PHYS 270-SPRING 2011

Dennis Papadopoulos

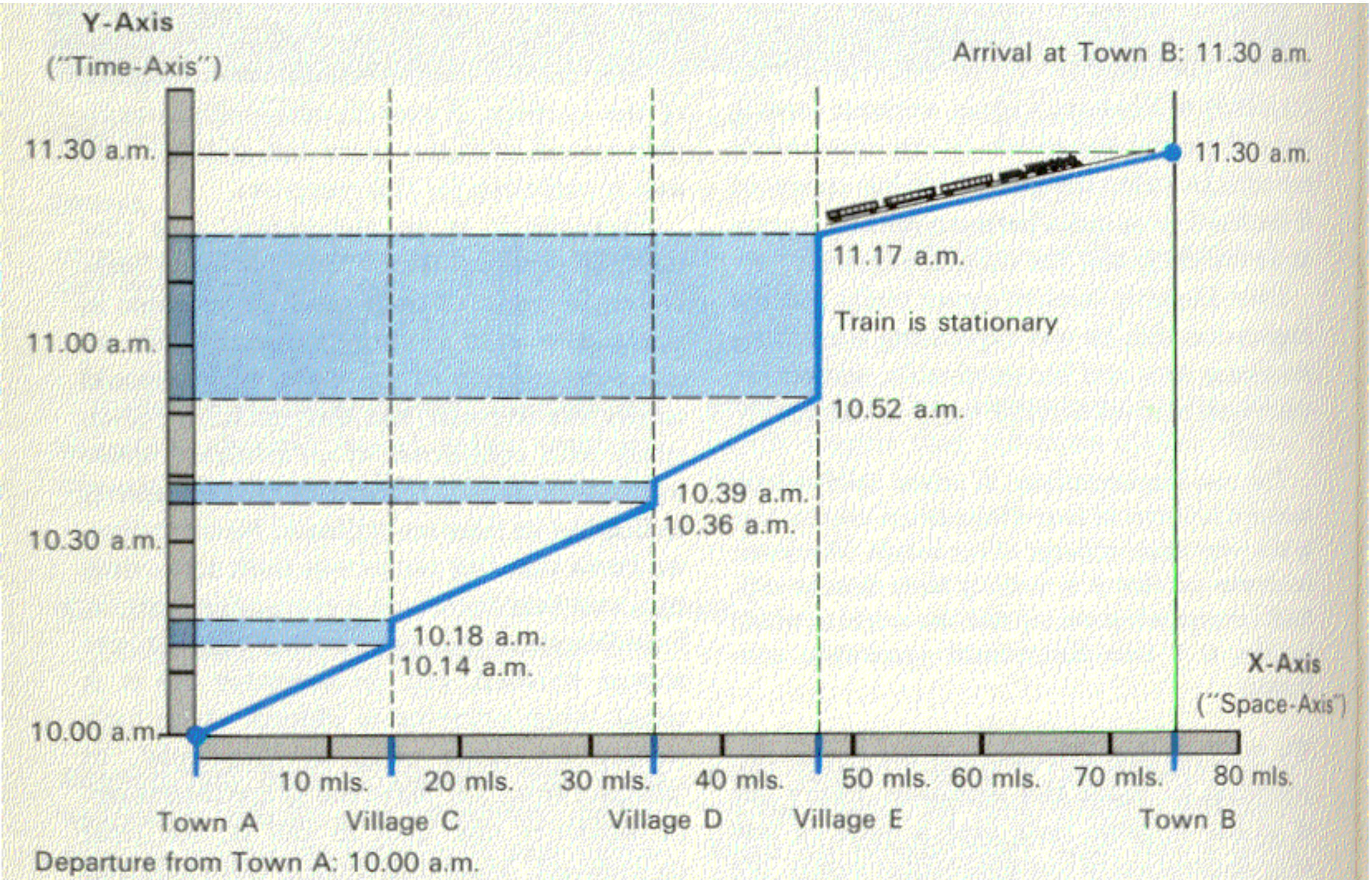
LECTURE # 22

RELATIVITY III

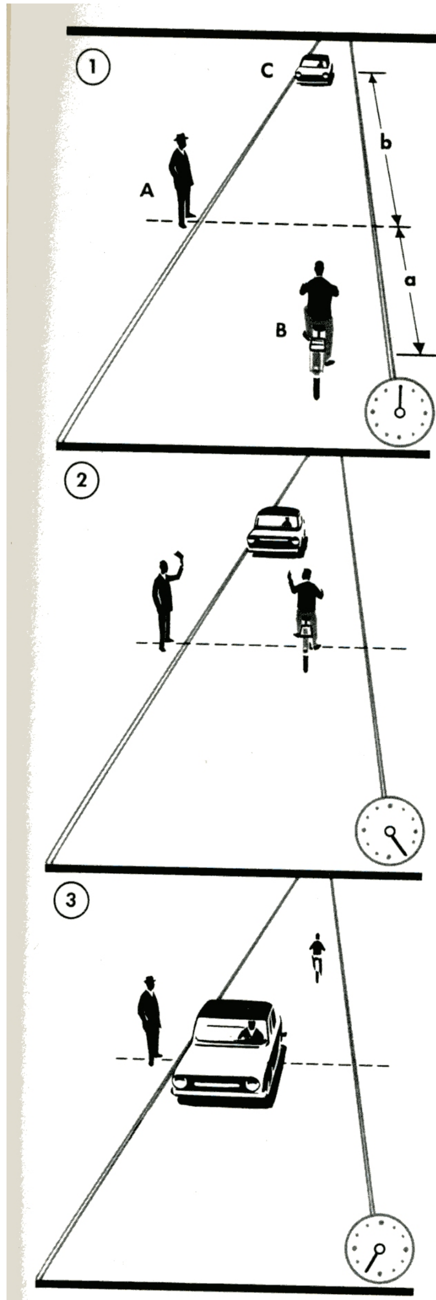
SPACE TIME DIAGRAMS

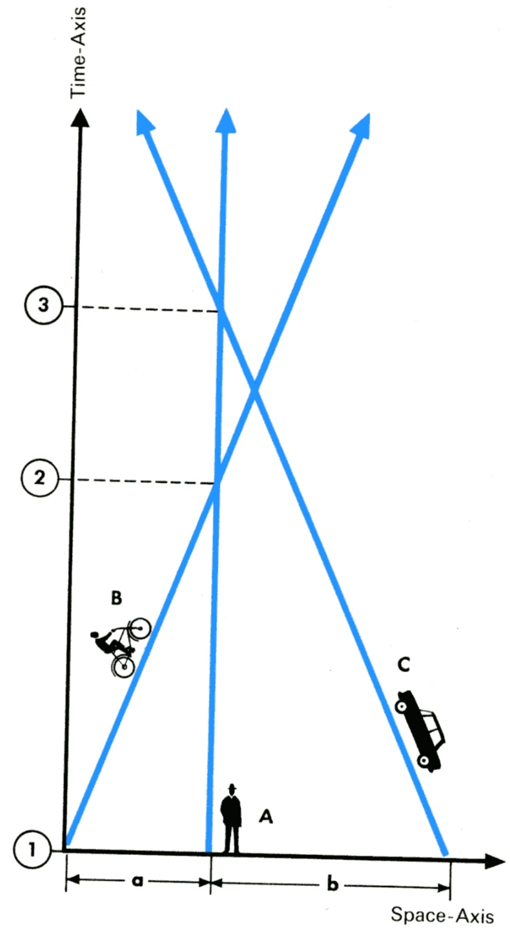
APRIL 28, 2011

Space-time Diagrams



The timetable of a train travelling from town A to town B can be presented in a space-time diagram.

