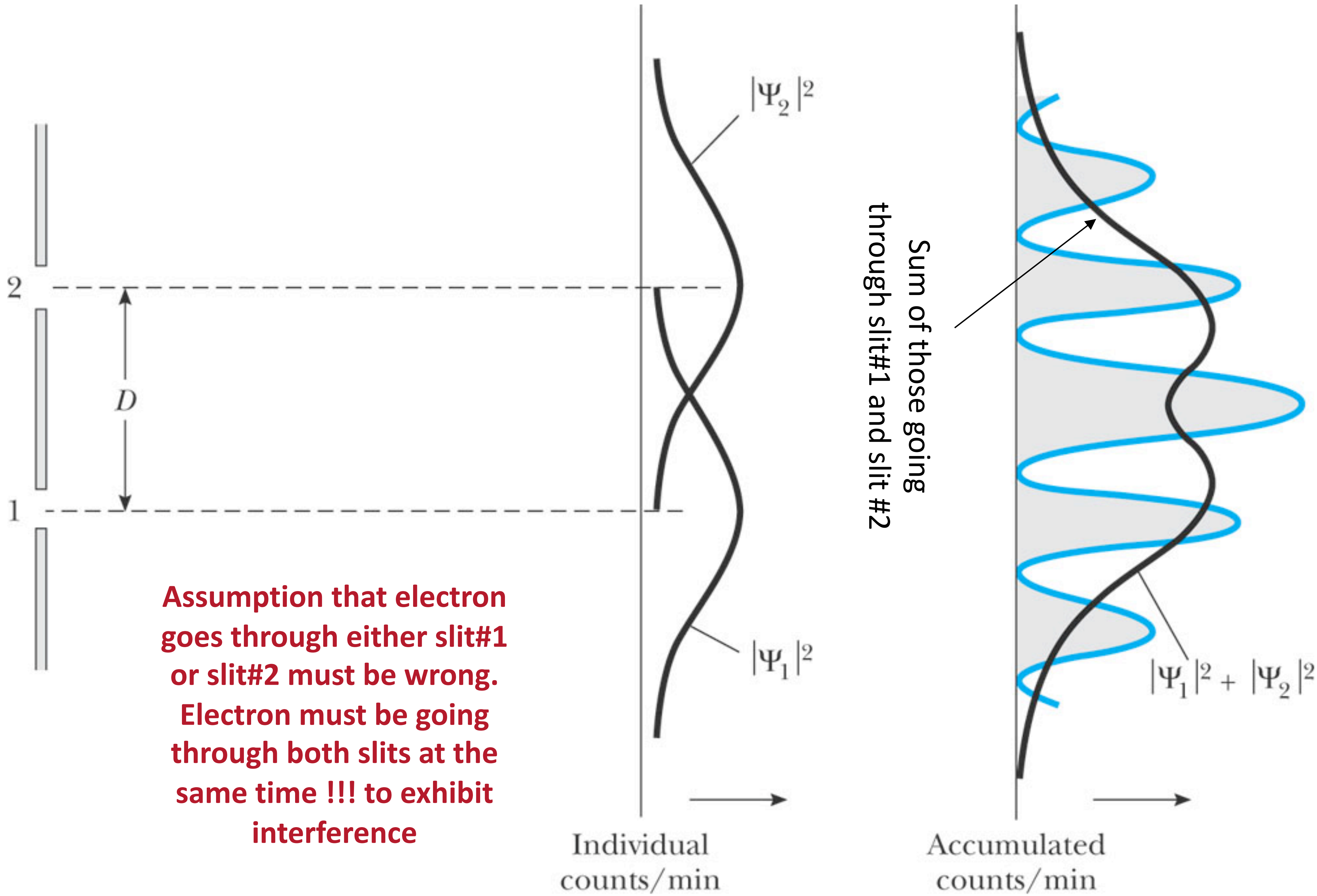
$$\text{Prob}(\text{in } \delta x \text{ at } x) \propto |A(x)|^2 \delta x$$

Double A four times probability

Relates a particle like event to a continuous classical wave

Electron localized in the hole and indivisible.
Clearly goes through slit #1 or slit #2

Fig. 5-30, p. 182



Assumption that electron goes through either slit#1 or slit#2 must be wrong. Electron must be going through both slits at the same time !!! to exhibit interference

What does it mean?

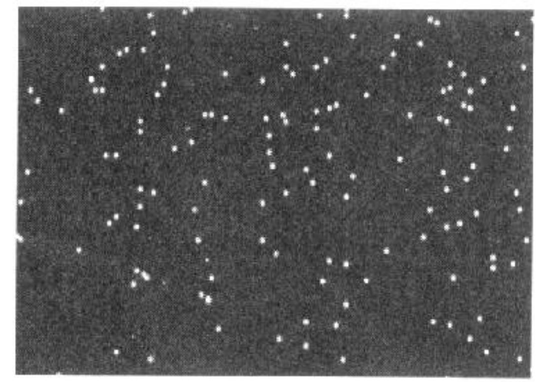
•A large number of electrons going through a double slit will produce an interference pattern, like a wave.

•However, each electron makes a single impact on a phosphorescent screen-like a particle.

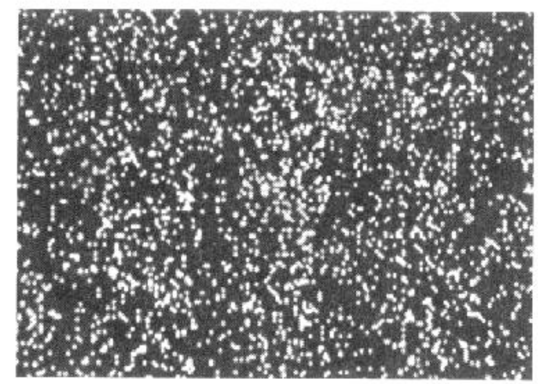
•Electrons have indivisible (as far as we know) mass and electric charge, so if you suddenly closed one of the slits, you couldn't chop the electron in half-because it clearly is a particle.

•A large number of electrons fired at two simultaneously open slits, however, will eventually, once you have enough statistics, form an intereference pattern. Their cumulative impact is wavelike.

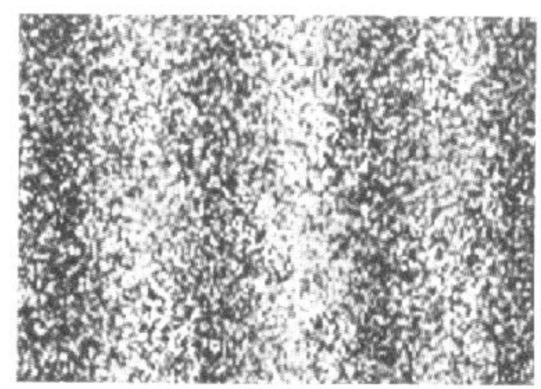
•This leads us to believe that the behavior of electrons is governed by probabilistic laws. --The wavefunction describes the probability that an electron will be found in a particular location.



(*b*) After 100 electrons



(*c*) After 3000 electrons



(*d*) After 70 000 electrons


a
b
c
d
e
f

Connecting the Wave and Photon Views

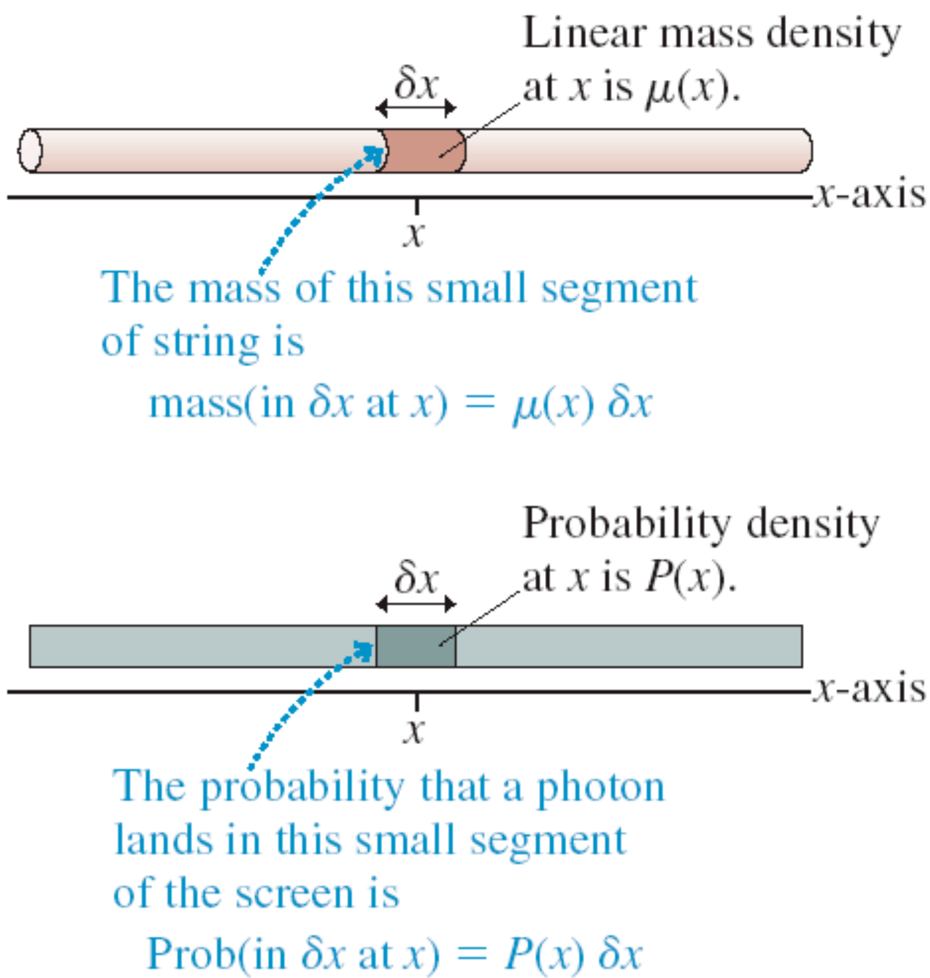
The *intensity of the light wave* is correlated with the *probability of detecting photons.* That is, photons are more likely to be detected at those points where the wave intensity is high and less likely to be detected at those points where the wave intensity is low.

The probability of detecting a photon at a particular point is directly proportional to the square of the light-wave amplitude function at that point:

$$\text{Prob}(\text{in } \delta x \text{ at } x) \propto |A(x)|^2 \delta x$$

$$\mathrm{Pr}\,ob(\delta x, x) = P(x)\delta x$$

FIGURE 40.4 The probability density is analogous to the linear mass density.



Probability Density

We can define the probability density $P(x)$ such that

$$\text{Prob}(\text{in } \delta x \text{ at } x) = P(x)\,\delta x$$

In one dimension, probability density has SI units of m^{-1}. Thus the probability density multiplied by a length yields a dimensionless probability.

NOTE: *P*(*x*) itself is *not* a probability. You must multiply the probability density by a length to find an actual probability.

The photon probability density is directly proportional to the square of the light-wave amplitude:

→ $$P(x) \propto |A(x)|^2$$

Electron behave in a way similar to photons.

