PHYS 270-SPRING 2011

Dennis Papadopoulos

LECTURE # 20

RELATIVITY I

NEWTONIAN RELATIVITY- GALILEAN TRANSFORMATIONS- SIMULTANEITY

APRIL 21, 2011

I: LAWS OF MOTION

Newton’s first law

Newton’s first law (N1) – If a body is not acted upon by any forces, then its velocity, *v*, remains constant

- N1 sweeps away the idea of “being at rest” as a natural state.
- N1 includes special case with $v=0$, i.e. a body at rest remains at rest if $F=0$, as part of more general law

Newton's second law

Newton's 2nd law (N2) – If a body of mass *M* is acted upon by a force *F*, then its acceleration *a* is given by *F=Ma*

- N2 defines "inertial mass" as the degree by which a body resists being accelerated by a force.
- Since momentum $p=mv$ and $a=$rate of change in v, $ma=$ rate of change in $(m\ v)$
- Thus, another way of saying N2 is that force = rate of change of momentum
- Alternate form of N2 is more general, since it includes case when mass is changing

Newton's third law

Newton's 3rd law (N3) - If body A exerts force $F_{A \to B}=f$ on body B, then body B exerts a force $F_{B \to A}=-f$ on body A.

- N3 is often phrased in terms of "equal" (in magnitude) and "opposite" (in direction) forces
- From N3, the *total force* on a closed system is *0*, i.e.

  $F_{tot}= F_{A \to B} +F_{B \to A} =f+(-f)=0$

- Combining with N2, this implies that the total momentum of a closed system is *conserved* [does not change] if there are no external forces, i.e.

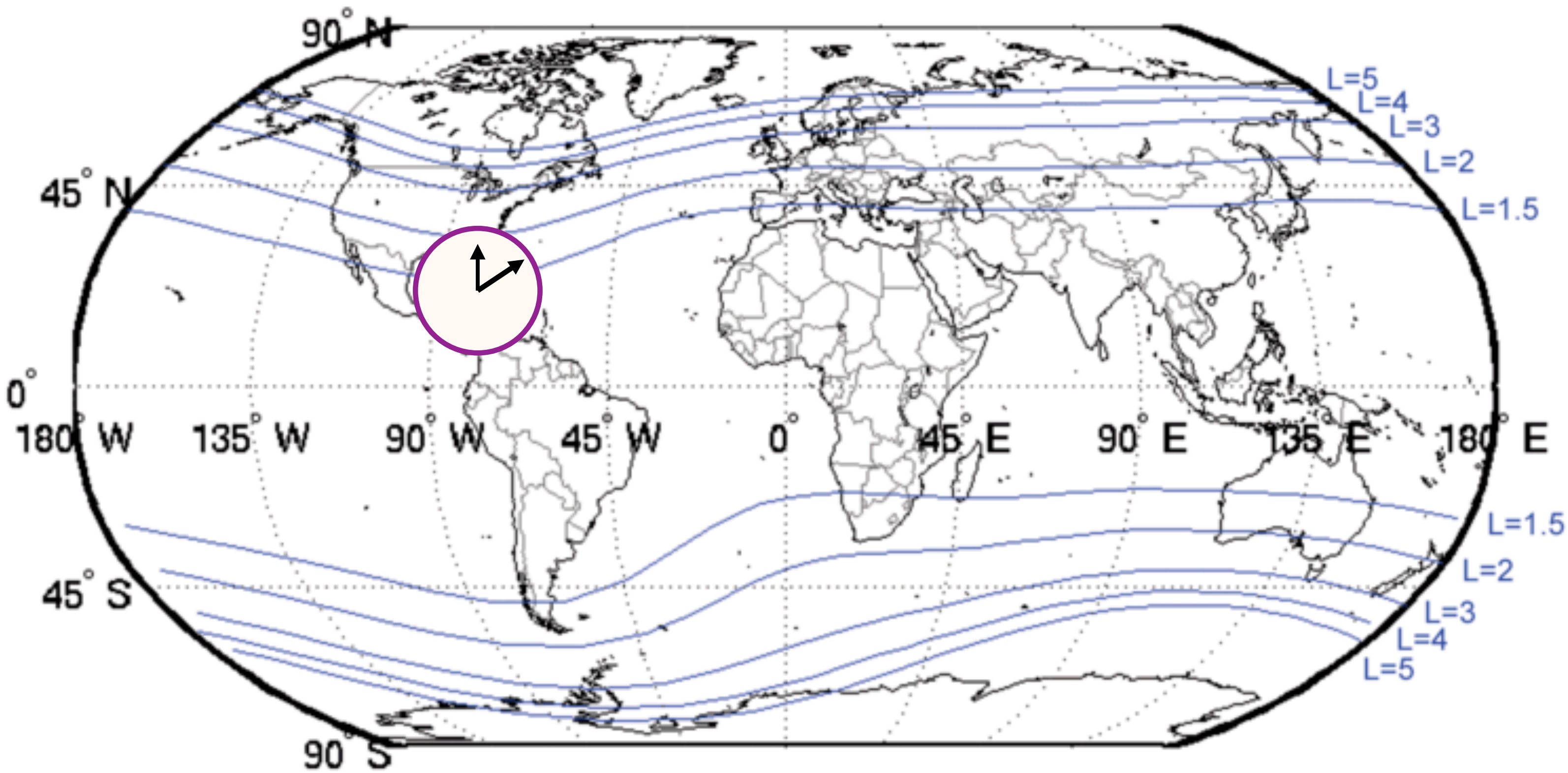
  $F_{tot}=0 \Rightarrow$ *(rate of change of p_{tot})=0* $\Rightarrow p_{tot}$ =constant

- Any momentum change of one part of a closed system is compensated for by a momentum change in another part, i.e.

  (rate of change of p_A)= - (rate of change of p_B)

Coordinate Systems and Events

Latitude, longitude, altitude, time



Events space time coordinates

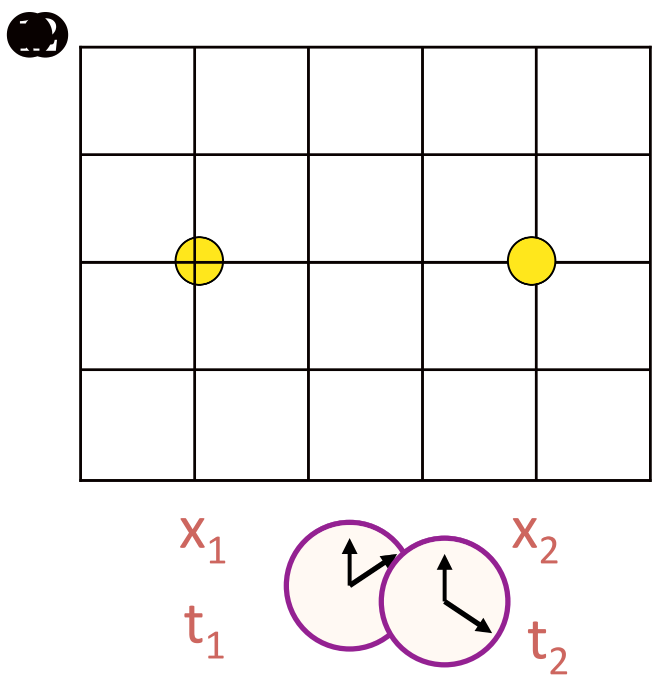
Coordinate systems

- Scientific observations involve making measurements
- fundamental measurements are always of events in terms of their coordinates in space and time
- Space-time coordinates are often written as (x,y,z,t)
- Coordinates are convenient labels, not fundamental attributes of space and time
  - We are free to choose whatever units we want (e.g. m, km,foot,…), and whatever coordinate origin we want
  - What matters is the *intervals* in time and space, not absolute numbers. For Event 1 at (x_1,y_1,z_1,t_1) and Event 2 at (x_2,y_2,z_2,t_2), the time interval is $\Delta t=t_2-t_1$, and using the Pythagorean theorem generalized to 3D, the space interval (distance) is

$$\Delta s = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Velocities and accelerations

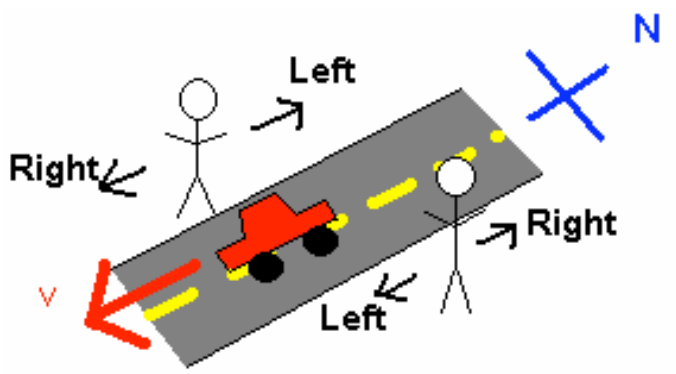
- Velocities are rates of change of vector positions
- Accelerations are rates of change of vector velocities
- For motion in a *given* direction, the velocity is equal to the change in position $\Delta x = x_2 - x_1$ divided by the corresponding change in time $\Delta t = t_2 - t_1$: $v = \Delta x / \Delta t$
- Similarly, $a = \Delta v / \Delta t$



II. FRAMES OF REFERENCE

WHAT IS A FRAME OF REFERENCE ?

THE PROSPECTIVE FROM WHICH A SYSTEM IS OBSERVED



Car moving to left for one observer and to the right for the other. Both agree that is moving south. To translate their observations we need a transformation. In this case a rotation by 180 degrees

A set of axis relative to which an observer can measure at any time the position, motion and orientation of all points in a system

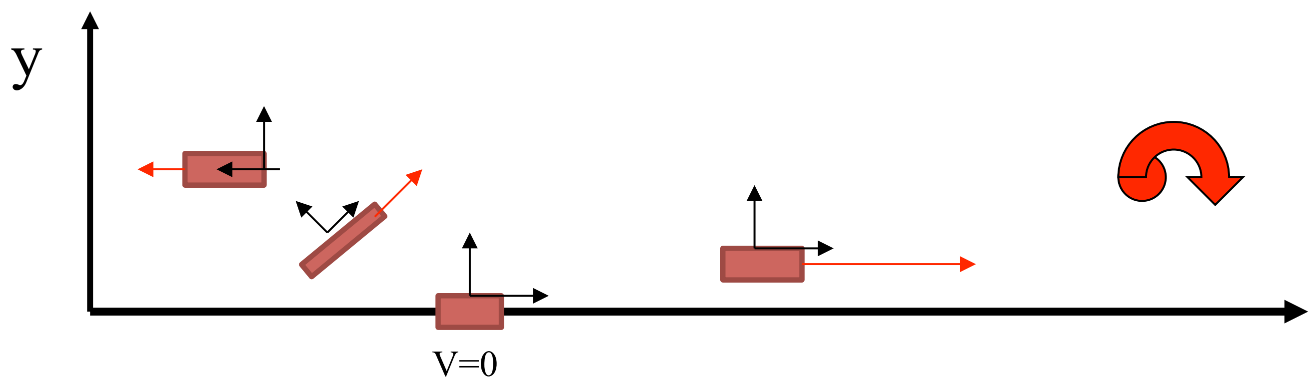
Types of Frames of reference

- The frame of reference in which a measurement is made consists of the spatial coordinates (the grid) and time coordinate (the clock) that are used to make the measurement
- Note that in general, we use a "clock" that is attached to the spatial coordinate system we are using (why this matters will become apparent soon!)
- The reference frame may potentially have any arbitrary motion and/or acceleration. However, reference frames that have $a \neq 0$ are fundamentally different from those with $a=0$
- "Inertial frame" = unaccelerated frame
- "Non-inertial frame" = accelerated frame
- *How can an observer inside the frame tell the difference?*
  - In an inertial frame, a free particle (no forces acting) has constant velocity (including $v=0$ special case)
  - In a non-inertial frame, a free particle's velocity (speed and/or direction) varies
  - Note that for humans, even if we don't have a free particle handy for experiments, we can sense accelerations physiologically