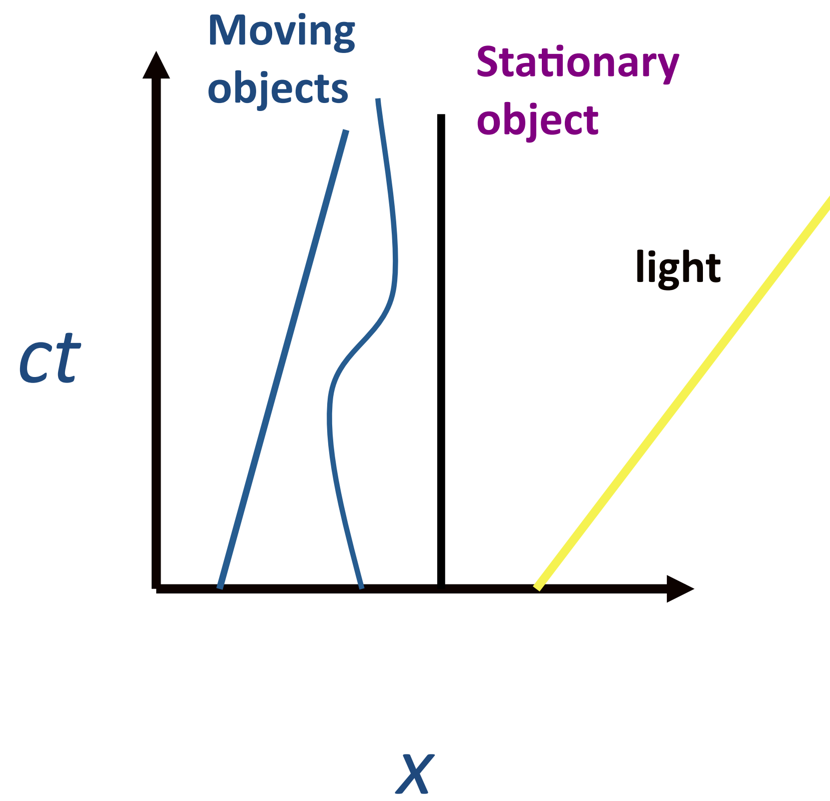




An everyday occurrence is represented in terms of a space–time diagram. An observer A stands at the edge of a street, a cyclist B passes him, then from the opposite direction a motorist C passes by. *Left:* the three situations during this process.
Above: the space–time diagram

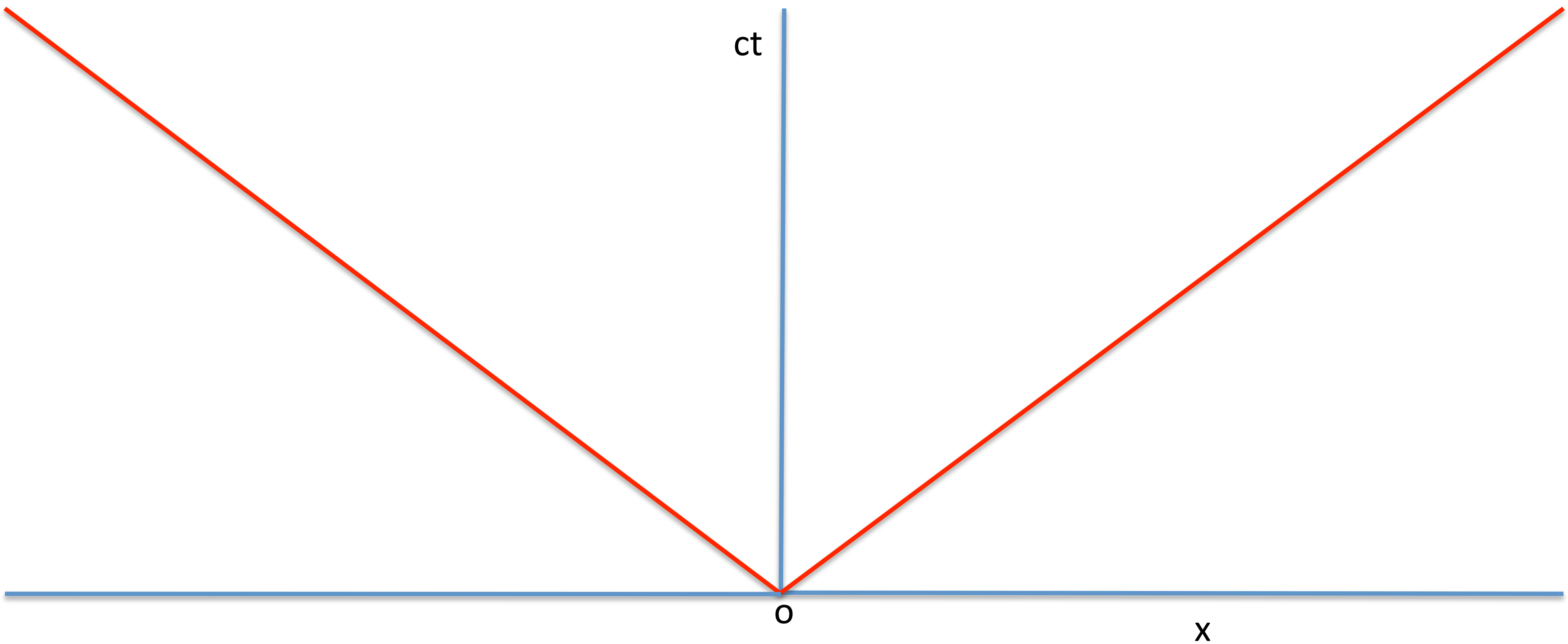
Space-time diagrams

- Because space and time are “mixed up” in relativity, it is often useful to make a diagram of events that includes both their space and time coordinates.
- This is simplest to do for events that take place along a line in space (one-dimensional space)
  - Plot as a 2D graph
  - use two coordinates: *x* and *ct*
- Can be generalized to events taking place in a plane (two-dimensional space) using a 3D graph (volume rendered image): *x, y* and *ct*
- Can also be generalized to events taking place in 3D space using a 4D graph, but this is difficult to visualize



World lines of events

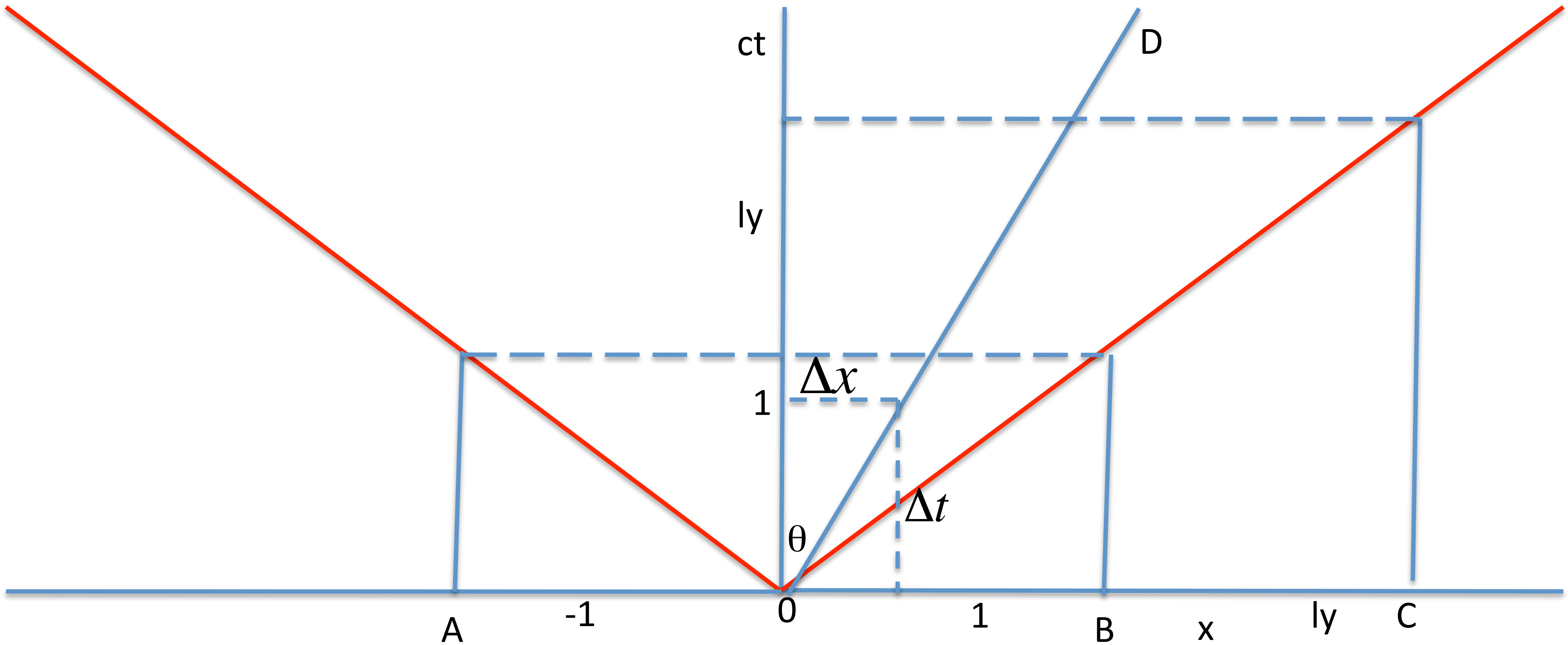
Units of Coordinates-Lightcone



1. The time axis ct has dimensions of distance, just as x.
2. If you select as units of time years the dimensions of the vertical axis are light-years. It is then convenient to select the same units of distance in the horizontal axis.