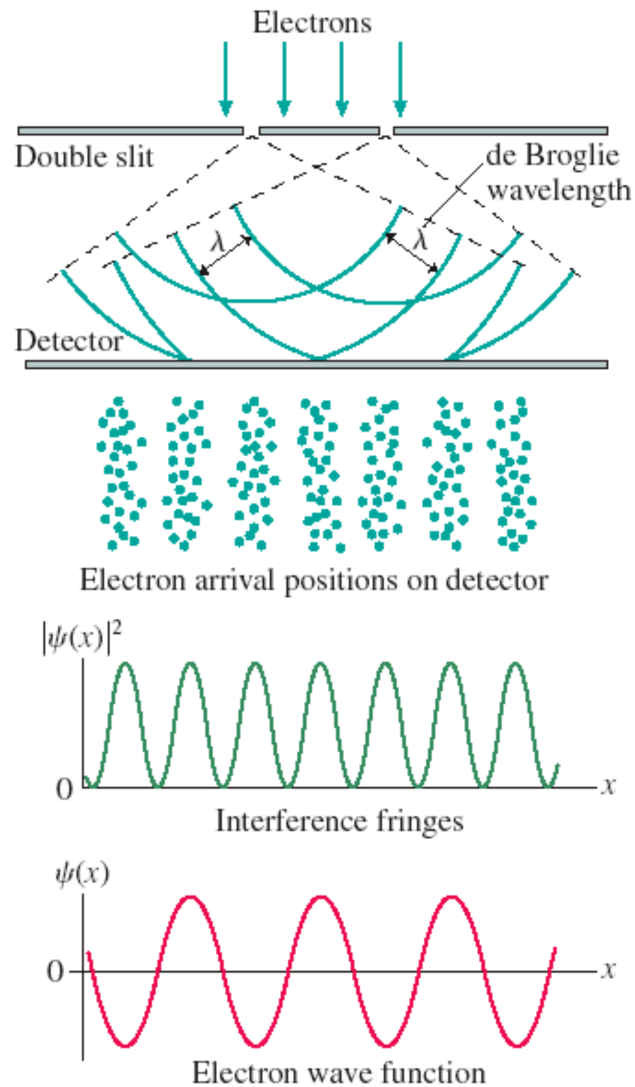
POSTULATE: There is a function $\Psi(x)$ for matter waves similar to the wave amplitude A(x). Note that in em A(x) is the electric field amplitude E.

$\Psi(x)$ is different.

Probability density is proportional to $|\Psi(x)|^2$

Just like photons $P(x) \sim |A(x)|^2$

FIGURE 40.5 The double-slit experiment with electrons.



Wave Function

Electron behave in a way similar to photons.

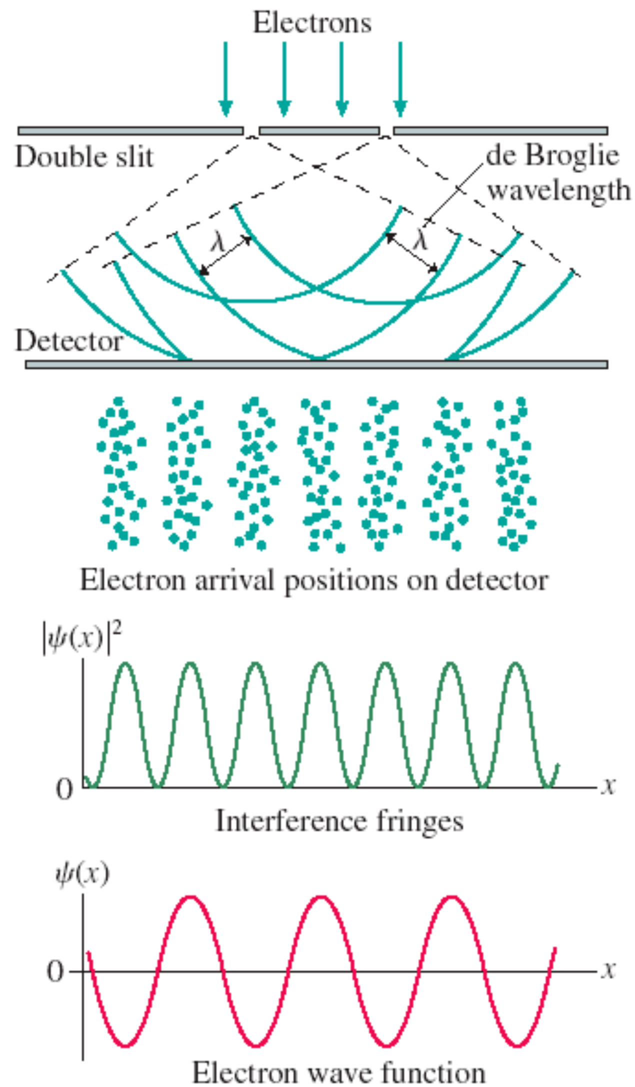
POSTULATE: There is a function Ψ(x) for matter waves similar to the wave amplitude A(x). Note that in em A(x) is the electric field amplitude E.

Ψ(x) is different.

Probability density is proportional to $|\Psi(x)|^2$

Just like photons $P(x) \sim |A(x)|^2$

FIGURE 40.5 The double-slit experiment with electrons.



• Descriptor of a process that obeys some laws (eqs)

• Newtonian x(t) obeys Newton's law

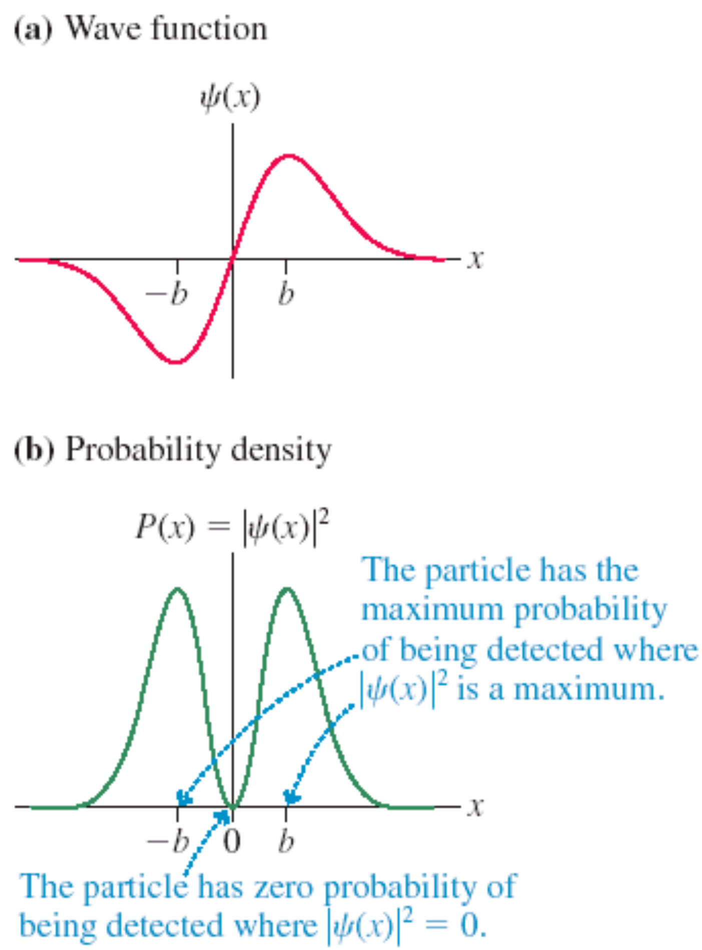
• x(t) completely determined but Ψ(x,t) not.

• In QM plays the role of x(t)

• Description passed all experimental tests

• It obeys the Schroedinger Equation

FIGURE 40.6 The square of the wave function is the probability density for detecting the electron at various values of the position x.



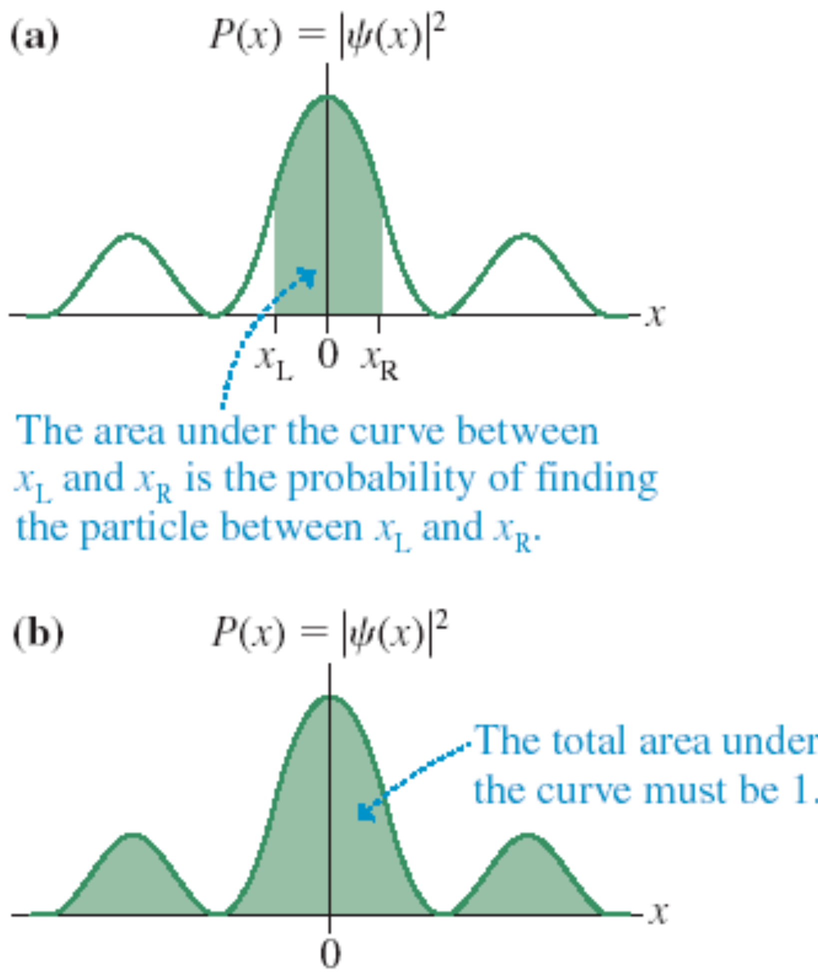
Normalization

- A photon or electron has to land *somewhere* on the detector after passing through an experimental apparatus.
- Consequently, the probability that it will be detected at *some* position is 100%.
- The statement that the photon or electron has to land *somewhere* on the *x*-axis is expressed mathematically as

$$\int_{-\infty}^{\infty} P(x)\,dx = \int_{-\infty}^{\infty} |\psi(x)|^2\,dx = 1$$

- Any wave function must satisfy this **normalization condition.**

FIGURE 40.8 The area under the probability density curve is a probability.