INERTIAL FRAME OF REFERENCE

A COORDINATE SYSTEM DEFINED BY THE NON-ACCELERATING MOTION OF OBJECTS THAT HAVE A COMMON DIRECTION AND SPEED

Red arrow →constant velocity

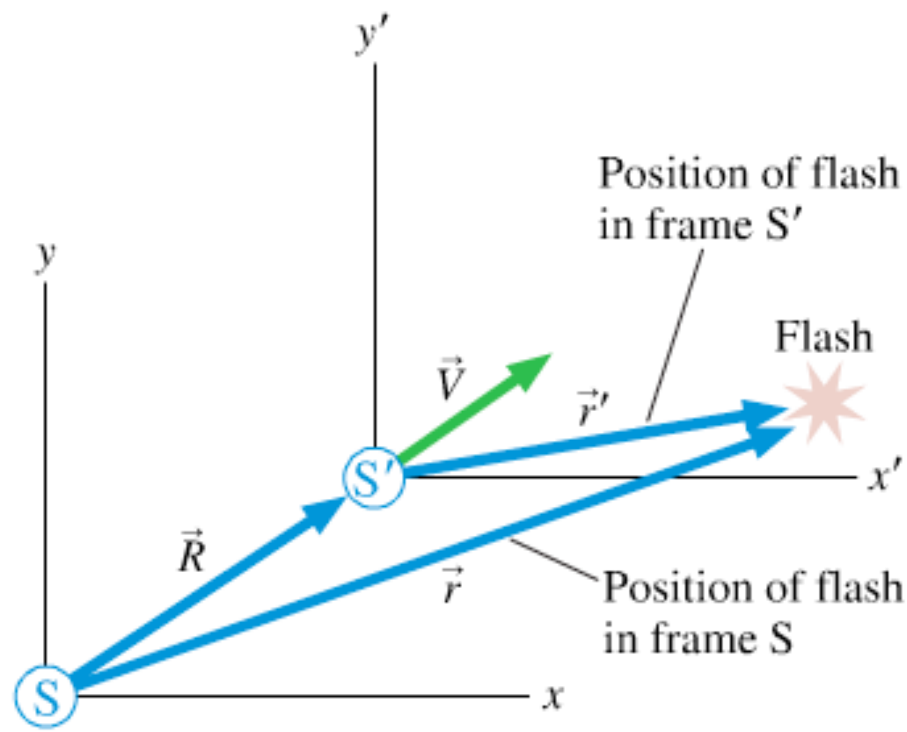


Brown inertial frames

Red non-inertial frame

Is the Earth an inertial frame ?

FIGURE 4.26 Measurements made in frames S and S′.



$$\vec{r} = \vec{r}' + t\vec{V} \quad \text{or} \quad \vec{r}' = \vec{r} - t\vec{V} \tag{4.20}$$

Equation 4.20 is called the **Galilean transformation of position.** It will be easiest for most purposes to write this in terms of components:

$$\begin{aligned} x &= x' + V_x t \\ y &= y' + V_y t \end{aligned} \quad \text{or} \quad \begin{aligned} x' &= x - V_x t \\ y' &= y - V_y t \end{aligned} \tag{4.21}$$

NEWTONIAN PRINCIPLE OF RELATIVITY

- THE LAWS OF MECHANICS ARE INVARIANT IN INERTIAL REFERENCE FRAMES

EG: Play ping-pong on a train moving with constant velocity same as playing on the ground.
No mechanical experiment can detect motion at constant speed

LAWS THAT EXHIBIT THE SAME MATHEMATICAL FORM FOR ALL OBSERVERS ARE CALLED COVARIANT

EINSTEIN: THE LAWS OF PHYSICS ARE COVARIANT IN INERTIAL REFERENCE FRAMES.

The Galilean Transformations

Consider two reference frames S and S′. The coordinate axes in S are x, y, z and those in S′ are x', y', z'. Reference frame S′ moves with velocity v relative to S along the x-axis. Equivalently, S moves with velocity −v relative to S′.

The *Galilean transformations of position* are:

$$\begin{aligned} x &= x' + vt \\ y &= y' \\ z &= z' \end{aligned} \qquad \text{or} \qquad \begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned}$$

The *Galilean transformations of velocity* are:

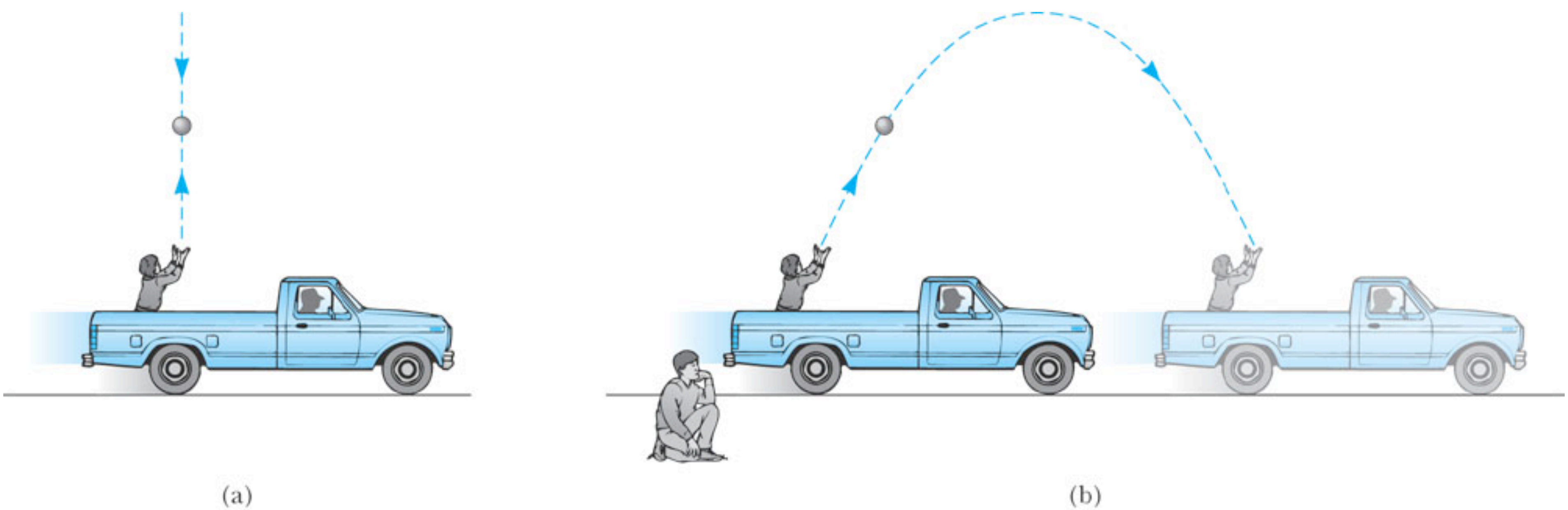
$$\begin{aligned} u_x &= u'_x + v \\ u_y &= u'_y \\ u_z &= u'_z \end{aligned} \qquad \text{or} \qquad \begin{aligned} u'_x &= u_x - v \\ u'_y &= u_y \\ u'_z &= u_z \end{aligned}$$

The Galilean transformations for accelerations are:

$$\vec{a} = \vec{a}'$$

Experiment at rest

Experiment in moving frame



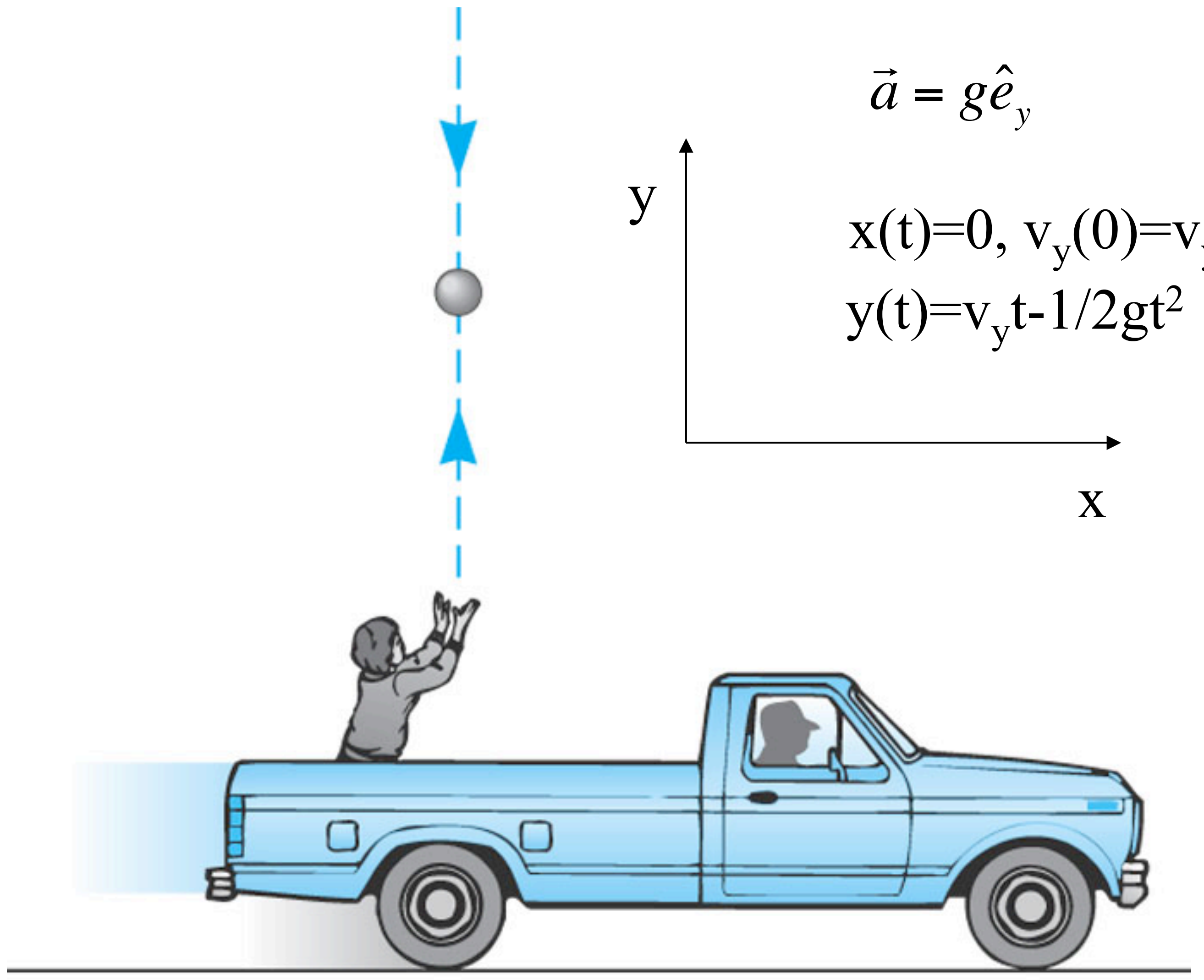
Same result. Ball rises and ends up in the thrower's hand. Ball in the air the same length of time.
Experiment looks different from ground observer (parabolic trajectory, speed as a function of time) and observer on the truck.
However, they both agree on the validity of Newton's laws.