Similarly if you select as time units seconds, the vertical axis will be light-seconds and the units of x will be light-seconds also. Etc., etc.

Units of Coordinates-Light-cone



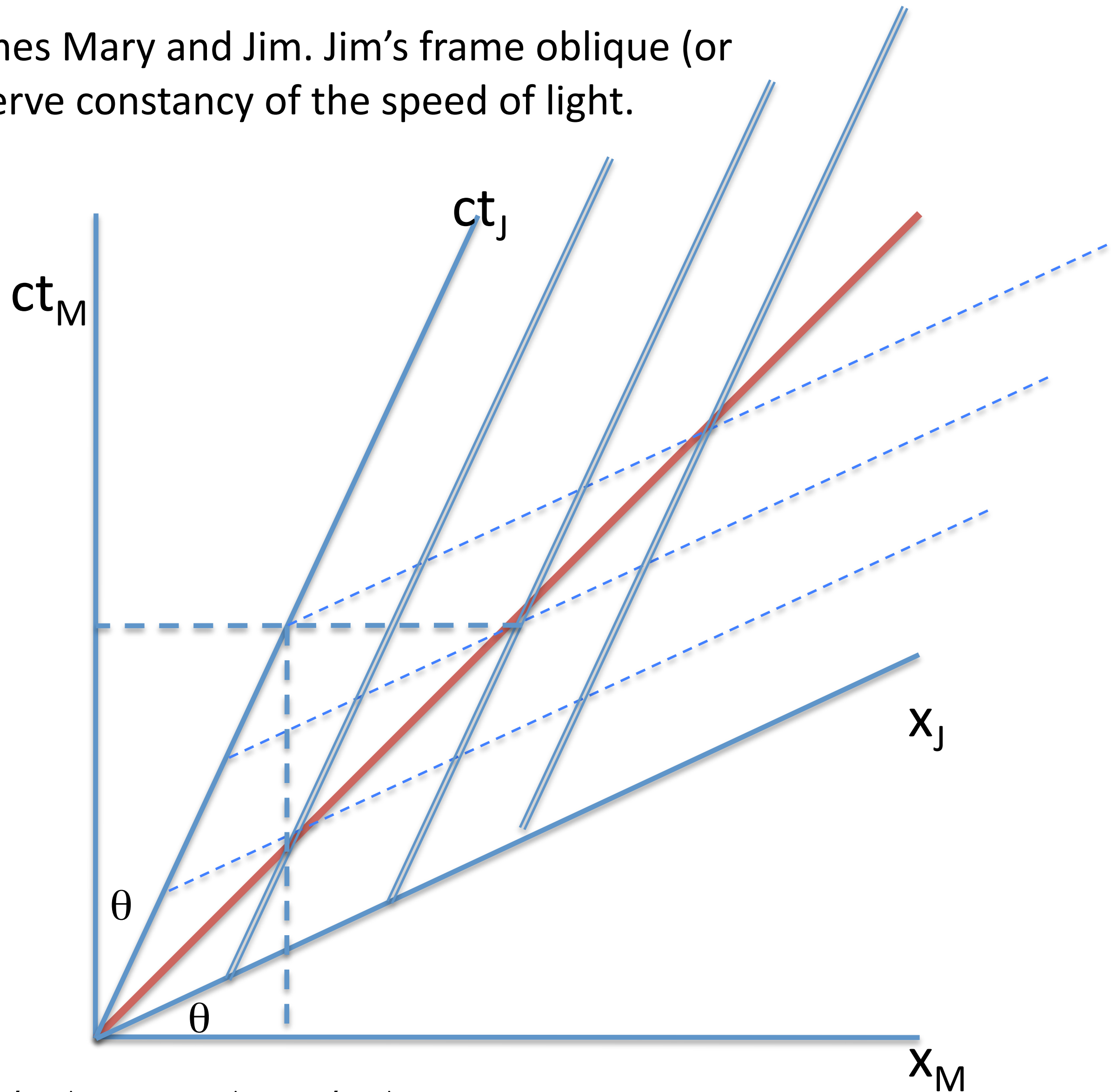
$$slope = \Delta x / c\Delta t = v / c = \beta = \tan\theta$$

$$\gamma^2 = 1/1 - \tan^2\theta = 1/(2 - \sec^2\theta)$$

Red lines are 45 degrees $\beta=1$ − world line of light or light cone. No information can move outside the light cone.

How to determine simultaneity and event observation

Two inertial frames Mary and Jim. Jim's frame oblique (or slanted) to preserve constancy of the speed of light.



$$\beta = 1 = \Delta x_M / c\Delta t_M = \Delta x_J / c\Delta t_J$$

Time dilation. Event A Jim's birthday. Coordinates $(ct_J, 0)$ and (ct_M, x_M)

$$(ct)^2 - x^2 = constant$$

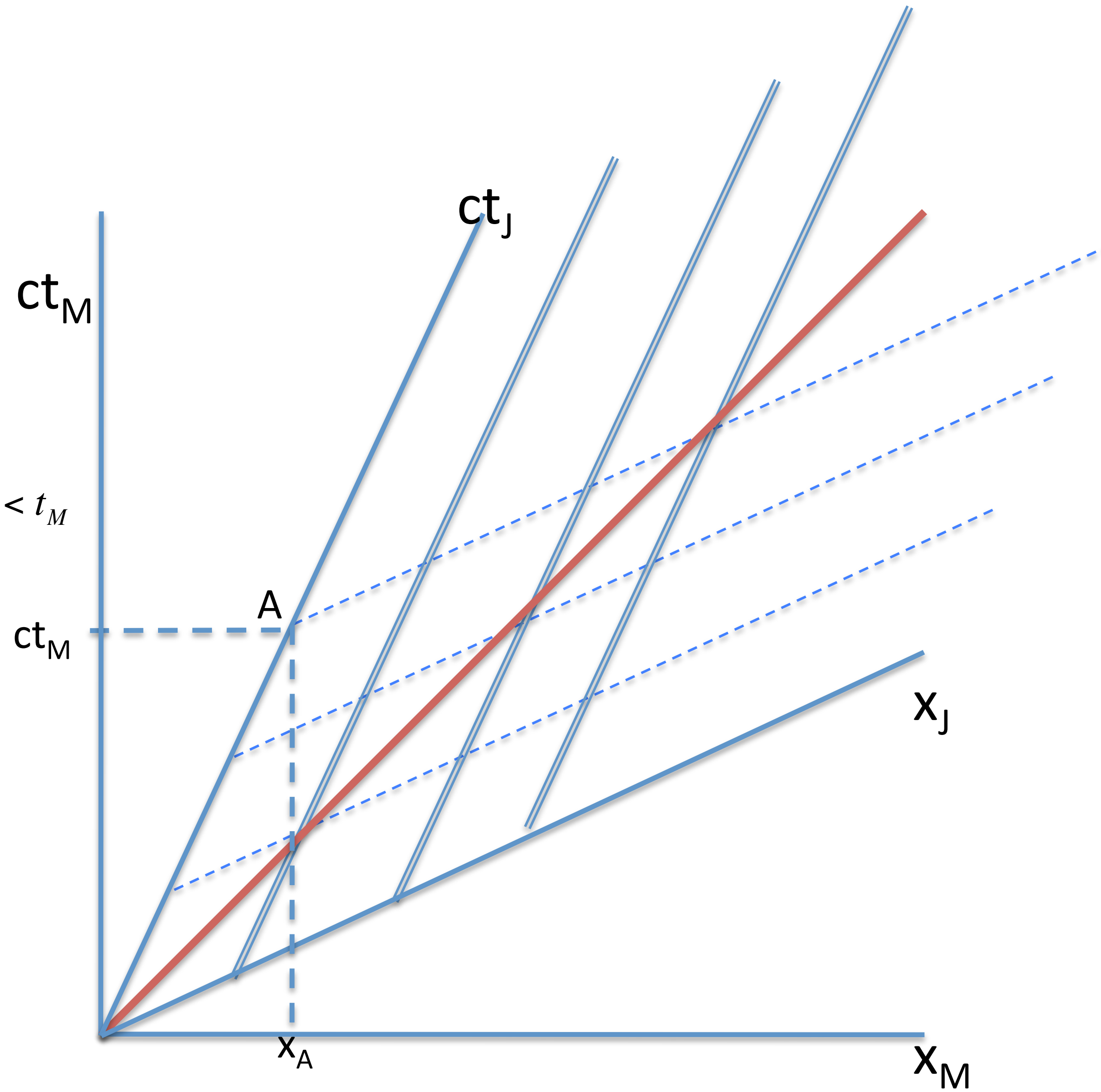
$$(ct_M)^2 - x_M^2 = (ct_J)^2 - x_J^2 = (ct_J)^2 . t_j < t_M$$