$$P(x,t)dx = |\Psi(x,t)|^2 dx$$

$$P(a \le x \le b) = \int_a^b |\Psi(x,t)|^2 \, dx$$

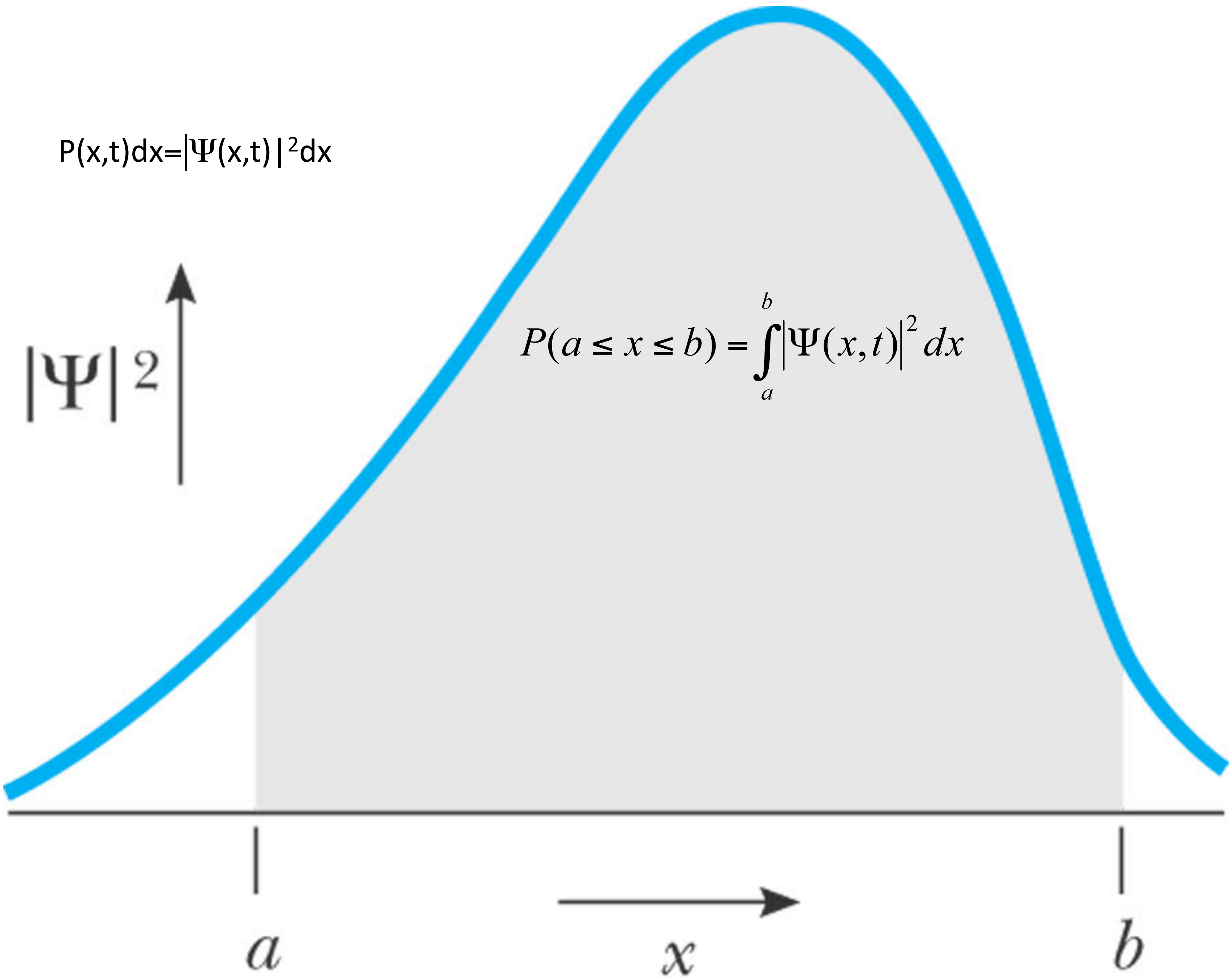


Fig. 6-1, p. 193

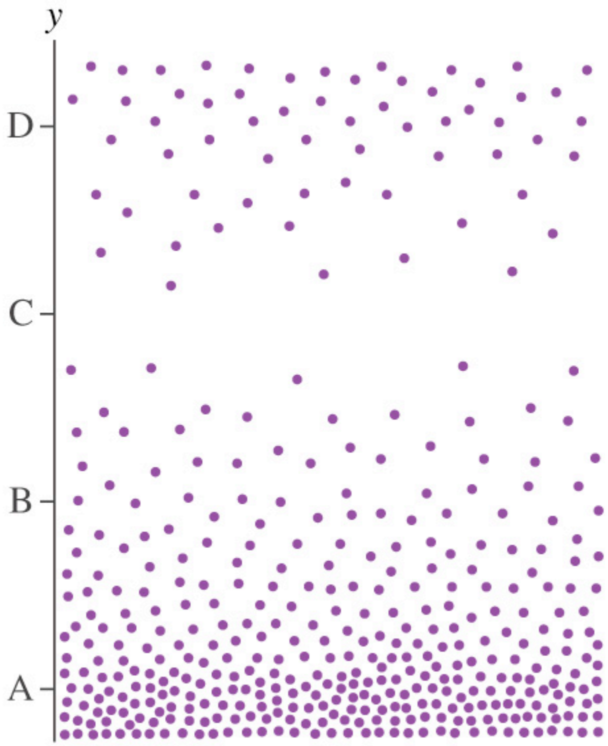
Suppose you roll a die 30 times. What is the expected numbers of 1's *and* 6's?

A. 12
B. 10
C. 8
D. 6
E. 4

Suppose you roll a die 30 times. What is the expected numbers of 1's *and* 6's?

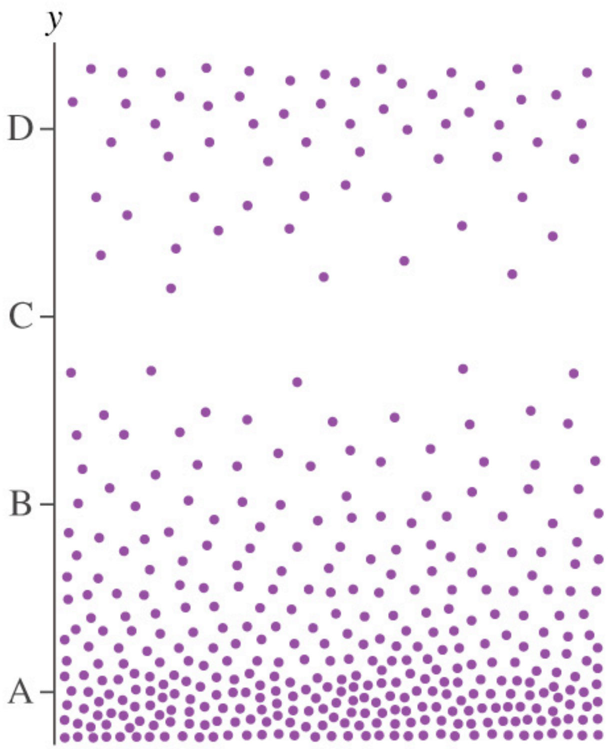
A. 12
B. 10 ✔
C. 8
D. 6
E. 4

The figure shows the detection of photons in an optical experiment. Rank in order, from largest to smallest, the square of the amplitude function of the electromagnetic wave at positions A, B, C, and D.



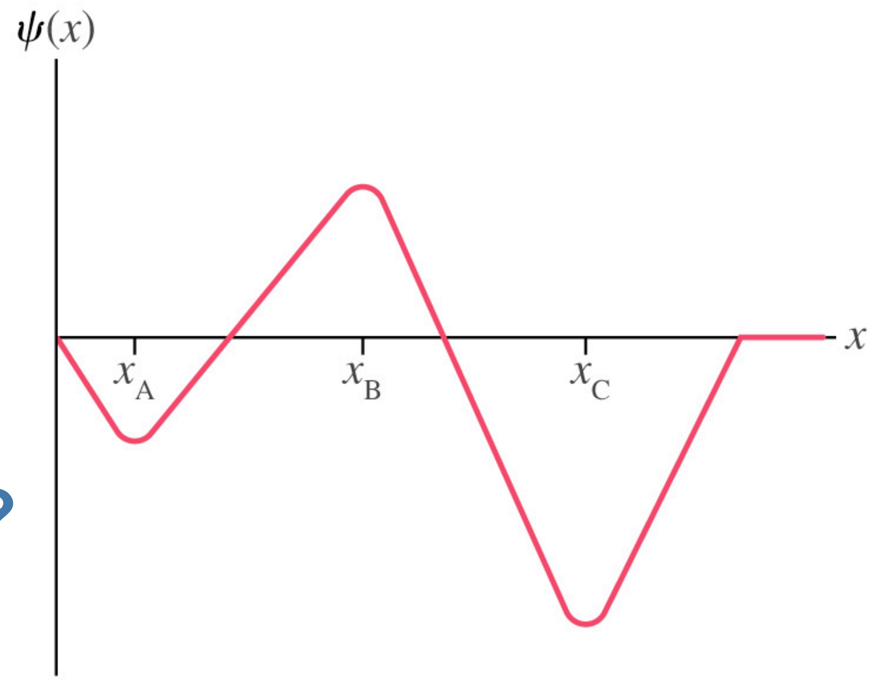
A. D > C > B > A
B. A > B > C > D
C. A > B = D > C
D. C > B = D > A

The figure shows the detection of photons in an optical experiment. Rank in order, from largest to smallest, the square of the amplitude function of the electromagnetic wave at positions A, B, C, and D.



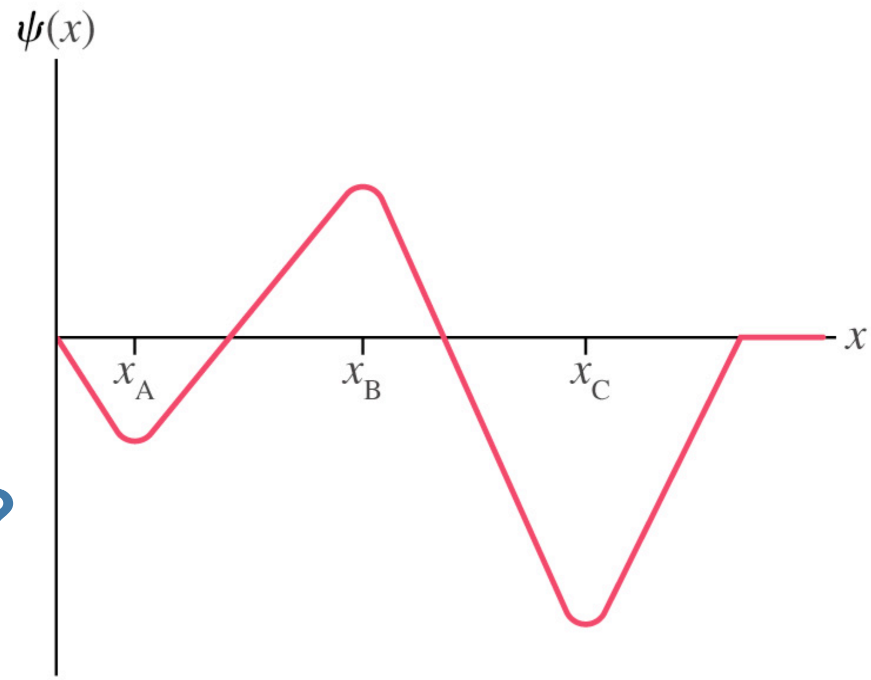
A. D > C > B > A
B. A > B > C > D
✔ **C. A > B = D > C**
D. C > B = D > A

This is the wave function of a neutron. At what value of *x* is the neutron most likely to be found?