Fig. 1-1, p. 4

$\vec{a} = g\hat{e}_y$

$x(t)=0,\ v_y(0)=v_y\ ,y(0)=0$

$y(t)=v_y t-1/2gt^2$



(a)

Fig. 1-1a, p. 4

$$x' = x + ut$$

$$y' = y$$

$$v_y' = v_y$$

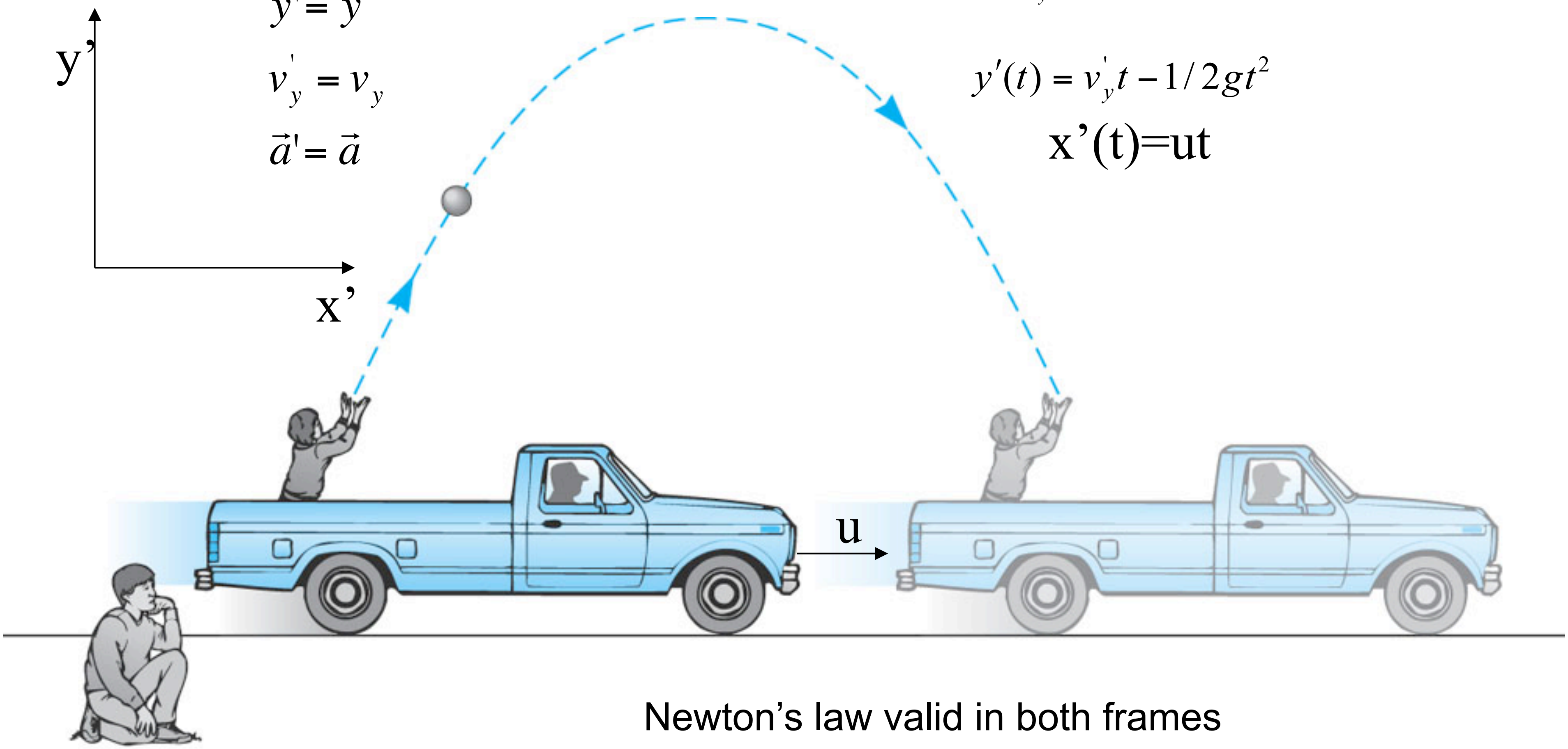
$$\vec{a}' = \vec{a}$$

$$x(t) = 0$$

$$y(t) = v_y t - (1/2)gt^2$$

$$y'(t) = v_y' t - 1/2gt^2$$

x’(t)=ut



Newton’s law valid in both frames

(b)

Assumed t invariant

Fig. 1-1b, p. 4

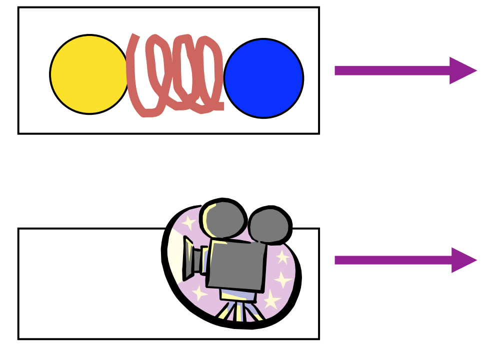
An illustration of Newton's laws
Momentum Conservation

- We can see that aspects of Newton's laws arise from more fundamental considerations.
- Consider two equal masses M at rest. Initial momentum is $p=0$. Masses are suddenly pushed apart by a spring… will move apart with same speed V *in opposite directions* (by symmetry of space!). Total momentum is $p=MV-MV=0$. Total momentum is unchanged.

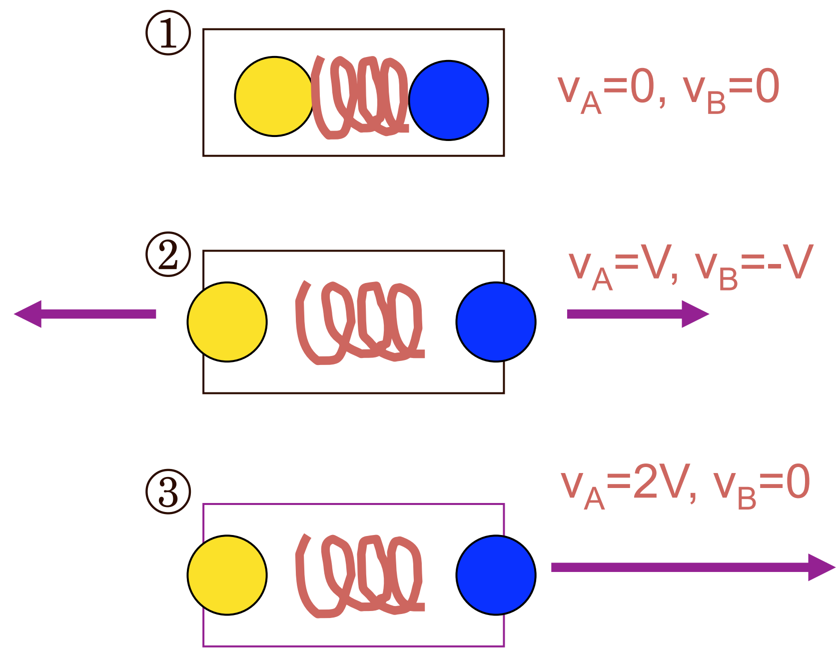
Before: $v_A=v_B=0 \Rightarrow p_{tot}=0$

After: $v_A=V$, $v_B=-V$
$\Rightarrow p_{tot}=Mv_A - M\,v_B=MV-MV=0$

- Same situation, but masses are now both initially moving at velocity V. Initial momentum is $p_{tot}=2MV$.
- Can turn into the previous situation by "moving along with them at velocity V".



1. Change of perspective [subtract V from all velocities] brings masses to rest…
2. Do same problem as before…
3. Change back to original perspective [add V to all velocities] …
4. Final velocity of one ball is $2V$; final velocity of other ball is 0. Final total momentum is $p_{tot}=2MV$. No change in total momentum.



Using Galilean Transformations

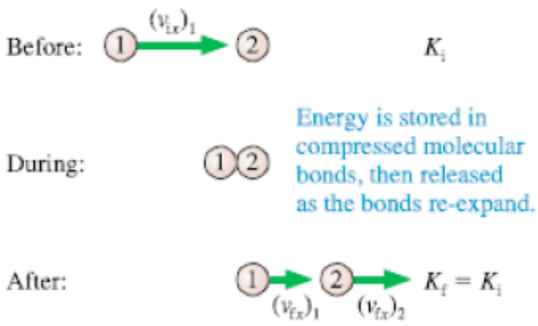
Chapter 10:

Elastic collisions and conservation of momentum

momentum conservation: $m_1(v_{fx})_1 + m_2(v_{fx})_2 = m_1(v_{ix})_1$

energy conservation: $\frac{1}{2}m_1(v_{fx})_1^2 + \frac{1}{2}m_2(v_{fx})_2^2 = \frac{1}{2}m_1(v_{ix})_1^2$

FIGURE 10.24 A perfectly elastic collision.



$$(v_{fx})_1 = \frac{m_1 - m_2}{m_1 + m_2}(v_{ix})_1$$

$$(v_{fx})_2 = \frac{2m_1}{m_1 + m_2}(v_{ix})_1$$

(perfectly elastic collision with ball 2 initially at rest)

Transform velocity to where one object is at rest w.r.t. other, solve collision problem, and then transform back

$$u'_{1f} = \frac{m_1 - m_2}{m_1 + m_2}u'_{1i}$$

$$u'_{2f} = \frac{2m_1}{m_1 + m_2}u'_{1i}$$

THE SPEED OF LIGHT PROBLEM

- Maxwell's equations:
  - Predict "waves" of electromagnetic energy – and it was quickly realized that these are light waves!
  - The speed of light "c" appears as a fundamental constant in the equations.
  - c=300,000 km/s
  - **BUT, what frame of reference is this measured relative to???**

MAXWELL'S EQUATIONS

Maxwell's highly successful equations…

Gauss' law
$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

no magnetic monopoles
$$\oint \vec{B} \cdot d\vec{A} = 0$$

Faraday's law
$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

Ampere's law
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \frac{1}{c^2} \frac{\partial}{\partial t} \int \vec{E} \cdot d\vec{A}$$