$$x_M = V_J t_M$$

$$(c^2 - V_J^2) t_M^2 = (ct_J)^2$$

$$t_M = t_J / \sqrt{1 - V_J^2 / c^2} = \gamma_J t_J \equiv \gamma_J \tau_J$$

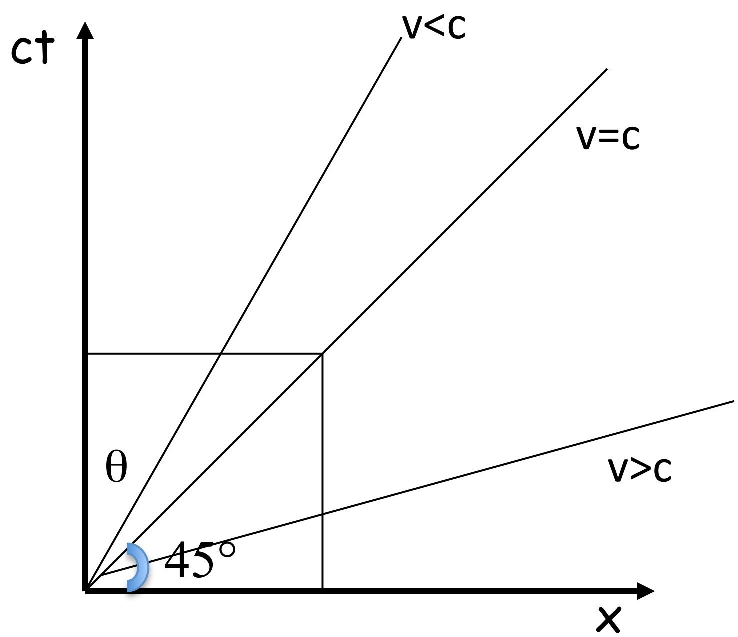
We derived time dilation



Time, the fourth dimension?

“Spacetime”

In x,y space the two space dimensions are interchangeable if they have the same units. A similar relationship can be used to express the relationship between space and time in relativity.

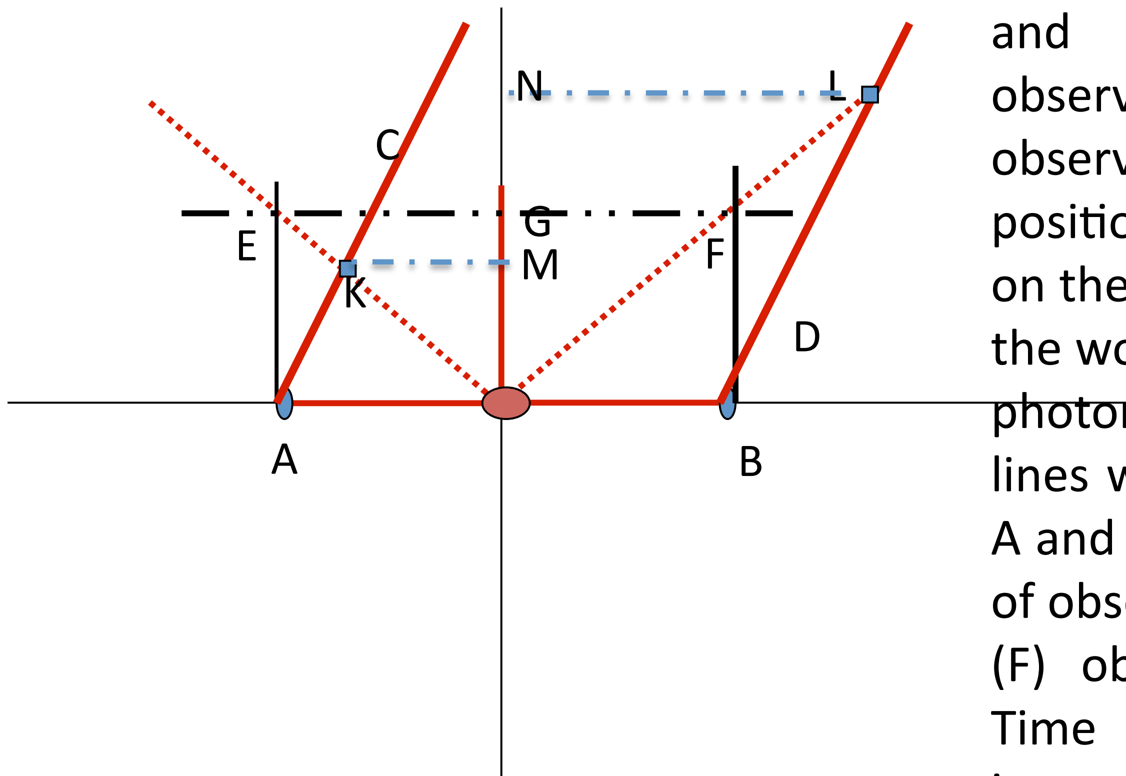


Light propagating in one dimension in a spacetime coordinate system as viewed from a frame S. The distance traveled is equal to the speed of light times the time elapsed.

t in years distance in lightyears
t in secs distance in lighteseconds

$\beta = v/c = \tan\theta$

$$\gamma^2 = \frac{1}{2 - \sec^2\theta}$$



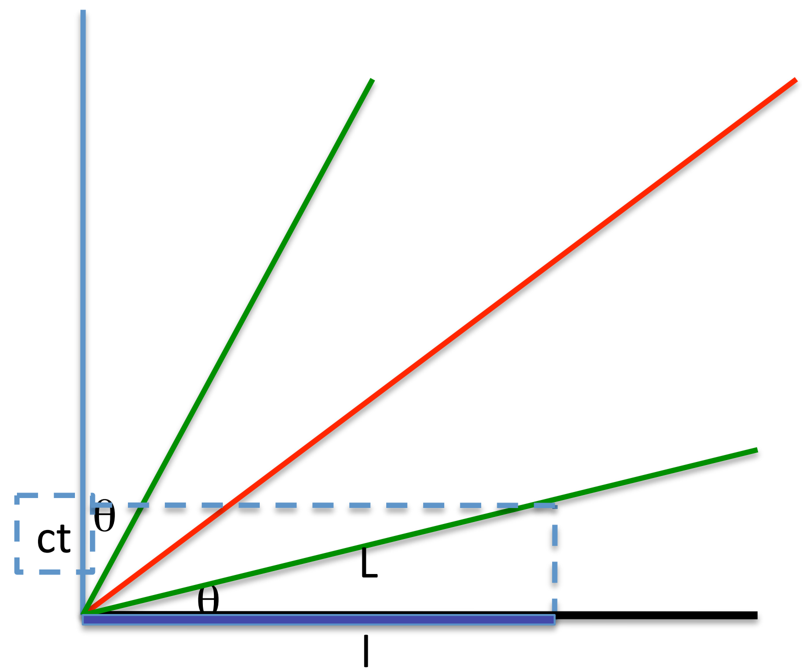
SIMULTANEITY

Event K first, events E and F next , event L last

Flashes emitted at the red point at t=0 and viewed by two stationary observers A and B and two moving observers C and D. At t=0 all observers located at their positions marked with blue on the x-axis. Dotted red lines the world lines of the emitted photons (light cones). Black lines world lines of observers A and B. Red lines world lines of observers C and D. Event E (F) observer A (B) sees flash. Time of events given by intercept G of dotted black line with time axis. Events simultaneous. Events K (L) when observer C (D) sees flash. Time of events M and N.