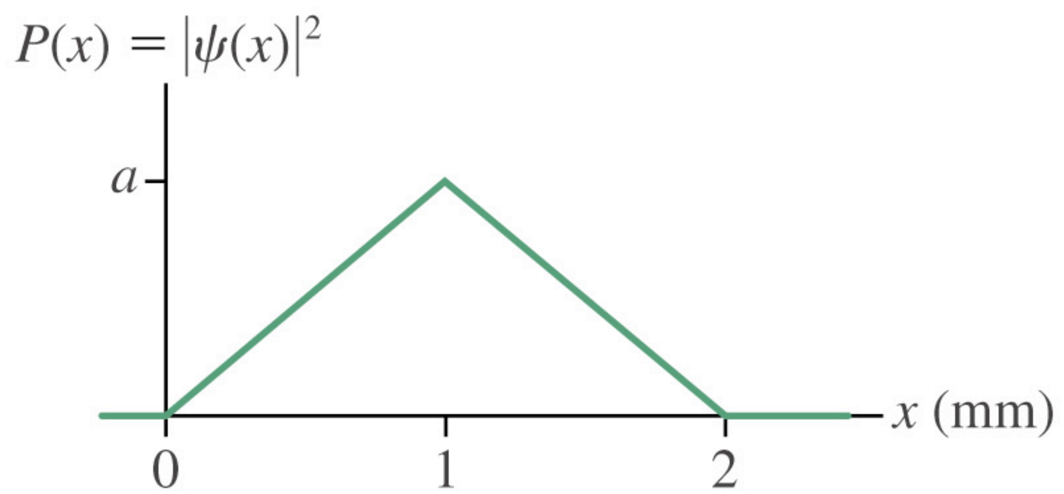
A. $x = 0$
B. $x = x_A$
C. $x = x_B$
D. $x = x_C$

This is the wave function of a neutron. At what value of *x* is the neutron most likely to be found?



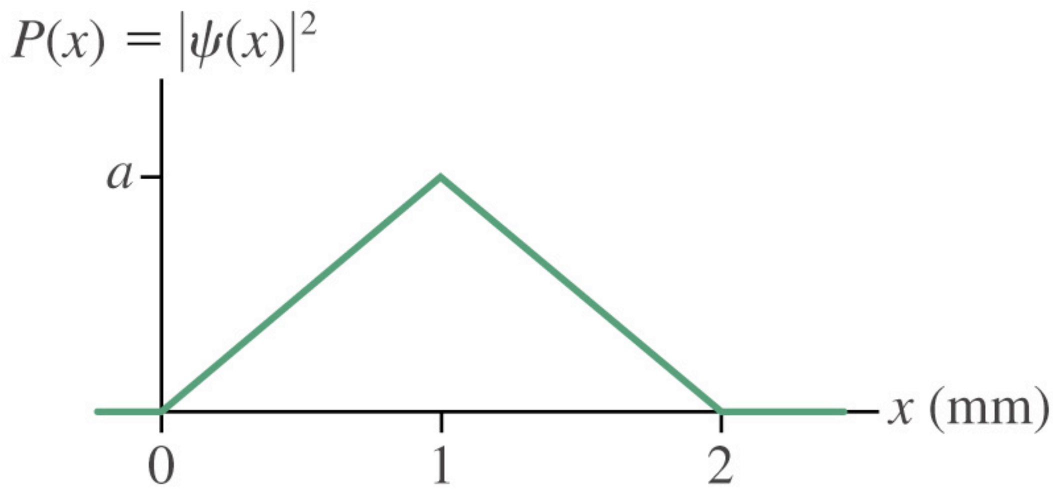
A. $x = 0$
B. $x = x_A$
C. $x = x_B$
✔ **D. $x = x_C$**

The value of the constant *a* is



A. a = 0.5 mm$^{-1/2}$.
B. a = 1.0 mm$^{-1/2}$.
C. a = 2.0 mm$^{-1/2}$.
D. a = 1.0 mm^{-1}.
E. a = 2.0 mm^{-1}.

The value of the constant *a* is



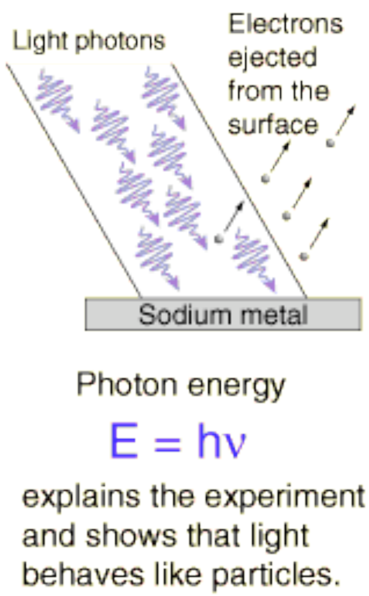
A. $a = 0.5\ \text{mm}^{-1/2}$.
B. $a = 1.0\ \text{mm}^{-1/2}$.
C. $a = 2.0\ \text{mm}^{-1/2}$.
✔ **D. $a = 1.0\ \text{mm}^{-1}$.**
E. $a = 2.0\ \text{mm}^{-1}$.

$$\frac{1}{2}a(2mm) = 1$$

$$a = 1mm^{-1}$$

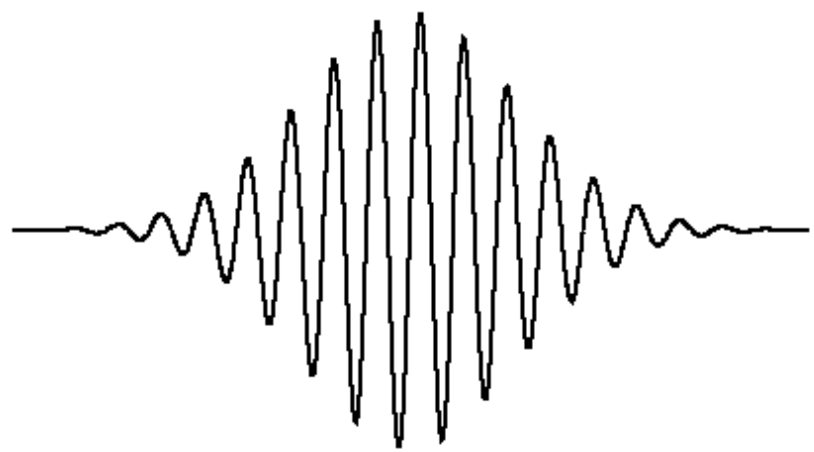
WAVE PACKETS

The word "particle" in the phrase "wave-particle duality" suggests that this wave is somewhat localized.



How do we describe this mathematically?

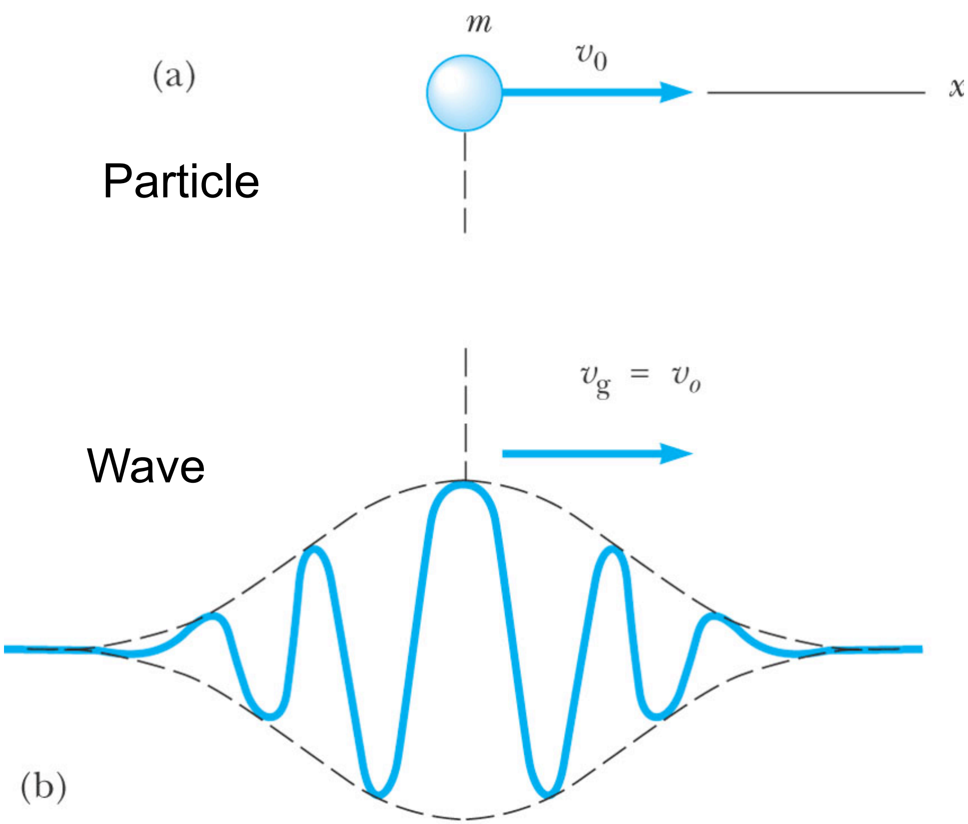
…or this

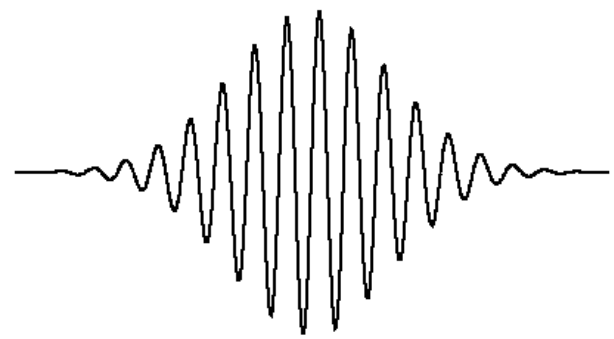


…or this

WAVE PACKETS

How do we describe this mathematically?