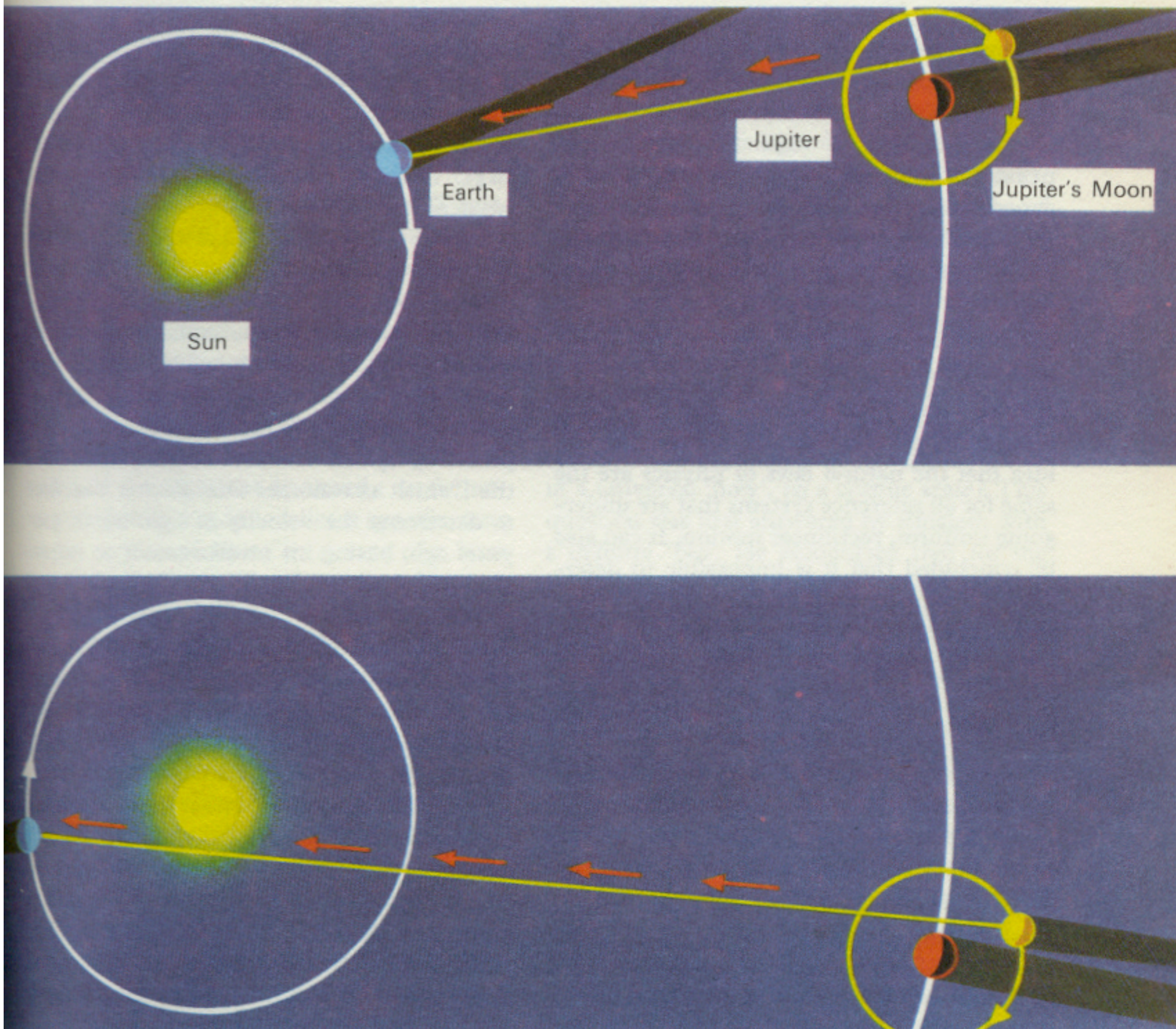
Not invariant under Galilean transformation

Contain a constant velocity!

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

In the 19th century, these equations were thought to hold only in the luminiferous ether!

The eclipse of one of Jupiter's moons occurs earlier than calculated when Jupiter is close to the Earth. However, it occurs later if the Earth and Jupiter are far away from each other. About 300 years ago, the Dane Olaf Römer calculated the speed of light on the basis of this irregularity.

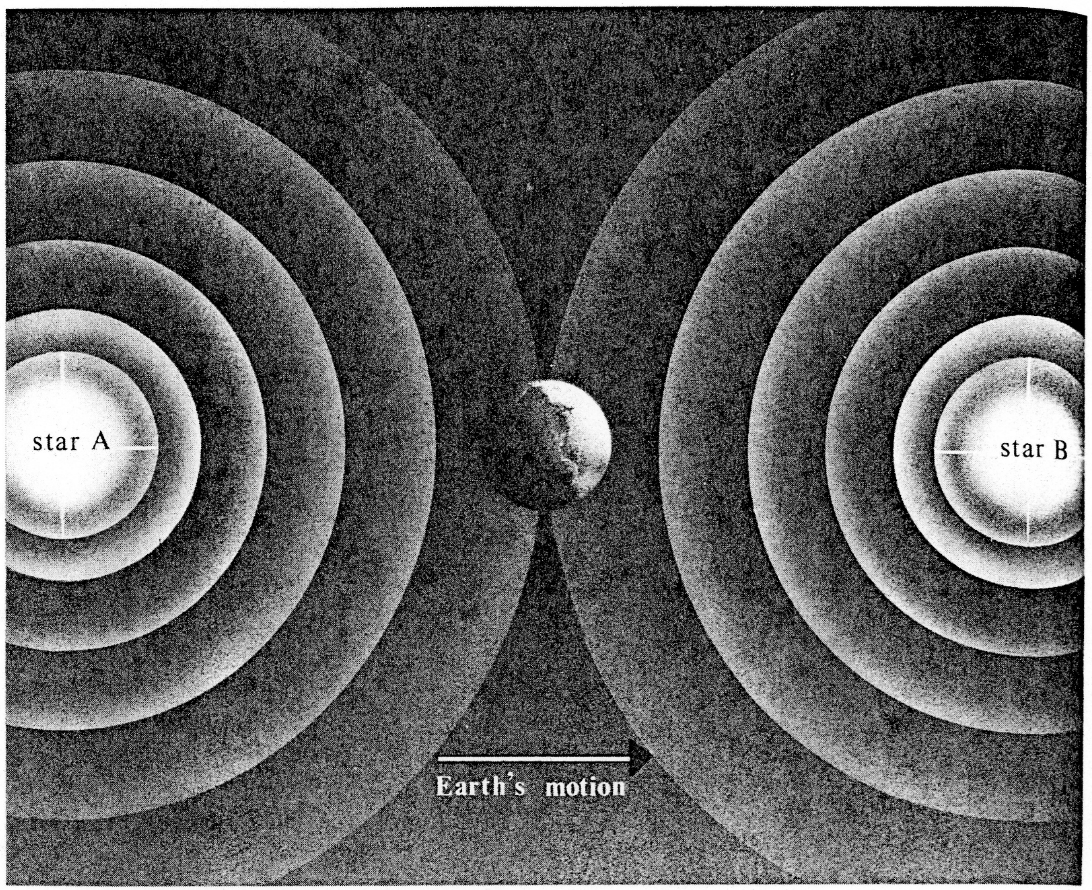


$$\Delta t \approx 2R_{SE} / v_{light}$$

Ether and light waves

- Luminiferous Ether (19th century)
  - Hypothetical substance that fills space - provides a "medium" through which light can travel.
  - Idea was that Maxwell's equations, as written, would apply only in frame of ether
  - This would explain why the speed of wave propagation "c" is a constant in the equations
  - If speed of light in ether is "c" , and *if Galilean relativity holds*, then speed of light measured in other frames would be different from "c"
  - Albert Michelson & Edward Morley attempted (1887) to measure motion of Earth through ether…

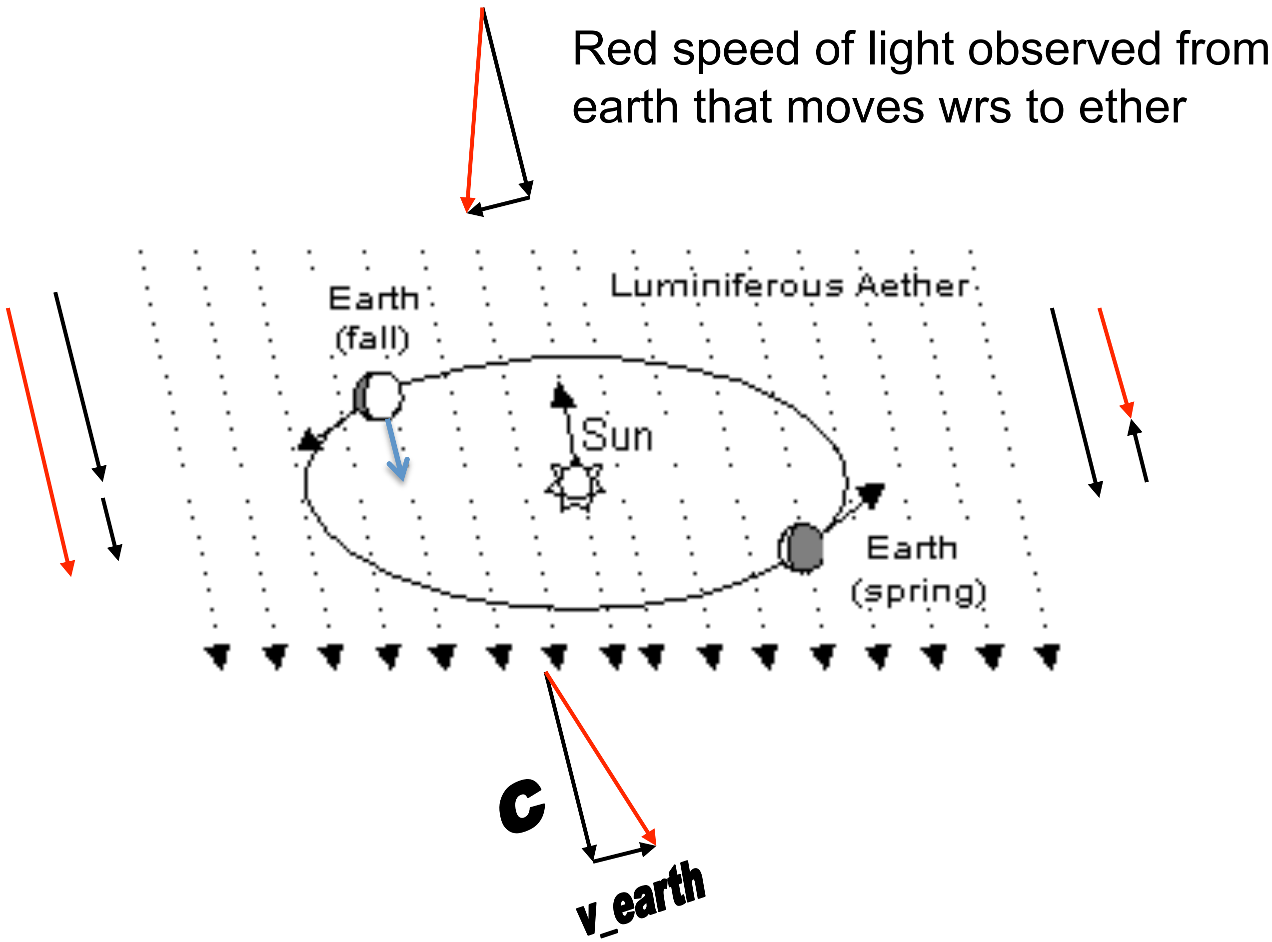
ETHER



Aether drift theory held that if the velocity of light was constant relative to a stationary, all-pervading aether, then when the earth in its orbit was moving away from star *A* and toward star *B*, the observed speed of the light coming from star *B* would be higher than that of the light coming from star *A*.