Length contraction

$$\tan\theta = \beta$$



$$(ct)^2 - l^2 = -L^2$$

$$\tan^2\theta = \beta^2 = \frac{(ct)^2}{l^2}$$

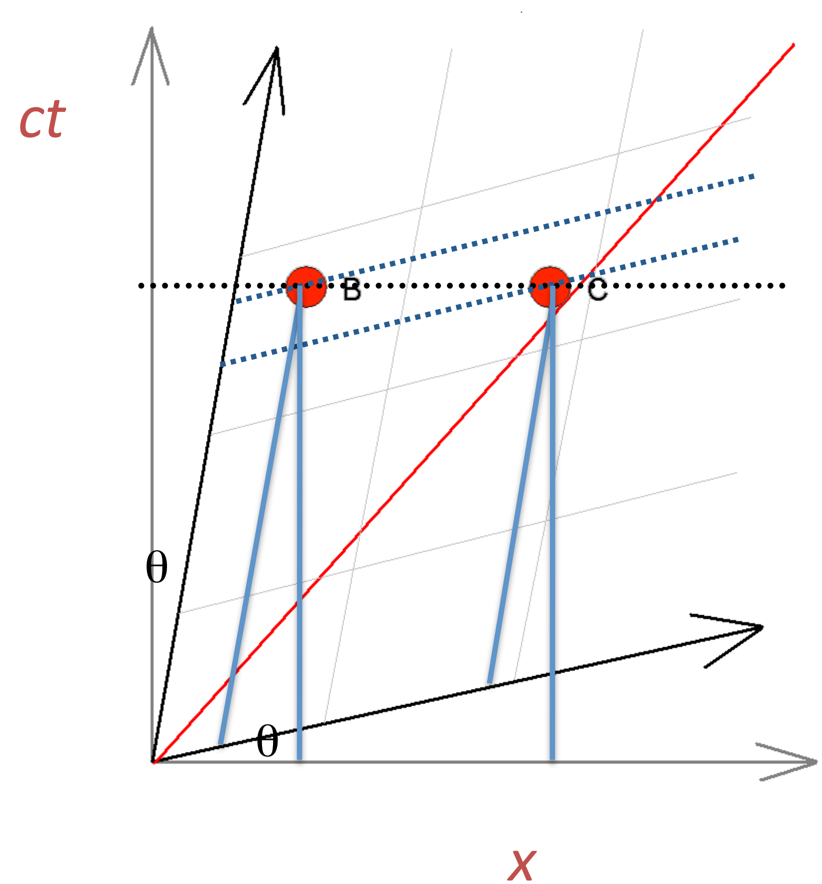
$$l^2(1-\beta^2) = L^2$$

$$L = \sqrt{1-\beta^2}\, l$$

$$l = L/\gamma$$

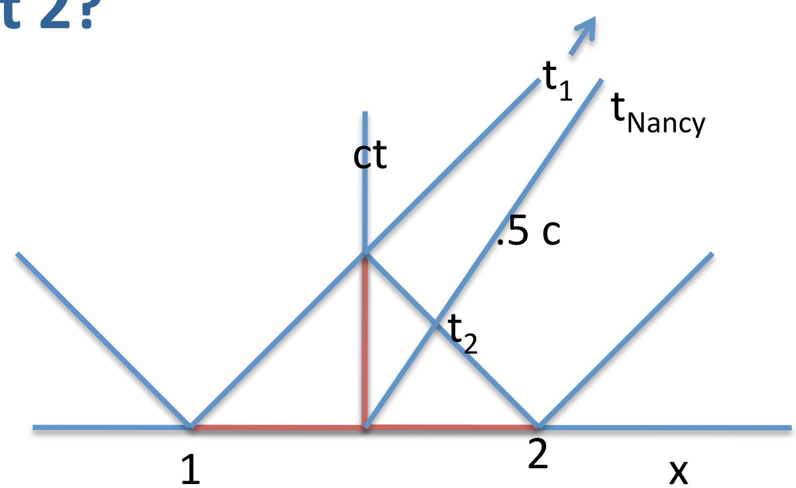
Spacetime diagrams in different frames

- Changing from one reference frame to another…
  - Affects time coordinate (time-dilation)
  - Affects space coordinate (length contraction)
  - Leads to a distortion of the space-time diagram as shown in figure.
- Events that are simultaneous in one frame are not simultaneous in another frame



A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Nancy is flying her rocket at $v = 0.5c$ in the direction from the tree toward the pole. The lightning hits the tree just as she passes by it. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Nancy, does event 1 occur before, after or at the same time as event 2?

A. before event 2
B. after event 2
C. at the same time as event 2

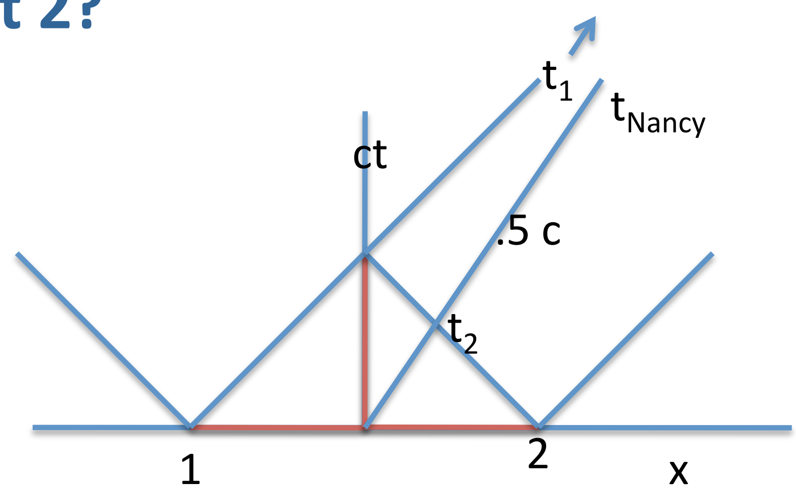


A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Nancy is flying her rocket at $v = 0.5c$ in the direction from the tree toward the pole. The lightning hits the tree just as she passes by it. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Nancy, does event 1 occur before, after or at the same time as event 2?

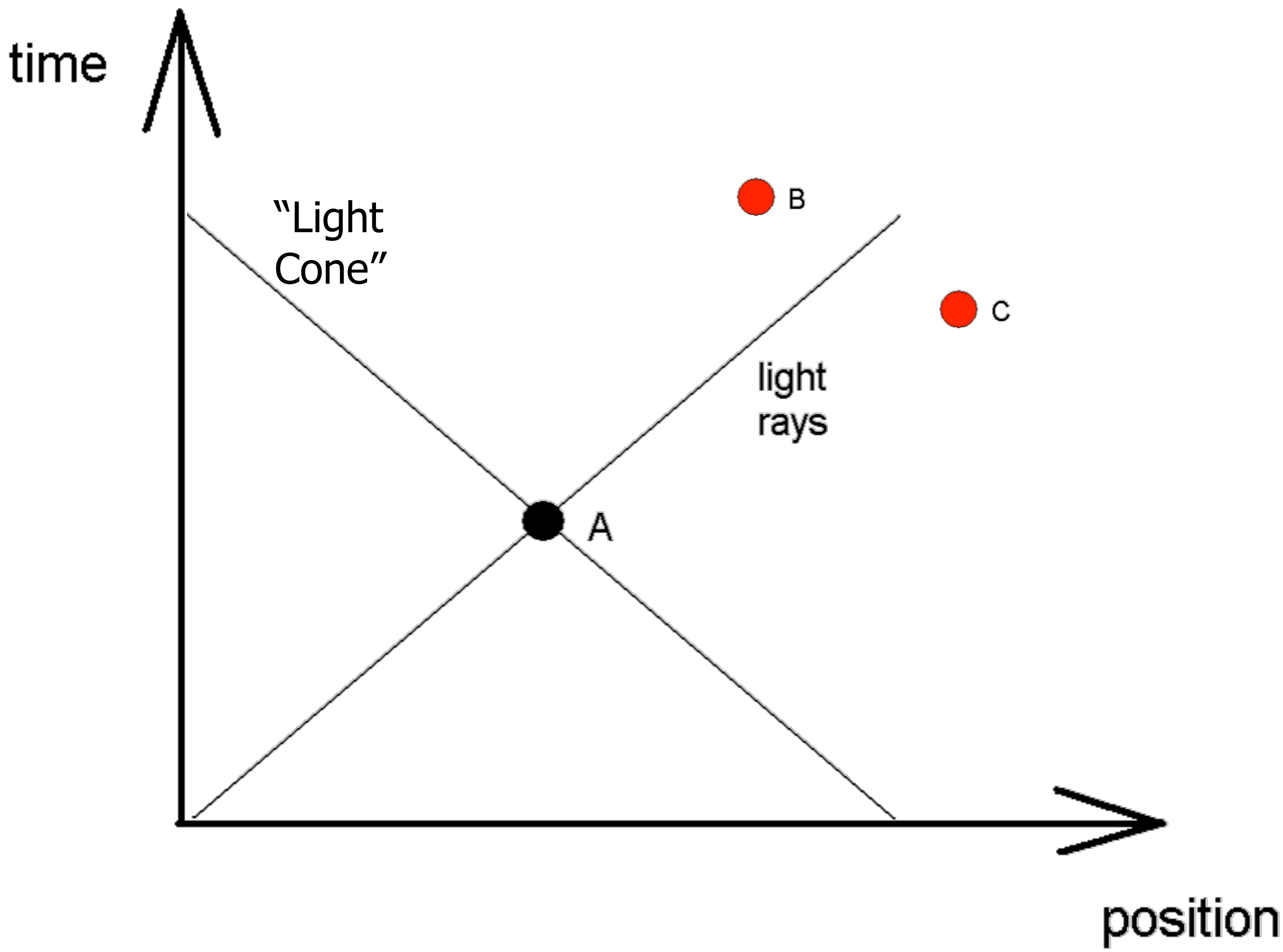
A. before event 2

✔ **B. after event 2**

C. at the same time as event 2



Light cone for event "A"