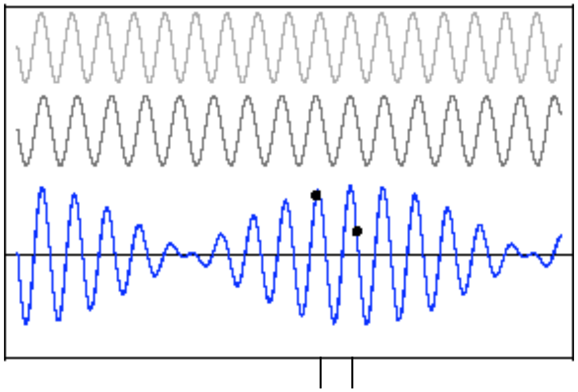


Interefering waves, generally…

$$y = y_1 + y_2 = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$\Downarrow$$

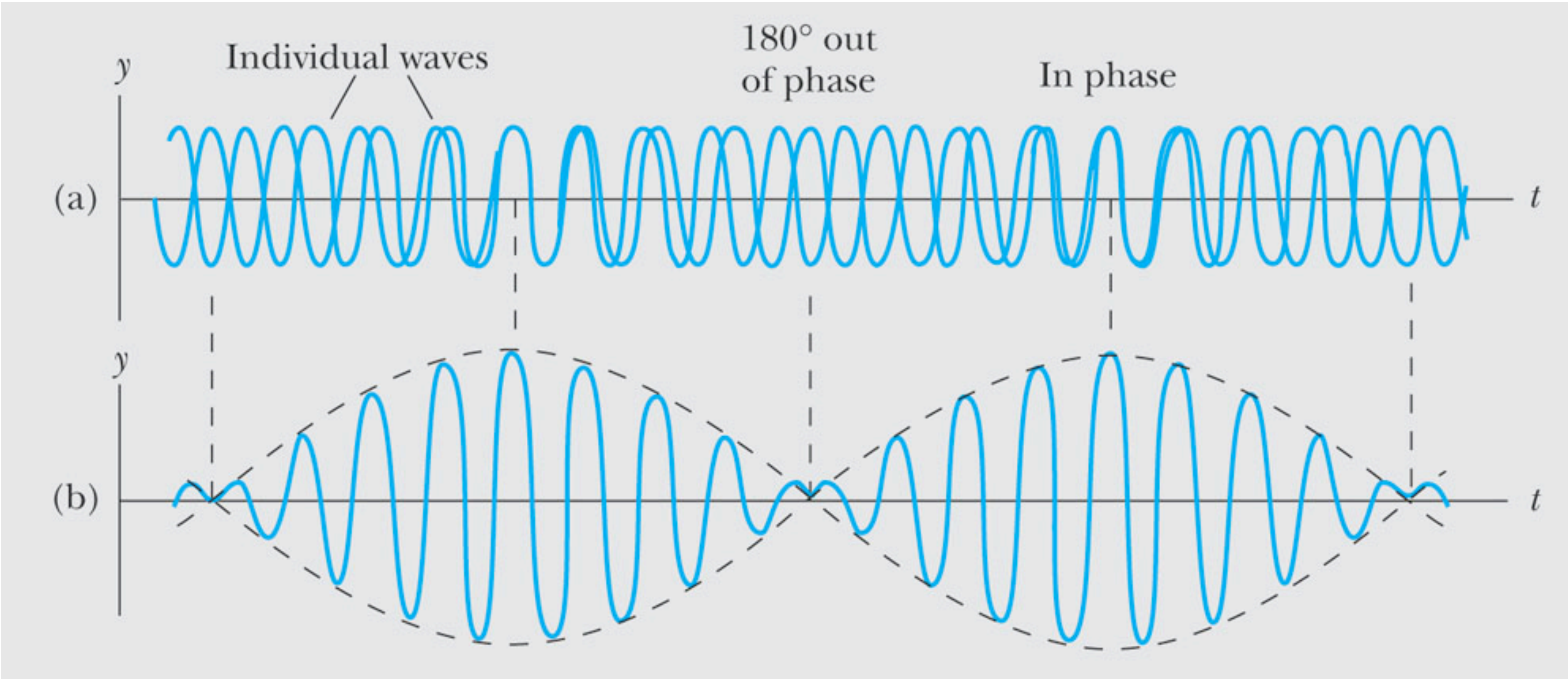
$$y = 2A\cos\frac{1}{2}\{(k_2 - k_1)x - (\omega_2 - \omega_1)t\} \bullet \cos\frac{1}{2}\{(k_1 + k_2)x - (\omega_1 + \omega_2)t\}$$



"Beats" occur when you add two waves of slightly different frequency. They will interfere constructively in some areas and destructively in others.

Can be interpreted as a sinusoidal envelope: $2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$

Modulating a high frequency wave within the envelope: $\cos\left[\frac{1}{2}(k_1 + k_2)x - \frac{1}{2}(\omega_1 + \omega_2)t\right]$