From T. Ferris : "Coming of Age in the Milky Way"

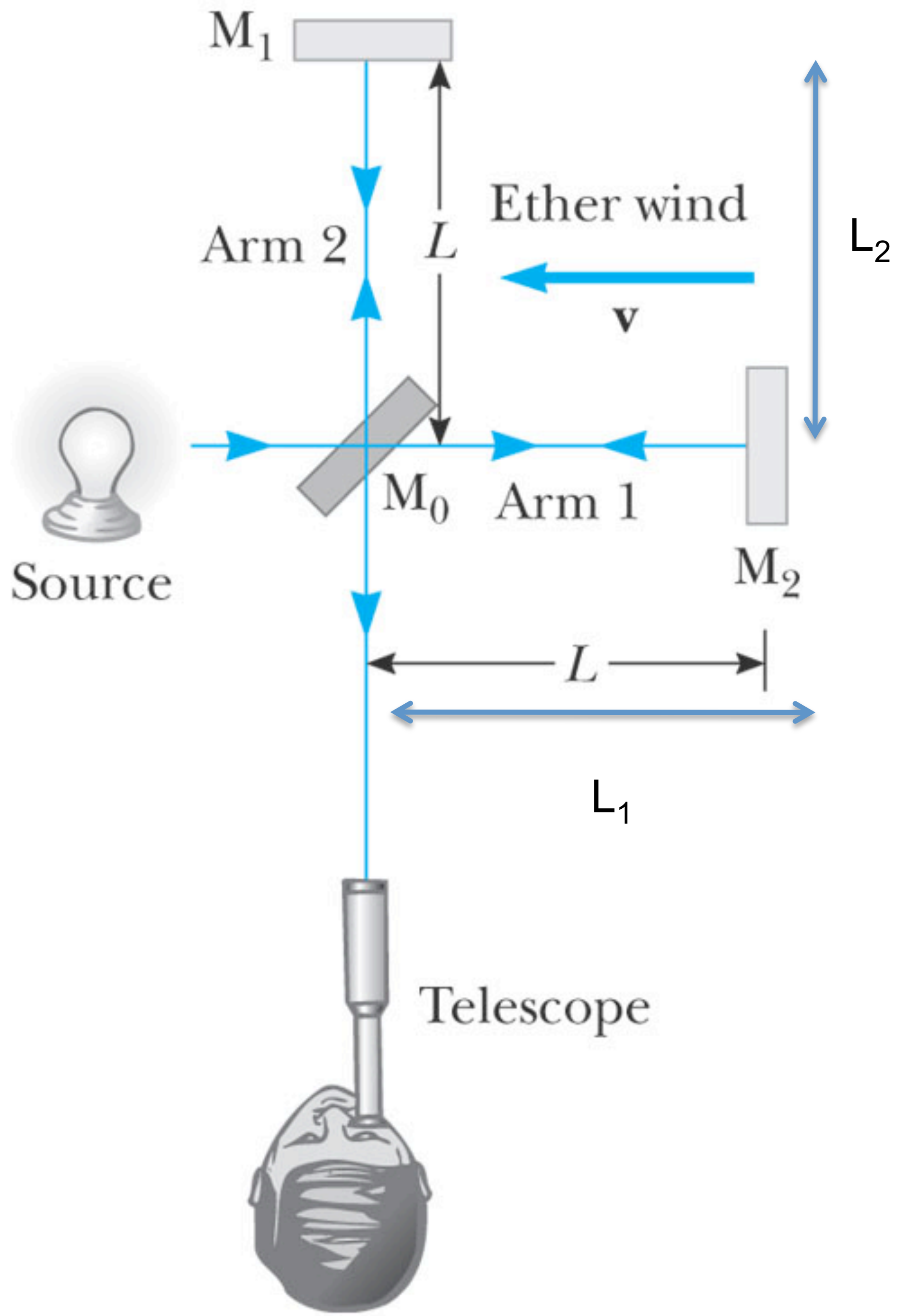
Light must travel through a medium: hypothesize that a "luminiferous ether" exists



Earth is moving with respect to the ether (or the ether is moving with respect to the earth), so there should be some directional/season dependent change in the speed of light as observed from the reference frame of the earth.

$\Delta r = 2L_1 - 2L_2$

If speed constant in both directions
Constructive interference if $L_2 - L_1 = m(\lambda/2)$, $m = 0, 1, 2$



M-M results

- Travel time difference would be measured using interference fringes of light from two paths
- Apparatus could be rotated to make sure no effects from set-up
- Repeated at different times of year, when Earth's motion differs; Earth's speed around the Sun is ~30 km/s
- Experiment performed in 1887
- Results
  - M-M showed that speed of light was same in any direction to within 5 km/s
  - Modern versions of the experiment show constancy to better than 1 micron/s
- So, what's going on??

Attempts to deal with M-M results

- Maybe the ether "sticks" to the Earth?
  - Gets "dragged" as Earth spins and orbits Sun…
  - Possibility at the time, but no-longer viable.
- Maybe the ether squeezes the arms of the M-M experiment and distorts the result? "Fitzgerald contraction" (1889)?
  - A contraction (in the direction parallel to motion through ether) would change the light travel time to compensate for the difference expected due to different speed of light

$$L = L_0 \sqrt{1 - V^2 / c^2}$$

- Major mystery ("crisis") in 19th century physics – two highly successful theories seemed incompatible!
  - Mechanics – Galilean Relativity and Newton's laws
  - Electromagnetism – Maxwell's equations

I: SPECIAL RELATIVITY WHY AND WHAT

" SR AROSE FROM NESSECITY, FROM SERIOUS AND DEEP CONTRADICTIONS IN THE OLD THEORY FROM WHICH THERE SEEMED NO ESCAPE" ***EINSTEIN 1905***

RELATIVITY POSTULATES:

1. THE LAWS OF PHYSICS ARE INVARIANT (COVARIANT) IN INERTIAL REFERENCE FRAMES.
2. SPEED OF LIGHT IN VACUUM IS CONSTANT INDEPENDENT OF MOTION OF SOURCE AND OBSERVER

NEWTONIAN PRINCIPLE OF RELATIVITY

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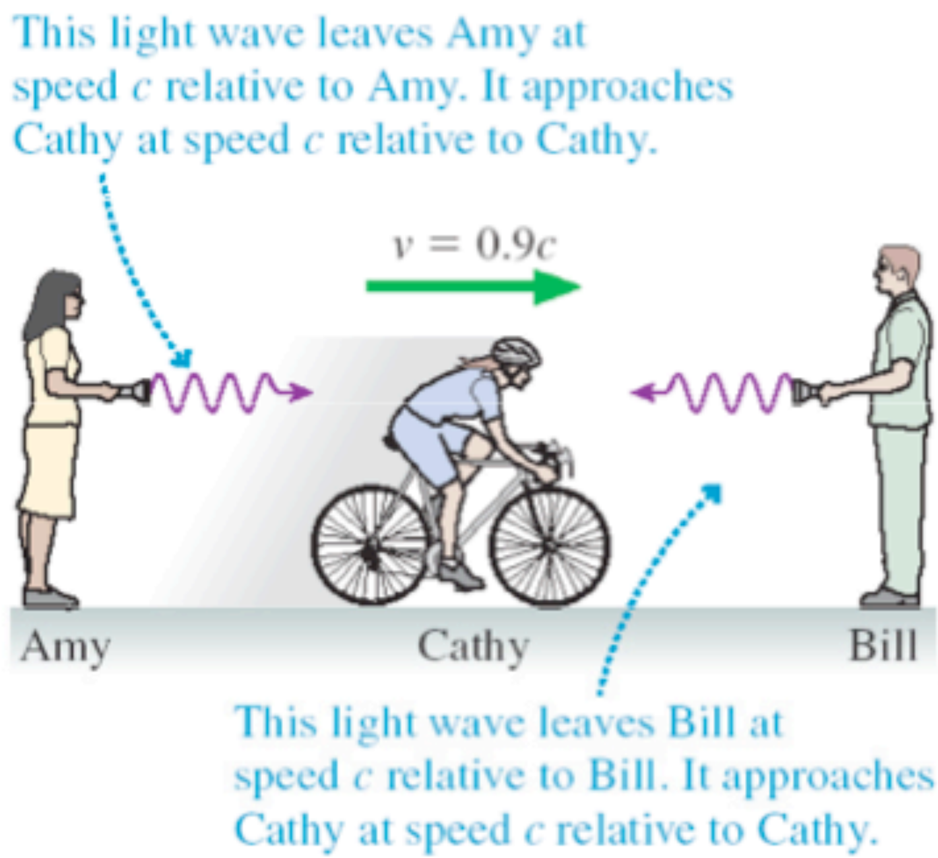
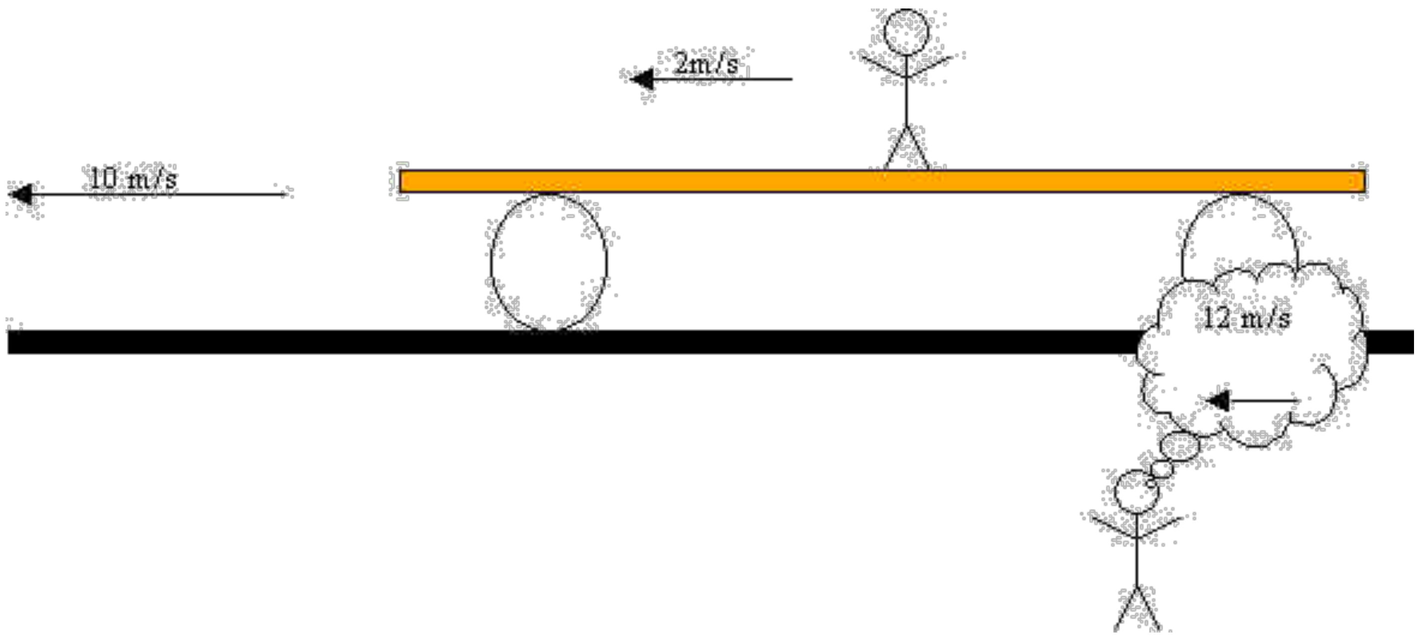
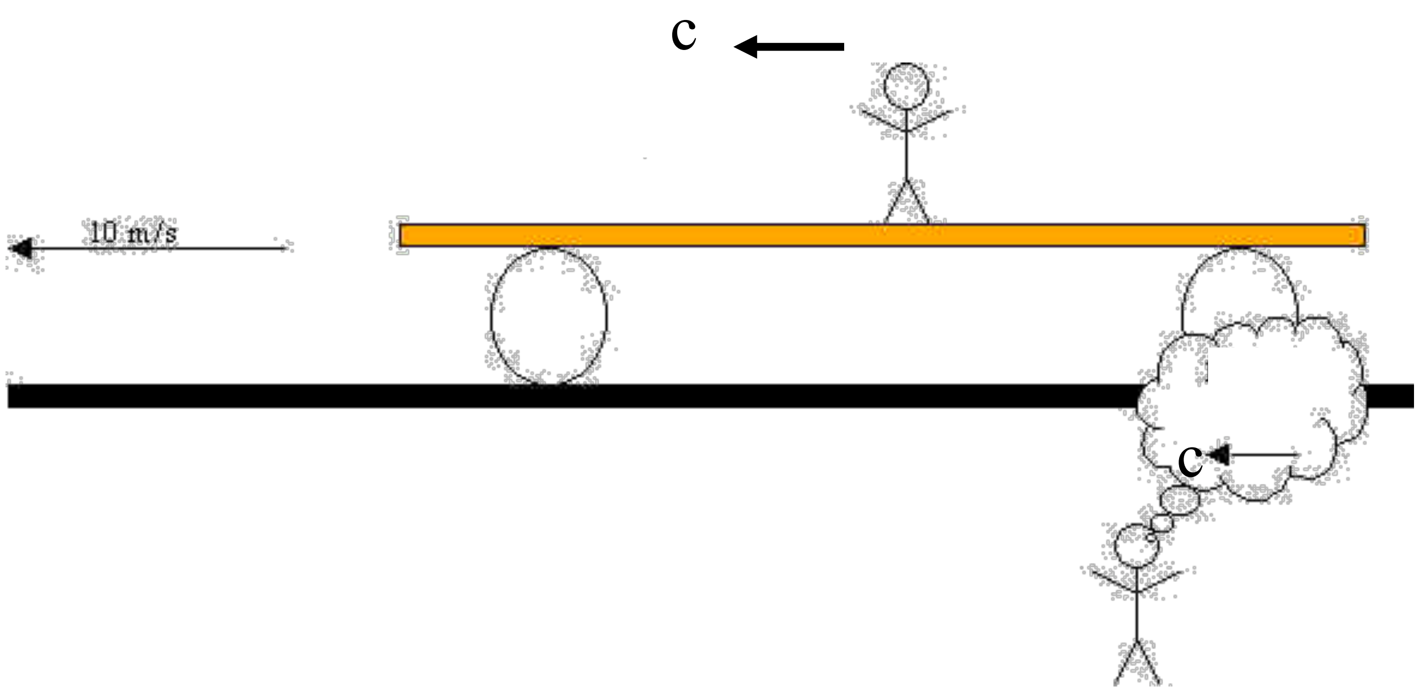
THE LAWS OF PHYSICS ARE COVARIANT IN INERTIAL REFERENCE FRAMES.