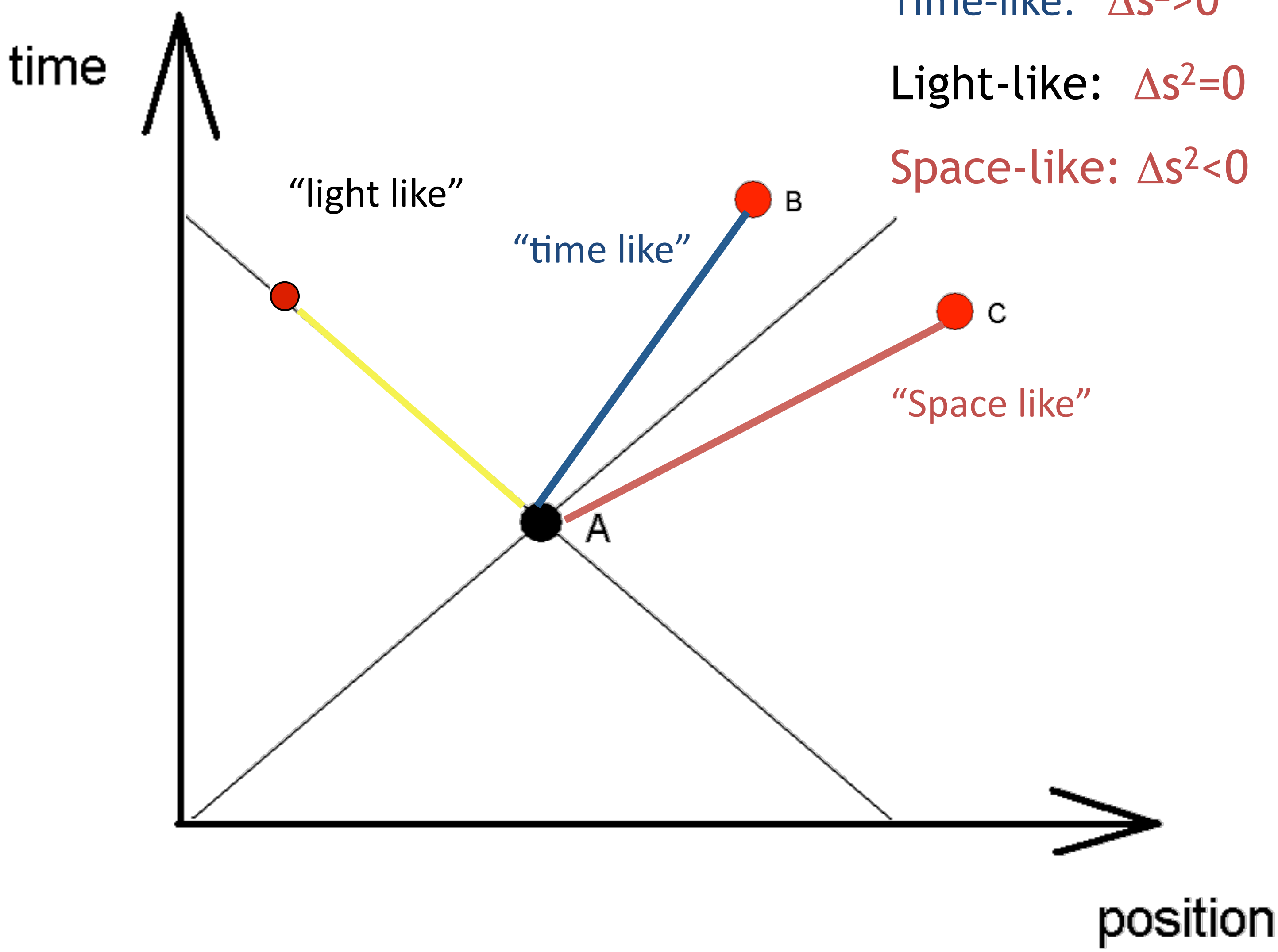
Different kinds of space-time intervals

$$\Delta s^2 = (c\Delta t)^2 - (\Delta x)^2 = inv$$

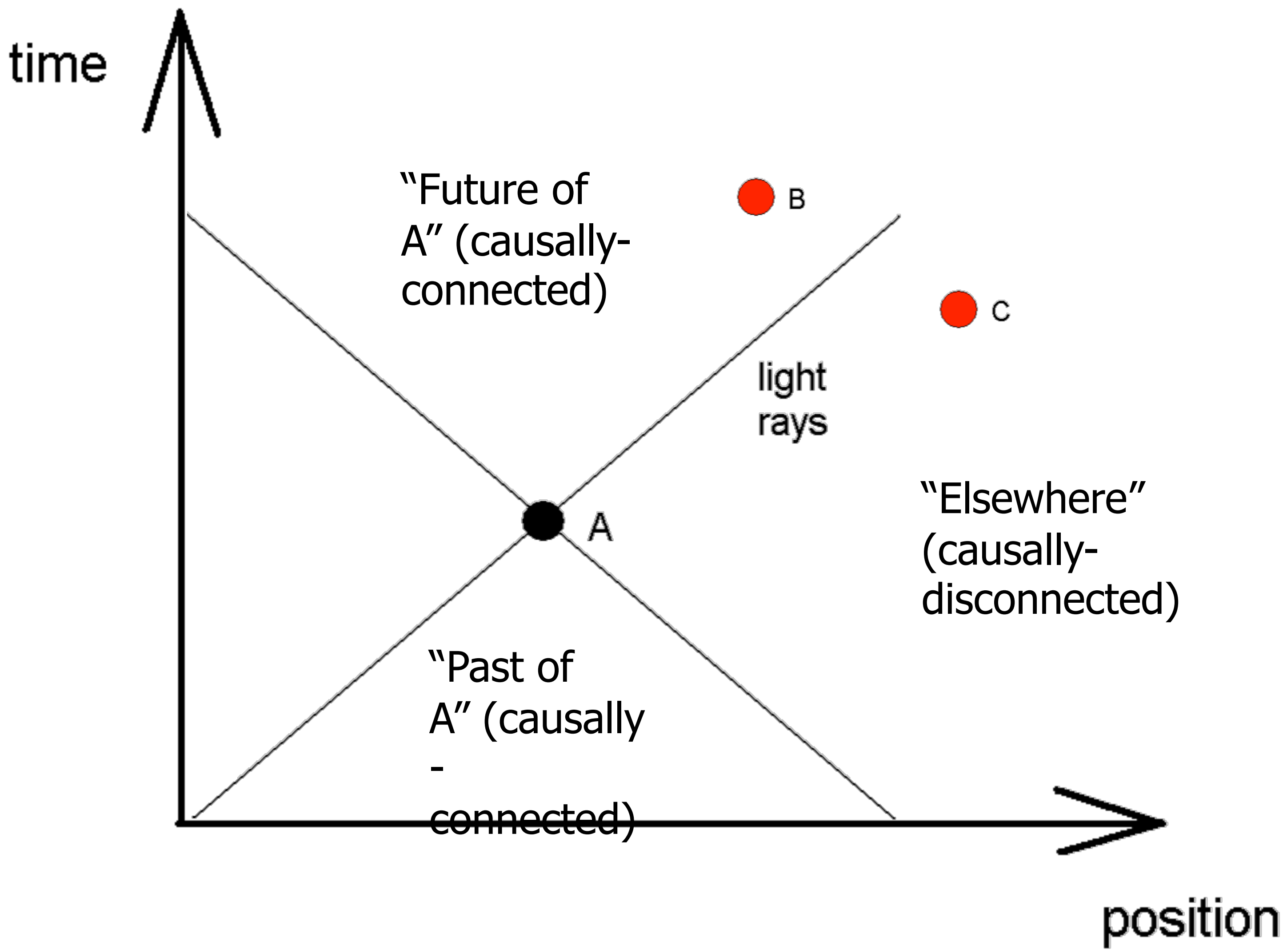
Time-like: $\Delta s^2 > 0$

Light-like: $\Delta s^2 = 0$

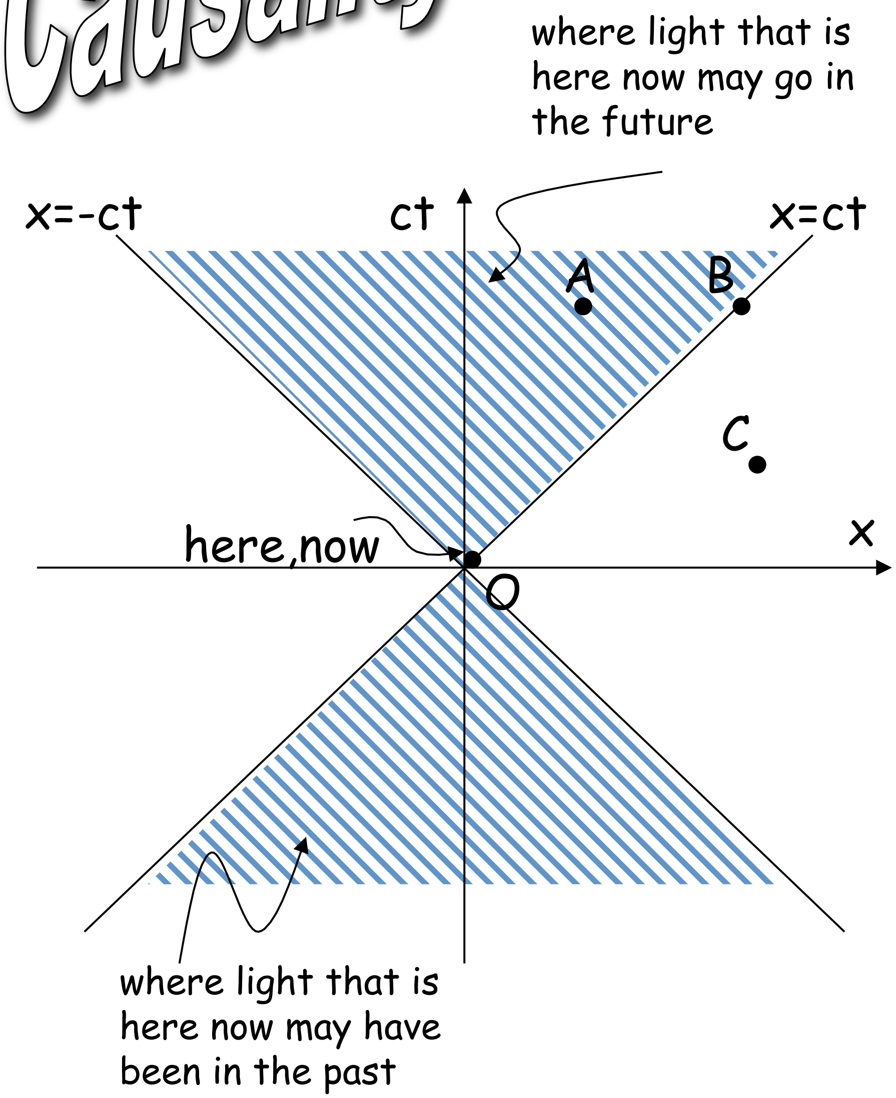
Space-like: $\Delta s^2 < 0$



Past, future and “elsewhere”.



Causality



Could an event at O cause A?

Yes, because a "messenger" at O would not have to travel at a speed greater than the speed of light to get there.

$$c\Delta t > |\Delta x|$$

Could an event at O cause B?

A light signal sent from O could reach B.

$$c\Delta t = |\Delta x|$$

Could an event at O cause C?

No, the spacetime distance between O and C is greater than could be covered by light. It would require time travel.

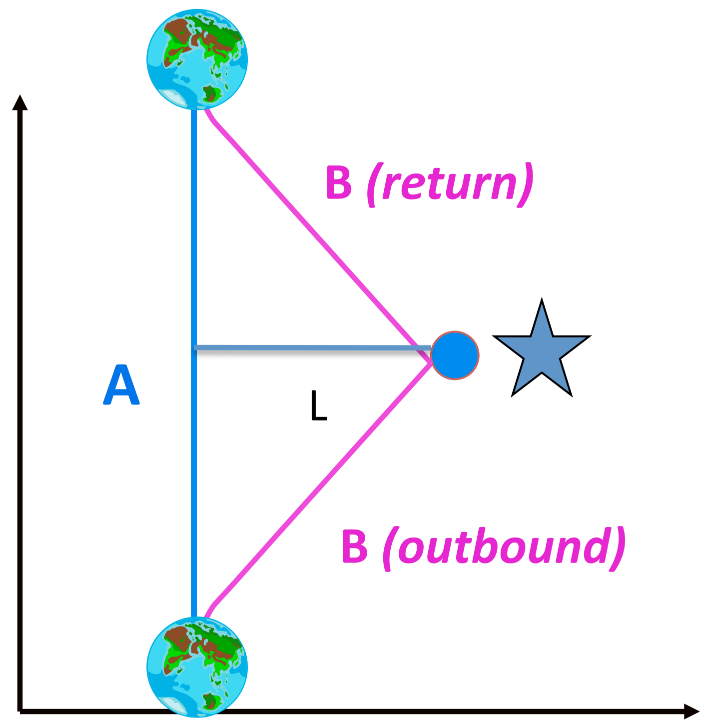
$$c\Delta t < |\Delta x|$$

The twin paradox

- Suppose Andy (A) and Betty (B) are twins.
- Andy stays on Earth, while Betty leaves Earth, travels (at a large fraction of the speed of light) to visit her aunt on a planet orbiting Alpha Centauri, and returns
- When Betty gets home, she finds Andy is greatly aged compared her herself.