At constant value of x

$$2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$$

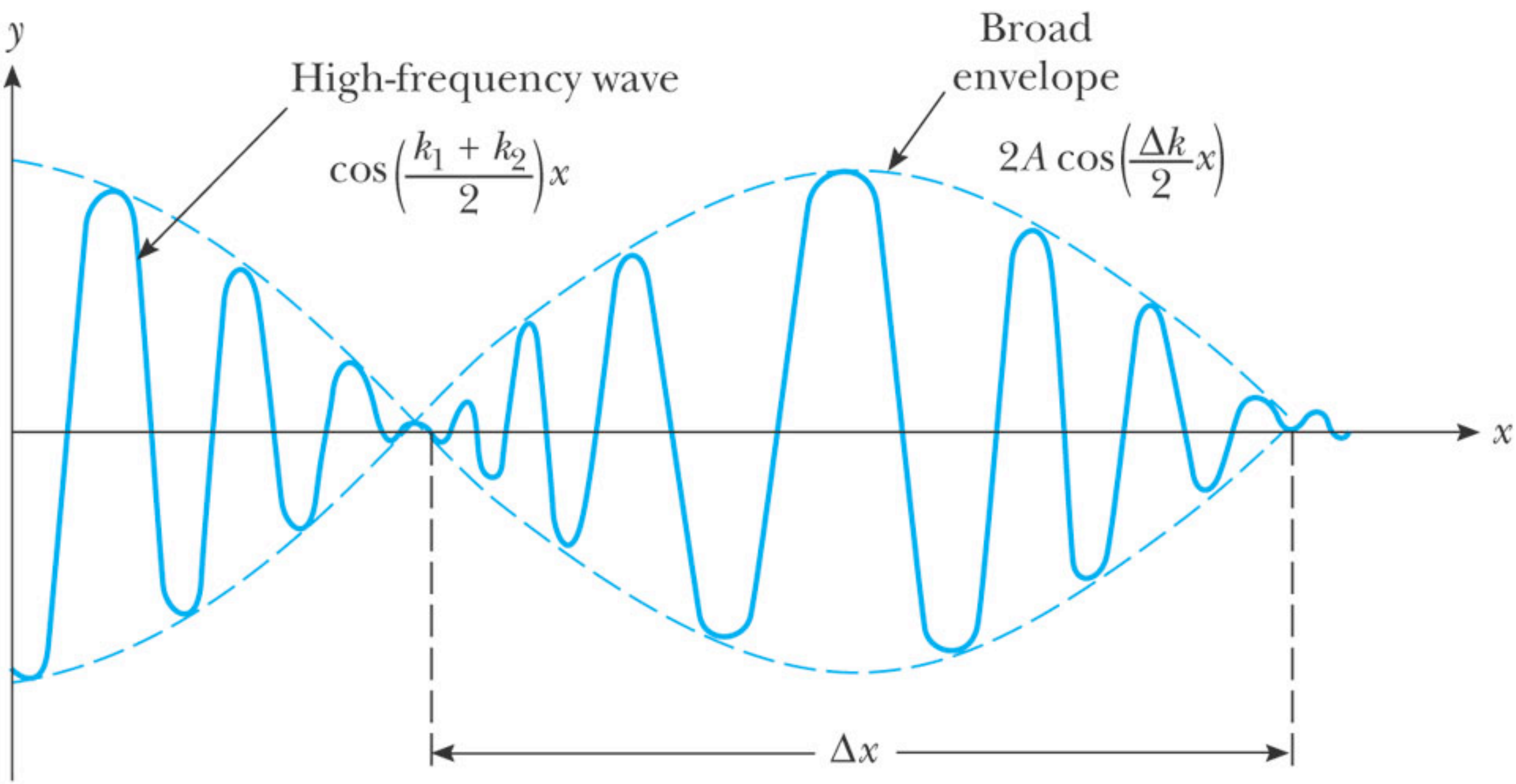
$$2A\cos(\Delta\omega t/2) =$$

$$= 2A\cos(\pi\Delta f t)$$

$$\pi\Delta f\Delta t = \pi$$

$$\boxed{\Delta f\Delta t = 1}$$

Fig. 5-18, p. 165

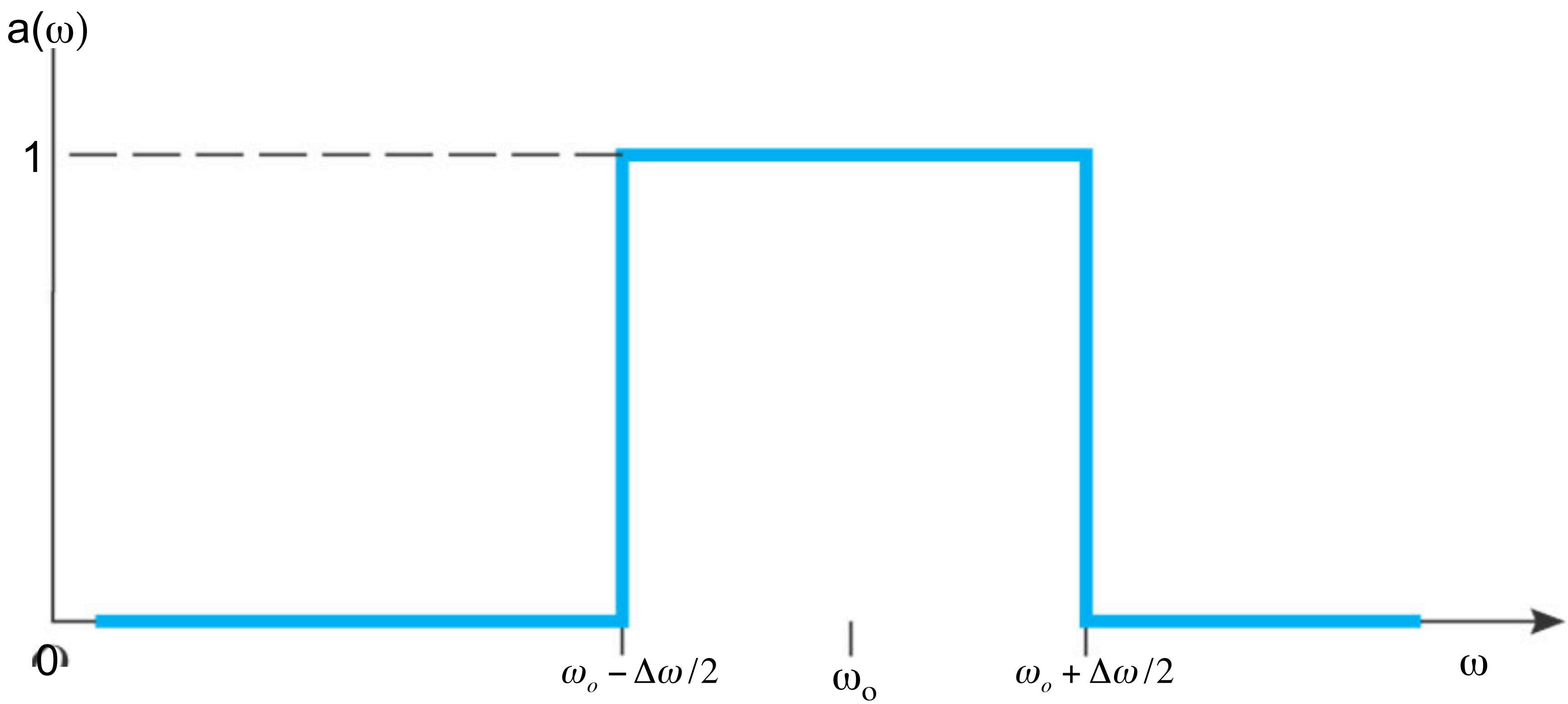


Look in space at any instant of time

$$(1/2)\Delta k \Delta x = \pi$$

Fig. 5-19, p. 166

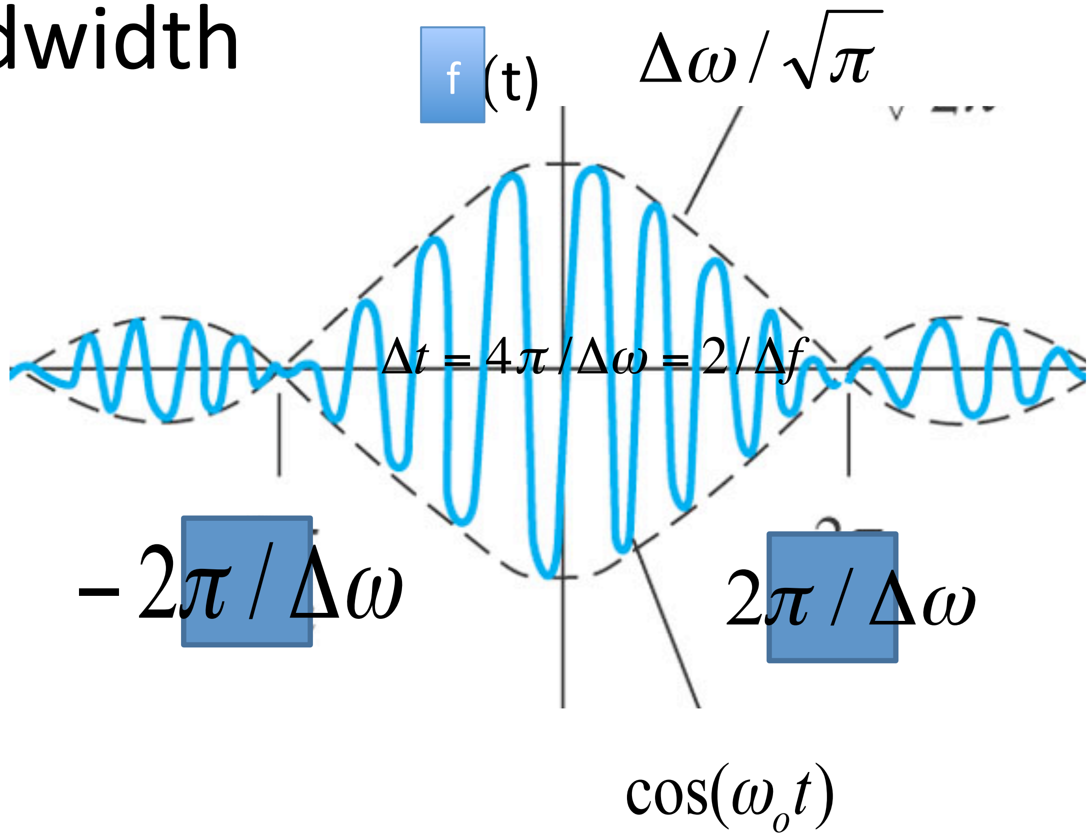
Take many close frequencies instead of two close frequencies



$$f(t) = A\int_0^\infty a(\omega)\cos\omega t d\omega$$

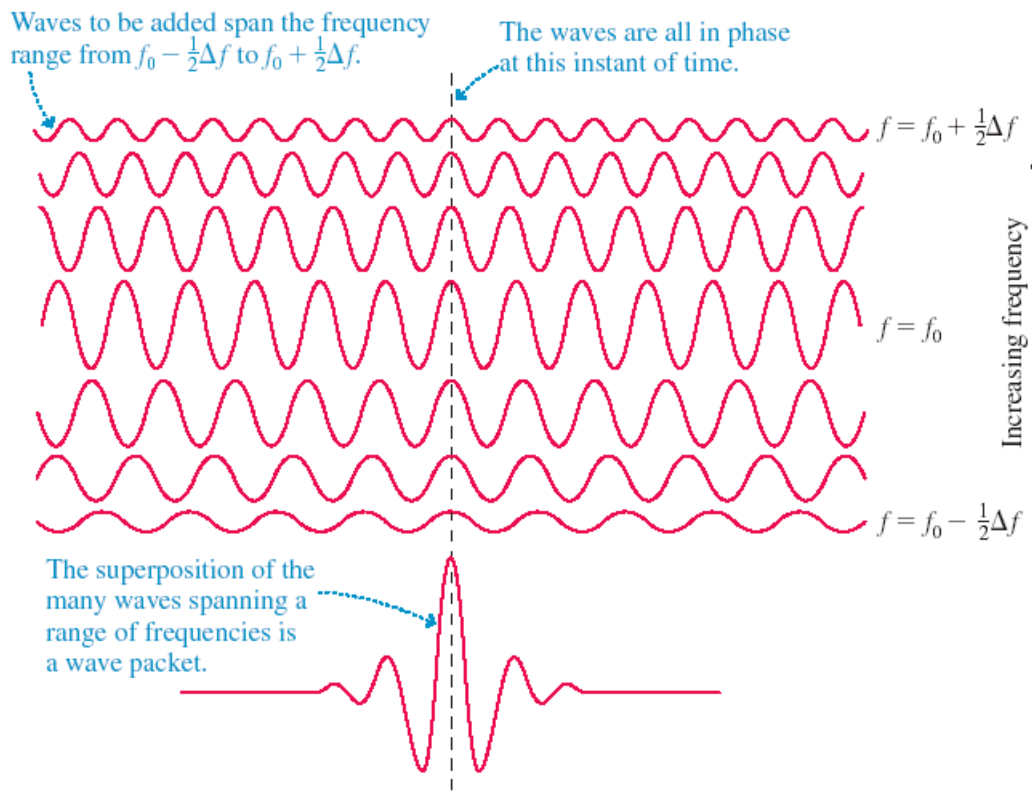
Fig. 5-23, p. 172

Bandwidth



$$\Delta f \Delta t \approx 1$$

FIGURE 40.14 A single wave packet is the superposition of many component waves of similar wavelength and frequency.



Wave Packets

Suppose a single nonrepeating wave packet of duration Δt is created by the superposition of *many* waves that span a range of frequencies Δf.

Fourier analysis shows that for *any* wave packet

$$\Delta f \Delta t \approx 1$$

We have not given a precise definition of Δt and Δf for a general wave packet.

The quantity Δt is "about how long the wave packet lasts," while Δf is "about the range of frequencies needing to be superimposed to produce this wave packet."

What minimum bandwidth must a medium have to transmit a 100-ns-long pulse?

A. 100 MHz
B. 0.1 MHz
C. 1 MHz
D. 10 MHz
E. 1000 MHz

$$\Delta f \Delta t \approx 1$$

What minimum bandwidth must a medium have to transmit a 100-ns-long pulse?

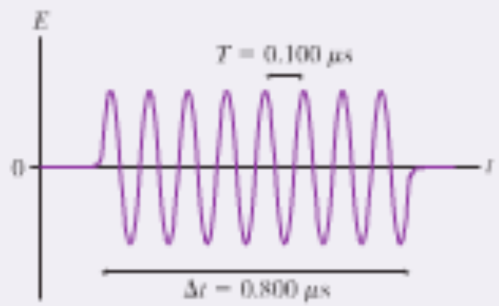
A. 100 MHz
B. 0.1 MHz
C. 1 MHz
✔ **D. 10 MHz**
E. 1000 MHz

EXAMPLE 40.4 Creating radio-frequency pulses

A short-wave radio station broadcasts at a frequency of 10.000 MHz. What is the range of frequencies of the waves that must be superimposed to broadcast a radio-wave pulse lasting 0.800 μs?

MODEL A pulse of radio waves is an electromagnetic wave packet, hence it must satisfy the relationship $\Delta f \Delta t \approx 1$.

FIGURE 40.15 A pulse of radio waves.



VISUALIZE FIGURE 40.15 shows the pulse.

SOLVE The period of a 10.000 MHz oscillation is 0.100 μs. A pulse 0.800 μs in duration is 8 oscillations of the wave. Although the station broadcasts at a nominal frequency of 10.000 MHz, this pulse is not a pure 10.000 MHz oscillation. Instead, the pulse has been created by the superposition of many waves whose frequencies span

$$\Delta f \approx \frac{1}{\Delta t} = \frac{1}{0.800 \times 10^{-6}\ \text{s}} = 1.250 \times 10^{6}\ \text{Hz} = 1.250\ \text{MHz}$$

This range of frequencies will be centered at the 10.000 MHz broadcast frequency, so the waves that must be superimposed to create this pulse span the frequency range