FIGURE 37.9 Light travels at speed c in all inertial reference frames, regardless of how the reference frames are moving with respect to the light source.

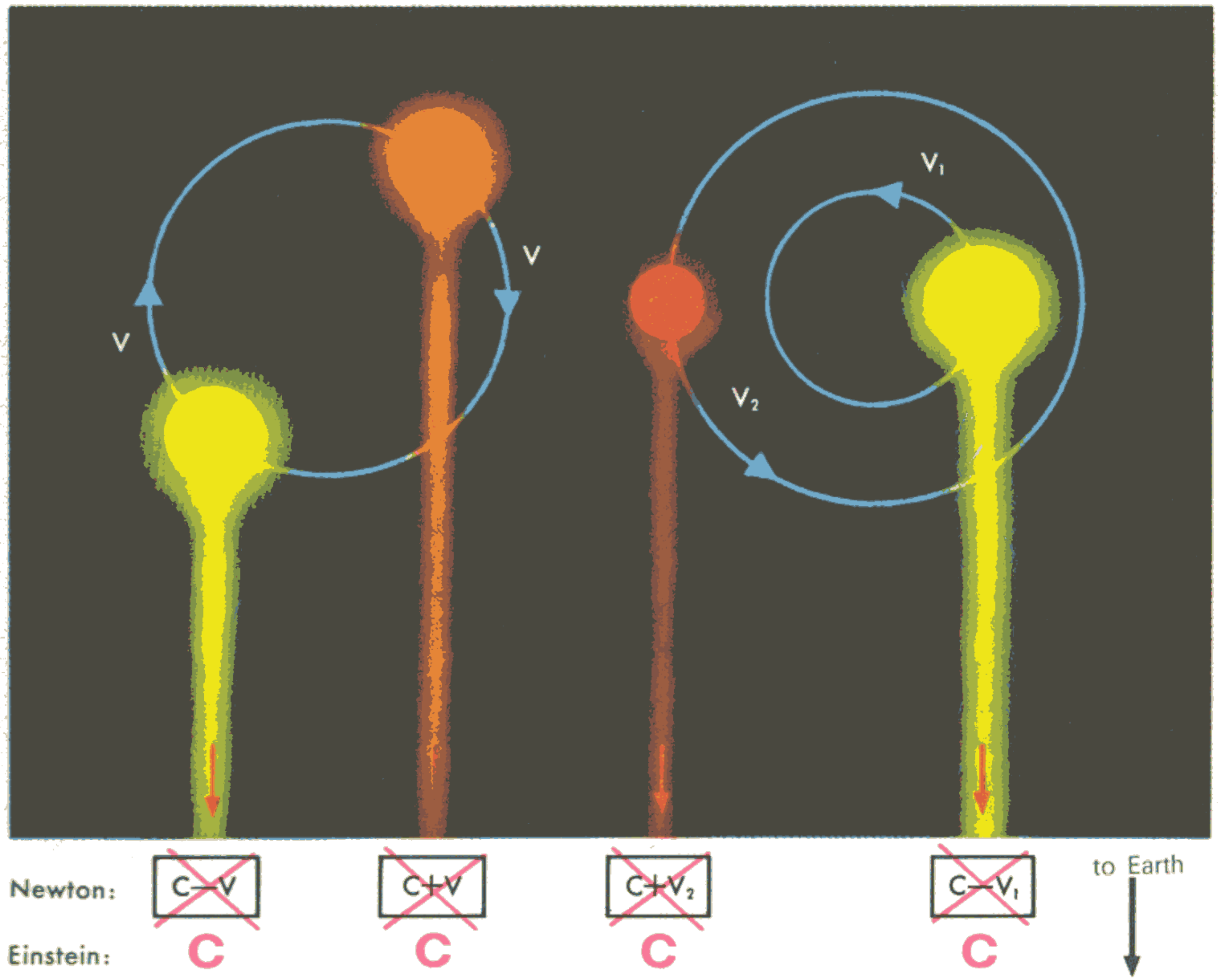


THE SPEED OF LIGHT PROBLEM




c
10 m/s
c

INVARIANCE OF SPEED OF LIGHT



Observations of double stars confirm Einstein's new formula for addition of velocities, in which the velocity of light represents a limiting value.

TRUTH AND CONSEQUENCES

WHAT IS SPEED ?

WHAT DISTANCE AN OBJECT WILL TRAVEL IN A GIVEN DURATION OF TIME V=DX/DT

DISTANCE IS A NOTION ABOUT SPACE – HOW MUCH SPACE IS BETWEEN TWO POINTS

DURATION IS A NOTION ABOUT TIME – HOW MUCH TIME ELAPSES BETWEEN EVENTS

SPEED IS A SPACE-TIME NOTION – CONSTANCY OF SPEED OF LIGHT REQUIRES THAT WE MODIFY CONVENTIONAL CONCEPTS OF SPACE AND TIME

The radical consequences

$$\textbf{Speed} = \frac{\textbf{distance traveled} \quad \text{(function of v)}}{\textbf{time elapsed} \quad \text{(function of v)}}$$

If the speed of light is a constant…then… length and time must be variables??

These effects are known as length contraction and time dilation.

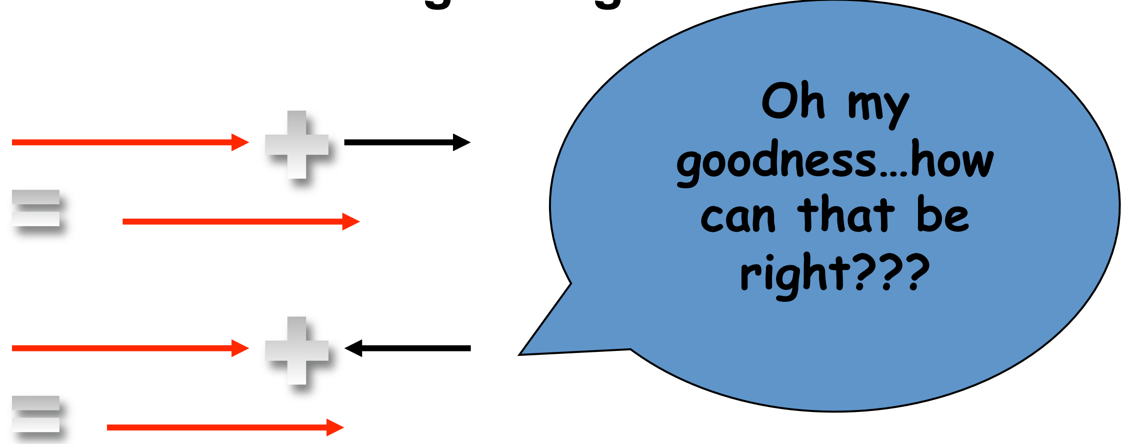
How come you never noticed this before, and how come *most of the time* I can get away with Galilean transformations in your calculations?

speed of light = 670 616 629 miles per hour

Most of the time the speed of the object whose motion you are calculating is so slow relative to the speed of light that the discrepancy due to relativity is negligible. (Most, but not all of the time)

The Solution???

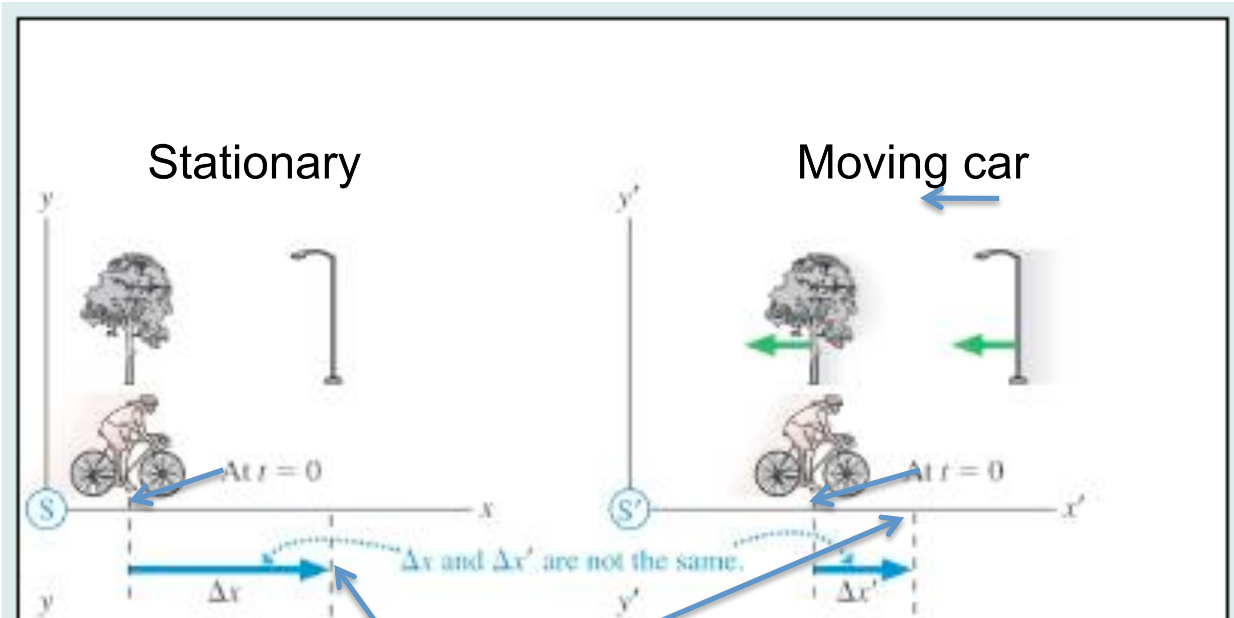
The speed of light in vacuum has the same value, c=300000000 m/s, in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.



All the laws of physics have the same form in all inertial reference frames.

Alright…we know that Newtonian mechanics worked in all inertial reference frames under Galilean transformations, but does the same hold true for Maxwell's equations of electromagnetism?