- Andy attributes this to the time dilation he observes for Betty's clock during her journey
- Is this correct?
- What about reciprocity? Doesn't Betty observe Andy's clock as dilated, from her point of view? Wouldn't that mean she would find him much older, when she returns?
- Who's really older?? What's going on???

Andy's point of view

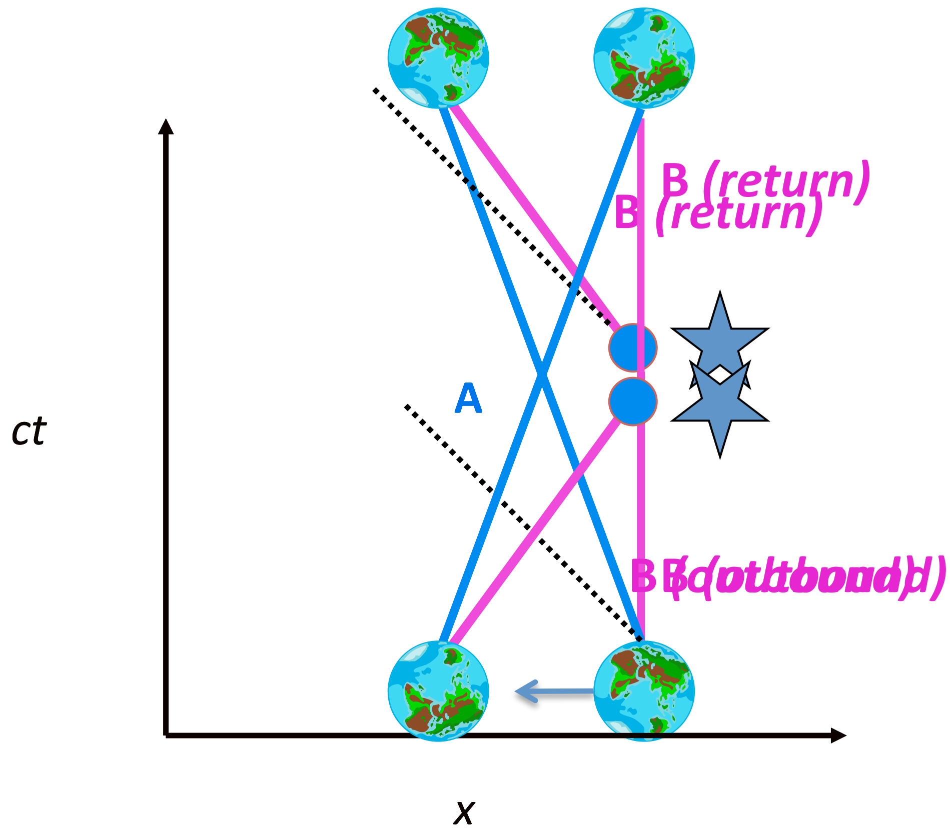
- Andy's world line, in his own frame, is a straight line
- Betty's journey has world line with two segments, one for outbound (towards larger x) and one for return (towards smaller x)
- Both of Betty's segments are at angles <45° to vertical, because she travels at v<c
- If Andy is older by Δt years when Betty returns, he expects that due to time dilation she will have aged by $\Delta t/\gamma$ years
- Since $1/\gamma = (1-v^2/c^2)^{1/2} < 1$, Betty will be younger than Andy, and the faster Betty travels, the more difference there will be



$$(c\Delta t_A/2)^2 - L^2 = (c\Delta t_B/2)^2$$
$$\Delta t_B < \Delta t_A$$
$$L = v\Delta t_A/2$$
$$(\Delta t_A)^2 = (\Delta t_B)^2/[1-(v/c)^2]$$
$$\Delta t_A = \gamma \Delta t_B$$