$$9.375\ \text{MHz} \le f \le 10.625\ \text{MHz}$$

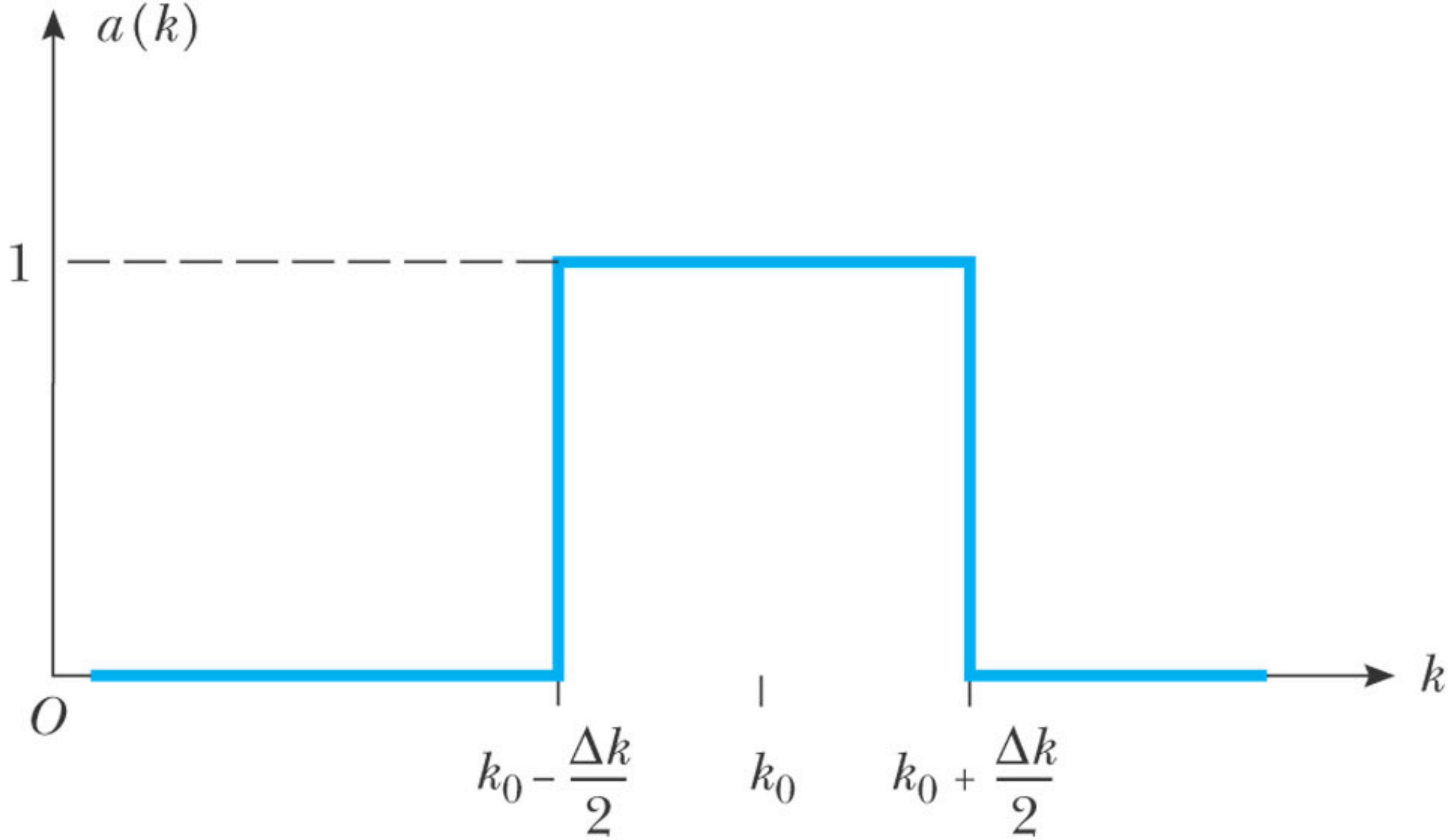


Fig. 5-23, p. 172

$$\lambda = 2\pi / k$$

$$\lambda = h / p$$

$$h / p = 2\pi / k$$

$$p = \hbar k$$

$\Delta k \Delta x = 4\pi$

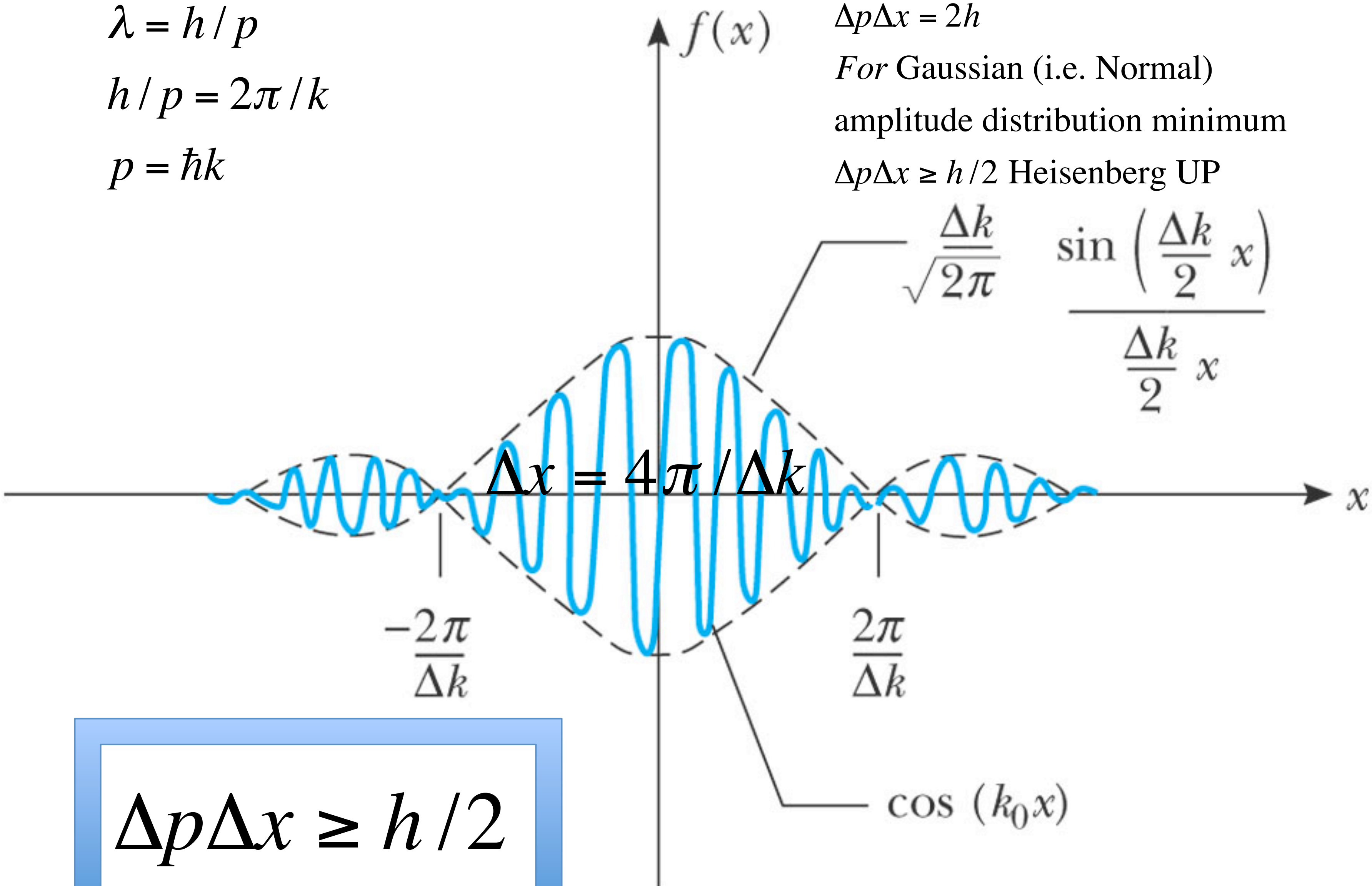
2

$\Delta p = \hbar \Delta k$

$\Delta p \Delta x = 2h$

For Gaussian (i.e. Normal) amplitude distribution minimum

$\Delta p \Delta x \geq h/2$ Heisenberg UP



$$\Delta p \Delta x \geq h/2$$

Group velocity

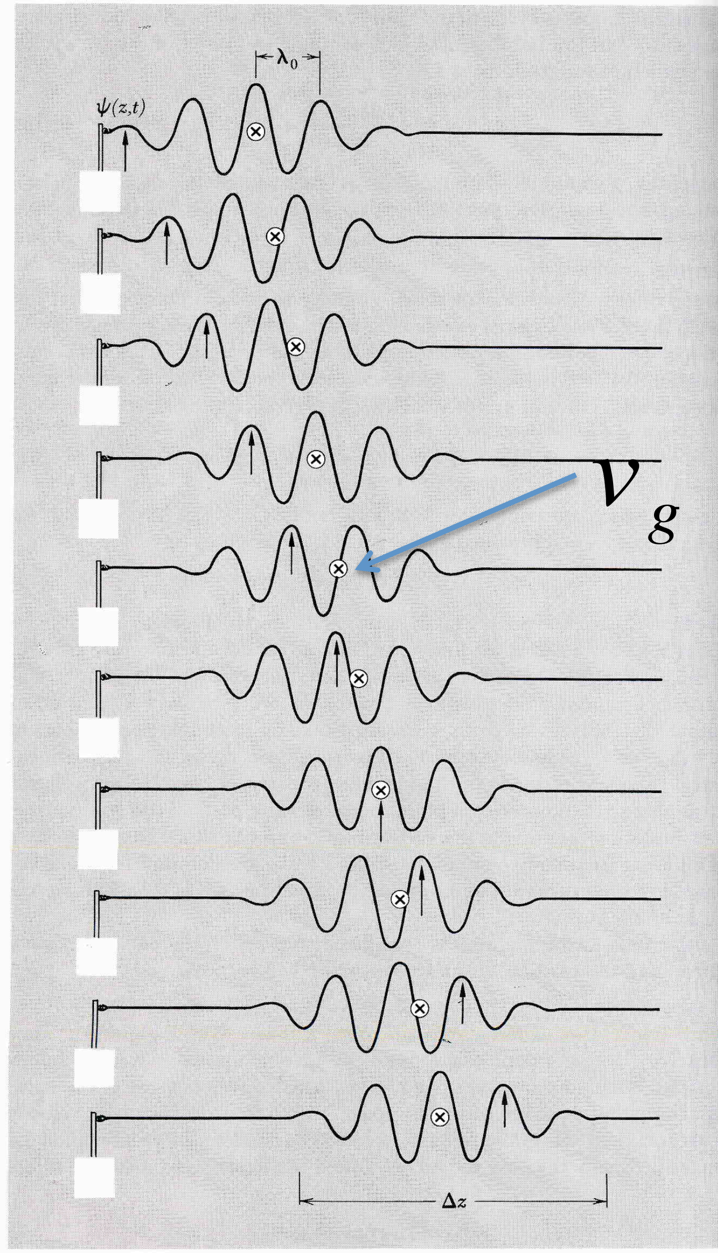


Fig. 6.7 Wave packet with phase velocity twice the group velocity. The arrow travels at the phase velocity, following a point of constant phase for the dominant wavelength. The cross travels at the group velocity with the packet as a whole.