Measure the velocity of a bicycle by a stationary observer and one moving in a car



At t=Δt

Next replace the bicycle with a light wave that moves from the tree to the lamppost. The experiment gives

u=c=u'

$$\Delta x > \Delta x' = \Delta x - V\Delta t$$

$$u = \frac{\Delta x}{\Delta t} > \frac{\Delta x'}{\Delta t'} = \frac{\Delta x - V\Delta t}{\Delta t} = u'$$

$$u + V = u'$$

$$\Delta x / \Delta t = \Delta x' / \Delta t' = c$$

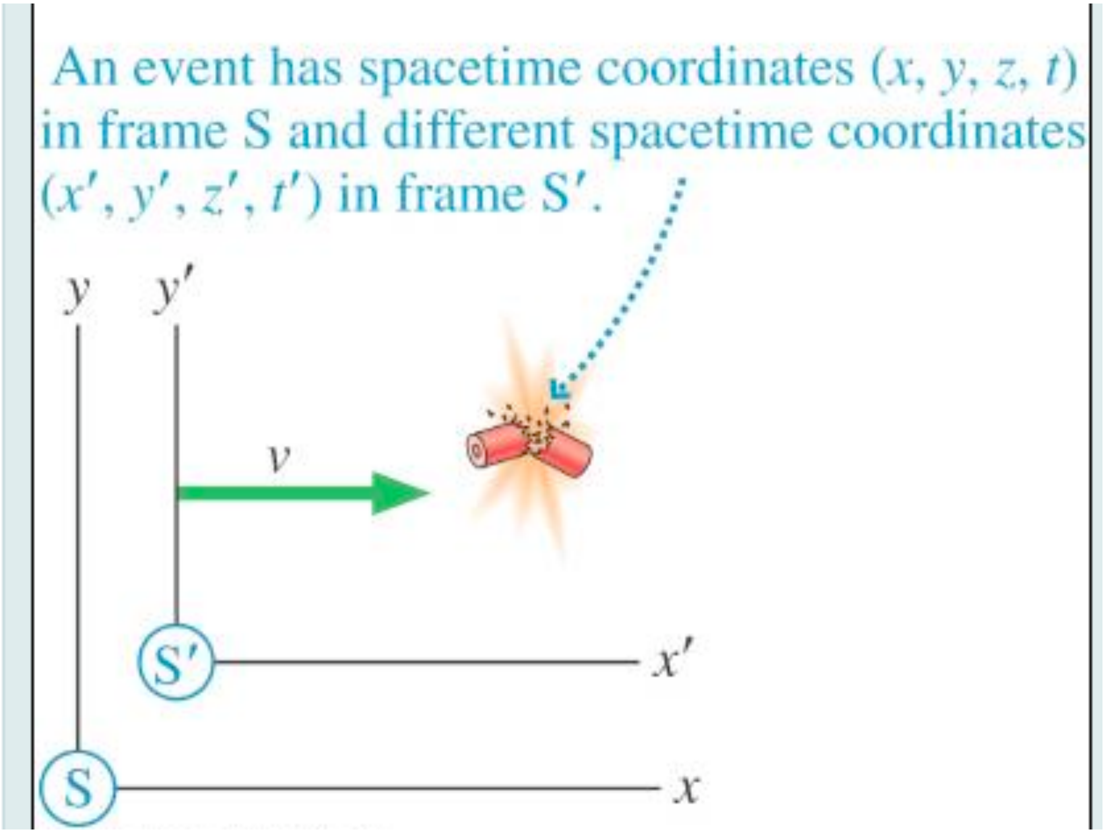
$$\Delta t / \Delta t' = \Delta x / \Delta x' = \Delta x / (\Delta x - V\Delta t) > 1$$

Since Δx>Δx' this can happen only if Δt>Δt'

Time dilation

EVENTS –SPACETIME COORDINATES

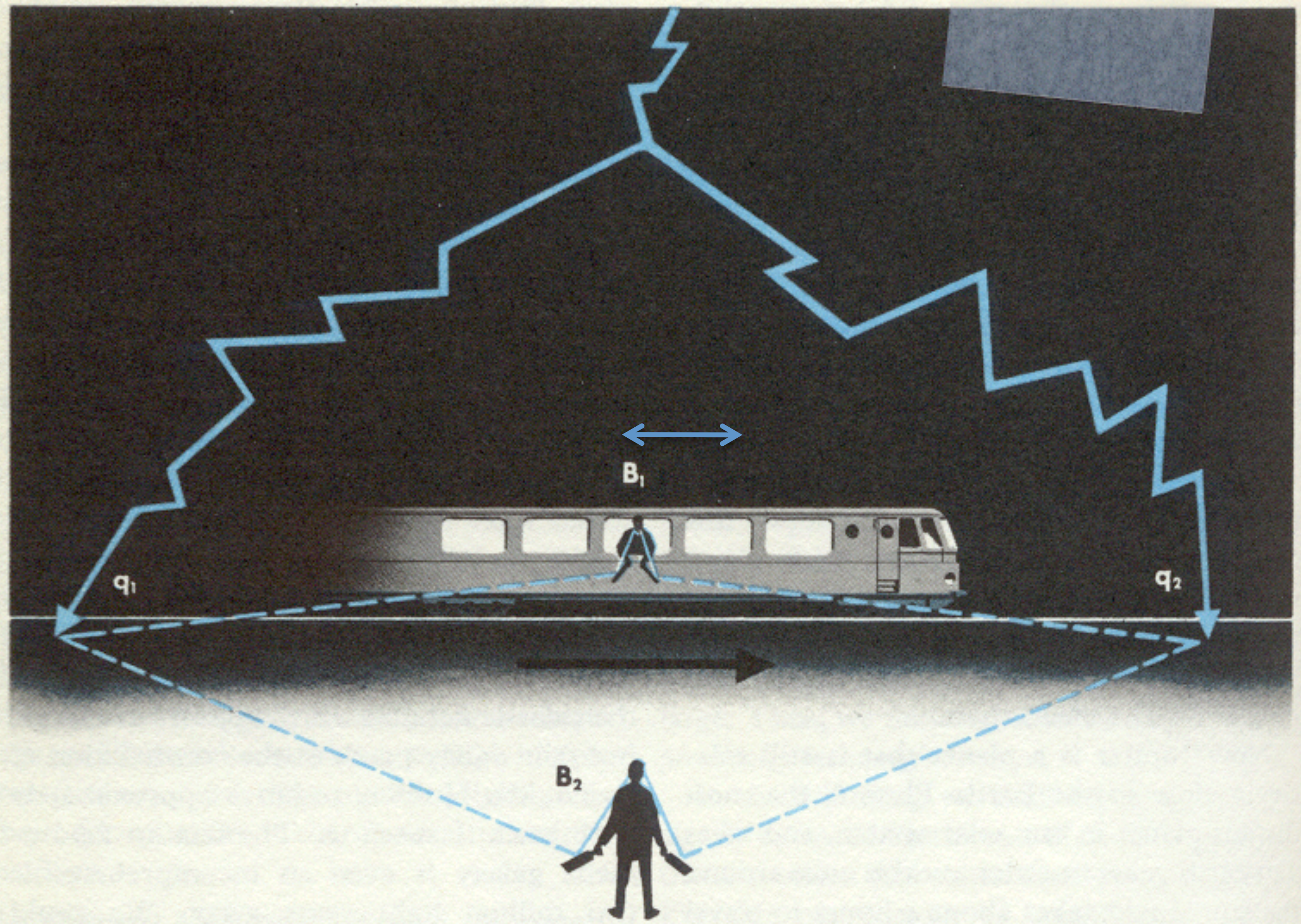
Event is something that happened in a location of spacetime



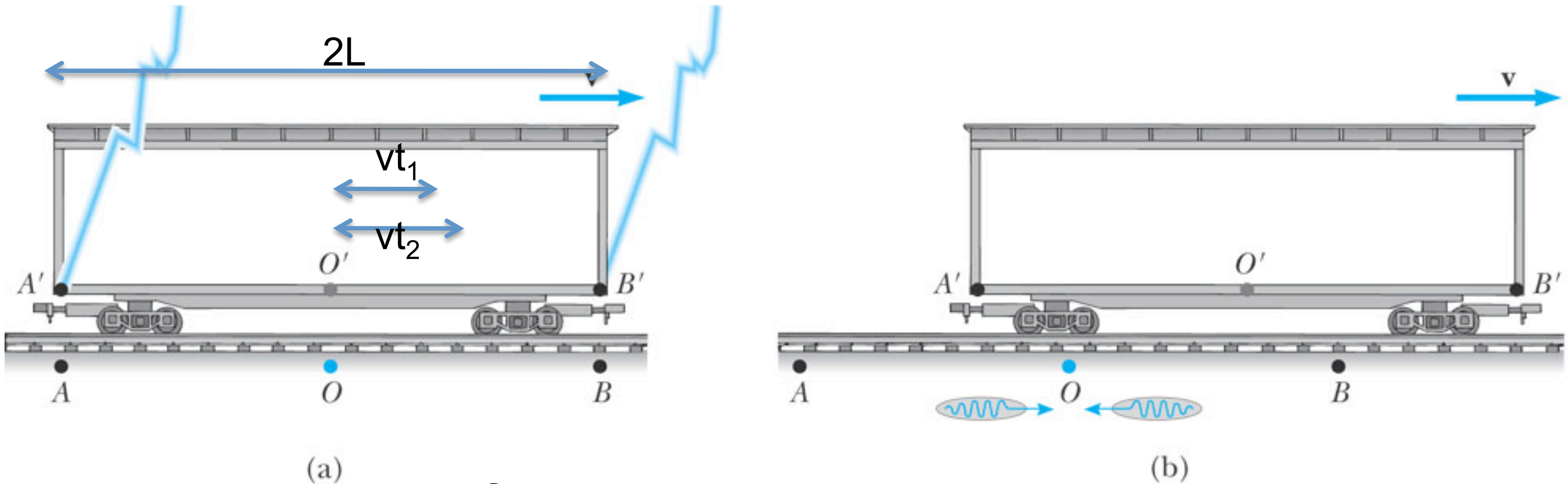
SIMULTANEITY

- NEWTON -> UNIVERSAL TIMESCALE FOR ALL OBSERVERS
  - **“Absolute, true time, of itself and of its own nature, flows equably, without relation to anything external”**
- EINSTEIN
  - “A time interval measurement depends on the reference frame the measurement is made”

SIMULTANEITY



A stationary observer B_2 on the railway embankment sees the forked lightning strike at two positions q_1 and q_2 simultaneously. The observer B_1 moving with the train undergoing uniform rectilinear motion, sees the lightning strike q_2 a fraction of a second before q_1. The reason for this is the finite signal speed of light.



$$t_1 = \frac{L + vt_1}{c}$$

$$t_1 = \frac{L/c}{1 - v/c}$$

$$t_2 = \frac{L - vt_2}{c}$$

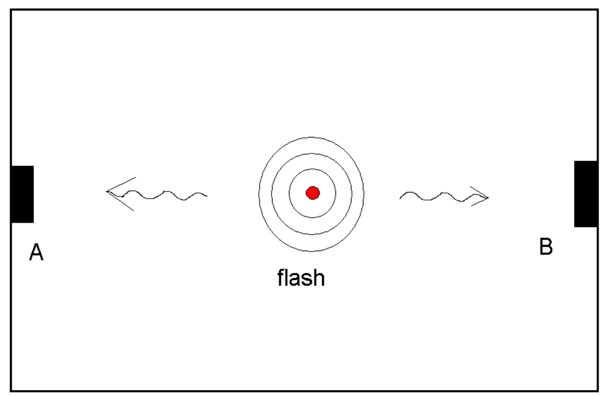
$$t_2 = \frac{L/c}{1 + v/c}$$

$$\Delta t = (L/c)[\frac{1}{1 - v/c} - \frac{1}{1 + v/c}] = \frac{(2Lv/c^2)}{1 - \frac{v^2}{c^2}} = (2L/c)(\beta\gamma^2)$$

$$\beta \equiv v/c$$

$$\gamma \equiv \frac{1}{\sqrt{1 - \beta^2}}$$

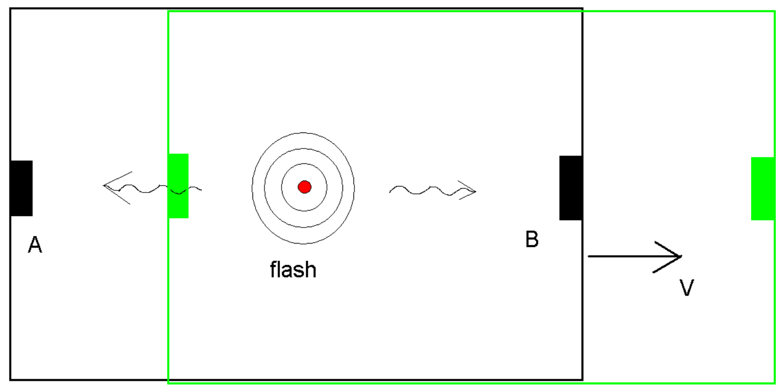
- Consider an observer in a room. Suppose there is a flash bulb exactly in the middle of the room.
- Suppose sensors on the walls record when the light rays hit the walls.