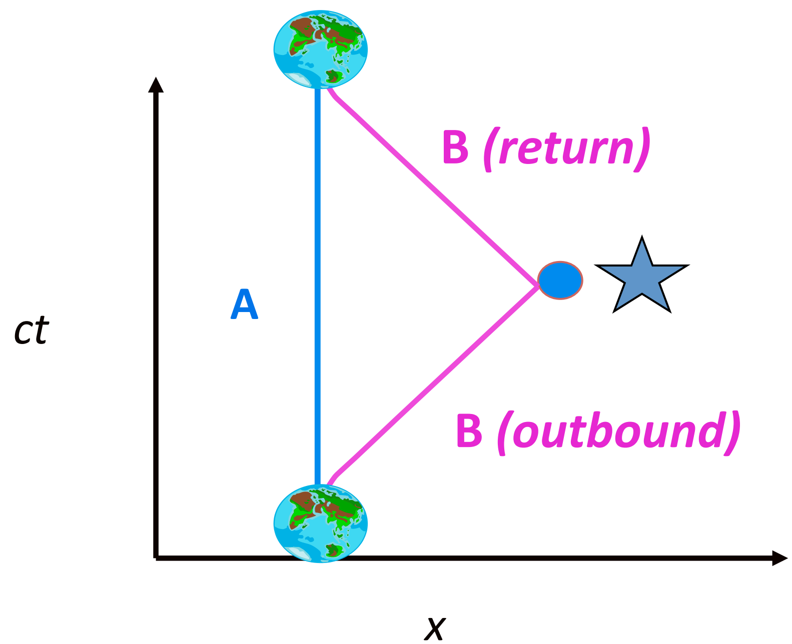
Betty's point of view

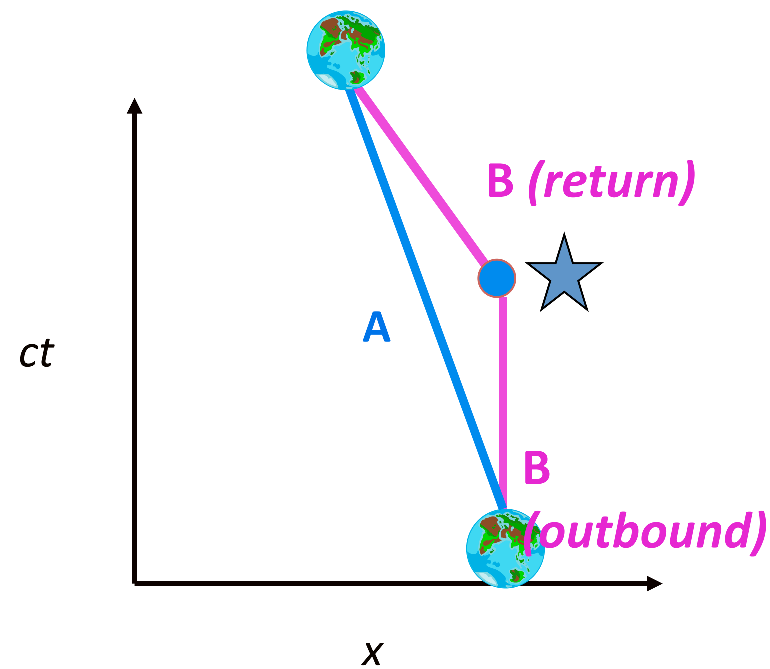
- Consider frame moving with Betty's outbound velocity
- Andy on Earth will have straight world line moving towards smaller *x*
- Betty's return journey world line is not the same as her outbound world line, instead pointing toward smaller *x*
- Both Andy's world line and Betty's return world line are at angles < 45 ° to vertical (inside of the light cone)
- Betty's return world line is closer to light cone than Andy's world line



- For frame moving with Betty's return velocity, situation is similar

Different kinds of world lines





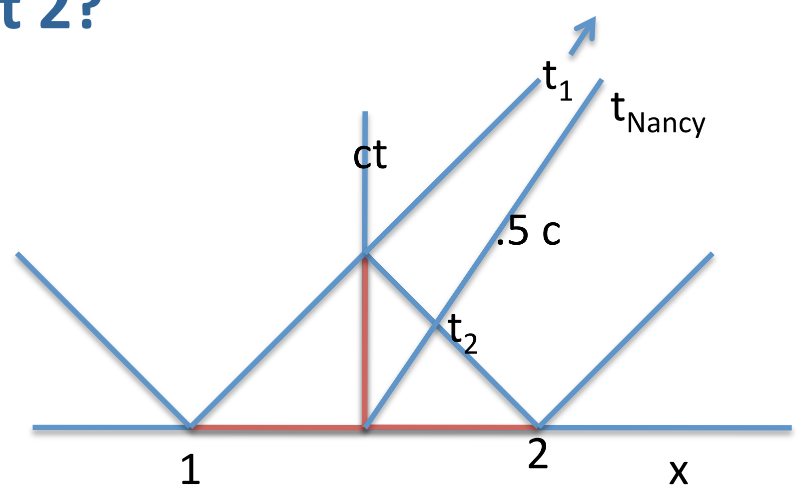
- Regardless of frame, Betty's world line does not connect start and end points with a straight line, while Andy's does
- This is because Betty's journey involves **accelerations**, while Andy's does not

A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Nancy is flying her rocket at $v = 0.5c$ in the direction from the tree toward the pole. The lightning hits the tree just as she passes by it. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Nancy, does event 1 occur before, after or at the same time as event 2?

A. before event 2
B. after event 2
C. at the same time as event 2

A tree and a pole are 3000 m apart. Each is suddenly hit by a bolt of lightning. Mark, who is standing at rest midway between the two, sees the two lightning bolts at the same instant of time. Nancy is flying her rocket at $v = 0.5c$ in the direction from the tree toward the pole. The lightning hits the tree just as she passes by it. Define event 1 to be "lightning strikes tree" and event 2 to be "lightning strikes pole." For Nancy, does event 1 occur before, after or at the same time as event 2?

A. before event 2
B. after event 2 ✔
C. at the same time as event 2



Molly flies her rocket past Nick at constant velocity *v*. Molly and Nick both measure the time it takes the rocket, from nose to tail, to pass Nick. Which of the following is true?

A. Nick measures a shorter time interval than Molly.
B. Molly measures a shorter time interval than Nick.
C. Both Molly and Nick measure the same amount of time.

Molly flies her rocket past Nick at constant velocity *v*. Molly and Nick both measure the time it takes the rocket, from nose to tail, to pass Nick. Which of the following is true?

✔ **A. Nick measures a shorter time interval than Molly.**

B. Molly measures a shorter time interval than Nick.

C. Both Molly and Nick measure the same amount of time.

Uses one clock

$$(c\Delta t_N)^2 = (c\Delta t_M)^2 - l^2 < (c\Delta t_M)^2$$

OPTIONAL

What is energy? **The capability to do WORK ->W**

What is work ?

$$dW = Fdx = \frac{dp}{dt}dx = vdp = vd(\gamma mv) = mv(vd\gamma + \gamma dv)$$

$$d\gamma = -\frac{1}{2}\frac{-2v/c^2}{(1-v^2/c^2)^{3/2}}dv = \frac{\gamma^3 v}{c^2}dv$$

$$\gamma v dv = \frac{c^2}{\gamma^2}d\gamma$$

$$v^2 d\gamma = c^2(1-\frac{1}{\gamma^2})d\gamma$$

$$v/c << 1 \rightarrow \gamma = 1/\sqrt{1-v^2/c^2} \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

$$K \approx \frac{1}{2}mv^2$$

$$dW = mc^2 d\gamma$$

$$K = \int_0^v dW = mc^2\int_1^\gamma d\gamma = mc^2(\gamma - 1)$$