$$v_g = \frac{\partial \omega}{\partial k}$$

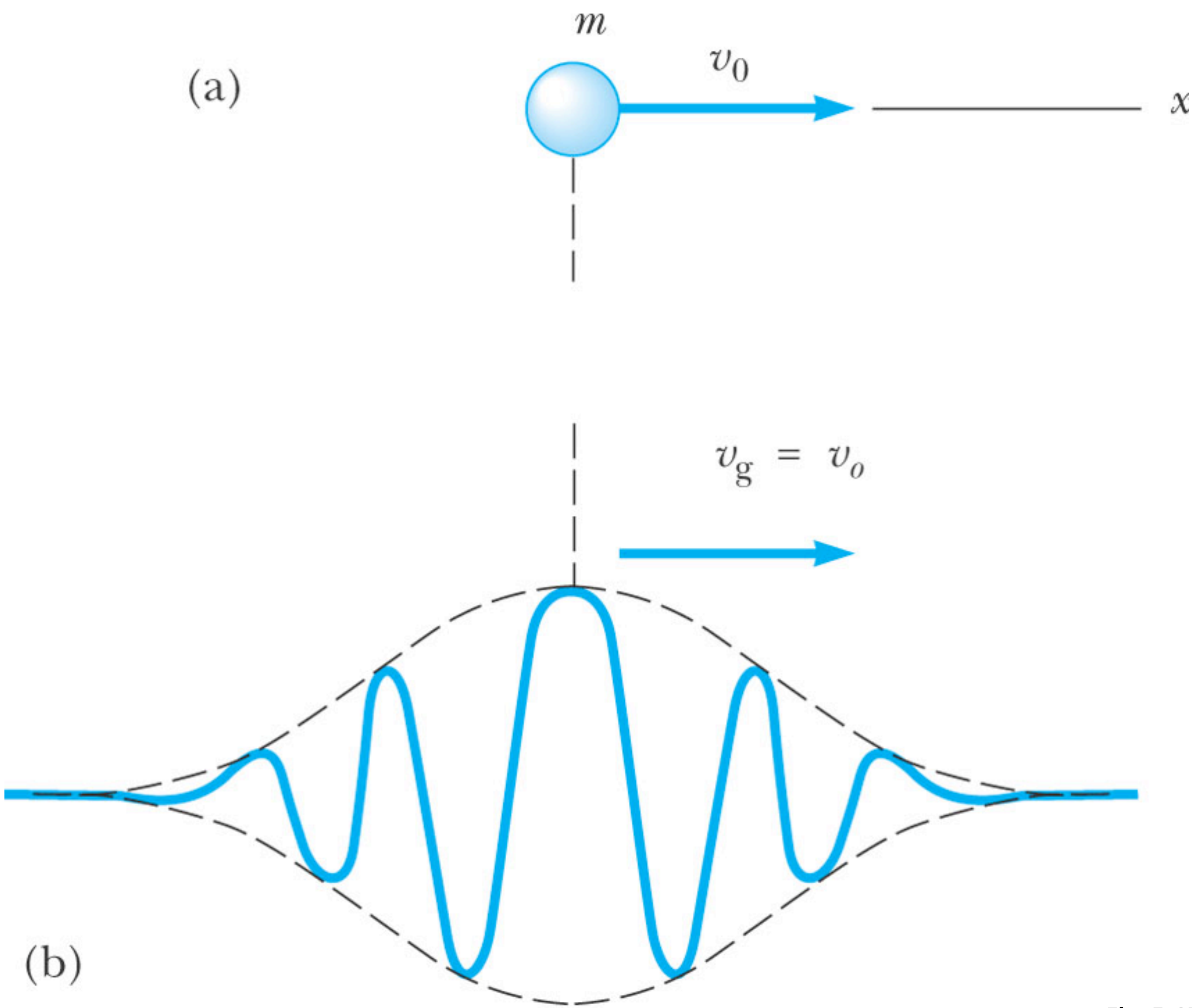


Fig. 5-17, p. 164

FIGURE 40.12 History graph of a wave packet with duration Δt.

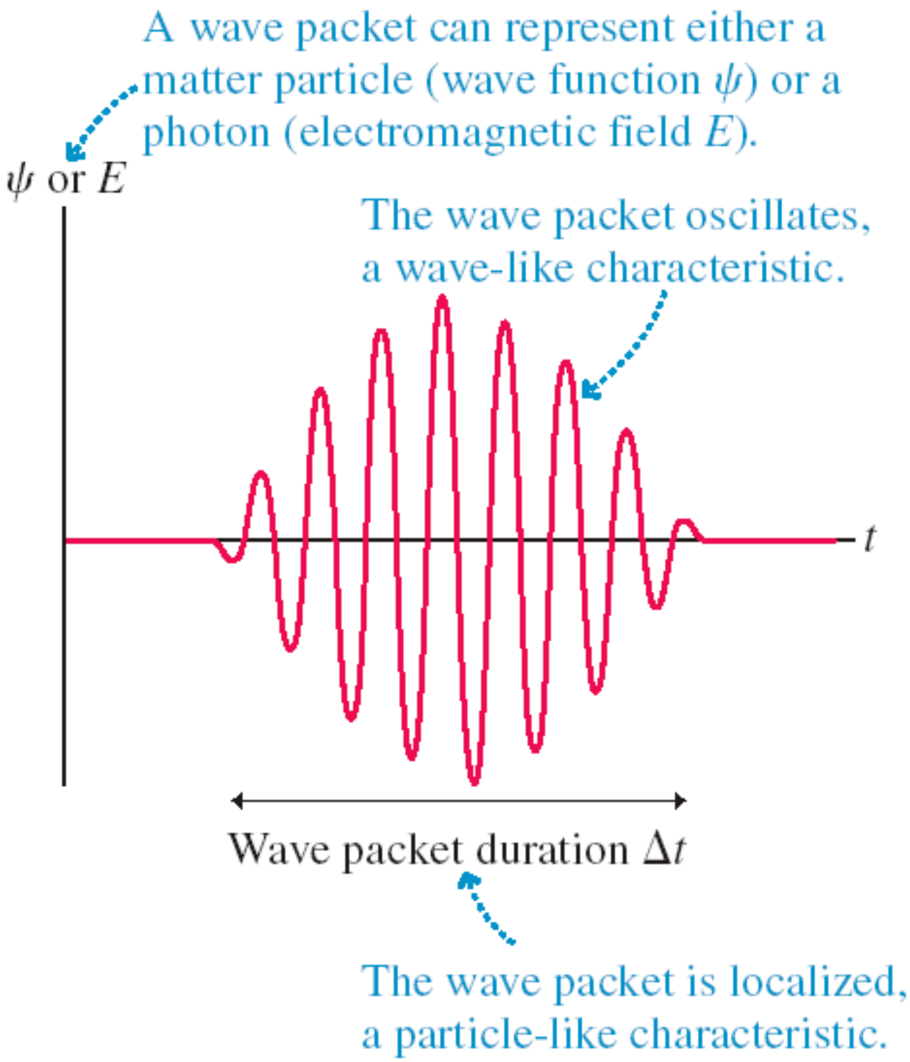
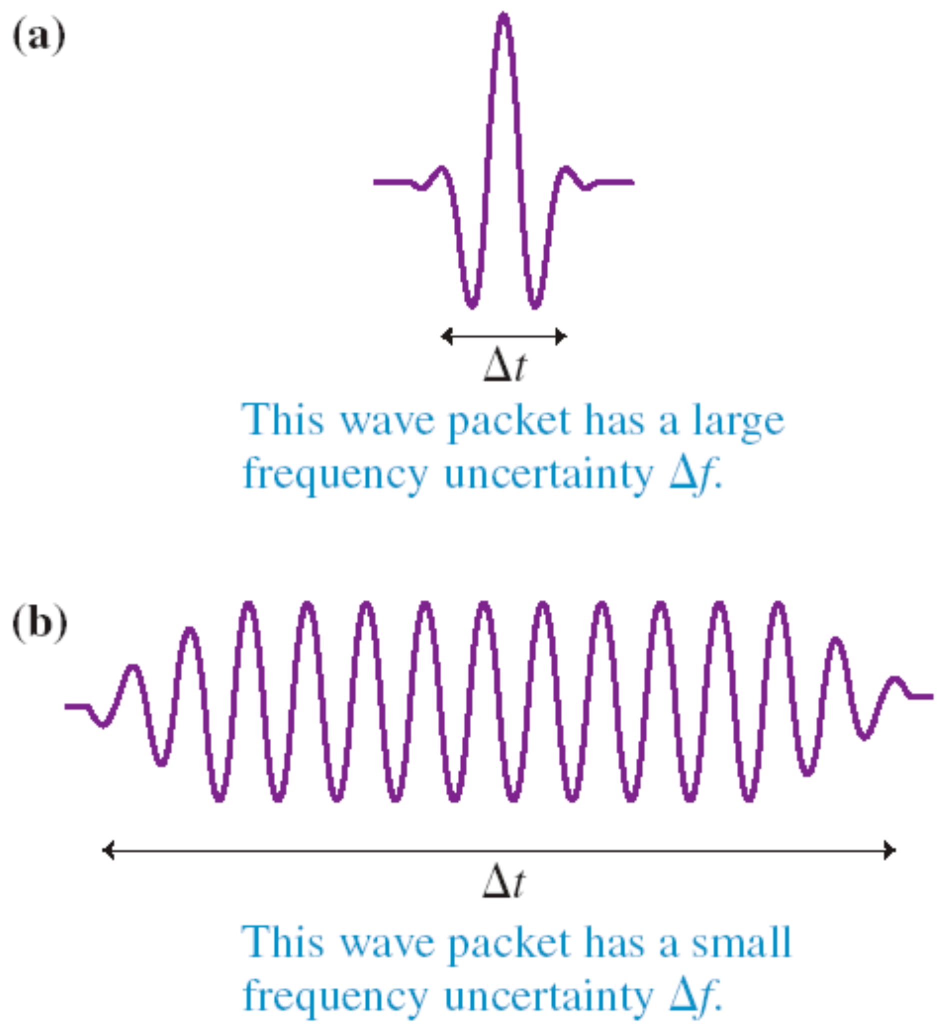
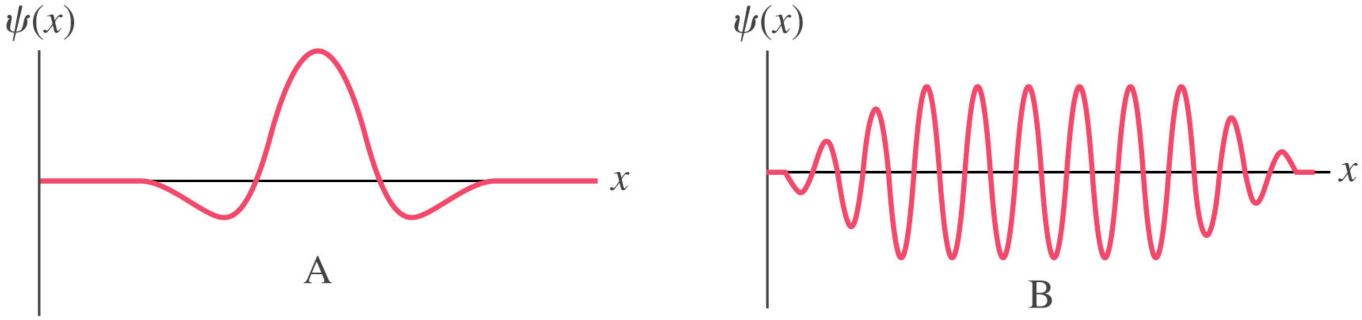


FIGURE 40.16 Two wave packets with different Δt.

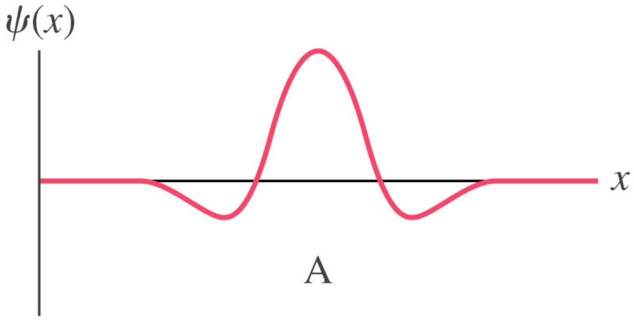


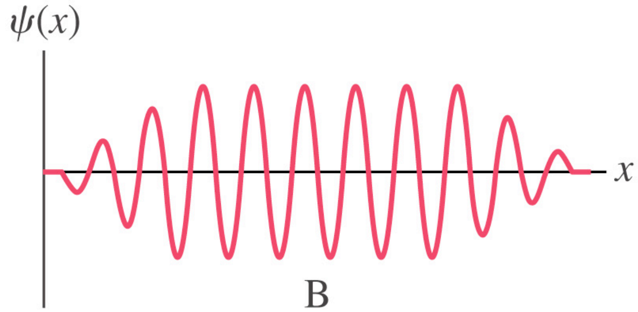
Which of these particles, A or B, can you locate more precisely?



A. A
B. B
C. Both can be located with same precision.

Which of these particles, A or B, can you locate more precisely?





A. A
B. B
C. Both can be located with same precision.