- **Since speed of light is constant**, light rays will hit opposite walls at precisely the same time. Call these events A and B.

Change frames…

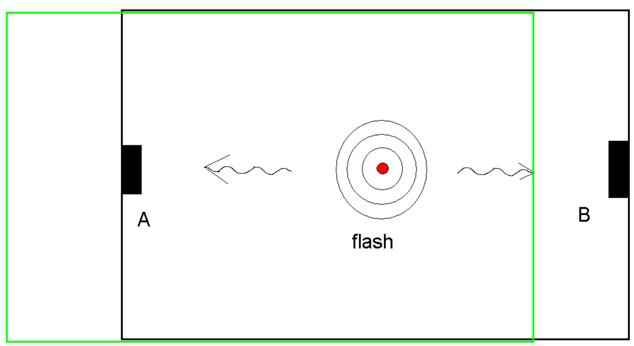
- Imagine performing same experiment aboard a moving train, and observing it from the ground.
- For the observer on the ground, the light rays will not strike the walls at the same time (since the walls are moving!). Event A will happen before event B.



- But observers riding on the train think the events are simultaneous.
- Concept of “events being simultaneous” (i.e. simultaneity) is different for different observers (**Relativity of simultaneity).**

Change frames again!

- What about perception of a 3rd observer who is moving faster than the train?



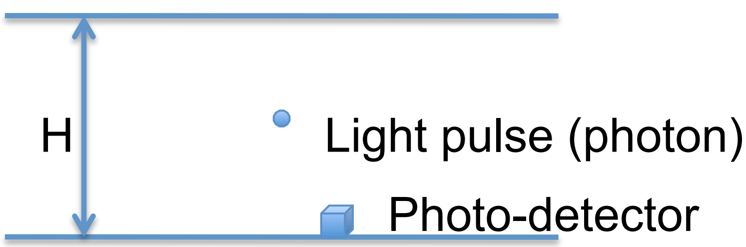
- 3rd observer sees event B before event A
- So, order in which events happen can depend on the frame of reference.

TIME DILATION

A light clock consists of two parallel mirrors and a photon bouncing back and forth over the distance H. An observer at rest with the clock will measure a click at times

$\Delta t_o = 2H/c$ (Unit of time)

One second

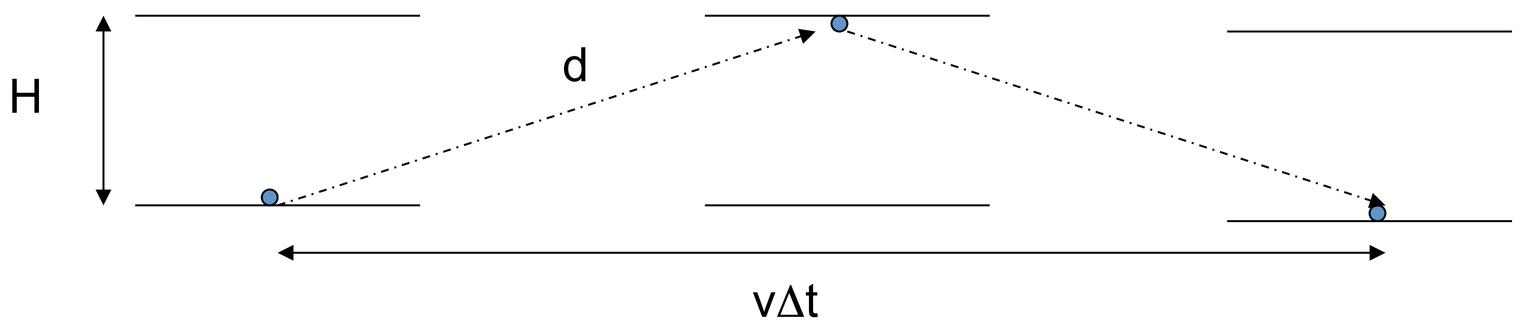


Call Δt_o PROPER TIME interval often denoted by $\Delta\tau$

TIME DILATION

Now suppose that we put the clock on a platform sliding at constant speed v. Use Einstein's postulate – speed of light constant=c. Measure the unit of time Δt as seen by a stationary observer

Proper time - observer sees events at the same spatial point – Rides with the clock->$\Delta\tau = 2H/c = \Delta t_o$



$d^2 = H^2 + (v\Delta t/2)^2$

Unit of time is shortest for a clock located in a reference frame in which the clock is at rest

$$\Delta t = \gamma \Delta\tau \geq \Delta\tau$$

since

$$\gamma \geq 1$$

$$\Delta t = 2d/c = 2\sqrt{H^2 + (v\Delta t/2)^2}\Big/c$$

$$\Delta t = (2H/c)\gamma = \Delta t_o \gamma$$

$$\gamma \equiv 1/\sqrt{1-(v/c)^2}$$

Time dilation

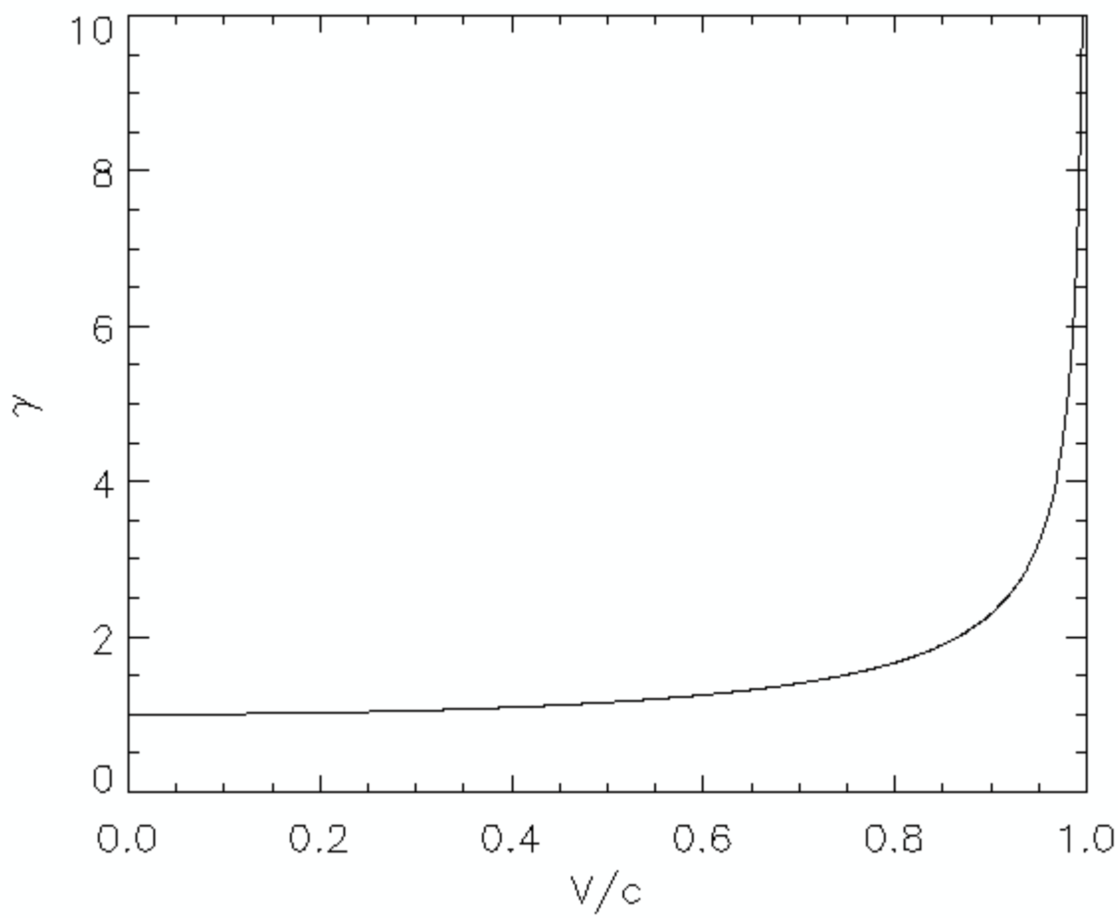
- This effect called **Time Dilation**.
- Clock always ticks most rapidly when measured by observer in its own rest frame
- Clock slows (ticks take longer) from perspective of other observers
- When clock is moving at V with respect to an observer, ticks are longer by a factor of

$$\Delta t \div \Delta t_o = \frac{D/c}{\sqrt{1 - V^2/c^2}} \div \frac{D}{c} = \frac{1}{\sqrt{1 - V^2/c^2}}$$

- This is called the **Lorentz factor,** γ

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \Delta t = \gamma \Delta t_o$$

Lorentz factor



Clocks and time

- Does this "time dilation" effect come about because we used a funny clock?
- No, any device that measures time would give the same effect!
- The time interval of an event as measured in its own rest frame is called the ***proper time***
- Note that if the astronaut observed the same "light clock" (or any clock) that was at rest on Earth, it would appear to run slow by the same factor γ , because the dilation factor depends on *relative speed*
- This is called the ***principle of reciprocity***

Why don't we ordinarily notice time dilation?

Some examples of speeds in m/s

- 0.0055m/s world record speed of the fastest snail in the Congham,UK
- 0.080 m/s the top speed of a sloth (= 8.0 cm/s)
- 1 m/s a typical human walking speed
- 28 m/s a car travelling at 60 miles per hour (mi/h or mph) or 100 kilometres per hour (km/h); also the speed a cheetah can maintain
- 341 m/s the current land speed record, which was was set by ThrustSSC in 1997.
- 343 m/s the approximate speed of sound under standard conditions, which varies according to air temperature
- 464 m/s Earth's rotation at the equator.
- 559 m/s the average speed of Concorde's record Atlantic crossing (1996)
- 1000 m/s the speed of a typical rifle bullet
- 1400 m/s the speed of the Space Shuttle when the solid rocket boosters separate.
- 8000 m/s the speed of the Space Shuttle just before it enters orbit.
- 11,082 m/s High speed record for manned vehicle, set by Apollo 10
- 29,800 m/s Speed of the Earth in orbit around the Sun (about 30 km/s)
- 299,792,458 m/s the speed of light (about 300,000 km/s)